MICROWAVE MULTIPACTOR BREAKDOWN IN HELIX ANTENNAS, THEORY AND SIMULATIONS

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ABSTRACT

Multipactor breakdown is a serious risk in all types of high power microwave equipment working in low pressure or vacuum environments. This is especially pronounced in the case of RF devices intended for operation in space, due to the limited testing ability, the high costs involved, and the near impossibility of repairing a system in orbit. This motivates the need for theoretical predictions and good design guidelines. Most studies of the multipactor breakdown mechanism have considered relatively simple geometries, where the field is homogeneous, surfaces are flat, and the only way that electrons are lost is through absorption by the conductive walls.

We investigate the basic mechanisms of the multipactor breakdown phenomena in a quadri-filar helix antenna (QFHA). The ECSS specifies the parallel plates model to be used for breakdown threshold estimates, but in contrast to the parallel plates, the helix system is characterized by an inhomogeneous field profile, curved surfaces, and an open geometry. Since the full system is too complicated to yield to theoretical analysis in the first instance, a simplified geometry in the form of two infinite parallel cylinders was chosen to represent the system. This choice can be justified due to the low pitch angle and long lengths (in terms of wavelength) of typical QFHA:s. We develop a model which takes into account the electron density dilution due to the curved surfaces, the random arrival phase of electrons, and the ponderomotive force, which tends to push electrons out of high field areas. The resulting lower breakdown voltage estimates show a strong dependence on cylinder radii, where double sided multipactor below certain radii becomes impossible. Monte Carlo simulations of the two wire system, taking into account the full electron dynamics, have been performed for different cylinder separations. The theoretical predictions and simulations show good agreement, and seem to provide a clear improvement compared to those corresponding to the parallel plates model.

INTRODUCTION

Multipactor has been studied for over 70 years for different reasons. The main studies have been done on the parallel plates geometry, mainly because the resonance conditions are analytically tractable. But other types of geometries have been handled as well, for example the coaxial, circular, elliptical, rectangular and wedge-shaped waveguides. In addition to this, studies have been done on waveguide irises, microstrip lines, and other more peculiar arrangements. But until recently, no research had been done on an open system, where electrons are allowed to travel outside the rf configuration. In this particular case we are interested in the dynamics of electron multiplication between the wires of a quadri-filar helix antenna. A theoretical study of this problem was published in 2010, [1], and a comparison between theory and simulations will be published in the autumn of 2011, [2]. In this paper we will present a summary of the theory, and the results of the comparison between theory and simulations.

The helix antenna consists of four cylindrical wires that are twisted into helix shape. The purpose is to produce a circularly polarized EM wave. At any one time, and at a specific height along the helix antenna, the major part of the electric field in the helix system will be localized between two of its four cylinders, meaning that a multipactor avalanche will take place mainly between neighbouring pairs of cylinders, and not in diagonal or by any other complicated trajectory. Until recently, no research had been performed on such systems, so the general recommendation by the European space standard (the ECSS [3]) states that when examining a pair of wires, one should model this system with two parallel infinite plates. It is however quite obvious that this will give a very restrictive voltage threshold, since it does not take into account the spreading of electrons when they travel outwards from the curved surface of cylindrical wire. In addition to this, the regions where the electric field will be at its strongest are quite localized on the two points where the cylinders are closest to each other.

To do something better than the parallel plates approach we chose to start the analysis of the full helix geometry by just looking at two cylinders. The great benefit of this system is that the electromagnetic field is known analytically, but the most important features of the original setup; the curved surfaces and the open geometry are still there. Further simplifications were made by approximating all electrons to be emitted at the most energetic phase, and assuming that the large frequency-gap product of the system allows one to treat the electron motion as non-resonant. [4]

ELECTRIC FIELD CONFIGURATION

As stated previously, the electric field for the simplified two-wire system can be found analytically (neglecting self-inductance), [5]. The geometry of the setup is shown in Fig. 1.

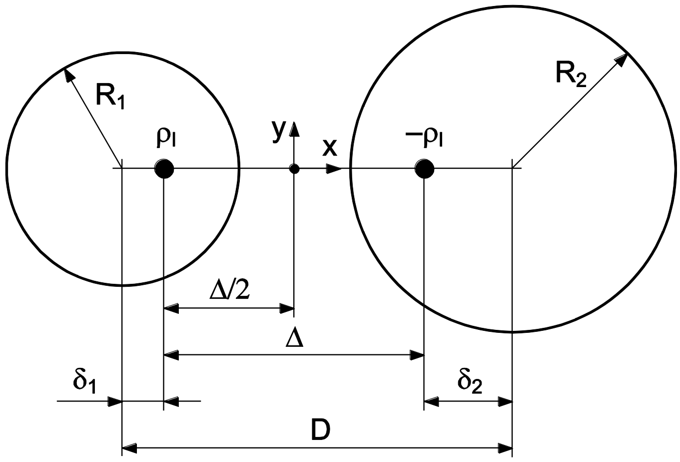

Fig. 1. The geometrical configuration of the two-wire system.

The electric field outside the cylinders is given by

$$\overline{E}(u,v) = \frac{E_0}{4}\frac{(v^2-u^2+1/4)\hat{x}-2uv\hat{y}}{(v^2+u^2-1/4)^2+v^2} \tag{1}$$

where $u = x/\Delta$, $v = y/\Delta$, and E_0 is the minimum electric field on a line drawn between cylinder centres. In this summary we shall only look into the case when the cylinders have equal radii (for unequal radii, see [1]), and in this case we have

$$\Delta = \sqrt{D^2 - R^2} \tag{2}$$

where $R = R_1 = R_2$ is the cylinder radii. The maximum electric field is found on the two points on the cylinders where they are closest to each other. The relation between the maximum electric field amplitude, $E_{\max}$, and the minimum between the cylinders, E_0, is given by

$$E_{\max} = \frac{E_0}{2}\left(1+\frac{D}{2R}\right) \tag{3}$$

The maximum field is related to the voltage between the cylinders via

$$E_{\max} = \frac{V}{R}\sqrt{\frac{D+2R}{D-2R}}\frac{1}{\cosh^{-1}\left(\frac{D^2}{2R^2}-1\right)} \tag{4}$$

where $\cosh^{-1}$ is the inverse of $\cosh$.

APPROXIMATE MODEL

To get an upper estimate on the breakdown threshold, we shall use the approximation that all electrons have the maximum energy possible in this system. We do this by assuming that all electrons are emitted at the most energetic phase, in the maximum electric field of the system. At the field maximum on the cylinder surfaces, the field is given by

$$E(t) = E_{\max} \sin \omega t \tag{5}$$

where ω is the field frequency in radians per second. Hence, the initial motion of the most energetic electrons emitted from this spot is given by

$$v(t) = v_{\omega}(1 + \cos \omega t) \tag{6}$$

where $v_{\omega} = eE_{\max} /(m\omega)$, e is the electron charge and m is the electron mass. The motion of the electron is composed of a drift velocity, and an oscillatory motion. Since the drift velocity is caused by the initial acceleration of the electron at the moment of emission, its direction will be perpendicular to the surface of the cylinder. But since the cylinder surfaces are curved, the electrons that are emitted just a small distance away from the field maximum will spread out in the tangential direction when crossing the gap distance between the cylinders. This will cause an enlargement of an initially small patch of electrons emitted at one cylinder, before they impact the other one. Hence, the geometry of the system will cause a dilution of the electron density between passages. This effect is equivalent to a loss of electrons from the system, and will have a big effect on the multipactor threshold. This effect is illustrated qualitatively in Fig. 2.

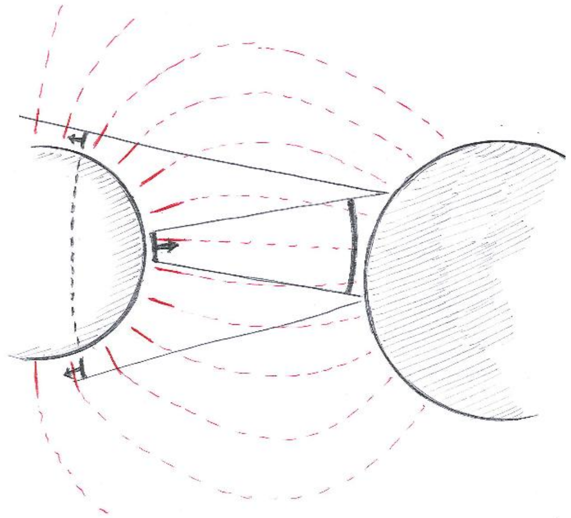

Fig. 2. A qualitative illustration of the geometrical spreading of electrons between successive passages.

Mathematically one can incorporate the geometrical dilution of the electron density by defining an effective SEY, viz

$$\sigma_{eff}(v_i) \equiv \sigma(v_i)\frac{R}{D-R} = \sigma(v_i)\frac{r}{1-r} \tag{7}$$

where $r = R/D$, and $\sigma(v_i)$ is the SEY at an impact velocity v_i.

From this simple consideration it is possible to draw a very important conclusion; if the effective SEY maximum is less than unity, double sided multipactor becomes impossible. Mathematically we can state the necessary criterion for possible existence of double sided multipactor as

$$\sigma_{\max} > \frac{1}{r} - 1 \tag{8}$$

For example, if we use the dimensions of an 8 GHz helix antenna used in space communication; $R = 0.6$ mm, and $D = 5$ mm, we get $\sigma_{\max} > 7.3$, which is a high value indeed.

The next issue to tackle is to estimate the impact velocity, v_i. During the traversal of the gap between the cylinders, the field amplitude that an electron experiences will change, leading both to a change in the oscillatory speed, and the drift speed of the electron. But upon the instant of impact on the other cylinder we shall treat all the electrons as if they were impacting on the other cylinder in the same place as they were emitted from the first one. This means that the drift velocity and oscillatory velocity will not change between emission and impact. Furthermore, the frequency-gap product of realistic helix systems is quite high, meaning that the electrons will oscillate many times between impacts. Since there will be a small inevitable spread in emission velocity of electrons, the arrival phase of electrons over the cylinder surface in the volume where they will impact during one period will be randomized. This means that we can use a statistical average for the impact velocity. In [1] we argue that the correct average for the case when the drift velocity has the same magnitude as the oscillatory velocity is

$$< v_i >_{nr} \approx \frac{3}{2} v_\omega \tag{9}$$

where the index nr stands for non-resonant. This is not very far from the approximate value of the impact speed in resonant multipactor, where

$$< v_i >_r \approx 2 v_\omega \tag{10}$$

[4], but it will raise the multipactor threshold somewhat.

To use these considerations practically, we define the multipactor threshold as the field strength where the effective SEY of the average impact velocity is equal to unity, viz

$$\sigma_{eff}(< v_i >) = 1 \tag{11}$$

The effective SEY is given by Eq. (7), and the average impact velocity can be chosen as either Eq. (9) or (10), depending on if the discharge is expected to be resonant or not.

Since the average impact speed is directly coupled to the maximum electric field strength, which in turn is connected to the voltage through Eq. (4), it is not hard to determine the breakdown threshold for any system where the SEY-function is known. In Fig. 3 a comparison is made between the breakdown threshold for non-resonant and resonant multipactor in a parallel plates geometry, and in a two-cylinder geometry. In the figure the Vaughan approximation is used for the SEY-curve [6], using the maximum SEY-value for Silver as recommended by the ESA standard [3]

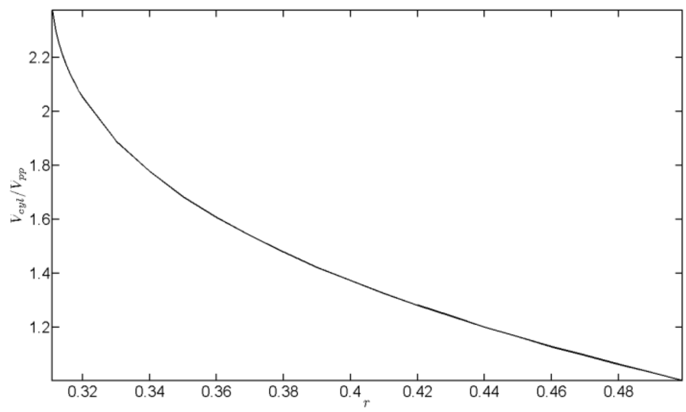

Fig 3. The breakdown voltage for the two-cylinder system divided by the voltage for parallel plates. The curves are independent on the choice of approximation for the impact velocity, as long as the same formula is used for both cases.

Fig. 3 shows the effect of the geometrical spreading of electrons, in that the voltage necessary to allow multipactor goes up with decreasing radius of the cylinders. The left edge of the curve actually represents the point where multipactor becomes impossible completely, for the maximum SEY is below the value of the necessary effective SEY for breakdown.

SIMULATIONS

A software tool was developed to validate the accuracy of the theoretical treatment. The code used a Monte Carlo algorithm where the random element was the direction of the emission velocity of secondary electrons. The emission energy of secondary electrons was fixed to 3 eV, but the angle of emission with respect to the cylinder surface normal was randomly distributed over π radians in the xy-plane (see Fig. 1). The software kept track of electrons by working with trajectories that could contain any decimal numbers of electrons. Upon every impact, the number of electrons in the trajectory was multiplied with the SEY corresponding to the impact velocity of that electron trajectory. The software monitored the number of trajectories, and the total number of electrons in a rectangle enclosing both cylinders. The multipactor threshold was defined as the voltage where the total number of electrons divided by the number of trajectories neither showed exponential decrease or increase. Three major simulation series over a range of cylinder radii were performed. The Vaughan approximation for the SEY was used in all, and they all used the field frequency 8 GHz. The gap width of the first series was small, only $d = 0.15$ mm, and it used the SEY-curve for Silver, $\sigma_{\max} = 2.22$, $W_1 = 30$ eV, where W_1 is the first cross over energy. The second series used a large gap width of $d = 3.8$ mm, corresponding to a realistic helix antenna, but kept the SEY-curve the same as the first series. The last series used the same gap as the second, but used an unnaturally large value for the SEY maximum, $\sigma_{\max} = 10$, but kept the first cross over energy the same as previous series. The results are summarized in tables 1, 2 and 3.

Table 1. Simulation results for $\sigma_{\max} = 2.22$, $W_1 = 30$ eV, and $d = 0.15$ mm.

Wire radii in mm	Voltage in Volts
30	70
10	70
3	70
1	80
0.5	90
0.3	100

Table 2. Simulation results for $\sigma_{\max} = 2.22$, $W_1 = 30$, and $d = 3.8$ mm.

Wire radii in mm	Voltage in Volts
30	3800
20	3800
15	4000
10	5800
8.5	7900

Table 3. Simulation results for $\sigma_{\max} = 10$, $W_1 = 30$, and $d = 3.8$ mm.

Wire radii in mm	Voltage in Volts
30	2800
20	3100
15	3300
10	3800
7.5	4400
5	5400

COMPARISON BETWEEN SIMULATIONS AND THEORY

We shall now briefly display the comparison between the simulations and the theoretical predictions. First of all we have the simulations for the small gap, $d = 0.15$ mm. The first cross over point was taken to be $W_1 = 30$ eV, and the frequency was $f = 8$ GHz, which actually means that the multipacting electrons will cross the gap in around one period, and the discharge should be resonant. This means that the non-resonant approach for the averaging of the impact velocity should give a too high value for the threshold, as compared with the simulations. A comparison between the simulations and the predictions of the dispersive theory (Eqs. (7) and (11)) using both the resonant (Eq. (10)) and non-resonant approximation (Eq. (9)) for the impact speed is displayed in Fig. 4.

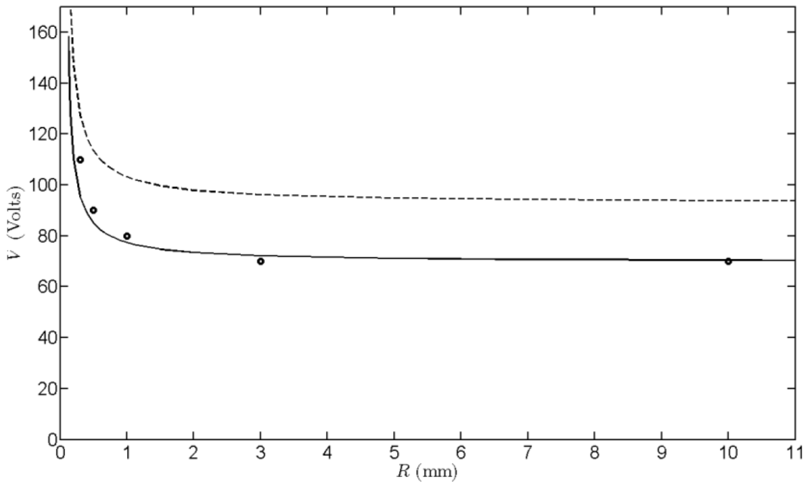

Fig. 4. A comparison between simulations and theory for the small gap width, $d = 0.15$ mm, using $\sigma_{\max} = 2.22$. The circles are the simulated values, the solid line is the theoretical prediction by the dispersive theory using the resonant approach for the average impact velocity, and the dashed line uses the non-resonant approach for the average impact velocity.

It is indeed seen in Fig. 4 that the non-resonant approximation for the average impact speed results in a too high value for the breakdown threshold when compared to the simulated values. On the other hand, the resonant approximation gives a good estimate of the voltage. The curvature of the lines is due to the dependence of the effective SEY on the cylinder radii, and is seen to match the simulated values quite closely.

The second simulation series was done for a larger gap width, $d = 3.8$ mm, corresponding to a realistic helix antenna. In this case the multipactor discharge should not be resonant, since the electrons will complete many cycles between impacts. In Fig. 5 a comparison is made between the simulated results and the dispersive theory, using both the non-resonant and resonant approximations for the average impact speed.

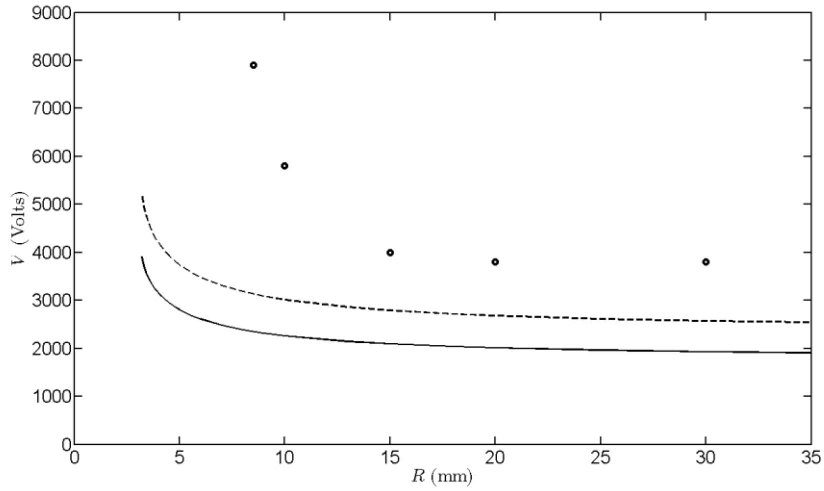

Fig. 5. A comparison between simulation and theory for the large gap width, $d = 3.8$ mm, using $\sigma_{\max} = 2.22$. The circles are the simulated values, the solid line is the theoretical prediction by the dispersive theory using the resonant approach for the average impact velocity, and the dashed line uses the non-resonant approach for the average impact velocity.

Fig. 5 shows quite clearly that the dispersive theory gives the correct curvature of the lines as the radii of the cylinders are decreased. However, the predicted voltage is a bit too low. The prediction using the resonant value for the average impact speed gives a too low value, which is natural, since the discharge should be non-resonant. But the non-resonant approximation also gives a too low value. To see if this is due to the fact that we use the average value for the impact velocity in the formula for the SEY, instead of using an average value of the SEY directly (which would be more complicated) we increased the SEY maximum to an extremely high value in the last simulation series, $\sigma_{\max} = 10$. This makes the SEY function more linear around the first cross over point, and would mean that the approximation $\sigma(< v_i >) \approx < \sigma(v_i) >$ should be more close to reality. Fig. 6 shows the results from a comparison between the non-resonant dispersive approximation and simulated result for the large gap width with a very high SEY maximum.

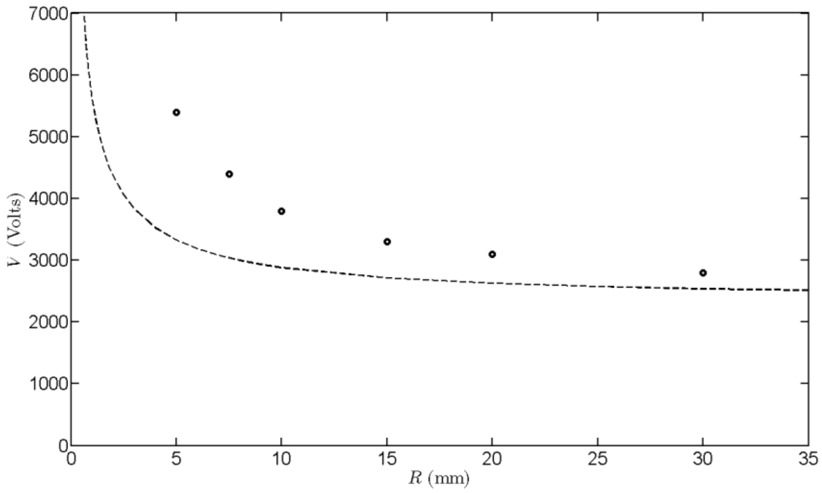

Fig. 6. A comparison between simulation and theory for the large gap width, $d = 3.8$ mm, using $\sigma_{\max} = 10$. The circles are the simulated values, the dashed line is the theoretical prediction by the dispersive theory using the non-resonant approach for the average impact velocity.

Fig. 6 shows a closer agreement between the predicted breakdown voltage and the simulated results. It seems to imply that the direct use of the average impact velocity in the SEY function does introduce an error. However, there is still a discrepancy between the simulations and theory. This is most likely due to the fact that in the theoretical treatment we regard all the electrons to have the maximum energy available. This is not correct. Actually, in a non-resonant discharge there should be a span of allowed emission phases. Taking this into account would further decrease the average impact speed, thereby increasing the threshold voltage.

CONCLUSIONS

In this paper we have summarized the results from a recent comparison made between theoretical predictions and simulations of the multipactor breakdown threshold in a two-wire antenna system, chosen to represent qualitatively a quadri-filar helix antenna. The approximate theory incorporated the geometrical spreading of electrons between cylinder gap passages by defining an effective SEY. It also used an approximation for the average impact speed that should be applicable when the discharge can be considered to be non-resonant. A Monte Carlo software was developed to simulate the multipactor electron dynamics in the two-wire system, and simulations were done for different values of cylinder gap width and maximum value of SEY.

The comparison between theory and simulation shows a good qualitative agreement, confirming that the dispersion of electrons due to the curvature of the cylinders is an important effect. Taking this factor into consideration, along with knowledge of the SEY maximum, makes it possible to determine a lowest radius which can allow double-sided multipactor for any system, which is a very important design consideration. A quantitative disagreement between the predicted and simulated voltage was also found. This seems to be due partly to the fact that we use the average value for the impact speed in the formula for the SEY, where we really should use the average SEY. But to some degree, the discrepancy should also be due to the fact that any real discharge would involve electrons that have a spread over emission phase, whereas in the approximate theory we assume that all electrons are emitted at the most energetic phase.

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