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Optimal bidding strategy for PHEV fleet aggregators - A bilevel model

Master's Thesis in Electric Power Engineering

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COVER
Photo: Volvo Cars

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Abstract

The shortage of fossil fuels and the climate change has forced humans to find new ways of transportation. A solution to this problem could be to start using plug-in hybrid vehicles(PHEV). This type of vehicles need to be charged in an organized way to minimize the impact on an already strained electric system. A load scheme for the PHEV aggregators reduces the impact on the system, by moving the charging to low load periods. In this thesis a bi-level model is created built up on to charge during low load periods, in ordered to find the optimal bidding strategy for the PHEV fleet. The upper level problem is the aggregators, which are placing bids in a market pool during a 24 hour period. The lower level problem is the market pool, which is returning the clearing price for the bids on an hourly basis.

The bi-level model is turned into a two-level optimization problem. The upper level problem is a cost function, minimizing the charging cost. The lower level problem are built up on welfare maximization and optimal power flow(OPF). By using different mathematical approaches, this problem is turned in to a mathematical problem with equality constraints(MPEC). The MPEC model was found not to be able to solve large systems. Therefore, where the bi-level model implemented into two embedded models, to find a solution for large systems.

The models were finally tested in a case study on four networks systems of different size and on a full scale model of the swiss power system. It was found that the MPEC was faster at solving the problem and also in some cases found other solutions then the embedded models. In these cases the MPEC found a better solution. The cause for this may have been that the MPEC found the global minimum, while the embedded models could have found a local minimum. When comparing the minimum charging cost models with a system cost minimization approach, it was found in a specific case that the charging cost was lower with the charging models. In this case the MPEC gave a lower charging cost than the embedded models. When the full scale test was done on the swiss system, only one of the embedded models was working. When comparing the two cost models the results showed that there was a very small difference in the charging cost. The MPEC didn't work on this system, which is a complication due to that it might give a larger difference compared to the supply driven cost model. If the MPEC would have worked, it would have been possible wheatear or not the aggregators had gained market power in the Swiss system.

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Abbreviations

$PHEV$	Plug-in-hybrid electrical vehicles.
KKT	Karush Kuhn Tucker.
$MPEC$	Mathematical program with equilibrium constraints.
OPF	Optimal power flow.

Indices

i	Index for generator units.
j	Index for load units.
b	Index for each load block .
t	Index for time periods.
n, m	Index for lines from 1 to N and from 1 to M.

Constants

$P_{Gi}^{t,max}$	Upper limit of generator i at time period t.
$P_{Gi}^{t,min}$	Lower limit of generator i at time period t.
n_{G_i}	Marginal cost of generation.
$P_{L_{jb}}^{t,max}$	Upper limit of block b of load j at time period t.
$P_{L_{jb}}^{t,min}$	Lower limit of block b of load j at time period t.
$P_{L_{jb}}^{t,ref}$	Reference load j at time period t.
$P_{V_{k,conn}}^{t}$	Charging capacity of all vehicles connected to node at time period t.
$E_{VB_j}^{t,max}$	Upper battery bound of virtual battery at node j at time period t.
$E_{VB_j}^{t,min}$	Lower battery bound of virtual battery at node j at time period t.
$E_{VB_{j,arr}}^{t}$	Positive energy contribution of arriving vehicles at node j at time period t.
$E_{VB_{j,dep}}^{t}$	Negative energy contribution of departing vehicles at node j at time period t.
$C_{V_{k,batt}}$	Battery capacity of individual vehicle k
$SOC_{V_{k,min}}$	Lower level state of charge of individual vehicle k
$SOC_{V_{k,req}}$	Required state of charge of individual vehicle k
$\bar{\eta}_{j,char}$	Average battery charging efficiency at node j.
B_{nm}	Susceptance of line n to m.
$P_{l_{nm}}^{max}$	Line limit for line from node n to m.

Variables

$\alpha_{L_{jb}}$	Demand bid of block b
δ_n^t	Angle at node.
P_{G_i}	Power produced by generator i.
$P_{L_{jb}}$	PHEV load at block b of load j.
$E_{VB_j}^t$	Energy content of virtual battery.

Dual variables

λ_n^t	The Lagrangian, Generation-demand equilibrium at node n at time t
$\mu_{Gi}^{t,min}$	Minimum production of generator i at time t
$\mu_{Gi}^{t,max}$	Capacity of generator i at time t
$\mu_{jb}^{t,min}$	Minimum power of demand of block b at load i at time t
$\mu_{jb}^{t,max}$	Capacity of demand of load b at load i at time t
$\nu_{nm}^{t,min}$	Transmission capacity of line n to m at period t, direction m to n
$\nu_{nm}^{t,max}$	Transmission capacity of line n to m at period t, direction n to m
$\xi_n^{t,min}$	Upper bound of the voltage angle at bus n at time t.
$\xi_n^{t,max}$	Lower bound of the voltage angle at bus n at time t.
$\xi_1^t = 0$	Voltage angle at bus $n = 1$ at time t

Linearization variables

$\omega_{G_i}^{t,min}$	Lower bound for generator i at time t
$\omega_{G_i}^{t,max}$	Upper bound for generator i at time t
$\omega_{L_{jb}}^{t,min}$	Lower bound for block b of load j at time t
$\omega_{L_{jb}}^{t,max}$	Upper bound for block b of load j at time t
$\psi_{nm}^{t,min}$	Transmission capacity of line n to m at period t, direction m to n
$\psi_{nm}^{t,max}$	Transmission capacity of line n to m at period t, direction m to n
$\varphi_n^{t,min}$	Lower bound for the voltage angle at time t
$\varphi_n^{t,max}$	Upper bound for the voltage angle at time t

Chapter 1

Introduction

Due to the up coming shortage of fossil fuels and the climate change the topic, of how to reform the human behavior of transportation has become of great importance. One solution to this problem is to substitute the classic combustion car with an electrical one, which is not run on fossil fuels. The technology in existing electrical cars has limitations for long distance transportation. A first step in this change is therefore to use Plug-in-hybrid electrical vehicle (PHEV), which has two engines. One electrical engine for short distance driving and one combustion engine for long distances transportation. The difference between a plug in hybrid and the normal hybrid used today (e.g. Toyota Prius), is that the PHEV has a larger battery capacity that can be charged while the cars is parked. The majority of all cars are parked most of the time and without any prepared charging schemes all this vehicles would start to charge at random times. This will cause a large impact on an already congested grid and thus increase the generation costs. A solution is to have controlled charging managed centralized or by the fleet aggregator. This will decrease the the additional load caused by PHEV during time periods when the load is already high. The load schemes for the PHEV will be shifted to the periods when the load normally is low in the net.

There have been several studies to evaluate the effects of optimal PHEV charging. Most of these studies have been done on centralized charging in [1] and [2], where the charging is controlled centrally and results in valley filling during low load hours. There have also been studies on decentralized charging where the Nash certainty equivalence was applied [3]. The result showed that the PEHV fleet acted homogenous and that all vehicles had the same loading pattern. This pattern was also filling the valleys during the low load hours.

Many electricity markets in the world builds up on spot market pricing. The participants, suppliers and consumers are placing offers and bids in a market pool, in form of capacity and price. From the bids and offers the market clearing price is determined, on an hourly interval. This is done on a day ahead basis. In this project the consumer bidding is studied where PHEV fleet aggregators participate in the electricity market by placing bids. The aim for the project is to find the optimal bidding strategy for the aggregators, which will give the lowest charging cost. To find the optimal strategy a bi-level model is created built up on to charge during low load periods. The upper level problem in this model is the aggregators, which are placing bids in a market pool during a 24 hour period. This bi-level model is turned in to a two level optimization problem. The upper level problem is cost minimization of the charging. The lower level problem is a market clearing problem built up on optimal power flow and welfare maximization.

This type of two level optimization problems for electricity markets has earlier been studied in [6]. There, an optimization model, which simulates the electricity markets, is presented. In the article the authors also mention demand side bidding, where the consumers will gain market power by having the ability to react on prices.

In this thesis the two-level optimization problem is turned into a mathematical problem with equality constraints (MPEC). This is mainly done using already existing techniques by other engineers and researchers. Large parts of the MPEC model is taken from [5], where the authors are creating a model to find the optimal bidding strategy for the producer. The model in [5] is changed to find the optimal bidding strategy for the PHEV aggregators. It was necessary to add conditions for PHEV fleet to this model, the framework for that is taken from [4]. In this paper a finished model already exists, where only small modifications had to be done to fit the strategy model. Most of the mathematical concepts are taken from these two papers to create the model. A lot of the mathematical theory and some of the assumptions are therefore relaying on other peoples research. In this report the focus is instead on the theory that was used to modify the existing MPEC to fit together with the PHEV model.

When the model was developed it was found that the MPEC couldn't solve large network problems due to the fact that computer was running out of memory, because of too many variables. Because of this it was necessary to implement the bi-level model in another way to reduce the number of optimization variables. Two embedded models were created, which is built up on the original problem, a genetic algorithm and a non-convex solver. The three minimum charging cost models were tested in a case study on system cost on four networks of different size and a full scale model of the swiss system. The results from the case study showed that the MPEC was faster than the embedded models. Although it has a weakness of not being able to solve large networks. Congestion in the lines was a factor that increased the solving time. To investigate the aggregators ability to gain market power by participating in the bidding on the market. The minimum charging cost models are compared with a minimum system cost model. This model is considering what is best for the society and not the consumer. It was found in one of the case that the aggregators were having a lower charging cost with the MPEC and the embedded models. From a simulation made on a full scale model of the swiss system, it was found that the charging cost was slightly lower with the embedded model.

Structure of the thesis

The first chapters of this thesis are describing the theory behind the models, economical frameworks and optimal power flows, which the market clearing is built up on. They are also containing the mathematical theory for the creation of the MPEC. These parts are followed by a chapter describing the derivation of the MPEC and the structure of the embedded models. After this part the results are presented as a case study, where the models are simulated on four different networks of different sizes and on a full scale model of the swiss power system. In every case there is a discussion part where the results are evaluated. In the last part of the report the conclusions from this master thesis can be found. There are two appendices; the first one describes the economical concepts and used terms and the second one contains data from the simulations.

Chapter 2

Mathematical Optimization

A traditional optimization problem focuses on minimizing or maximizing the objective function f(x) with respect to x. The goal is to find the action which gives the optimal outcome. An optimization problem has an amount of feasible action to choose between to find the solution. These actions are called the constraints and are boundaries or equalities for the variables. The problem can be written mathematically on the form:

$$\begin{gathered} min\ f_0(x) \\ subject\ to : f_i(x) \leq b_i \end{gathered}$$

Where $f(x)$ is the objective function that describes the return or payoff to the decision maker. The vector $x = (x_1, x_2, ...x_n)$ is the optimization variables, which is a subset C of the finite-dimensional real vector space R^n. The function f_i is the constrains or feasible actions a decision maker can choose between. x* is the desired outcome of the problem, the optimal solution [11].

2.1 Lagrange

To find the optimal solution for a optimization problem the Lagrange method can be used. This method creates the Lagrange function where the constraints are added to the objective function together with the Lagrange multipliers.

A standard optimization problem with an objective function and equality constraints is written on following form:

$$\begin{gathered} min\ f_0(x) \\ subject\ to : g_i(x) = b_i \end{gathered}$$

To this type of problem there are added conditions. To find a maximum or minimum of this problem the Lagrange method can be used, this gives addition unknown variables. These variables λ_i, which are called Lagrange multipliers, are used to find a new optimum value for the problem. The Lagrange is an extended function of the objective function and are defined as:

$$\mathcal{L}(x, \lambda) \;=\; f(x) \;+\; \sum_i \lambda_i g_i(x)$$

This is the primary problem as a function of the constraints. By taking the first order partial derivatives of the Lagrangian function, the first order optimal conditions can be found which are the new constraints for the extended problem [8].

$$\nabla_{x_i} f = \frac{d\mathcal{L}(x, \lambda)}{dx_i} \quad and \quad \nabla_{\lambda_i} f = \frac{d\mathcal{L}(x, \lambda)}{\lambda_i}$$

The Lagrange multipliers state how the optimal solution will react if there is a marginal change in any of the constraints.

2.2 The Lagrange dual function

The Lagrange dual function is similar to the previous method, adding the constraints to the objective function. For this function there are two types of Lagrange multipliers. The first comes from the inequality constraints and the other from the equality constraints. These are called the dual variables and are used as a function of the primal problem. The new problem is called the dual problem or the dual function. To find the this function two steps need to be done; derive the Lagrangian function and minimizing the problem.

A standard optimization problem written as an objective function, equality and inequality constraints is written of following form:

$$\begin{gathered} min\ f_0(x) \\ subject\ to: g_i(x) = 0 \\ h_i(x) \leq 0 \end{gathered}$$

The optimization variables $x \in \mathcal{R}^n$, where the domains for g_i and h_i are in domain $\mathcal{D}$ which is non empty and includes the optimal value p^*. This type of problem is similar to the previous, but has additional inequality constraints. To find a solution the Lagrangian duality is used and the objective function is increased by adding the Lagrange variables. This variables are a weighted sum of the constrain functions. The Lagrange is defined as before but with an additional multipliers:

$$\mathcal{L}(x, \lambda, \mu) \;=\; f(x) \;+\; \sum_{i=1} \lambda_i g_i(x) \;+\; \sum_{j=1} \mu_j h_j(x) \tag{2.1}$$

Where $\mathcal{L} : \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R}^p \to \mathcal{R}$. The two multipliers (the dual variables) are the λ_i and μ_j, which are associated with inequality and the equality constraints respectively.

The Lagrangian dual function where $\mathcal{G} : \mathcal{R}^m \times \mathcal{R}^p \to \mathcal{R}$ is defined as the minimum value of the Lagrangian over x: for $\lambda \in \mathcal{R}^m$ and $\mu \in \mathcal{R}^p$

$$\mathcal{G}(\lambda, \mu) = inf\ \mathcal{L}(x, \lambda, \mu)\ = inf\left(\ f(x) + \sum_i \lambda_i g_i(x) + \sum_j \mu_j h_j(x)\right) \quad (2.2)$$

This function gives for each pair of (λ, μ) with $\lambda\ \succeq\ 0$ a lower bound on the optimal value x^* of the optimization problem. The lower bound is now dependent on these parameters and to find the best point within this new bound an additional optimization problem is created, the Lagrange dual problem:

$$\begin{aligned} Maximize &\quad \mathcal{G}(\lambda, \mu) \\ Subject\ to &\quad \lambda\ \succeq\ 0 \end{aligned}$$

The Lagrange dual problem is convex because the objective function is concave and the constraint is convex. This is independent from the primal problem if it is convex or not [8].

Strong duality theorem States that optimal solution from the primal problem is equal to the optimal solution for the dual problem.

$$p^* = d^*$$

This means that the duality gap between the primal and dual problem is zero and this condition holds if the primal problem is convex. This is not always the case but if the optimization problem is convex and on the following form:

$$\begin{gathered} min\ f_0(x) \\ subject\ to : Ax = b \\ f_i(x) \leq 0 \end{gathered}$$

strong duality is valid for most cases [8].

2.3 Karush Kuhn Tucker

The KKT can be seen as a "standardized" way to represent the first order optimality condition and is necessary to find an optimal solution for a non-linear optimization problem. To find the KKT conditions, the Lagrange function is set up:

$$\mathcal{L}(x, \lambda, \mu)\ =\ f(x)\ +\ \sum_i \lambda_i g_i(x)\ +\ \sum_j \mu_j h_j(x)$$

Let x^* and (λ, μ) be any primal and optimal points with zero duality gap, when minimizing $\mathcal{L}(x, \lambda, \mu)$ with respect to x, the gradient disappears over x^* which gives [8]:

$$\nabla f(x^*) \;+\; \sum_i \lambda_i \nabla g_i(x^*) \;+\; \sum_j \mu_j \nabla h_j(x^*) = 0$$

The Karush Kuhn Tucker conditions are then given by:

$$\begin{aligned} g_i(x^*) &\leq 0 \quad i = 1, ... \\ h_j(x^*) &= 0 \quad j = 1, ... \\ \lambda_i^* &\geq 0 \quad i = 1, ... \\ \lambda_i^* g_i(x*) &= 0 \quad i = 1, ... \\ \nabla f(x^*) \;+\; \sum_i \lambda_i \nabla g_i(x*) \;+\; \sum_j \mu_j \nabla h_j(x^*) &= 0 \end{aligned}$$

where the first and second conditions are the states of the constraints on the primal problem. The third and fourth conditions are the complementary slackness conditions. The last one is the partial derivatives of the Lagrangian which are equal to zero at the optimum point[9].
An optimization problem with differentiable objective function and constraint function, for which strong duality holds, obtains any pair of primal and dual optimal points, which satisfies the KKT conditions. For a convex problem the the KKT conditions are satisfied if $f(x)$ is convex and $h(x)$ is affine[1] [8].

2.4 Complementary slackness

A vector $\mathcal{G}(\lambda, \mu)$ is the optimal solution for the problem $f(x^*)$. The only remaining difficulty for the solution is the complementary slackness conditions; $\lambda g(x^*) = 0$ and $\mu h(x^*) = 0$. These conditions consist of a sum of products of two variables, and when strong duality holds, we know that $\lambda g(x^*) = 0$. If $\lambda_i^* \geq\ 0$, then the primal constraint is binding $g(x^*) = 0$, or if the constraint is not bidding, its shadow price is zero. This can be used to transform the optimization problem f(x) into an equivalent mixed integer quadratic program, by modifying the constraints $\lambda g(x^*) = 0$ and $\mu h(x^*) = 0$. Add for each of these products a zero-one variable, η_i and a large positive constant M, which gives the new constraints [12]:

$$\begin{aligned} \lambda_i &\leq \eta_i M \\ f_i(x) &\leq (1 - \eta_i) M \end{aligned} \tag{2.3}$$

[1]Affine: A function is affine if the proportions are preserved after a geometric transformation.

2.5 Bi-Level program

A bi-level program is a mathematical program, which contains an optimization problem. The program is a two level problem where the upper level determines the response from the lower level. The upper level contains two vectors, x and y, where x is determining the strategy and y represents the response from the lower level. The problem can be formulated:

$$\begin{aligned} min \quad & F(x,y) \\ subject\ to: \quad & (x,y) \in C \\ y\ solves \quad & Q(x) \\ Q(x): minimize \quad & f(x,y) \\ Subject\ to: \quad & y \in T(x) \end{aligned} \tag{2.4}$$

$$\tag{2.5}$$

where $F = \mathcal{R}^n + \mathcal{R}^m \rightarrow \mathcal{R}$ and $C \subseteq \mathcal{R}^n + \mathcal{R}^m$ are non empty closed subsets. $T(x)$ is the feasible set for the lower problem and $Q = \mathcal{R}^n \rightarrow 2\mathcal{R}^m$ is a set of valued mapping such that $Q(x)$ is a set of solutions to the lower level problem for each x. $2\mathcal{R}^m$ denotes the set of all subsets of $\mathcal{R}^m$ and for each $x \in \mathcal{R}^n$, $Q(x) \subseteq T(x)$. We assume that $T(x)$ is closed and non-empty for each $x \subseteq \mathcal{R}^n$.

In some economic and engineering problems, where an equilibrium condition is to be found, the lower level problem can be replaced with an expression describing this equilibrium. A typical economic case is the two person Stackleberg game theory problem, where the leader anticipates the results from the followers and adjusting its strategy thereafter. It can be mathematically formulated as:

$$\begin{aligned} min \quad & f(x,y) \\ subject\ to: \quad & x \in X, y \in Q(x) \\ Q(x):\ minimize \quad & h(x,y) \\ Q(x): minimize \quad & f(x,y) \\ Subject\ to: \quad & y \in T(x) \end{aligned} \tag{2.6}$$

$$\tag{2.7}$$

where f is the leader's objective function (cost function) and h is the follower's objective function (cost function). It is assumed that h is smooth and convex in y and $T(x)$ is closed and convex [10]. This problem is similar to finding the optimal bidding strategy for the PHEV fleet.

2.6 MPEC

A Mathematical Program with Equilibrium Constraints (MPEC) is a bi-level problem, which is rewritten as a one level problem. This is done by adding the KKT conditions for the lower level problem to the upper level problem [10].

2.7 Power flow calculations

Power flow analysis is a method to do numerical studies on electricity networks during steady state operation. It is mainly used as a tool for planning the usage of a power system and for future expansions, but can also be used for economic and fault analysis.

The method calculates the currents and voltages on every bus and the real and reactive power flows to every bus. The focus is mainly on AC power flows but it is also possible to calculate DC flows. The linear parameters in the network such as loads, transformers and lines, have constant value. But the relationships between voltages and currents are non-linear and the same for active and reactive power consumption at each node. A network can be written on matrix form:

$$[Y_{ij}][V_i] = [I_i] \tag{2.8}$$

where I_i is the injected current in bus i, V_i is the voltage at bus i and Y is the bus admittance matrix. Y_{ii} is the self admittance which is the sum of all admittances connected to the bus. Y_{ij} is the negative of branch admittance between bus i and j.

The power flow equation for the active power can be written as:

$$P_i = \sum_{i=1}^{N} |V_i||V_j|(G_{ij}\cos(\theta_{ij}) + B_{ij}\sin(\theta_{ij})) \tag{2.9}$$

where P_i is the injected power at bus i, V_i the voltage at bus i, G_{ij} is the real part of Y_{ij} and B_{ij} imaginary part of the admittance matrix. θ_{ij} is the voltage angle difference between bus i and j.

The power flow equation for the reactive power can be written as:

$$Q_i = \sum_{i=1}^{N} |V_i||V_j|(G_{ij}\sin(\theta_{ij}) - B_{ij}\cos(\theta_{ij})) \tag{2.10}$$

where Q_i is the injected reactive power at bus i but the rest of the parameters are the same as in the active power flow equation.

When the power flow model is set up for a network there are mainly three types of buses defined.

- PV Bus, the real power P and the magnitude of the voltage is known. There is a generator connected to the bus.
- PQ bus, the active power P and reactive power Q are known. It is normally a load but can also be a generator with fixed values.

- Slack bus or swing bus is the network reference bus where the voltage angle is zero. Normally only one of these buses exist in a network. These buses are used as reference when the value of the other buses is calculated. Often a bus with a connected generator is chosen because it has two unknown parameters, the voltage angle and the voltage magnitude.

By knowing the type of bus all the parameters can be calculated from that particular bus[9].

2.8 Solution methods for power flows

To solve these nonlinear power flow problems, there are different iterative methods. The most common one is the Newton Raphson method, but there are also other methods such as the Gauss Seidel and the Fast Decoupled (a simplified and fast method which is not so accurate). All these methods are used for analysis of AC networks. When doing power market analysis, the networks are often very large and it is not very important to have high precision in the calculations, but rather high calculation speed. Therefore DC power flow is used, which gives linear equations and thus several iterations are not needed. To create the algorithm, the following assumptions are made [9]:

1. All voltage magnitudes are equal to 1.
2. The resistance is ignored at all branches and only the susceptance is used in the admittance matrix:
   $B_{ij} = -1/x_{ij}$
3. The angle differences between the branches is very small:
   $\sin(\theta_{ij}) = \theta_i - \theta_j$
   $\cos(\theta_{ij}) = 1$
4. All ground branches are ignored:
   $B_{i0} = B_{j0} = 0$

The DC model:

$$\begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_{n-1} \end{bmatrix} = \begin{bmatrix} B' \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \vdots \\ \Delta\theta_{n-1} \end{bmatrix} \tag{2.11}$$

Where:

$$B' = -1/x_{ij} \tag{2.12}$$

and the DC power flowing on each line is:

$$P_{ij} = -B_{ij}(\theta_i - \theta_j) = \frac{\theta_i - \theta_j}{x_{ij}} \tag{2.13}$$

Chapter 3

Optimization of the electricity market

The electricity market is a spot price built market, where individuals place a supply or demand bid to a central operator for the system. The system operator is determining the price of the day ahead market by matching the bids on a hourly basis. This is done by running an optimal power flow (OPF), with the objective to maximize the social welfare. An optimization model for the electricity market consists of both economical and technical factors, where the objective function is mostly a social welfare maximization problem built up on economical functions. The problem is limited by different technical parameters e.g. generation capacities, transmission capabilities and load limits. The aim for this type of optimization problems is to find the price at each node according to the present demand.

3.1 Welfare maximization

Welfare maximization is a method built up on the microeconomic supply and demand model. The goal is to maximize the surplus for the market participants, consumers and producers. The surplus is the monetary amount, which the consumer has left when purchasing a good. For the producer the surplus is the profit made by selling a good. The supply and demand model describes how a market condition changes depending on the price and the amount of a good. The model is mathematically represented by two equations. The first equation describes the consumer behavior of the willingness to purchase a good to a certain price. The second equation describes the producer's willingness to produce and sell at a certain price. This can be represented by a diagram showed in figure 3.1. A more in depth description of the microeconomic theory can be found in Appendix A.

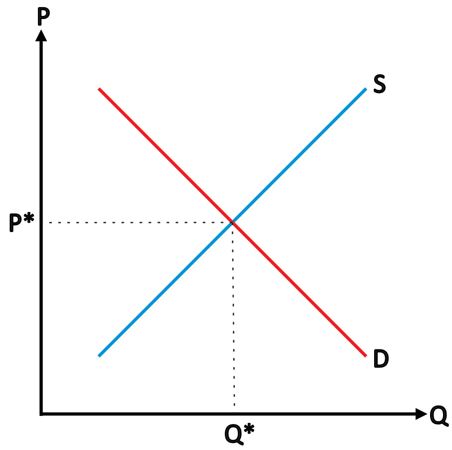

Figure 3.1: The supply and demand model, where the intersection point shows the market equilibrium P^* and Q^*.

3.2 Problem formulation

When formulating an electricity market problem, the supply and demand model is represented by two equations. The demand equation calculates the marginal benefit a consumer gets by purchasing an additional unit. The supply function is defining the marginal cost of the generators. These two functions can be turned into an optimization problem by taking the consumer benefit minus the producer cost. By maximizing this problem, the maximum social welfare will be found. This is formulated mathematically:

$$max \sum_j B(P_{Lj}) - \sum_j C(P_{Gi}) \tag{3.1}$$

where $B(P_{Lj})$ is the consumer benefit and $C(P_{Gi})$ is the producer benefit. P_{Lj} is the demand at each node and P_{Gi} is the generation at each node. This optimization problem is subjected to inequality and equality constraints from an OPF of the system. This OPF is built up on the characteristics of the net in form of transmission, generation and load capacity. The equality constraint is the power balance in the system, where the sum of all generating power units, loads and transmission lines are equal to zero. This is described by the following equation:

$$\textstyle\sum_i P_{G_i} - \sum_j P_{L_j} - \sum_l P_{l_{nm}} = 0 \tag{3.2}$$

where P_{G_i} is the generation and P_{L_j} is the load at each node. $P_{l_{nm}}$ is the transmitted capacity between the nodes.

The inequality constrains are defining the capacity boundaries for each generator, limits of each load and flow limits in each line between the nodes.

$$P_{Gi}^{min} \leq \quad P_{Gi} \quad \leq P_{Gi}^{max} \tag{3.3}$$
$$P_{Lj}^{min} \leq \quad P_{Lj} \quad \leq P_{Lj}^{max} \tag{3.4}$$
$$-P_{l_{nm}} \leq \quad B_{nm}(\delta_n^t - \delta_m^t) \quad \leq P_{l_{nm}} \tag{3.5}$$

where B_{nm} is the susceptance of a line and δ is the transmission angle $(B_{nm}(\delta_n^t - \delta_m^t) = P_{l_{nm}})$ [7].

3.3 Nodal prices

To find the nodal prices (spot price), the Lagrangian of the optimization problem is set up:

$$\begin{aligned} max \quad & \textstyle\sum_j B(P_{Lj}) - \sum_i C(P_{Gi}) - \lambda_n\Big(\sum_i P_{Gi} - \sum_i P_{Lj}\Big) \\ & -\mu_l\Big(|P_{l_{nm}}| - P_{l_{nm}}^{max}\Big) - \mu_l\Big(P_{Gi} - P_{Gi}^{max}\Big) \end{aligned} \tag{3.6}$$

The nodal prices can be found by differentiating the Lagrangian with respect to the load:

$$p_n \quad = \quad \lambda_n + \sum_l \mu_l \frac{\delta P_{l_{nm}}}{\delta P_{Lj}} \tag{3.7}$$

where p_n is the nodal prices, λ_n and μ_l are the Lagrangian multipliers. μ_l is describing the scarcity of the transmission capacity.
If there is a change in some of the constraints, the Lagrangian multipliers will adjust the optimal solution of the problem. At major changes in the constraints for example in the available capacity or demanded price, the Lagrangian will change the equilibrium solution, which leads to a price change. If the change dosen't reach a constraint limit, the multiplier will be zero and there is no change in the price.
When using a monetary objective function, the multiplier will create a threshold. This is the price that a producer or consumer is willing to pay to purchase an additional unit of capacity. This threshold, which is the optimal solution of the Lagrangian multiplier, is called the shadow price [13].

Chapter 4

PHEV fleet model

The following part describes how the PHEV fleet is modeled. The fleet consists of a set of aggregated storage resources where the capacity and energy content is changing when a vehicle is arriving or departing at a node. This is modelled by a virtual battery. It is built up on a simulation of the normal day driving pattern of a fleet of 1 million vehicles. The characteristics of this model is that the battery level is decreasing during those time periods when the vehicles are used for transportation to work and home, i.e. in the morning and in the evening. $E^t_{VB_J}$ is the stored energy content of the connected batteries to node j at time t. The energy content which is needed for usage for the driving patterns of a vehicle during a normal day, can vary between a minimum and maximum.

$$E^{t,min}_{VB_j} \leq E^t_{VB_j} \leq E^{t,max}_{VB_j} \quad \forall j \forall t \tag{4.1}$$

which is the constraint for the battery. The lower battery bound of an individual vehicle is given by either its minimum state of charge or by its required state of charge before departure ($SOC_{V_k,min}$ and $SOC_{V_k,req}$). These are summed up at each node.

$$E^{t,min}_{VB_j} = \sum_{k \in \Omega^t_j} max(SOC_{V_{k,min}}, SOC_{V_{k,req}})C_{V_{k,batt}} \tag{4.2}$$

The upper battery bound is given by the sum of all $P_{V_k,conn}$ which represents the capacities connected to the node.

$$E^{t,max}_{VB_j} = \sum_{k \in \Omega^t_{L_j}} C_{V_{k,batt}} \tag{4.3}$$

The content of a virtual battery is given by its content in the previous time step: the charging power on each node is $P^t_{L_{jb}}$, the average charging efficiency of the fleet $\bar{\eta}_{j,char}$ and the positive and negative departure of PHEVs arriving and departing at each node $E^t_{VB_{j,dep}}$ and $E^t_{VB_{j,arr}}$ respectively.

$$\begin{aligned} E^{t+1}_{VB_j} &= E^t_{VB_j} + \sum_{t,j,b} P^t_{L_{jb}} \cdot \Delta t \cdot \bar{\eta}_{j,char} \\ &- E^t_{VB_{j,dep}} + E^t_{VB_{j,arr}} \end{aligned} \tag{4.4}$$

The initial stored energy $E^0_{VB_j}$ in the battery is equal to the final value $E^T_{VB_j}$. Basically the energy content is the same at the start and at the end of every time period [4].

$$E^0_{VB_j} = E^T_{VB_j} \tag{4.5}$$

Chapter 5

Derivation of the first model - MPEC

A bi-level program is a mathematical program which is containing a two level optimization problem, where the upper level determines the response from the lower level. The upper level contains two vectors, variables x and y, where x is the decision variable (the strategy) and y represents the response from the lower level. The problem can be formulated:

Minimize F(x,y)
Subject to:(x,y)$\in$ C
y solves Q(x)
Q(x):minimize f(x,y)
Subject to :y $\in$ T(x)

The lower level problem in our case is continuous and convex, and can therefore be replaced by its Karush-Kuhn-Tucker (KKT) conditions, which give a mathematical program with equilibrium constraints (MPEC). The MPEC is non-linear and needs to be linearized. This is done by adding the dual variables as a function of the primal problem by using the Lagrange dual function. The new function can be simplified by applying the strong duality theorem on the new function. When solving the non-linearities in the KKT conditions, the complementary slackness theorem is applied.

5.1 Upper Level Problem

The purpose of the upper level problem is to find the ultimate offering strategy for the PHEV fleet aggregators. This is done by minimizing the charging cost of the PHEV. The load is using blocks, this function is dividing every hour into a number of periods which is equal to the number of blocks:

$$\min_{\alpha^t_{L_{jb}}} \sum_{t,n} \lambda^t_n \sum_{j,b} P^t_{L_{jb}} \tag{5.1}$$

where λ_n^t is the price at node n and $\sum_{j,b} P_{L_{jb}}^t$ is the charging load at each node.

Subject to
The minimum and maximum charged power:

$$P_{L_{jb}}^{t,min} \leq P_{L_{jb}}^t \leq P_{L_{jb}}^{t,max} \quad (5.2)$$

where $P_{L_{jb}}^t$ is the charged load at each node.
The upper bound of the inequality constraint is the sum of all connected capacities at each node, divided by number of blocks.

$$P_{L_{jb}}^{t,max} = \frac{\sum_{V_k \in \Omega_{L_j}^t} P_{V_k}}{nrBlocks} \quad (5.3)$$

The lower bound of the inequality constraint is the minimum load, which in our case is equal to zero.

$$P_{L_{jb}}^{t,min} = 0 \quad (5.4)$$

The initial value for the virtual battery $E_{VB_J}^0$ is equal to its final value $E_{VB_J}^{(T)}$.

$$E_{VB_j}^0 = E_{VB_j}^T \ \forall j \quad (5.5)$$

The bounds for the virtual battery:

$$E_{VB_j}^{t,min} \leq E_{VB_j}^t \leq E_{VB_j}^{t,max} \ \forall j \forall t \quad (5.6)$$

The upper battery bound is given by the sum of all capacities connected to the node.

$$E_{VB_j}^{t,max} = \sum_{k \in \Omega_{L_{jb}}^t} C_{V_{k,batt}} \quad (5.7)$$

The lower battery bound of an individual vehicle is given by either its minimum state of charge or by its required state of charge before departure ($SOC_{V_k,min}$ and $SOC_{V_k,req}$). These are summed up at each node.

$$E_{VB_j}^{t,min} = \sum_{k \in \Omega_j^t} max(SOC_{V_{k,min}}, SOC_{V_{k,req}}) C_{V_{k,batt}} \quad (5.8)$$

The content of a virtual battery is given by its content in the previous time step:

$$\begin{aligned} E_{VB_j}^{t+1} &= E_{VB_j}^t + \sum_t \sum_{j,b} (P_{L_{jb}}^t) \cdot \Delta t \cdot \bar{\eta}_{j,char} \\ &- E_{VB_{j,dep}}^t + E_{VB_{j,arr}}^t \end{aligned} \quad (5.9)$$

5.2 Lower Level Problem

The lower level problem is the marking clearing, which is built up on a economic social welfare maximization problem and a technical optimal power flow:

$$\min_{P^t_{G_i}, E^0_{V_j}, P^t_{L_{jb}}} \sum_{t,i} n_{G_i} P^t_{G_i} \; - \; \sum_{t,j,b} \alpha^t_{L_{jb}} P^t_{L_{jb}} \tag{5.10}$$

where n_{G_i} is the marginal cost for the generation, $P^t_{G_i}$ is the generated capacity, $\alpha^t_{L_{jb}}$ is the demand bid and $P^t_{L_{jb}}$ is the PHEV load.

Subject to
Power Balance at each node in the network:

$$\sum_{jb \in \Omega_n} P^t_{L_{jb}} + \sum_{j \in \Omega_n} P^{t,ref}_{L_j} \; + \sum_{m \in \theta_n} P^t_{l_{nm}} - \sum_{G_i \in \Omega_n} P^t_{G_i} \; = \; 0 \; : \lambda^t_n, \forall t, \; \forall n \tag{5.11}$$

where $P^{t,ref}_{L_j}$ is the reference load, which is the normal system outage. Generator capacity at each node.

$$P^{t,min}_{G_i} \; \leq \; P^t_{G_i} \; \leq P^{t,max}_{G_i} \; : \mu^{t,min}_{G_i}, \; \mu^{t,max}_{G_i} \; \forall t, \; \forall i \tag{5.12}$$

The upper and lower load demand at each node.

$$P^{t,min}_{Lj} \; \leq \; P^t_{L_{jb}} \; \leq P^{t,max}_{Lj} \; : \mu^{t,min}_{Lj}, \; \mu^{t,max}_{Lj} \; \forall t, \; \forall j \tag{5.13}$$

Network flows for each branch in the network:

$$P^t_{l_{nm}} \; = \; B_{nm}(\delta^t_n - \; \delta^t_m) \; \; \forall t, \forall n, \; \forall m \in \theta_n \tag{5.14}$$

Transmission capacity of each line:

$$-P^{max}_{l_{nm}} \; \leq \; B_{nm}(\delta^t_n - \; \delta^t_m) \; \leq \; P^{max}_{l_{nm}} : \nu^{t,min}_n, \; \nu^{t,max}_n \; \forall t, \forall n, \; \forall m \in \theta_n \tag{5.15}$$

Bounds for angles:

$$-\pi \; \leq \; \delta^t_n \; \leq \pi \; : \xi^{t,min}_n, \xi^{t,max}_n, \; \forall t, \; \forall n \tag{5.16}$$

Reference bus, fixed angle at bus 1:

$$\delta^t_1 = 0 \; \; \forall t \tag{5.17}$$

5.3 The KKT conditions

To add the lower level problem to the MPEC, it is replaced by its KKT conditions. To set up the KKT conditions, first the Lagrangian of the problem must be determined :

$$\mathcal{L} \;=\; F(x) + \sum_i \lambda_i g_i(x) + \sum_i \mu_i h_i(x) = F(x) + \lambda^T g(x) + \mu^T h(x) \quad (5.18)$$

where:
F(x) Objective function
g(x) ≤ 0 Inequality constraints
h(x) $= 0$ Equality Constraints

The Lagrangian for this problem can be formulated as:

$$F(x) \;= \textstyle\sum_{t,i} n_{G_i} \cdot P^t_{G_i} \;-\; \sum_{t,j,b} \alpha^t_{L_{jb}} \cdot P^t_{L_{jb}} \quad (5.19)$$

$$\lambda^T h(x) \;=\; \sum_{t,n} \lambda^t_n \Bigg(\sum_{j\in\Omega_n} P^t_{L_{jb}} + \sum_{j\in\Omega_n} P^{t,ref}_{L_j} + \sum_{m\in\theta_n} B_{nm}(\delta^t_n - \delta^t_m) - \sum_{Gi\in\Omega_n} P^t_{Gi} \Bigg) + \sum_t \xi^t_1 \delta^t_1 \quad (5.20)$$

$$\begin{aligned}
\mu^T g(x) \;=\;& \sum_i (P^t_{Gi} - P^{t,max}_{Gi})\mu^{t,max}_{Gi} - \sum_i (P_{Gi} - P^{t,min}_{Gi})\mu^{t,min}_{Gi} \\
+\;& \sum_j (P^t_{L_{jb}} - P^{t,max}_{L_{jb}})\mu^{t,max}_{L_{jb}} - \sum_j (P^t_{L_{jb}} - P^{t,min}_{L_{jb}})\mu^{t,min}_{L_{jb}} \\
-\;& \sum_{t,n,m\in\Theta_n} (P^{t,max}_{l_{nm}} - B_{nm}(\delta^t_n - \delta^t_m))\nu^{t,max}_n - \sum_{t,n,m\in\Theta_n} (P^{t,max}_{l_{nm}} + B_{nm}(\delta^t_n - \delta^t_m))\nu^{t,min}_n \\
-\;& (\pi - \delta^t_n)\xi^{t,max}_n - (\pi + \delta^t_n)\xi^{t,min}_n
\end{aligned} \quad (5.21)$$

5.3.1 The KKT-Conditions for the lower level problem

For a convex problem, the KKT conditions are satisfied if $f(x)$ is convex and $h(x)$ is affine (connected to $f(x)$). By using the Lagrangian function of the problem from the previous chapter, the KKT conditions can be derived by using the rules from chapter 2.3.

Optimality Conditions:

$$n^t_{G_i} - \lambda^t_n + \mu^{t,max}_{Gi} - \mu^{t,min}_{Gi} = 0 \quad (5.22)$$

$$-\alpha^t_{L_{jb}} + \lambda^t_n + \mu^{t,max}_{L_{jb}} - \mu^{t,min}_{L_{jb}} = 0 \quad (5.23)$$

$$\sum_{m\in\Theta_n} B_{nm}(\lambda_n^t - \lambda_m^t) + \sum_{m\in\Theta_n} B_{nm}(\nu_{nm}^{t,max} - \nu_{mn}^{t,max}) + \sum_{m\in\Theta_n} B_{nm}(\nu_{mn}^{t,min} - \nu_{nm}^{t,min}) + \xi_n^{t,max} - \xi_n^{t,min} + \xi_1^t = 0 \quad \forall t, \forall n \tag{5.24}$$

Equality Constraints:

$$\sum_{t,i} P_{G_i}^t - \sum_{t,j,b} P_{L_{jb}}^t - \sum_{t,j} P_{L_j}^{t,ref} = \sum_{t,m\in\Theta_n} B_{nm}(\delta_n^t - \delta_m^t) \quad \forall t, \forall j, \forall b, \forall n \tag{5.25}$$

$$0 = \delta_n^t \forall t, n = 1 \tag{5.26}$$

Complementary conditions and inequality constraints for primal and dual variables.

$$0 \leq P_{G_i}^{t,max} - P_{Gi}^t \perp \mu_{G_i}^{t,max} \geq 0 \ \forall t,\ \forall i \tag{5.27}$$

$$0 \leq P_{L_{jb}}^{t,max} - P_{L_{jb}}^t \perp \mu_{L_{jb}}^{t,max} \geq 0 \ \forall t,\ \forall j \tag{5.28}$$

$$0 \leq P_{G_i}^t - P_{G_i}^{t,min} \perp \mu_{G_i}^{t,min} \geq 0 \ \forall t,\ \forall i \tag{5.29}$$

$$0 \leq P_{L_{jb}}^t \perp \mu_{L_{jb}}^{t,min} \geq 0 \ \forall t,\ \forall i \tag{5.30}$$

$$0 \leq P_{l_{nm}}^{max} + B_{nm}(\delta_n^t - \delta_m^t) \perp \nu_n^{t,min} \ \forall t,\ \forall n, \forall m \in \Omega_n \tag{5.31}$$

$$0 \leq P_{l_{nm}}^{max} - B_{nm}(\delta_n^t - \delta_m^t) \perp \nu_n^{t,max} \ \forall t,\ \forall n, \forall m \in \Omega_n \tag{5.32}$$

$$0 \leq \pi - \delta_n^t \perp \xi_n^{t,max} \geq 0 \ \forall t,\ \forall n \tag{5.33}$$

$$0 \leq \pi + \delta_n^t \perp \xi_n^{t,min} \geq 0 \ \forall t,\ \forall n \tag{5.34}$$

5.4 MPEC

The overall problem is a mathematical program with equilibrium constraints, where the lower level problem is replaced by its KKT conditions.

The MPEC contains some non-linearities, the upper level problem has two unknown variables, λ_n^t and P_{Lj}^t. These two variables are multiplied with each other and make the equation non-linear. By rewriting the lower level problem as a Lagrangian dual problem and substituting it into the upper level problem, these non-linearities can be solved. Also some of the complementary slackness conditions are non-linear, (5.27)-(5.34) and thus need to be linearized. This is done by introducing integer variables.

The upper level problem:

$$\min_{\alpha_{L_{jb}}^t} \sum_{t,n} \lambda_n^t \sum_{j,b} P_{L_{jb}}^t \tag{5.35}$$

Subject to (5.5)-(5.9) and to KKT conditions (5.22)-(5.34)

5.5 Linearization of the objectiv function

To linearize the objective function, the term $\lambda_n^t P_{L_{jb}}^t$ can be substituted by the Lagrangian dual problem of the lower level problem. The Lagrangian dual problem is found by first setting up the Lagrangian of the lower level problem.

$$\mathcal{L}(x, \lambda, \mu) = F(x) \ + \ \lambda^T g(x) \ + \ \mu^T h(x) \tag{5.36}$$

The Lagrangian of the lower level problem:

$$\sum_{t,i} n_{G_i} \cdot P_{G_i}^t \ - \ \sum_{t,j,b} \alpha_{L_{jb}}^t \cdot P_{L_{jb}}^t \tag{5.37}$$

$$+ \ \sum_{t,n} \lambda_n^t \Bigg(\sum_{L_j \in \Omega_n} P_{L_{jb}}^t + \sum_{L_j \in \Omega_n} P_{L_j}^{t,ref} + \sum_{t,m \in \Omega_n} B_{nm}(\delta_n^t - \delta_m^t) - \sum_{Gi \in \Omega_n} P_{Gi}^t \Bigg)$$

$$+ \ \sum_i (P_{Gi}^t - P_{Gi}^{t,max}) \mu_{Gi}^{t,max} - \sum_i (P_{Gi} - P_{Gi}^{t,min}) \mu_{Gi}^{t,min} \tag{5.38}$$

$$+ \ \sum_j (P_{L_{jb}}^t - P_{L_{jb}}^{t,max}) \mu_{L_{jb}}^{t,max} - \sum_j (P_{L_{jb}}^t - P_{L_{jb}}^{t,min}) \mu_{L_{jb}}^{t,min}$$

$$- \ \sum_{t,n,m \in \Theta_n} (P_{l_{nm}}^{t,max} - B_{nm}(\delta_n^t - \delta_m^t)) \nu_n^{t,max} - \sum_{t,n,m \in \Theta_n} (P_{l_{nm}}^{t,max} + B_{nm}(\delta_n^t - \delta_m^t)) \nu_n^{t,min}$$

$$- \ (\pi - \delta_n^t) \xi_n^{t,max} - (\pi + \delta_n^t) \xi_n^{t,min} + \sum_t \xi_1^t \delta_1^t \tag{5.39}$$

The Lagrangian dual problem is defined by: $\mathcal{G}(\lambda, \mu) = inf \ \mathcal{L}(x, \lambda, \mu)$. In this case the problem is minimized by setting the optimization variables to zero. By also using the strong duality theorem, where the primal and dual problem have the same value at the optimal point. The new equation is given by the primal variables of the lower level problem as a function of the dual variables:

$$\sum_{t,i} n_{G_i} P_{G_i}^t \ - \ \sum_{t,j,b} \alpha_{L_{jb}}^t \cdot P_{L_{jb}}^t = \sum_{t,n} \lambda_n^t \sum_{j \in \Omega_n} P_{L_j}^{t,ref}$$

$$- \ \sum_i P_{Gi}^{t,max} \mu_{Gi}^{t,max} + \sum_i P_{Gi}^{t,min} \mu_{Gi}^{t,min} - \sum_j P_{L_{jb}}^{t,max} \mu_{L_{jb}}^{t,max} + \sum_j P_{L_{jb}}^{t,min} \mu_{L_{jb}}^{t,min}$$

$$- \ \sum_{t,n,m \in \Theta_n} P_{l_{nm}}^{t,max} \nu_n^{t,max} - \sum_{t,n,m \in \Theta_n} P_{l_{nm}}^{t,max} \nu_n^{t,min} - \pi \xi_n^{t,max} - \pi \xi_n^{t,min} \tag{5.40}$$

$$-\sum_{t,jb}\alpha^t_{L_{jb}}P^t_{L_{jb}} = -\sum_{t,i}n_{G_i}P^t_{G_i} + \sum_{t,n}\lambda^t_n\sum_{j\in\Omega_n}P^{t,ref}_{L_j} - \sum_{t,G_i}\mu^{t,max}_{G_i}P^{t,max}_{G_i}$$
$$+\sum_{t,G_i}\mu^{t,min}_{G_i}P^{t,min}_{G_i} - \sum_{t,jb}\mu^{t,max}_{L_{jb}}P^{t,max}_{L_{jb}} - \sum_{t,m\in\Omega_n}\nu^{t,max}_{nm}P^{max}_{l_{nm}}$$
$$-\sum_{t,m\in\Omega_n}\nu^{t,min}_{nm}P^{max}_{l_{nm}} - \sum_{t,n}\xi^{t,min}_n\pi - \sum_{t,n}\xi^{t,max}_n\pi \quad (5.41)$$

To substitute the new function into the upper level problem, we want to obtain a relation between the bidding variable α^t_{Lj} and the Lagrangian λ^t_n. This is done by first substituting α^t_{Lj} with equation (5.23), which gives us a new linear objective function:

$$\sum_{t,jb}\alpha^t_{L_{jb}}P^t_{L_{jb}} = \sum_{t,jb}(\lambda^t_n + \mu^{t,max}_{L_{jb}} - \mu^{t,min}_{L_{jb}})P^t_{L_{jb}} \quad (5.42)$$

$$\sum_{t,jb}\alpha^t_{L_{jb}}P^t_{L_{jb}} = \sum_{t,jb\in\Psi_n}\lambda^t_nP^t_{L_{jb}} + \sum_{t,L_{jb}}\mu^{t,max}_{L_{jb}}P^t_{L_{jb}} - \sum_{t,jb}\mu^{t,min}_{L_{jb}}P^t_{L_{jb}} \quad (5.43)$$

This can be rewritten with some of the KKT conditions.

$$\mu^{t,max}_{L_{jb}}P^t_{L_{jb}} = \mu^{t,max}_{L_{jb}}P^{t,max}_{L_{jb}} \quad (5.44)$$

$$\mu^{t,min}_{L_{jb}}P^t_{L_{jb}} = \mu^{t,min}_{L_{jb}}P^{t,min}_{L_{jb}} = 0 \quad (5.45)$$

Substituting this relation in (5.43) gives:

$$\sum_{t,Lj}\alpha^t_{L_{jb}}P^t_{L_{jb}} = \sum_{t,jb\in\Psi_n}\lambda^t_nP^t_{L_{jb}} + \sum_{t,jb}\mu^{t,max}_{L_{jb}}P^{t,max}_{L_{jb}} - \sum_{t,jb}\mu^{t,min}_{L_{jb}}P^{t,min}_{L_{jb}} \quad (5.46)$$

By substituting this into (5.41) we can express α^t_{Lj} in terms of λ^t_n ,which gives:

$$\sum_{t,jb}\lambda^t_nP^t_{L_{jb}} = \sum_{t,i}n_{G_i}P^t_{G_i} - \sum_{t,n}\lambda^t_n\sum_{j\in\Omega_n}P^{t,ref}_{L_j} + \sum_{t,i}\mu^{t,max}_{G_i}P^{t,max}_{G_i} - \sum_{t,G_i}\mu^{t,min}_{G_i}P^{t,min}_{G_i}$$
$$+\sum_{t,m\in\Omega_n}\nu^{t,max}_{nm}P^{max}_{l_{nm}} + \sum_{t,m\in\Omega_n}\nu^{t,min}_{nm}P^{max}_{l_{nm}} + \sum_{t,n}\xi^{t,max}_n\pi + \sum_{t,n}\xi^{t,min}_n\pi \quad (5.47)$$

$$\Rightarrow \min_{\lambda^t_n, P^t_{L_{jb}}}\Bigg(\sum_{t,i}n_{G_i}P^t_{G_i} - \sum_{t,n}\lambda^t_n\sum_{j\in\Omega_n}P^{t,ref}_{L_j} + \sum_{t,i}\mu^{t,max}_{G_i}P^{t,max}_{G_i} - \sum_{t,Gi}\mu^{t,min}_{G_i}P^{t,min}_{G_i}$$
$$+\sum_{t,m\in\Omega_n}\nu^{t,max}_{nm}P^{max}_{l_{nm}} + \sum_{t,m\in\Omega_n}\nu^{t,min}_{nm}P^{max}_{l_{nm}} + \sum_{t,n}\xi^{t,max}_n\pi + \sum_{t,n}\xi^{t,min}_n\pi\Bigg) \quad (5.48)$$

5.5.1 Linearization of the KKT conditions

The KKT conditions can be linearized by using the theorem for complementary slackness from chapter 2.3. This gives additional constraints with a linearization optimization variable and a constant M, which is deciding the size of the variable and this one should be large. The new conditions:

$$
\begin{aligned}
P_{Gi}^{t,max} - P_{Gi}^{t} &\geq 0 \; \forall t, \; \forall i && (5.49)\\
P_{L_{jb}}^{t,max} - P_{L_{jb}}^{t} &\geq 0 \; \forall t, \; \forall j && (5.50)\\
\mu_{G_i}^{t,max} &\geq 0 \; \forall t, \; \forall i && (5.51)\\
\mu_{Ljb}^{t,max} &\geq 0 \; \forall t, \; \forall j, b && (5.52)\\
P_{Gi}^{t,max} - P_{Gi}^{t} &\leq (1 - \omega_{Gi}^{t,max}) M^{P} \; \forall t, \; \forall i && (5.53)\\
P_{L_{jb}}^{t,max} - P_{L_{jb}}^{t} &\leq (1 - \omega_{L_{jb}}^{t,max}) M^{P} \; \forall t, \; \forall j, b && (5.54)\\
\mu_{G_i}^{t,max} &\leq \omega_{G_i}^{t,max} M^{\mu^P} \; \forall t, \; \forall i && (5.55)\\
\mu_{L_{jb}}^{t,max} &\leq \omega_{G_i}^{t,max} M^{\mu^P} \; \forall t, \; \forall j, b && (5.56)\\
\omega_{G_i}^{t,max}, \omega_{L_{jb}}^{t,max} &\in \{0, 1\}
\end{aligned}
$$

$$
\begin{aligned}
-P_{G_i}^{t,min} + P_{G_i}^{t} &\geq 0 \; \forall t, \; \forall i && (5.57)\\
-P_{L_{jb}}^{t,min} + P_{L_{jb}}^{t} &\geq 0 \; \forall t, \; \forall j, b && (5.58)\\
\mu_{G_i}^{t,min} &\geq 0 \; \forall t, \; \forall i && (5.59)\\
\mu_{L_{jb}}^{t,min} &\geq 0 \; \forall t, \; \forall j, b && (5.60)\\
-P_{G_i}^{t,min} + P_{G_i}^{t} &\leq (1 - \omega_{G_i}^{t,min}) M^{Q} \; \forall t, \; \forall i && (5.61)\\
-P_{Lj}^{t,min} + P_{L_{jb}}^{t} &\leq (1 - \omega_{Lj}^{t,min}) M^{Q} \; \forall t, \; \forall j && (5.62)\\
\mu_{G_i}^{t,min} &\leq \omega_{G_i}^{t,min} M^{\mu^Q} \; \forall t, \; \forall i && (5.63)\\
\mu_{L_{jb}}^{t,min} &\leq \omega_{G_i}^{t,min} M^{\mu^Q} \; \forall t, \; \forall j, b && (5.64)\\
\omega_{G_i}^{t,min}, \omega_{L_{jb}}^{t,min} &\in \{0, 1\}
\end{aligned}
$$

$$
\begin{aligned}
P_{l_{nm}}^{max} - B_{nm}(\delta_n^t - \delta_m^t) &\geq 0 \ \forall t, \forall n, \ \forall m \in \Omega_n && (5.65)\\
P_{l_{nm}}^{max} + B_{nm}(\delta_n^t - \delta_m^t) &\geq 0 \ \forall t, \forall n, \ \forall m \in \Omega_n && (5.66)\\
\nu_{nm}^{t,max} &\geq 0 \ \forall t, \forall n, \ \forall m \in \Omega_n && (5.67)\\
\nu_{nm}^{t,min} &\geq 0 \ \forall t, \forall n, \ \forall m \in \Omega_n && (5.68)\\
P_{l_{nm}}^{max} - B_{nm}(\delta_n^t - \delta_m^t) &\leq (1 - \psi_{nm}^{t,max})M^C \ \forall t, \forall n, \ \forall m \in \theta_n && (5.69)\\
P_{l_{nm}}^{max} + B_{nm}(\delta_n^t - \delta_m^t) &\leq (1 - \psi_{nm}^{t,min})M^C \ \forall t, \forall n, \ \forall m \in \theta_n && (5.70)\\
\nu_{nm}^{t,max} &\leq \psi_{nm}^{t,max} M^{\nu^C} \ \forall t, \forall n, \ \forall m \in \theta_n && (5.71)\\
\nu_{nm}^{t,min} &\leq \psi_{nm}^{t,min} M^{\nu^C} \ \forall t, \forall n, \ \forall m \in \theta_n && (5.72)\\
\psi_{nm}^{t,max}, \psi_{nm}^{t,min} &\in \{0, 1\}
\end{aligned}
$$

$$
\begin{aligned}
\pi - \delta_n^t &\geq 0 \ \forall t, \ \forall n && (5.73)\\
\pi + \delta_n^t &\geq 0 \ \forall t, \ \forall n && (5.74)\\
\xi_n^{t,max} &\geq 0 \ \forall t, \ \forall n && (5.75)\\
\xi_n^{t,min} &\geq 0 \ \forall t, \ \forall n && (5.76)\\
\pi - \delta_n^t &\leq (1 - \varphi_n^{t,max})M^D \ \forall t, \ \forall n && (5.77)\\
\pi + \delta_n^t &\leq (1 - \varphi_n^{t,min})M^D \ \forall t, \ \forall n && (5.78)\\
\xi_n^{t,max} &\leq \varphi_n^{t,max} M^{\xi^D} \ \forall t, \ \forall n && (5.79)\\
\xi_n^{t,min} &\leq \varphi_n^{t,min} M^{\xi^D} \ \forall t, \ \forall n && (5.80)\\
\varphi_n^{t,min}, \varphi_n^{t,min} &\in \{0, 1\}
\end{aligned}
$$

5.6 Adding the constraints for the PHEV to the model

To find the equality constrains for the PHEV, the following equation needs to be rewritten, by using the condition for initial value and the final value for the virtual battery:

$$E^t_{VB_j} = E^0_{VB_j} + \sum_{l=1}^{t} P^l_{L_{jb}} \Delta t \eta_{jb,char} - \sum_{l=1}^{t} E^l_{VB_{j,dep}} + \sum_{t=1}^{l} E^l_{VB_{j,arr}} \tag{5.81}$$

For the last time step:

$$E^T_{VB_j} = E^0_{VB_j} \tag{5.82}$$

which gives the equality constraint per node:

$$0 = \sum_{l=1}^{T} P^l_{L_{jb}} \Delta t \eta_{j,char} - \sum_{l=1}^{T} E^l_{VB_{j,dep}} + \sum_{l=1}^{T} E^l_{VB_{j,arr}} \tag{5.83}$$

$$\sum_{l=1}^{T} P^l_{L_{jb}} = \frac{\sum_{l=1}^{T} E^l_{VB_{j,dep}} - \sum_{l=1}^{T} E^l_{VB_{j,arr}}}{\Delta t \eta_{jb,char}} \tag{5.84}$$

By inserting (5.81) into (5.6) the inequality constraint can be found

$$E^t_{VB_j,min} \leq E^0_{VB_j} + \sum_{l=1}^{t} P^l_{L_{jb}} \Delta t \eta_{jb,char} - \sum_{l=1}^{t} E^l_{VB_{j,dep}} + \sum_{l=1}^{t} E^l_{VB_{j,arr}} \leq E^l_{VB_j,max} \tag{5.85}$$

$$\sum_{l=1}^{t} P^t_{L_{jb}} \Delta t \eta_{j,char} + E^0_{VB_j} \leq E^t_{VB_j,max} + \sum_{l=1}^{t} E^l_{VB_{j,dep}} - \sum_{l=1}^{t} E^l_{VB_{j,arr}} \tag{5.86}$$

The new objective function:

$$\sum_{t=1} t P^{t,PHEV}_{L_{jb}} \Delta t \eta_{jb,char} + E^0_{VB_j} \geq E^t_{VB_j,min} + \sum_{t=1}^{t} E^t_{VB_{j,dep}} - \sum_{t=1}^{t} E^t_{VB_{j,arr}} \tag{5.87}$$

Chapter 6

Second model - The embedded models

The MPEC was developed for a small network with three buses and thus it was shown that the model was working properly. When the model was tested on a larger network with more buses, the number of optimization variables in this problem was to big for Matlab and thus no solution was found. To decrease the number of optimization variables, two new programs were created. Both new models are built up on the bi-level model, but are using a genetic algorithm and non-convex solver. The problems are embedded, which means that one of the problems are inside of the other. The outcome of the upper level problem is used as input for the lower level problem.

The genetic algorithm builds up on an evolutionary algorithm, where the optimization problems consist of a population. For every iteration of the problem, the best individuals are stochastically selected and these individuals constitute the new population. The algorithm runs until the population have reached a satisfied level of fitness [16].

The solve the upper level problem the standard Matlab tool Fmincon is used. This is a non-convex solver, which is based on a Nelder-Mead algorithm. This is a heuristic search method that can converge to a non-stationary point [17]. It can solve a constrained non linear metavariable function and is using the cost as input. The cost is gotten from the OPF, which is embedded in the upper level objective function. The OPF is also linear.

The upper level problem is the same as in the first model, but instead of getting the demand bids as in the MPEC, the result from these two programs will be the derived charging profile.

$$\min_{\alpha^t_{L_{jb}}} \sum_{t,n} \lambda^t_n \sum_{j,b} P^t_{L_{jb}} \tag{6.1}$$

This problem is using same constraints as the upper level problem. The program calls the lower level problem which is the market clearing built on OPF framework placing the bids:

$$\min_{P^t_{G_i}, E^0_{V_j}, P^t_{L_{jb}}} \sum_{t,i} n_{G_i} P^t_{G_i} - \sum_{t,j,b} \alpha^t_{L_{jb}} P^t_{L_{jb}} \tag{6.2}$$

where n_{G_i} is the marginal cost for the generation, $P^t_{G_i}$ is the generated capacity, $\alpha^t_{L_{jb}}$ is the demand bid and $P^t_{L_{jb}}$ is the PHEV load. This problem has the same objective function and constraints as the lower level problem in the first model. The only difference is that the OPF is using the Power transfer distribution factor (PTDF) instead of voltage angles. By solving the problem with embedded models, the number of optimization variables Matlab is handling simultaneously will be reduced. The dual variables and the integers for the complementary constraints are not used, which is also reducing the number of variables. It is therefore possible to find solutions for larger network problems. The problems is not convex, therefore there is no guarantee that the embedded models will find the global maximum point rather then finding a local maximum point.

Chapter 7

Case study

In the process of creating the optimization model, it was first tested on a three bus network to to make certain that the model was working properly. The model has been run on a computer with two Intel Xenon 2.66GHz processors and 24GB ram-memory and the used software is Matlab with the optimization toolbox Tomlab. There are four different optimization models that are used in this case study; the MPEC, the embedded non-convex solver(Fmincon), the embedded genetic algorithm and the minimum system cost model. The three first models have the objective to minimize the charging cost, while the fourth has the objective to minimize system cost.

In this part the results from four case studies will be presented. Each part is followed by a discussion where the results are evaluated. In the first case study the solution from the models are investigated to see if the load schedule seems to be correct and logic. In the second case the overall performances are tested on the four models and a cost comparison between the cost models. The third case investigates how congestions in the grid are effecting the results. The last case is the test on the Swiss grid , where the results are showing how the PHEV fleet is affecting the system and a comparison between the cost models.

7.1 Case 1 - Load schedule

The purpose of this part is to see which periods the PHEVs are charging and to investigate the hourly prices. This is mainly done to validate that the results are logical. The results come from the four bus network and it consists of two generators, four loads and four lines. The reference load represents the normal load during a winter day. The PHEV load is the demand for each hour during a 24 hour period. The data is the simulated behavior of the PHEV fleet and consists of, the required energy, the total connected capacity and departing and arriving energy.

The network is simulated with four different set ups; the embedded models, the MPEC with three and ten load blocks. There are no observable differences in the results from the MPEC with three and ten blocks, therefore only one of the figures is presented. In figure 7.1 the reference load on all four nodes is shown. The reference load is the same for all three simulations.

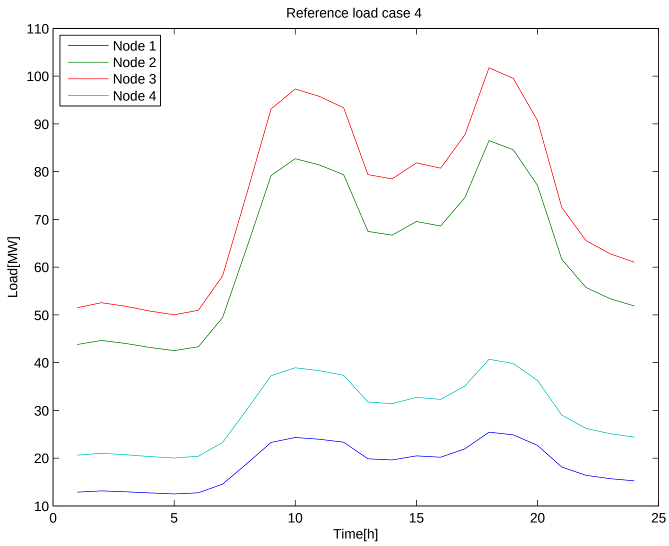

Figure 7.1: Reference load per node.

In figure 7.2 the nodal price at each bus is shown for the MPEC ten block model and the nodal prices are the same for the other models. In figure 7.3

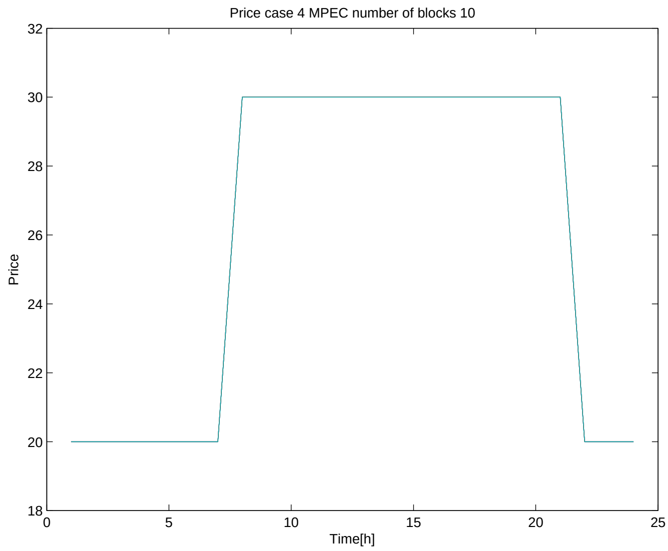

Figure 7.2: MPEC 10 block model: Price [CHF/MW].

to 7.4 the charged load of the PHEV at each node is shown. All the results are plotted and there is no difference between the MPEC with three blocks and ten blocks. There is no difference between the embedded models, but between the MPEC and the embedded models the charging pattern differs .

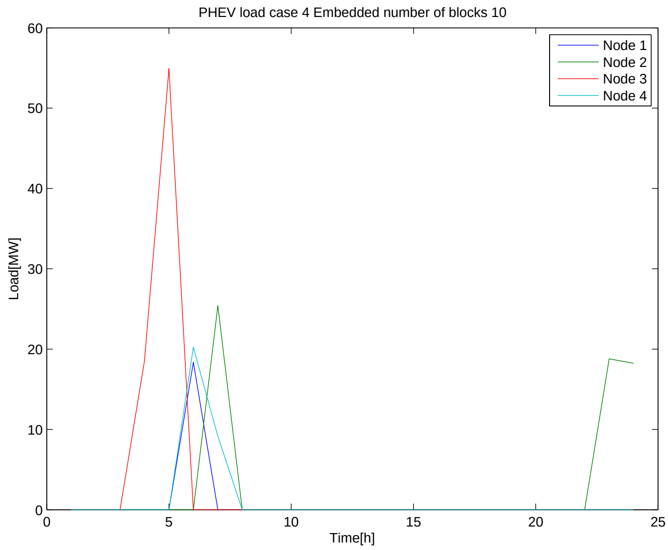

Figure 7.3: Embedded model, Fmincon: PHEV charged load.

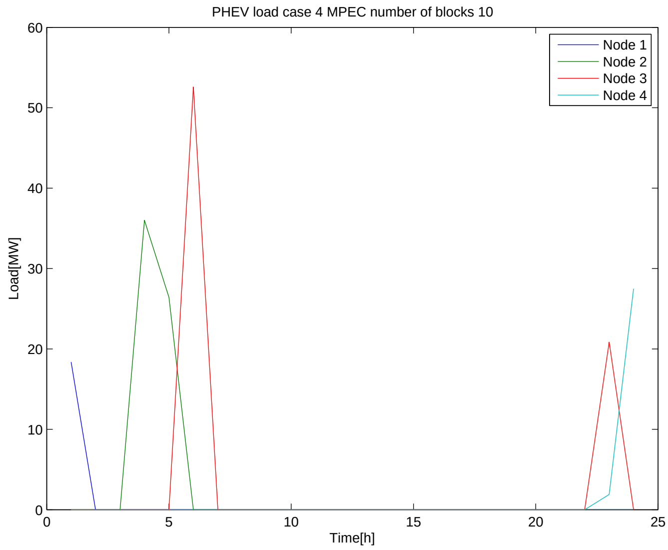

Figure 7.4: MPEC 10 block model: PHEV charged load.

7.1.1 Comments on the load schedule

When comparing the reference load (figure 7.1) with the nodal prices (figure 7.2) it can be seen that the prices are higher during those periods when the demand is high. In figure 7.3, the PHEVs charging load is shown. Here it is observable that the charging takes place during the time when the reference load and the price are low. Further more there is a difference between the MPEC and the embedded models when the PHEV fleet is charging. By plotting the total system load it can be seen that the load pattern is different between the embedded models and the MPEC. This is shown in figure 7.5 where all the four models; MPEC, Fmincon, genetic algorithm and the minimum system cost model are plotted together with the reference load. The embedded models are following the same load curves as the minimum system cost model. This is because of that the embedded models are using the results from the minimum system cost model as start values. A clearer plot of this figure B.1 appendix B, where the MPEC load curve is removed.

The total system cost and the charging cost for the four bus network

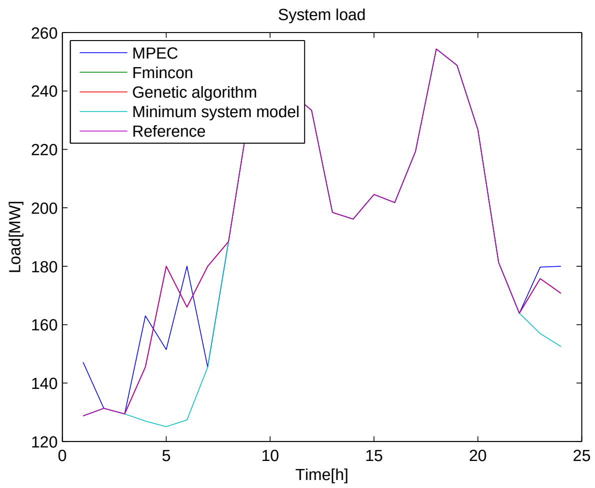

Figure 7.5: Reference load and the total system load computed with all four models.

is the same, apart from a very small difference compared to the genetic algorithm. In the end it seems like both models find the lowest cost but with different solutions.

7.2 Case 2 - Tests on 4 different networks

All the minimum charging cost models and the minimum system cost model are tested on all four different systems with 4, 9, 14 and 30 buses. The MPEC is tested twice, with three and ten load blocks. For each networks the models performance will be investigated. The outputs from each model are:

- Computational time in seconds it takes to find the solution for the problem.
- Computational time in seconds it takes for the solver to optimize the problem.
- Computational time in seconds for solving the optimization problem with different sizes of the linearization variable M.
- System cost
- Charging cost

The system cost and charging cost are used to compare the cost differences between the minimum charging cost models and the minimum system cost model. All the data from these tests can be found in Appendix B. From the test it was found that the embedded models and the MPEC were able to solve all networks.

7.2.1 Comments on test results

When comparing the data from the four different cases it can clearly be seen that the MPEC seems to be faster than the embedded models for solving each case. This can be seen in figure 7.6 and 7.7. The genetic algorithm seems to be faster at solving larger networks, while the Fmincon is faster for smaller networks, figure 7.7. The number of blocks for the MPEC is also a factor that determine the solving speed, figure 7.6. The solving speed is also dependent on the size of the linerazation variable M. This can be seen in appendix B for case nine where the computational time is shown for different values of M. In the MPEC a relationship between number of buses and solving time can be seen, figure 7.6. The Fmincon seems to be dependent on the number of loads, the genetic algorithm seems not. This can be seen in figure 7.8 where the computational time for the Fmincon is increasing with the number of loads. The genetic algorithm's solving time is higher for three loads than for four loads.

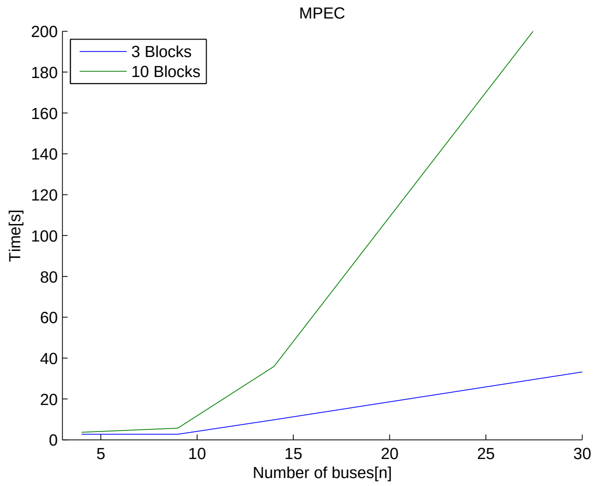

Figure 7.6: Solving time MPEC

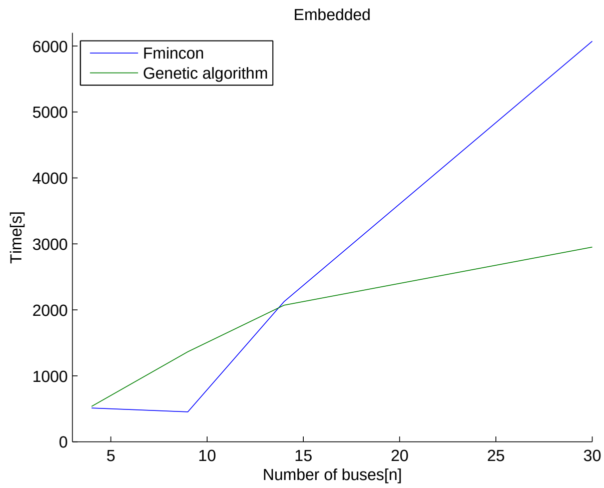

Figure 7.7: Computational time embedded models [time/bus], where the genetic algorithm seems to be faster at solving larger networks, while the Fmincon is faster for smaller networks.

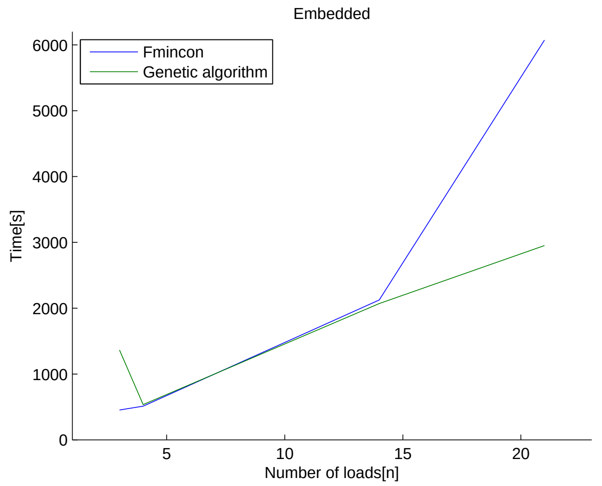

Figure 7.8: Computational time embedded models [time/load], where the Fmincon model seems to dependent on the number of loads.

Blocks	3	10
Computational time [s]	2.6832	3.6816
Optimization time [s]	1.3572	1.6224

Table 7.1: MPEC: Computational time for 9 bus network.

Table 7.1 shows the computational time for the MPEC to solve the nine bus network. The compute time is the total time it takes to solve the problem and optimization time is the time it takes for the Tomlab solver to find the optimal solution. By taking the computational time minus the optimization time it yields the time it takes for Matlab to set up the different matrices for the solver. This time can be reduced by writing the code more efficiency, which in the end will give a faster solution to each network.

Total system cost				
Buses	4	9	14	30
Genetic Algorithm	1	1	1	1
Fmincon	1	1	1	1
MPEC	1	1	1	1
Charging cost				
Buses	4	9	14	30
Genetic Algorithm	1	1	0.7522	1
Fmincon	1	1	0.7522	1
MPEC	1	1	0.6757	1

Table 7.2: The cost differences in per unit between the models, with the minimum charging cost models as reference.

In table 7.2 the cost differences in per unit have been calculated between the models, with the minimum charging cost models as reference. It can be seen that there is almost no differences in the total system cost for the two types of cost models. Although there are different charging costs in the 14 bus network, where the charging cost is much lower with the minimum charging cost models, especially with the MPEC. An explanation for this cost difference is that the embedded models may have found a local minimum point, while the MPEC may have found the global minimum. A conclusion to be drawn from these results is that the MPEC model is superior at finding the optimal point in the feasible space.

7.3 Case 3 - Congestion

Two of the test networks are containing congestions; the 9 and 14 bus network. The presented results here are the nodal prices for the 9 (figure 7.9) and the 14 bus network (figure 7.10) both simulated with the MPEC 3 block model. Different prices at every node can be observed in these figures. Congestions may be found when a line is maximally loaded, this is shown in figure 7.11 and 7.12 where the load flows are plotted in per unit. The two networks were also tested without any congestion and this by increasing the capacity of the congested lines. This was effecting the nodal prices which can be seen for the nine bus network in figure 7.13.

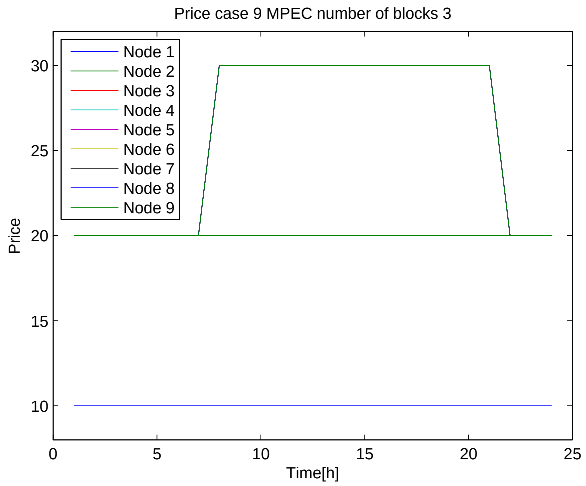

Figure 7.9: MPEC 3 Block model: Nodal prices[CHF/MW] for 9 bus network.

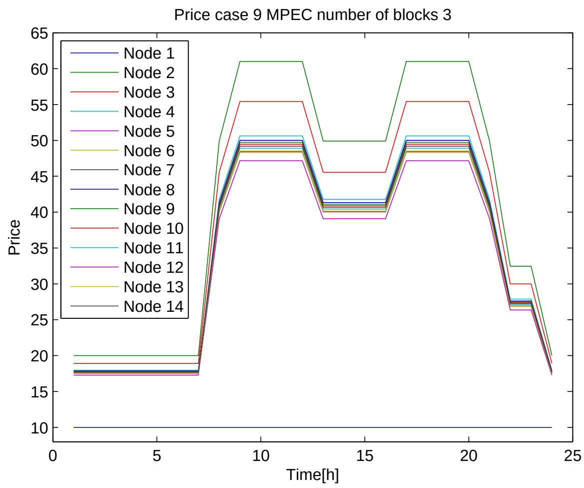

Figure 7.10: MPEC 3 Block model: Nodal prices[CHF/MW] for 14 bus network.

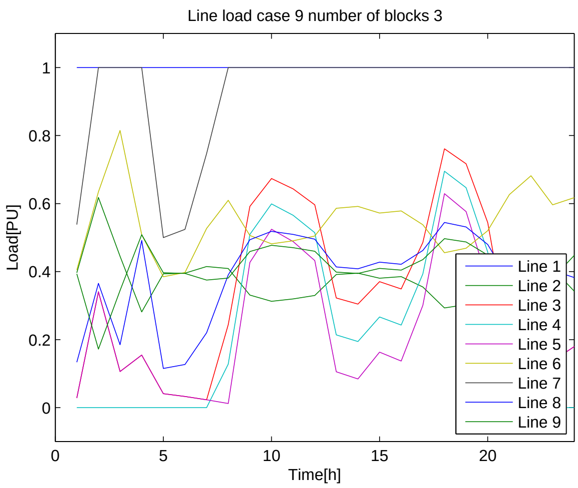

Figure 7.11: MPEC 3 Block model: Line flows for the 9 bus model, where congestion can be seen on line 1 and 7.

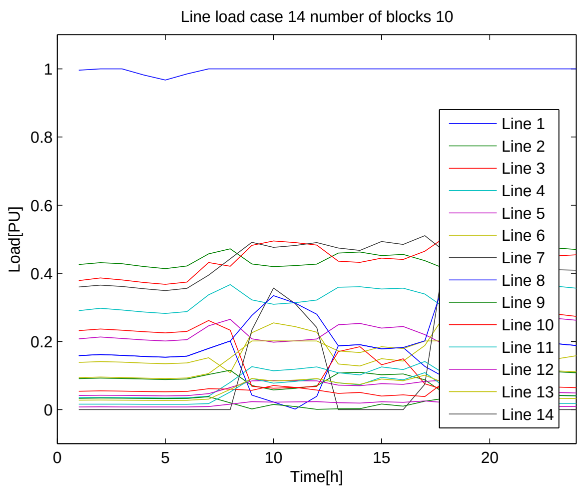

Figure 7.12: MPEC 3 Block model: Line flows for the 14 bus model, where congestion can be seen on line 1.

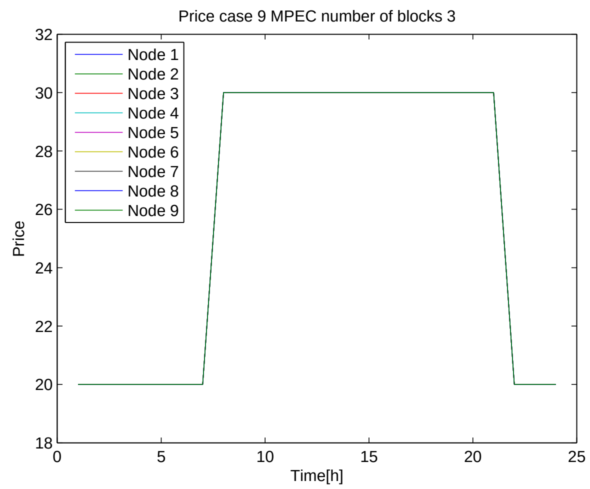

Figure 7.13: MPEC 3 Block model: Nodal price for case 9 without congestion, the prices are homogenous on all nodes.

Model	Fmincon	MPEC 10 Blocks
Computational time [s]	452.0440	5.6784
Computational time without congestion [s]	182.8175	2.8548

Table 7.3: Computational time with and without congestion

It was also found that the solving time was faster when the congestions were removed. The new computational time for the nine bus network can be found in table 7.3.

Also remarkable in this case, is the solution found by the embedded models for the 14 bus network. It can be seen that the price is higher around time period five, figure 7.14. There are also different prices for the embedded models and the MPEC between time 20 and 24 (MPEC figure 7.10). Both these observations may show that the models finds different solutions, which is affecting the cost.

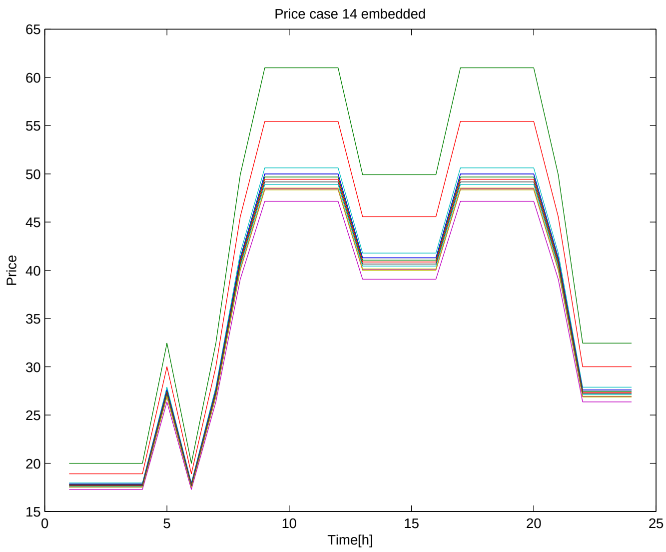

Figure 7.14: Embedded model: Nodal price for case 14.

7.3.1 Comments on congestion

In the 9 and 14 bus networks different nodal prices exist. This can be seen in figure 7.9 and 7.10. The reason for price differences is that the competition on the market is not perfect. This non perfect state is caused by the congestions on the lines, where power can't flow freely between all regions (buses). The line flows are shown in per unit in figure 7.11 and 7.12. An indication of congestion is that there is a steady state condition at the lines' maximum capacity (1 p.u.). This can be seen at line 1 and 7 in the 9 bus network and at line 1 for the 14 bus network. When increasing the capacity of the lines for both networks, the congestions are removed and there are homogenous prices on the market. This can be seen at the 9 bus network in figure 7.13.

When solving the 14 bus network the nodal price are different between the MPEC and the embedded models. This can be observed in figure 7.14 where the price is higher around time period five with the embedded model. There are also different prices for the models between time 20 and 24 (MPEC figure 7.10). When comparing the line flows, it can be seen that the line for the MPEC is not congested around time period five (figure 7.12), while the embedded is (figure B.2). The charging is also different for these periods which can be seen in figure 7.15 where the total system load is plotted.

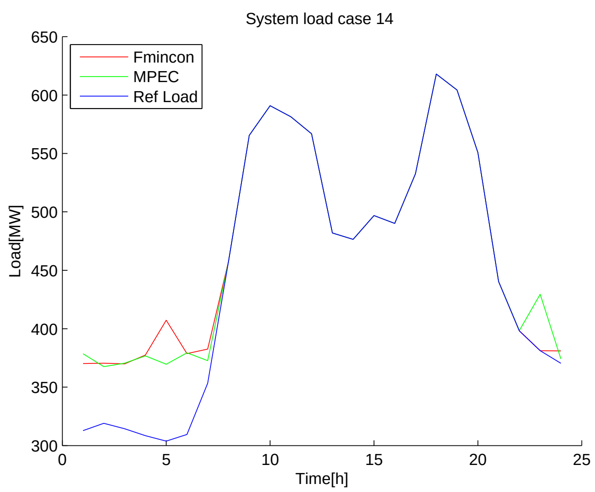

Figure 7.15: Total system load for case 14.

The conclusion of this is that the embedded models are finding different solutions then the MPEC. The MPEC may have found the global minimum while the Fmincon model only could have found a local minimum. This is discussed earlier in case two as it was found that the charging costs here not the same for the models due to different solutions.

Total system cost		
Congestion	none	with
Fmincon	1	1
Charging cost		
Fmincon	1	0.7522

Table 7.4: The cost differences in per unit between the models, with the minimum charging cost models as reference. If the value is smaller than one, the cost is lower the minimum charging cost model. One the cost is equal

The results from the 14 bus network shows that the system cost is higher with congestion. This could give the producer the ability to raise the electricity price by congesting the lines. By investigating how the cost between the two cost models is changing with congestion, it can be seen whether or not the aggregators will gain any market power with the minimum charging cost models. There are two cases with and without congestion, where the cost models are compared, table 7.4. In the case of congestion the consumers have a lower charging cost than the case without. The conclusion to be drawn from this result is that the consumer may gain market with the minimum charging cost models.

7.4 Case study - Swiss system

This is the final test of the swiss power system where PHEV load is simulated with one million vehicles. The energy usage is due to a normal day driving pattern of a swiss citizen. The MPEC and the genetic algorithm were not able to solve a system of this size, so the results come from the embedded model with the Fmincon solver.

Number of vehicles	1 million
Total charged power	8.374GW

Table 7.5: PHEV fleet data

Total system cost	
Fmincon	1
Charging cost	
Fmincon	0.9991

Table 7.6: Cost comparison

The results that are presented here are a cost comparison in per unit between the minimum system cost model and the demand bid model (the minimum system cost model is used as reference), table 7.6. In figure 7.16 different nodal prices due to congestions are shown and the load curve with and without PHEV fleet in figure 7.17. Data for the fleet the simulated fleet can be found in table 7.5.

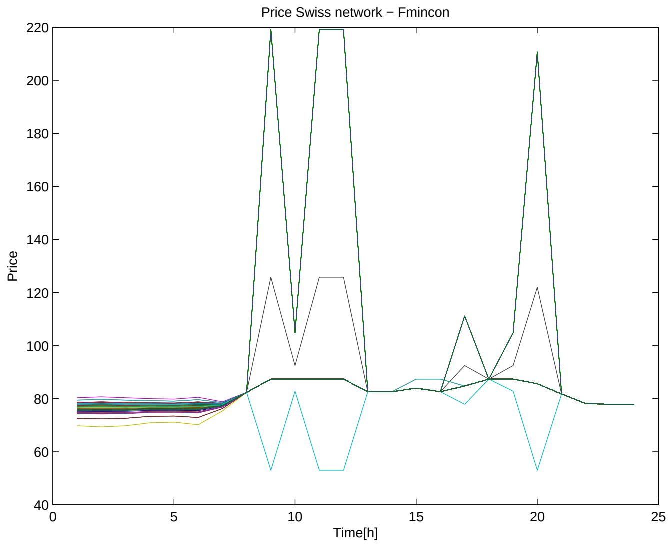

Figure 7.16: Fmincon model: Nodal prices [CHF/MW].

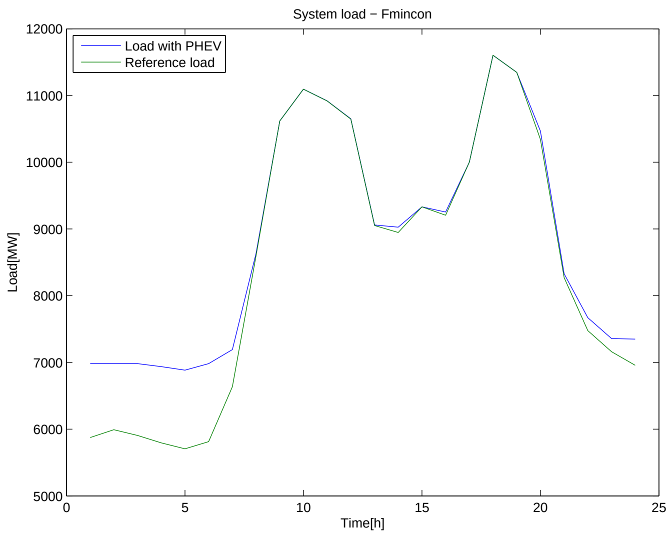

Figure 7.17: Fmincon model: Total load with and without PHEV fleet.

7.4.1 Comments on the Swiss system

From the results of the simulations on the swiss system, it can be seen that total load in the system is increased with 4.12% by the PHEV fleet.

When comparing the two cost models, the minimum system cost and the minimum charging cost, it appears that the charging is slightly lower with the charging model, table 7.6. The total system cost is more or less the same, consequently there is not a large difference between this two cost models. However, there can probably be a difference if the MPEC would have worked in this system. Thus is due to previous cost comparison of the smaller networks.

In the Swiss system congestions also do exist. This can be seen due to the cost differences between a large number of nodes, figure 7.17.

The charging of the vehicles is mainly done during the low load hours at evening and night, which is called Valley filling. This can be seen in figure 7.17 when comparing the reference load with the total load.

Chapter 8

Conclusion

In this part the conclusions from the development process and results of this master project are presented.
Conclusions about the MPEC:

- The MPEC model can find a charging schedule for the PHEV fleet.
- The MPEC model can't solve large networks due to too many optimization variables.
- By making the Matlab code more efficient, the set up of the matrices for the solver is made faster and the total time thus is decreased.

Conclusions from the results by the MPEC and the embedded models:

- The MPEC model is faster at solving problems then the embedded models for the four networks.
- The size of the linearization variable M is affecting the computational time for the MPEC. A larger M is prolonging the time. Also congestion in the system increases the computational time, which was found in the 9 bus network.
- The MPEC and the embedded models find different solutions. In one case the MPEC is better at finding an optimal solution then the embedded models, which was found when comparing the charging cost for the 14 bus network. Due to that, the MPEC finds the global minimum point and the embedded models find a local minimum.
- Compared with the minimum system cost model, the MPEC and the embedded modes give lower charging costs in some cases, which subsequently gives the PHEV aggregators market power.

- Congestion in the system is affecting the prices.
- For the Swiss system there is almost no cost difference between the embedded model, with the Fmincon solver, and the minimum system cost model.

8.1 Future work

The MPEC

The future work of this thesis would be to test the MPEC on a computer with larger capacity. Parts of the code for the MPEC can be made more efficient in order to decrease the computational time. The MPEC and embedded models ought to also be tested on more networks and compared to the supply bid model, to further investigate the opportunities of market power with the demand bid model.

Model development

In this model the PHEVs consist on fixed loads on every node. The loads are simulated from the driving pattern by the vehicles. A future work is to create a stochastic model for these loads, where the PHEVs are randomly charging at different nodes. Another further work could be to optimize which loading node the PHEV fleet is choosing depending to the nodal price and see how load is changing in the system.

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Appendix A

Basic micro economics

To analyze questions about the price and quantity in a market, the supply and demand model is used. This model is a graph with two linear functions, which are showing the demand and the supply in the market.

A.1 Supply and demand

The demand is defined as the amount of good that a consumer is willing to buy at given price. A consumer tends to demand a larger quantity at a lower the price. To get an overview of the problem the function can be plotted in a diagram, to shows the relationship between quantities demanded at each possible price. It describes how many units a consumer is willing to buy at a certain price. There are two causes that can change the demand and there is a clear distinction in how to determine these. When the underlying price is changing there will be a movement along the demand curve to find the new demanded quantity. When there is a change in other variables the demand curve is shifted to the right or to the left in order to get the new demanded quantity. For instance when the gasoline price goes up on, consumers search a substitute and start to use electric cars. The effect of this is that the demand for electricity is rising and the demand curve is shifted to the right. The demand function's mathematical model can either be a linear function or a non-linear function. Individual demand curves can be summed up to one curve to determine a group's behavior. The curve can be described by the following equation:

$$D = m - kq \tag{A.1}$$

To determine the market price we also need to know the quantity that firms are willing to supply at a given price. The supply is dependent on the cost of production and can also be affected by governmental restrictions and taxes. The supply curve shows the quantity supplied at each possible price. When the price of the underlying is changing the supply is reflected by a movement along the curve. When a change in costs, government rules, or other factors

changes the supply curve is shifted to the right or left [14].
The supply function can be written:

$$S = kq + m \tag{A.2}$$

Individual supply curves can be aggregated into one curve.

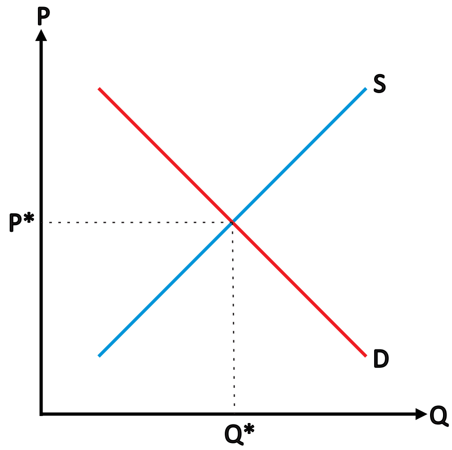

Figure A.1: The supply and demand model, where the intersection point shows the market equilibrium P^* and Q^*.

A.2 Economical terms

In this theses some economical terms will be used. The definitions for them is found in this part [14].

Market equilibrium is the state which appears when traders are able to buy and sell as much as they want in a market. There is an equilibrium price when the supply and demand doesn't change.

Price elasticity is the percentage change in the demand or supply made by a change in the price. The term inelastic is used when the demand or supply are unaffected by a price change, whereas elastic price don't.

Utility is a term used by the economist to describe the consumers' behavior on a market. The consumer preference is to choose the amount of a good which gives the largest possible utility. This can be modeled by a utility function, which shows the utility for each quantity. The utility function U is written:

$$U(z, b)$$

where y is the quantity y and b is the price. **Marginal utility** is the extra utility that a consumer gets by purchasing an additional unit of goods.

$$MU = du/dz$$

Short run cost To make a profit maximization decision a firm needs to know how their cost varies with the output. To get an overview of the costs, the firm has some concepts to be used. To produce an amount of output, there are two types of costs to use as input in the calculation. Fixed cost is the cost of the production, which is not changing in the short term with the level of output. A typical fixed cost is the one for capital goods, e.g. machines and facilities. The second one is the variable cost, which changes with the level of output, the amount of labor and material that is used in the production.

$$C = VC + F$$

where C is the total cost, VC the variable cost and F the fixed cost.

The marginal cost is how much a firm's cost is increasing by producing an additional unit of output.

$$MC = dC/dq$$

Perfect competition is a description of a state of the market, with the following properties;

- The market contains a large number of firms. If there are enough sellers in the market, no individual firms can rise or lower the price in the market.
- Identical products, if the sellers has identical or homogenous products it is difficult for the consumers to notice any difference between the product and therefore no firm can increase the price.
- Full information, consumers and producers have all information about the quantity and quality of the product in the market.
- No entry barriers, companies can enter and exit the market easy without any obstacles.
- Transactions cost, very low transaction costs for a buyer or seller to exchange goods.

The opposite to perfect competition is monopolistic competition or oligopolistic competition. Monopolistic competition takes place when there is one firm on the market that has the power and no additional firms can enter and earn profits. Oligopolistic competition is when there are a few firms that have the power in the market and there are large barriers, which make it difficult for additional firms to enter.

Welfare maximization Consumer welfare from a good is the benefit the consumer gets from purchasing the good at the market price minus what the consumer is ready to pay for the good. The utility a customer gains can be measured in a monetary amount by using the supply and demand curve. The difference between what the consumer is willing to pay and the actual cost, is the consumer surplus.

Producer welfare is what a supplier gains by participating in the market, determined by using the supply and demand curve. Monetarily this is calculated by taking the difference between the price the good is sold for and the minimum amount the producer is willing to sell at. The result will be the producer surplus. These surpluses are presented in A.2.

By maximizing the producer surplus and the consumer surplus the society's total welfare is maximized, which is described with the welfare equation:

$$W = CS + PS$$

Where W is the welfare, CS the consumer surplus and PS the producer surplus [14].

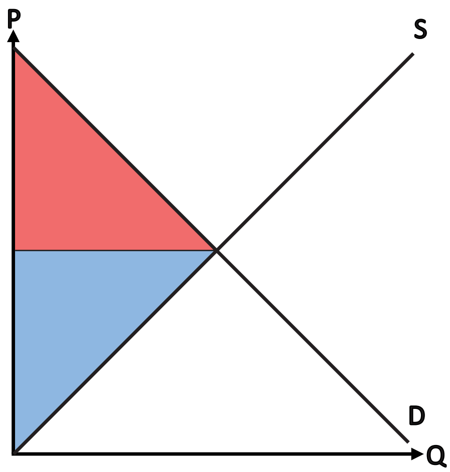

Figure A.2: The red area is the consumers surplus and the blue area is the suppliers surplus, by maximizing these two areas the welfare in the market will be maximized.

Appendix B

Data

B.1 4 bus network

Data

Buses	4
Loads	4
Generators	2
Lines	4
PHEV charging nodes	4

Minimum system cost model

System cost	98292.666505
Charging cost	3671.711274

Embedded models

Model	Fmincon	Genetic algorithm
System cost	98292.66650	98292.66650
Charging cost	3671.711274	3671.711274
Computational time[s]	512.042082	536.191037

MPEC

Number of blocks	3	10
System cost	98292.666505	98292.666505
Charging cost	3671.711274	3671.711274
Computational time[s]	2.683217	3.681623
Optimization Computational time[s]	1.357208	1.622410

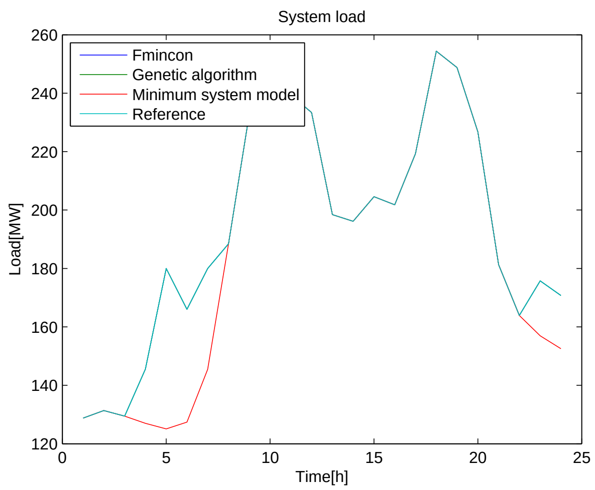

Figure B.1: Total load in the network where the solution from the MPEC is removed and it can be seen that the embedded models are following the minimum system cost

B.2 9 bus network

Data

Buses	9
Loads	3
Generators	3
Lines	9
PHEV charging nodes	3

Minimum system cost model

System cost	208390.209228
Charging cost	9467.934734

Embedded models

Model	Fmincon	Genetic algorithm
System cost	208390.209228	208390.209228
Charging cost	9467.934734	9467.934734
Computational time[s]	452.044097	1364.915149
Computational time[s][1]	182.817571	-

MPEC

[1]Without congestion

Number of blocks	3	10
System cost	208390.209228	208390.209228
Charging cost	9467.934734	9467.934734
Computational time[s]	2.730017	5.678436
Computational time[s][1]	2.854818	-
Optimization Computational time[s]	1.138807	2.184013

MPEC with different M

M	Compute time
100	2.636416
10000	2.901618
1000000	2.995219
100000000	3.494422
10000000000	3.697223

[1]Without congestion

B.3 14 bus network

Data

Buses	14
Loads	11
Generators	5
Lines	20
PHEV charging nodes	11

Minimum system cost model

System cost	193667.296433
Charging cost	12826.197522

Embedded models

Model	Fmincon	Genetic algorithm
System cost	193667.296433	193667.296433
Charging cost	9648.096392	9648.096392
Computational time[s][1]	2123.953615	2070.242470

MPEC

Number of blocks	3	10
System cost	193667.296433	193667.296433
Charging cost	8667.201168	8667.201168
Computational time[s]	9.796862	35.958230

Generator	*Bus*	Capacity	Cost
1	1	332.4	10
2	2	50	20
3	3	50	30
4	6	100	40
5	8	240	50

Table B.1: Generator cost and bus connection

[1] Without congestion

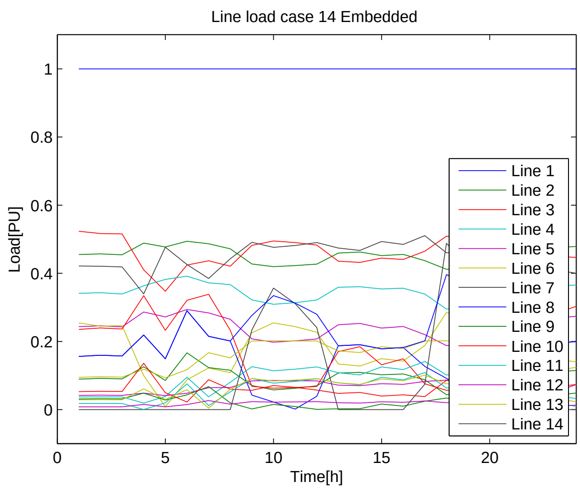

Figure B.2: The line flows in the 14bus network with congestion.

B.4 30 bus network

Data

Buses	30
Loads	21
Generators	6
Lines	41
PHEV charging nodes	21

Minimum system cost model

System cost	296399.418833
Charging cost	10393.944936

Embedded models

System cost	296399.418833	296399.418833
Charging cost	10393.944936	10393.944936
Computational time[s]	7291.159137	2950.088110

MPEC

Number of blocks	3	10
System cost	296399.418833	296399.418833
Charging cost	10393.944936	10393.944936
Computational time[s]	33.181412	231.146681

B.5 Swiss system

Data

Buses	191
Loads	126
Generators	134
Lines	267
PHEV charging nodes	126

Minimum system cost model

Model	Fmincon	Genetic algorithm
System cost	9614870.397442	-
Charging cost	642710.446951	-

Embedded models

Model	Fmincon	Genetic algorithm
System cost	9614870.675528	-
Charging cost	642145.079237	-
Computational time[s]	388083.250897 (4d, 11h, 48s)	-