# On the thermal balance of a small spherical microwave breakdown region in atmospheric Air

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The microwave breakdown of the gas inside transmission systems and other types of RF equipment involves the exponential growth of the number of free electrons in the system due to ionizing collisions of electrons with the neutral molecules of the gas. To this day it remains a serious failure mechanism for any type of high power microwave equipment [1]. Not only because of the noise generated and the changed electromagnetic characteristics of the equipment, but under atmospheric pressure conditions because of the intense heat that is generated in the breakdown region. This heat is caused by the motion of electrons in response to the field coupled with the high frequency of collisions with the neutral molecules. The result is not only the heating of the conducting elements and the direct damage to them, but also the heating of the surrounding gas, the concomitant lowering of the breakdown threshold, and the possible expansion of an initially small breakdown region.

Typically, when there are small protrusions in the conducting parts of the microwave device, these are the most vulnerable sites from the point of view of the risk for microwave breakdown, simply due to the resulting field intensification in the vicinity of the these protrusions. Examples of such irregularities can be tuning screws, welding points, sharp corners etc [2,3]. In atmospheric pressure Air, microwave breakdown is achieved at 30 kV/cm/Torr, corresponding to the high pressure part of the Paschen curve. The initial breakdown region might be localized completely in an area of field intensification, and initially be quite small. The electron density will rapidly saturate at a level where the internal field in the breakdown region will correspond to the breakdown value. On short timescales the situation might be stable. However, the motion of the electrons will generate heat and lower the surrounding breakdown threshold, allowing the small breakdown region to expand and consequently generate more heat. The full analytical treatment of this type of non-linear problems is extremely complicated. On short timescales, the electrodynamics of the system will be determined by the intricate interplay between the breakdown region and the external electromagnetic field. And on longer timescales, the heating of the gas will change the dynamics completely. Couple this with suitable boundary conditions and the problem becomes formidable indeed. Computer simulations can indeed model the situation, but for physical understanding simplified scenarios should be analyzed. In Ref. [4] we considered the situation where initially there is a spherical breakdown region in an otherwise homogeneous electric field. The volume of the sphere is assumed to be small in comparison with the wavelength, which allows us to use the quasi-steady approach for the field. At the same time, the sphere is assumed to be much larger than the attachment length $L_a \equiv \sqrt{D/\nu_a}$, where $D$ is the diffusion coefficient, and $\nu_a$ is the attachment frequency [5]. This allows us to neglect the process of diffusion inside the sphere, and the electrical field can be approximated as the breakdown threshold field in high pressure Air. This value for the electric field has a temperature dependence, since increasing the gas temperature locally decreases the neutral density and allows the electrons to reach higher energies. The temperature dependence is of the form $E_a(T) \approx E_{a0}T_0/T$, where $E_{a0}$ is the breakdown field at room temperature, $T_0$. The electric permittivity, $\varepsilon$, inside the breakdown region is a function of the electron density, and the field is known analytically from $E_a = |3E_0/(\varepsilon+2)|$, where $E_0$ is the externally applied field. This allows us to express the electron density as a function of the electric field strength and the gas temperature. Consequently, we can derive a heating term valid inside the breakdown region. This heating term will only depend on the temperature, and the equation for the evolution of the gas temperature looks like

$$\frac{\partial}{\partial t}(\rho c_v T) = \nabla(\kappa(t)\nabla T) + q_p(T) \qquad (1)$$

Where $\kappa(T) \approx \kappa_0 (T/T_0)^{3/4}$ is the thermal conductivity of Air [6], $\rho$ is the Air density, $c_v$ the heat capacity of Air at constant volume, and $q_p(T) \approx 3\varepsilon_0 \omega E_0^2 \frac{T_1}{T}\sqrt{1-(\frac{T_1}{T})^2}$ is the heating term for the breakdown region. Outside the sphere the heating is zero. The temperature $T_1 = T_0 \frac{E_{a0}}{E_0}$ corresponds to the temperature necessary to achieve breakdown by heating in the external field $E_0$. $\varepsilon_0$ is the vacuum permeability, and $\omega$ is the field frequency in rad/s.

The question is whether the sphere is thermodynamically stable or not. To investigate this we integrate Eq. (1) over the sphere radius, $R$, and employ the fact that the temperature on the edge of the sphere must be $T_1$, and the temperature far away must be $T_0$. We get $\frac{\partial W_{total}}{\partial t} = -Q_{loss} + Q_{Joule}$, where $Q_{loss} \propto R$ is the heat lost over the edge of the sphere, and $Q_{Joule} \propto R^3$ is the heat generated in the sphere. The two terms are plotted in Fig. 1.

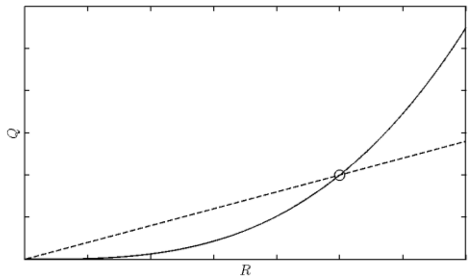


Fig. 1: The dependence on radius for the heat loss term (dashed line), and the heat generation term (solid line).

It is clear from the figure that any deviations from the point of equilibrium (marked by a circle) will lead to instability. If the radius decreases below the equilibrium value, less heat will be generated than what is lost over the edge and the sphere will cool down and contract. If the radius increases from the critical value, more heat is generated inside than what can be transported over the edge, and the sphere will expand.

The critical radius can be expressed as

$$R_{crit} = \sqrt{\frac{8\kappa_0 T_0}{7\varepsilon_0} \frac{((E_{a0}/E_0)^{7/4} - 1)}{\omega E_{a0}{}^2} (E_{a0}/E_0)^2}$$

and evaluated for any combination of parameters. For example, in room temperature at atmospheric pressure, when $E_{a0}/E_0 \approx 3$, and $\omega = 10^5$ Hz we get $R_{crit} \approx 0.1$ cm. Increasing the frequency decreases the radius, and for GHz values the radius will be below the attachment length. This implies that in the range of sizes between the attachment length and the wavelength in a microwave system, no stable breakdown region can exist inside a homogeneous field. However, it does not tell us if there might be stable situations at smaller sizes, where diffusion is important, or what happens close to very small field enhancements, where experiments seem to imply the existence of stable discharges.

Although the model is approximate, we can use the values given by the critical radius to determine safe sizes for areas of intensified heating or field strength inside otherwise homogeneous microwave systems. This can be a very useful consideration in the design of RF equipment, in order to avoid full scale breakdown.

## References


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