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Non-Pixel Based Reduction of the Jacobian, in a Multi Frequency Reconstruction Algorithm for Microwave Imaging

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Preface

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Abstract

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Master of Science

Microwave imaging shows great future potential in improving several diagnostic applications. Of particular advantage is that it is harmless and low cost in comparison to similar methods of imaging. However a large amount of calculations are required in order to reconstruct images from measured data.

The purpose of this project is to combine two different strategies in order to reduce the computational complexity and keep it within manageable time and size solutions and still generate images of acceptable quality. The main two methods investigated here are multi frequency image reconstruction and non pixel based reduction of the Jacobian. The image is then reconstructed with a Newton based inversion method. The multiple frequencies adds regularization and reduced the likelihood of getting trapped in local minima during the reconstruction procedure whereas the words non pixel based reduction is a filtration in order to reduce the problem size without apparent loss of image quality. In the non-pixel based reduction strategy the a spatial Fourier transform of the Jacobian is taken and weak Fourier coefficients are discarded. The imaging problem is then formulated in terms of the remaining coefficients, thus the computational cost is reduced. In order to test the this idea, many different objects have been tested in numerical simulations and in particular a simulated breast model has been used. It is shown that it is possible to effectively reduce the problem size as much as 75 % with only little loss in the imaging quality.

Introduction

Microwave imaging is a technique aiming at generating images of permittivity and conductivity inside an object. A biological object consists of different tissue, each with different dielectric properties .For example, water and fat which are essential almost in every parts of the body have different dielectric properties. The advantages of microwave imaging in biomedical application are not only the lower cost of imaging system in comparison to conventional techniques as MRI or CT. Further it is non ionizing and thus harmless for humans.

The imaging technique is based on transmitting a wave into the object under test. The wave is propagating through the object and registered at the receivers. Then this data is analyzed in order to reconstruct the dialectic properties of the object.

Potentially, valuable medical information can be obtained from the images of tissue dialectics. In [1] it is said that "The biological tissues present a significant contrast in dielectric properties between tissues with high water content such as soft tissue and low water-content tissues, such as fat and bone".

The geometry of the antenna arrangement is such that it is surrounding the object under test. The fundamental problem is that microwaves will be scattered at interfaces between different mediums. The complete scattering picture could become very complex and one must therefore make sure that this is captured in the measurements. Related to the geometrical implementation, to generate an image each antenna is sending and the others are receiving. The Maxwell equations govern the relation the receiver data and the dielectric properties. In microwave imaging the goal is to estimate the permittivity and conductivity of the mediums.

This procedure can be divided into two parts, the first is to solve the Maxwell equations in order to find the propagation pattern of the electromagnetic field. This task is also called the forward method and can be solved for example with FDTD, MOM or FMM. The second part is called the inverse method and denotes the problem of finding the dielectric properties of the object. This often turns into an optimization problem where the Newton iterative method (used in this project), or the conjugate gradient method can be used.

For medical purposes, according to [1] microwave frequencies around 1GHz are feasible for the whole body targets based on the penetration properties and signal to noise ratio of the measurements. Thus in this project the reconstruction frequencies are selected in 1 GHz range.

This imaging procedure is computationally very demanding and requires that the originally ill posed matrix equation is turned into a well posed problem. Further it requires the solving of a minimization problem using the Newton iterative method which requires inversion of a large matrix. These calculations will consume a large amount of CPU time on the computer. For example the cost of one matrix division is $O(n^3)$ where n is the number of matrix elements. This problem gets aggravated when the number of iterations increases, for example if twenty iterations are needed for convergence this will cause $20*O(n^3)$ operations.

According to the discussion so far we have to solve two big problems in microwave imaging. It is the problems of turning the ill-posedness into a well-posed problem and the large amount of calculations. In this project these two problems will be analyzed and a solution will be proposed and investigated. Here we will investigate the use of multi frequencies in order to improve the well-posedness if the problem. We will also investigate a non pixel based method for reducing the computational burden in solving the inversion problem. In the results section we will show imaging examples where these two techniques has been exploited for reconstructing a simple breast model with an included breast tumor.
It is important to mention that the more complex the objects are the more complex the pattern of scattered waves will be. This will also affect the convergence rate of the reconstruction problem, which becomes nonlinear. For example reconstructing an image with two object is in principle harder than reconstructing one simple object. Due to this fact different reconstructions with different objects are selected in order to illustrate the behavior of the convergence and how the techniques investigated here are solving this problem.

Moreover the question about how the reconstruction frequencies (multi frequencies) and the antennas quantity related to the convergence behavior will be investigated with different examples.
One group of examples is including a breast model with a tumor which is an example of a high contrast object which results in a highly non-linear behavior. By imaging reconstruction examples we can test how the multi frequency and non pixel based aspect are improving the reconstruction problem. Also as the non pixel based method means reconstruction based on the frequency response of the pixels in each iteration, showing its relation to special domain and its flexibility regarding image size, object shape, relative error analysis and reconstruction frequencies is the central task of this project which has been done step by step.

It is important to understand the relation between microwave imaging and X-ray imaging from the perspective of wave propagation in the medium. Microwaves are non ionizing radiations but the wave propagation from one medium to other one will lead to a significant loss of energy due to diffraction and scattering of the transmitted wave. Below figure shows a comparison between wave propagation for x-ray and microwaves within an object .Thus unlike x-rays, the image reconstruction in microwaves is getting more complicated due to scattering and diffraction of waves in the object, figure I.1.

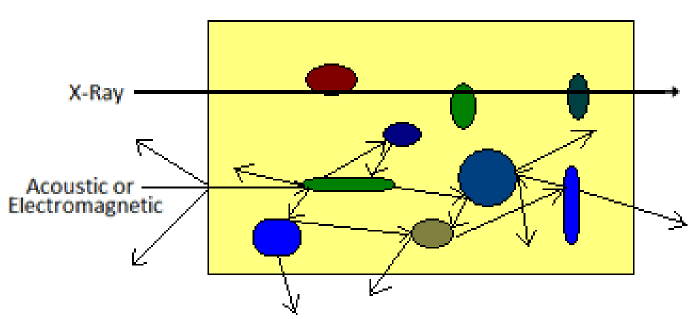

Figure .I.1, The X-ray behavior in comparison to electromagnetic wave behavior

As illustrated in the image, X-rays will pass the object on a straight path without getting scattered, however the microwaves are scattered in a surface where the dielectric properties are changed. Furthermore, even the scattered wave can again propagate into a new object property and get scattered again. On the other hand microwave propagation can undergo more interaction between different soft tissues than what is the case in X-ray systems. The reason is that different material properties are determining the propagation in the two cases.

So far it can also be concluded that imaging with this method is much more complicated in comparison to any X-ray technique due to a higher level of scattering. The purpose of the work presented here is to discuss ideas that can potentially reduce the calculation time. In other words this project is about reducing the reconstruction time by improving the convergence trajectory within iterations by feeding more information to the system with multiple frequencies participation in generating the imaging algorithm, and deleting the redundant frequencies about the less drastic details to decrease the computation.

In order to reach this goal a simulated microwave imaging system is designed which circularly surrounded with 10 to 20 antennas regarding experiments with Spatial grids equal to 30 by 30 and the grid size in X , Y-direction set as Spatial Step Length equal to 4.0 mm, which is simulated for a time period of 10 nano seconds. Related to this simulation model different experiments in this project has been conducted and the results reported .The first and second sections are dedicated to a brief review of electromagnetic with description of the numerical solver of the wave evaluation in the field .Also the three next sections following to them are dedicated to how the simulation implemented by C++ that can separate into two main sections as the forward simulation describing FDTD utilization to evaluate the propagated wave regarding each transmission and to estimate the possible combination of the dielectric values in different location to reduce the receiver differences from measurement which is the inverse section responsibility. Thus the important discussion about optimization placed in subsequent sections which ended with practical object reconstructions of the breast tumor. In other words, overcoming to the computational cost with acceptable relative error is the target of this project, which is achieved by selecting the frequencies containing the highest proportion of valuable information related to the imaging object, and how this selection is configured by microwave imaging algorithm (here FDTD and Newton based method utilized), is embedded in multiple frequency and non pixel based method that are used together to tackle the problem in this project.

Finally it should be mention that in this project the microwave imaging algorithm is described step by step followed by many reconstruction examples. The feasibility of multi frequency method will be discussed and followed by the non pixel based reduction algorithm and its capability in improving the breast model reconstruction.

1. Microwave imaging and the brief review of Electromagnetics

In order to open the portal of microwave imaging, it is essential to understand the fundamental mathematics behind, which is described by the well-known Maxwell equations. Therefore in the first part a brief review of the wave propagation behavior modeled by Maxwell equations will be discussed. Moreover numerical computational schemes for solving these equations are described in subsequent sections.

This review contains a brief description of the microwave modeling technique called equivalent problem, which is essential for the understanding of the tomography algorithm discussed in Methodology section.

1.1 Background, Review of Electromagnetic

In order to understand the electromagnetic wave propagation in any given geometry we need to solve the Maxwell equation with proper boundary conditions for the problem. Many different numerical field solvers can be used to evaluate the field in the selected locations (this will be discussed next).

Assume to have a domain which is characterized by its permittivity (ε) and conductivity (σ) properties. The propagated electric and magnetic fields should satisfy the Maxwell equations.[3][4]

$$\nabla * E = -\partial B/\partial t \tag{1.1}$$

$$\nabla * B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \tag{1.2}$$

$$\nabla . D = q_e \tag{1.3}$$

$$\nabla . B = 0 \tag{1.4}$$

Further we have the relations $D = \varepsilon E$, $B = \mu H$ and ω represent the wave frequency. From these equations one can derive the conditions for the boundary conditions which the wave propagating from one medium to another medium.

$$-n*(E_2 - E_1)=M_s \tag{1.5}$$

$$n*(H_2 - H_1)=J_s \tag{1.6}$$

$$n.(D_2 - D_1)=q_e \tag{1.7}$$

$$n.(B_2 - B_1)=q_m \tag{1.8}$$

Where $\hat{\mathbf{n}}$ is the normal vector from the boundary surface between the two different mediums. A common use of the electromagnetic theory is to find the electromagnetic field everywhere in a computational domain, and regarding this goal the four Maxwell equations can be manipulated to obtain descriptive equations separately for the electrics field and magnetic wave separately .Combining equations (1.1) and (1.2) gives equations (1.9) where the magnetic field is discarded from the equation.

$$\nabla(\nabla . E) - \nabla^2 \mathrm{E} - \mathrm{k}^2 \mathrm{E} = -\mathrm{j}\omega\mu \mathrm{J} \quad (1.9)$$

In a similar way the wave equation can also be expressed in terms of the electric filed (1.10).

$$\nabla^2 \mathrm{E} + \mathrm{k}^2 \mathrm{E} = \mathrm{j}\omega\mu \mathrm{J} - \frac{1}{j\omega\varepsilon}\nabla(\nabla . J) \quad (1.10)$$

Here J is the current density in three dimensions and k is the wave number. It can be represented in two related forms:

$$k = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda \quad (1.11)$$

Now it is time to discuss the Maxwell differential equations solver that is shown in (1.12) which is the Green's function, satisfying the Helmholtz equation with a delta source.

$$\nabla^2 \mathrm{G}(\mathrm{r}, \mathrm{r}') + \mathrm{k}^2 \mathrm{G}(\mathrm{r}, \mathrm{r}') = -\delta(\mathrm{r}, \mathrm{r}') \quad (1.12)$$

It is practical to show equation (1.10) in integral form while relating it to equation (1.12). Due to the fact that integration in a program easily can be made by loops, using the integration format of Maxwell equations is very feasible and in (1.13) it is shown how the x-direction coordinate of the current is computed.

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$$E_x(r) = -\mathrm{j}\omega\mu \iiint G(r, r')[J_{x(r')} + \frac{1}{k^2}\frac{\delta}{\delta x}\nabla'.J(r')]\mathrm{d}r' \quad (1.13)$$

After above descriptions it remains to calculate the Green's function, especially since the two dimensional Green's function will be used here in the microwave image reconstruction method.

Solving equation (1.12) for a homogenous medium results in the Hankel function the first and second kind zero order. Therefore it can be written as in (1.14) where A is j/4. For better understanding it is possible to write the Green's function as

G((x,y) ,(x',y')) and k is the wave number.

$$G(\rho,\rho')=AH_0^2(K|\rho-\rho'|) \tag{1.14}$$

An approximation which is very practical when it comes to reducing the complicated mathematical equations of electromagnetic , in numerical calculation of the electric and magnetic field is the Fast multipole method. When the measured location is posed far enough from the transmitting source (kR>>1, where k is the wave number and R is the distance) some approximation in calculation can be applied which are very effective regarding the calculation cost in programming. Particularly in field computations in microwave imaging it become useful when optimizing the reconstruction algorithm for speed. It can be explained as follows.

Assume the Green's function again , it is possible to expand the inner argument of the Hankel function whit the Taylor expansion then discarding the orders above minus two or three ($1/r^2$) related to the expecting error caused by the approximation .Thus after some modification the far field equation will be reduced as what is in equation (1.1.1) .

$$E(r)=-j\omega A(r) \tag{1.1.1}$$

Moreover the reduced form of the far field approximation with the first order Green's function modified for three dimension in equation (1.1.2) This can easily be reduced to two dimension too, by using the two dimensional green function expansion, equation (1.1.3) .

$$E(r)=-\frac{j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}\iiint J(r')e^{jkr'.r}dr' \tag{1.1.2}$$

$$E(\rho)=-\omega\mu\sqrt{\frac{j}{8\pi k}}\;\frac{e^{-jk\rho}}{\sqrt{p}}\int J(p')e^{jkp'.\rho}d\rho' \tag{1.1.3}$$

Before ending this part, it is practical to mention that due to the physics of the microwave propagation described by Maxwell equations, it is possible to understand some useful characteristics to gain variety of information in order to model the wave behavior.

Especially in microwave imaging that where in one iteration all the antennas should transmit and receive the propagated wave, the equivalent modeling is one of the useful methods in order to form the equations of the system .In the following its description is coming which its application can be fully understood and discussed in section 3.1 and 3.2 when the structure of imaging is discussed.

1.2. Equivalent problem

The equivalent problem is feasible in microwave imaging because it is about modeling the scattering points as the new sources and it is very drastic to analyze because the wave scatterings are increased when confronting a higher dielectric medium like the breast model included the tumor. Thus modeling this situation is beneficial to gain more information with easier mathematical manipulation.

In many cases it is possible to use an equivalent model instead of real one to simplify the solving procedure. Especially that in programming it is easier to write some functions as a simulated situation then change the different problems as combination of these simulated functions, in the other words it is not practical to write many different solving approaches regarding variety of problems, when it is possible to deal with it as a configuration of simpler problems. One of the best examples relating to this discussion is the wave scattering. When a transmitted wave is passing through mediums with different properties some amount of wave will pass it and other parts will be scattered. Simulation of these scattering positions can be easier when using the equivalent model because, it is possible to consider the scattering locations as new sources in that locations, then deal with them as new transmitted waves from that parts . It is very feasible to use this model because now it is possible to deal with the receivers as the transmitters in each transmission of wave from one and just one transmission of wave. In other words by this modeling if one antennas sending and other receiving, it is equal to say that, if these receivers sending the same signals that they stored, the field values remain the same .The application of this modeling will be shown in section 3.1 and 3.2 where extracting the information from the receivers to generate the equations that describing the field behavior is discussed .

An other simple example can be considered like, electric and magnetic currents J and M that cause the radiation fields E1 and H1 with properties epsilon_1 and sigma_1, in a object . Now to model this by an equivalent source ,first assume one surface that separate the object from outer space by equivalent Js and Ms on the surface ,which can be modeled by the boundary equations and treated as source .[3][4]

Js= $\hat{n}$*(H1-H) and Ms= $\hat{n}$*(E1-E)

The resulting electrical field regarding above discussion and the obtained boundary conditions can be calculated according to equation (1.2.2) [3][4]:

$$E^s = -j\omega\mu \iint G(r,r')[J(r') + \frac{1}{k^2}\nabla'\nabla'.J(r')]dr' \qquad (1.2.2)$$

1.3. Numerical Algorithms to solve Maxwell equations

One of the challenges is how to solve the Maxwell equation numerically, especially in applications where, the occupied memory and computational time is critical and important .Thus this point of view obliges the project to describe and compare the different numerical solver of the Maxwell equations.

How to solve the electromagnetic equation numerically has a long history, yet the main applicable idea regarding programming was the FDTD, in which a discretized computational grid was used and the Maxwell equations were solved step by step in the time domain. Following this method, many other numerical techniques from different aspects have been created, that aiming at solving the electromagnetic field in time or in frequency domain.

Although many numerical differential method existed which can be used to evaluate the Maxwell equations, the possibility to optimize it for the desired application is very important. This is important in microwave imaging since the calculation time and the usage of memory could easily grow out of control. Thus there is a need for dedicated methods where these problems can be reduced or eliminated. A good candidate for this is the Fast multipole method, FMM, that not only is a very fast method to solve and evaluate the electromagnetic wave propagation in mediums with different dielectric property , but also the memory consumption is much better in comparison with other methods. Another possibility is to combine different numerical methods. For example the FMM technique can be combined with the MOM.

At last it is practical to know that in microwave imaging, the numerical solution can be called the forward method. This labeling coming from the fact that microwave imaging contains two main computations, one is the forward method that is exactly the numerical solution of the Maxwell equation in a field with determined properties. The other part in microwave imaging is the inverse method that aim at finding the dielectric values in the area of wave propagation.

2. Numerical methods used in microwave imaging

As discussed in the previous section, solving the Maxwell equations needs a numerical solver. Therefore we will here introducing a few different numerical methods and discuss their pros and cons with respect to microwave imaging. Moreover, the FDTD method is selected as the numerical solver in this project, but its mathematical structure is compared with MOM and FMM to clarify their features, especially the difference between time domain and frequency domain is discussed.

2.1. FDTD (Finite-difference time-domain method)

FDTD is a computational method in the study of wave propagation in complex dielectrics.
This method is a time base differential equation solver, which used to evaluate (simulate) the electromagnetic wave in different location between the source of transmission and the receiver position. The (FDTD) method, utilizes the method of finite differences to solve Maxwell’s equations in the time domain. The FDTD method is very straight forward to implement in different problems and easy for computer programming, the solution domain is typically discretized into small cubic or voxels in three dimensional problems. Maxwell's equations are relating the magnetic and electric field then, the changes in the E-field in different time samples is related to the variations of the H- vector field that rotating within the area , which is the definition of the curl. The FDTD time iteration algorithm determines that , in every position in calculating the field , the new value (t_{n+1}) is obtained from the (t_n) value and the H-field curl value on this location .By this procedure the electric and magnetic fields will be calculated related to each other and their curl. Remembering that, the important point is the time steps (sampling time) which the simulation values constructed regarding it pixel by pixel.

The FDTD is solved in selected volume, built up by cubic cells that segmenting the area , then calculating the Maxwell differential equation regarding each cubic in every time step. The electric and magnetic field vector are then computed step by step based on the previous values in the entire computational field. The computational cell is shown in figure (2.1.1), also the E and H-field will be evaluated at any time step regarding different positions of the each cubic.

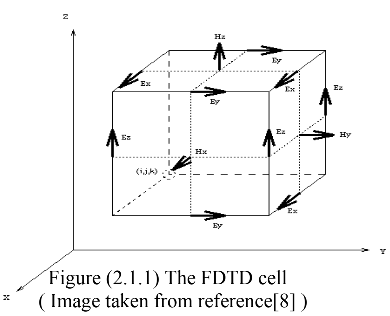


Figure (2.1.1) The FDTD cell
(Image taken from reference[8])

A big disadvantage with this method is the memory usage which is high because all of the area should be discretized and evaluated in each time step. It is clear that with this situation a long time is needed even when just one location value in time frame is needed, in other words in each time step all of the area should be evaluated . The worst case is where the calculation of a far point is become the goal of the computation, thus this distance requires the computations to be excessively large. However FDTD have one big positive feature, which makes it interesting for certain applications. In FDTD the simulation is made in the time domain, thus one can obtain multi frequency data from a single time domain calculation. For example if a pulse is transmitted, then by finding the electric and magnetic field for a cubic cell in one assumed position within a frame of time steps, the simulation contains different frequency values of the electromagnetic wave in this location.[8][4] Individual frequencies can then be analyzed through the Fourier transform. In order to understand how simple FDTD is working assumes the Maxwell equations again in their expanded format regarding each coordinate like below:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right) \tag{2.1.1}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \tag{2.1.2}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} - \sigma E_z\right) \tag{2.1.3}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{2.1.4}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) \tag{2.1.5}$$

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) \tag{2.1.6}$$

These equations are written in terms of the field components in figure (2.1.1), the Yee cell. The implementation of this algorithm is very straight forward in terms of finite differences. However as is discussed before the time domain formulation makes it very appealing in broad band microwave imaging as multi frequency information can be obtained with one single FDTD simulation.

Finally it should be mentioned that there exists some other forward methods, which are attempting to make new perspective of faster and reliable field numerical computation that can be constructed based on FDTD in time domain. One example of other computational electromagnetic techniques which is utilized to simulate the propagation of wave in the field is, Multiscaling method that optimize the screen grid, regarding the acceptable relative error. [21]

2.2. The method of the moments (MOM)

The basic idea in MOM is to convert the functional equations to a matrix equation. The general formulation of this method is described by the equations *L(f)=g* ,where L is the operator ,*g* is the source and *f* is the response .

Converting the functional equation to matrix form allows for formulating the solution a matrix inversion .Also the matrix calculation is constructed in linear space, which results in a multivariable deterministic problem. The deterministic situation is defined in matrix situation by the linear independency of each matrix row and the matrix rank.

In order to describe the MOM method again, begins with *L(f)=g* , here L is differential operator (in our problem L is ∇^2,f is electric filed and g is transmitting signal from antenna like $\delta(r)$.[3][4]

If we choose *f=Σ an*fn* , where *fn* is segmentation of the field (better to be triangular for electromagnetic case , because here we have second order derivation and the result of two times derivation of triangles is simple direct function) , it is resulted to a new linear equation as *Σ an*L(fn)=g*. The samples *fn* are the basis functions, can be selected with any kind of segmentation but due to above reason the triangle is selected for MOM.

On the other hand the area of the *fn* can be selected regarding the expected error. The wider area if chosen for each Fn the summation and respected matrix size will be lower, in the other words if the areas represented by Fn become smaller the error of using MOM to solve the problem is smaller, yet the matrix size is higher which cause the operation cost increased.

To make the linear conversion defining a new operator is essential. Assume to define the operator < > as < f1, f2 > $=\int(f1 * f2)dx$, which is the simple case in software implementation. Relating again to *Σ an*L(fn)=g* , it is possible to combine it with the new operator which will results to *Σ an<wm*L(fn>)=<wm ,g>* where *Wn* are any linear perpendicular functions but for simplicity in electromagnetic problem *wn=fn*. Now it is time to simply convert the sigma equation to its matrix form in (2.2.1), it is obvious that the matrix multiplication can model the vector multiplication in a sigma equation like above. [9][4]

[Lmn][an]=[gm] (2.2.1)

To describe it, equation (2.2.2) is expanded to the form of matrix array and multiplication.

$$\begin{bmatrix} < w1, Lf1 > & \cdots & < w1, Lfn > \\ \vdots & \ddots & \vdots \\ < wn, Lf1 > & \cdots & < wn, Lfn > \end{bmatrix} \begin{matrix} a1 \\ \ldots \\ an \end{matrix} = \begin{matrix} < w1, g > \\ < \cdots > \\ < w2, g > \end{matrix} \qquad (2.2.2)$$

[an]=[Lmn^{-1}][gm] (2.2.3)

It is clear that the differential equation changed to matrix multivariable solution, thus here is an example of electromagnetic wave to clarifying the idea. As discussed in the previous section about the basic of electromagnetic, the goal in this discussion is to solve the equation in (2.2.4).[9]

$$\nabla^2 E + k^2 E = j\mu\omega J \qquad (2.2.4)$$

Regarding (2.2.2) ,changing the (2.2.4) to its integral model is essential ,because of the linear operation *<w1,Lf1>* it can easily handled with integral form .

$$Ez(r) = \frac{-kn}{4} * \iint J(r') H0^{(2)}(K|r-r'|)\, ds' \qquad (2.2.5)$$

The field segmented by triangles called ΔCn , according to previous section triangles in a two dimensional area are the best selection, because the electromagnetic solution containing a second order derivation which convert the triangles to a Dirac function .Also the electric current in one coordinate represented and converted as ***Jz*** as ***Jz****=Σ JzΔCn ,* and the gm in (2.2.3) equation can be written gm=Ez(xm,ym).

Now it is the time to show the final calculation in (2.2.6) which is based on MOM understanding key.

$$Lmn = \frac{-kn}{4} * \int_{C1}^{Cn} H0^2 \left(K \sqrt[2]{(x-xm)^2 + (y-ym)^2}\right))ds \qquad (2.2.6)$$

The further action is to solve simple matrix equation, also as L matrix is reversible it is possible to take it into other side for other type of problem.

One important step to reduce the calculated cost when using the triangular segmentation in MOM is to estimate the Hankel function in this area. Equation (2.2.7) showing the idea about the second order Hankel function which should be used in two dimensional electromagnetic problems.

$$H0^2(r) = 1 - j\frac{2}{\pi} log(\gamma r) \qquad (2.2.7)$$

Utilizing (2.2.7) in (2.2.6) will result in a very optimum estimation of the integration in the (2.2.6) that cost very low calculation time to evaluate it, (2.2.8) shows the final equation.

$$Lmn = \frac{-kn}{4} Cn * [1 - j\frac{2}{\pi} log(\gamma r)] \text{ Cn is the area of one triangle segment [9][4]} \qquad (2.2.8)$$

The final words in this section are that the conversion from the electromagnetic functional equation to matrix form is simple and make the MOM technique very popular and easy in programming. The cost of calculation is lower especially in comparison with previous techniques like FDTD.

MOM have one problem especially in two dimensional computation regarding Hankel function that coming in (2.2.7) which shows a singularity behavior around the(γr)when goes near to zero (because of the logarithm argument) thus estimation around the source (near to zero) have higher error in comparison with points far from the source .[9][4]

2.3 Fast multipole method (FMM)

In many two-dimensional scattering problems, it is usual to simplify the scalar Helmholtz equation to the second-kind of its integral equation. The resulted equation can be considered to be solved with different numerical techniques.

One basic method for the numerical problem of scattering is to discretize the second kind integral equation utilizing an appropriate quadrature technique for example Nyström lemma. Results of this type of discretization is to make a systems of linear algebraic equations that can be solved by Gaussian elimination or iterative methods such as conjugate gradient or generalized conjugate residual .[19]

The operation cost changes between O(N) (the best case)and O(N^2)(the worse case without using the FMM) . With the assumption that there are N multipole points and for each segment the two adjacent segments are "nearby"(these require direct calculation) All other segments are considered "far points ".

In the other worlds in FMM method the far points will be estimated by spherical harmonics expansion (in Helmholtz problem) and the near points calculated without the expansion. It is obvious that this estimation cause more numerical errors, yet there is some optimization strategies to limit these errors. Below example is taken from Rokhlin article [MUST INCLUDE REFERENCE] and are showing that the multipole expansion resulted in a calculation with complexity O($N^{4/3}$) .
In order to discuss the technique on electromagnetic issue, again consider the well known Maxwell equation $\nabla^2 E + K^2 E$=0 with boundary condition E=0 on the field margins at the boundary of the scatterer.

Also scattered wave can be concluded based on previous notation as (2.3.1) which is deducted from the Maxwell equation and the integration in (2.3.1) that is happened on the determined area regarding boundary condition.

$$Ez_{scat}(r)=\int_{c0}^{cn}\left(\frac{\partial G(k|r-r'|)}{\partial n(r')}K(r')\right)dl' \qquad (2.3.1)$$

$$Ez_{scat}(r)=\left(\frac{i}{4}\right)*\int\left(\frac{\partial H^1(k|r-r'|)}{\partial n(r')}K(r')\right)dl' \qquad (2.3.2)$$

Remembering the Green function in (2.3.1) and its solution in two dimensional situation that is the Hankel function (2.3.2). The integration area notified in (2.3.1) by the letter Cn , on the other hand it is possible to consider this Cn with its triangular segmentation . On this Cn the Hankel function H_0^1(k (p – p')) can be expanded in terms of higher order Hankel and Bessel functions coming in (2.3.3).

$$H_0^1(k|r-r'|)=\sum_{m=-\infty}^{m=+m} H_m^1(kr)J_m(kr').\exp(\theta-\theta') \qquad (2.3.3)$$

Now it is possible to combine these three equation together which will resulted the (2.3.4) that is the fundamental key of the FMM.

$$Ez_{scat}(\rho,\theta)=\left(\frac{i}{4}\right) * \int \left(\frac{\sum_{m=-\infty}^{m=+\infty} H_m^1(k\rho)\partial J_m(k\rho').exp(\theta-\theta')}{\partial n(\rho',\theta)} K(\rho',\theta)\right)dl' \qquad (2.3.4)$$

The important point is that FMM is usually based on the computational MOM method. Although most feasible techniques in electromagnetics are based on the MOM for solving integral equations, some of them used Nystrom's method. "The main difference is that in MOM, the currents are expanded in a series of basic functions (pulse), and the unknowns are the weights multiplying these basis functions. In Nystrom's method, the integrals are discretized using convergent quadrature formulas for a given kernel, and the unknowns are the current values at the sample points on the surface of the scatterer.
Nystrom's method has the advantage that it is easier to develop higher order quadrature formulas than it is to employ higher order basis functions (as done in moment methods). In addition, matrix fill is more efficient with Nystrom's method than with moment methods." [18][19]

Assume a discretized model on Cn (like MOM).Then the integration can be represented as two sigma, where dl is the discretized element .The problem can also be formulated using an equivalent physical electric surface current. The important point is if we reduce the calculation of infinite summation to N point then, it is a case with N multipoles estimation. The next step is that the estimation dependent on units, which is showing up at the (2.3.5), FFM is faster but also contains some inherent error sources depending on the adjusted estimation before programming (depending on the calculations which have been decided to evaluate outside or inside the circle of radius (α) in (2.3.5)).

$$E_z^{scat}(\rho,\theta) = \sum_m \alpha_m H_m^1(k\rho)\exp(im\theta) \quad if\ \rho > \alpha$$
$$E_z^{scat}(\rho,\theta) = \sum_m \beta_m J_m(k\rho)\exp(im\theta) \quad if\ \rho < \alpha \qquad (2.3.5)$$

Also it is essential to describe the multipole expansion from mathematical base to understand the relation of it with FMM. M**ultipole expansion** is a mathematical series generating a function that depends on angles (in this application the two angles on a sphere) . These series are practical as the strong truncation is one of their features, in the other words, only the first few coefficients need to be retained for a acceptable approximation in comparison with original function. Multipole expansions are very frequently used in the study of electromagnetic and gravitational fields, where the fields at far points respected as sources in a small region. [10]

The series is written as a sum of spherical harmonics. Thus, we might write a function f(θ,ρ) as the summation model in (2.3.6).

$$f(\theta,\rho) = \sum_{l=0}^{\infty}\sum_{m=-l}^{+l} C_l^m Y_l^m(\theta,\rho)\ ,\ Y_l^m(\theta,\rho)=\mathrm{N}e^{im\rho}P_l^m\cos(\theta) \qquad (2.3.6)$$

$Y_l^m(\theta, \rho)$ is signifying a spherical harmonic function of degree ℓ and order m, P_l^m have near relation to Legendre polynomial, N is a normalization constant, and θ and ρ represent colatitudes and longitude, respectively (assuming a spherical geometry).
On the other hand, the MOM comprises the calculation of forces on a particle by N other particles. It is possible to assume a circle around each one of N point, then for those N points which are in the circle use the exact method of MOM and for other distant points it is faster to use expanded method of FMM as above.

As the last part of the description of FMM ,it is helpful to summarize the procedure .The FMM utilizes a controlled approximation (by dividing the computational domain in near and far field locations) .On the other hand regarding the system Green's function ,which let the response of N separated points respected as summation of single points .The multipole expansion let to the far points response estimated with reduced calculation procedures .The error related to this estimation can be controlled by filed separating circle diameter around each discretized areas .Clearly, the less approximation cause more calculation cost and less error (trade off).
[10-19]

3. Methodology of microwave imaging

In this section the mathematical relation between physics of propagated wave and the dielectric combination of the object will be discussed .On the other hand by introducing the linear operator included some estimation of the field behavior the mathematical overall relations modeled as a group of linear equation in the matrix from where the left side of the matrix are the variables multiplied by the jacobian of them and the right side the differences between the measured signal and simulated signal will be posed .Finally , the algorithm will be completed by combining it with multi frequency idea and describing how to optimize the algorithm with non pixel based method ,which fulfills the discussion from the theoretical views (in subsequent sections).

3.1. Geometry of microwave imaging

Microwave imaging algorithm can be divided into two parts, the forward method and the inverse method, and these are used to converge the target dielectric property within different iterations. In other words the image will be described by the permittivity and conductivity of the pixel's in the imaging area.

In order to reach this target based on the scattering the transmitter and receivers are constructed around the imaging area as in figure (3.1.1) (the imaging area with eight antennas around it).

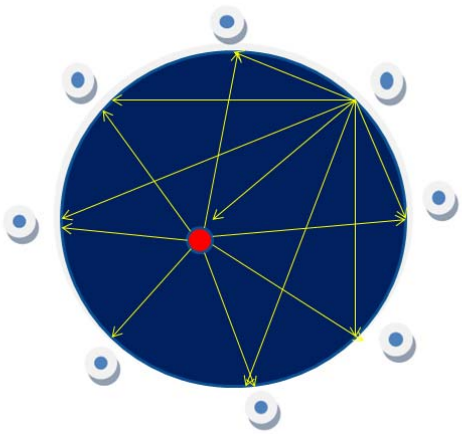


Figure (3.1.1), the microwave imaging system, the small circles represent the antennas.
The microwave propagation and interacting with high dielectric object as red color is depicted.

Each antenna is acting both as sending and as receiving antennas for the microwaves from the imaging plate. The goal is to find the unknown property of the different mediums of the object in the imaging area, it is possible to use different wave forms to send .For example pulse and one single frequency wave, also in one experiment all of the antennas sending and storing the receiving waves which let the further analysis conducted on these information to generate the image. The number of antennas can be changed. In principle increasing, the number of antennas will improve the image quality as there will be more data to be analyzed. This system can also be implemented in both two and three dimension to generate the 2D and 3D images.

Nevertheless there are some restrictions about the geometrical parameters of the imaging system which force to use optimization in order to gain higher proportion of information in the imaging algorithm(that is the goal of this project) , for instance the number of the antennas, the more antennas are used the forward calculation in each iteration of the imaging will take much longer time to be completed also if the antennas positions selected too near to each other the transmission behavior is somehow the same ,therefore both antennas providing almost the same information.

Simple mathematics in the next part shows the idea that if buy using the FDTD the consumed time for a single sending and saving the receivers data , be equal to **(α)** and the number of antennas equal to N , then α*N is the taken time for the forward method in each iteration . Also the memory consumption increased by N*N ,because regarding each action of sending ,N receiving data should be stored (each N transmitters require N data memory section) .The case become worsen when the imaging grids are increased , as the FDTD seek to find the value of electromagnetic wave for all of the grids in each iteration . The above discussion state that optimizing the of mathematical computation is very important and required on this area of imaging .In other words to delineate the optimization first the basic structure of imaging algorithm should be described here ,where it is the best place to introduce how image generated based on its backbone fundamental formulas and its geometrical design . Then in next sections it will be shown that how this fundamental relation can be configured and be optimized according to the application.

3.2. The imaging algorithm

In order to describe this algorithm first begin with an example, assume the ith transmitter sends a wave and the antenna with number j receives the data. The small changes to its received electric field according the small changes of the epsilon value $\delta\varepsilon$ from the complex dielectric properties can be calculated by the equation (*3.2.*1) also the operator $\widehat{D}$ is defined regarding the (3.2.1) in to (3.2.2).

$$\delta\text{Eij}=\frac{K0^2}{\varepsilon 0} * \int Ei(\vec{r})\, Gj(\vec{r})\delta\varepsilon(\vec{r})d\vec{r} \qquad (3.2.1)$$

$$\widehat{D}\delta\varepsilon = \delta\text{Eij} \qquad (3.2.2)$$

Where k0 $=2\pi\omega\sqrt{\varepsilon_0}$/c and ε_0 are the wavenumber and epsilon value of the background medium, ω is frequency, and c is the speed of light. The Green's function G_j is calculated from the Helmholtz's equations in (3.2.3)

$$(\nabla^2 + k^2)\text{Gj} = -\delta(\vec{r} - \overrightarrow{rj}) \qquad (3.2.3)$$

It is important to remember that (i) and (j) are the coordinates of the transmitter and receiver, thus the ($\overrightarrow{rj}$) is the location of the jth transmitter. Also the solution of the Green's function is the Helmholtz's equations which are the Hankle function regarding 2D differential equation problem (3.2.4) and it will be clear form what is shown in (3.2.5).

$$\text{G(r)} = \frac{i}{4} H_0^1(k|r|) \qquad (3.2.4)$$

$$H_a^1(x) = \frac{(J_{-a}(x)) - (e^{a\pi i} * J_{+a}(x))}{i sin(a\pi)} \quad \text{and} \quad J_n(x) = \frac{1}{\pi} * \int_0^{\pi} \cos(n\tau - x sin n\tau)\, d\tau \qquad (3.2.5)$$

Based on the above discussions the goal is to estimate how much the permittivity values should be changed in relation with stored signals in receivers after each transmission. The first task is to measure the stored values in the receivers related to the different experiment by using the (3.2.1) equation, that make it possible to assign permittivity values (epsilon values) to the object in the imaging plate ,and these epsilon values generate the image. Measuring the received signals and the electromagnetic field evaluation in different locations are called forward step of testing .In order to simulate the imaging system ,at the fist action an object with determined permittivity and conductivity should be placed in the imaging area then each antennas sends a wave ,and the receivers store the incoming wave as a signal with a determined frame of time , regarding each transmission .

It is proper to call the received information by these setting, the measured data, as they have taken when an object is placed inside the area.

The second step is to conduct the experiment with the blank area (the sigma and epsilon values become equal to the background values) and now it is time to store the receiver data once again which can be called the simulation data. After these procedures the differences of received data regarding these two experiments is possible to be calculate, which can be regarded as the delta electric field (δEij) regarding different transmissions in equation (3.2.1).

Equation (3.2.6) evaluates the variations of the electric waves by different transmissions which is one of the final matrix equation, the φ will represent the simulated (RS)and measured (RM)data differences .

$$\varphi(\varepsilon) = \left\| RS_{ij} - RM_{ij} \right\|^2 \quad , \tag{3.2.6}$$

Remembering the $\widehat{D}$ operator which let the Newton modeling of the equations to find the epsilon values by what is written in (3.2.7) and (3.2.8).

$$\varepsilon^{n+1} = \varepsilon^n + \delta\varepsilon^n \tag{3.2.7}$$

$$(\widehat{D}_t\widehat{D} + \gamma) * \delta\varepsilon^{n+1} = \widehat{D}_t * \varphi(\varepsilon) \tag{3.2.8}$$

In above equations $\widehat{D}_t$ is the transpose of $\widehat{D}$ and remembering that this operator causes the calculation of δEij regarding ε values according to (3.2.1) and (3.2.2). It is obvious that $\widehat{D}$ make the two dimensional Jacobian vector regarding the electric filed and epsilon values. Also the Jacobian matrix is two dimensional, consisted of each row generated by each sending and receiving.[1]

Now it is essential to discussed about how to calculate the Jacobian elements by the $\widehat{D}$ vector. In order to obtain each element of the two dimensional Jacobian vectors which can be called Jacobian matrix , (3.2.9) equation formulates the elements value clearly.[2]

$$\frac{d\varphi_r^t}{dk_l^2} = \frac{Rc<E^t,E^r>_l}{2j\omega V_t V_r} \tag{3.2.9}$$

$$k^2(\mathrm{r}) = \omega^2\mu\varepsilon(r) + j\omega\mu\sigma(r) \tag{3.2.10}$$

Where the K is the complex dialectic property based on (3.2.10) , thus the $\frac{d\varphi_r^t}{dk_l^2}$ means that the variation of epsilon and sigma(values, discussed more in inverse method section) related to the differences of measured and simulated data according to each transmission and receiving (noted by φ_r^t) .

One important discussion before the final imaging equation is the formation of the Jacobian matrix regarding receiver's data. In microwave discussion it is possible to consider the scattering points as the new source points. On the other hand when the object placed in the imaging area, yields wave scattering because of any changes in dielectric properties create the scattering in that locations .Thus it is possible to respect the scattering locations as the new sources. Relating to previous equations, utilizing another electromagnetic feature helps for better modeling. This feature let the source and receivers are respected reciprocal, in other words the signals of receivers and sources in transmission can remain the same if their positions become changed by each other. The reciprocal feature let the advantage of respecting the receivers as sources (regarding the scattering) ,become useful in constructing the Jacobian matrix .The procedure is to assume the receiver data regarding each transmission as a source voltage then by using (3.2.9) , it is possible to define new Jacobian elements as in (3.2.10.A) when RtRr are the regarding receiver information . These new Jacobian elements should be added as new rows to the Jacobian matrix. This has the effect that it reduces the degrees of freedeom for the overall solution [27].

$$\frac{d\varphi_r^t}{dk_l^2} = \frac{Rc<E^t,E^r>_l}{2j\omega R_t R_r} \qquad (3.2.10.A)$$

In order to describe the free degrees, consider that the numbers of unknown variables in the matrix are much larger than the number of equations. The number of variables is equal to the number of image pixels, but on the other hand the number of equations is equal to the number of receiving/transmitting antenna combinations. Thus the overall equation quantity for solving is antennas quantity multiplied with itself or if the antennas quantity is N there are N^2 solution equation existed to be evaluated.

Free degrees are the independent variables in a linear system equation which in this modeling are equal to the quantity of the pixels in the resulted image .The first problem happened is the number of equations is much lower than the image pixels, therefore some new techniques should be applied to make these equations evaluated .There is two basic methods which will be used to solve and improve this problem, that are low pass filtering and making Jacobian matrix with multi frequency idea.

In order to discuss about the low pass filtering again there is a need to remember the equation (3.2.8) and the Jacobian calculation part ($\widehat{D}_t\widehat{D} + \gamma$). The γ in this equation is a constant that mathematically doing the regularization task .The regularization is used when the rank of the matrix is low regarding the variables and in this case its effect on the image is exactly like to imply a low pass filter on the target object image .Without regularization the calculation of the $\widehat{D}_t\widehat{D}$ is not beneficial as the matrix rows represent the repetitive values or the same equations to solve multivariable problem.

The matrix rank problem has not been solved yet, and need to introduce some new ways to add new equations. One of the effective techniques to add more equation is making the Jacobian calculation with different frequencies. For example the Jacobian can be processed by equation (3.2.10.A), however this relation have different values regarding different frequencies.[27]

The FDTD program has one advantage that its processing occurre in the time domain and the electric field in different locations of the field can be determined with time stepping procedure. The good point here is as the time domain electric field signal have the range of frequencies samples too and each frequencies sample not only can generate a new Jacobian row in Jacobian matrix ,but also new differences vector by using the equation (3.2.6). It is easy to conclude that each frequency can generate the N*N equations (N is the number of the antennas), moreover if the M different frequency are used to create the solving equations then generally there are $(M * N^2)$ equations existed. Therefore the problem of number of equations regarding Jacobian matrix rank cab be solved and what is required for the next step is a matrix division to obtain the epsilon (and sigma) values. The best place to discuss the matrix equation is now, because the rank problem in Jacobian matrix can be solved. The first step is to combine (3.2.6) and (3.2.8) in to matrix equation in (3.2.11).

$$\begin{pmatrix} j[1,1] & \cdots & j[1,n] \\ \vdots & \ddots & \vdots \\ j[1,n] & \cdots & j[m*n*n,n] \end{pmatrix} * \begin{matrix} \varepsilon 1 \\ \dots \\ \varepsilon n \end{matrix} = \begin{matrix} \varphi 1 \\ \dots \\ \varphi[m*n*n] \end{matrix} \tag{3.2.11}$$

The first matrix in (3.2.11) is the Jacobian and each row is calculated with respect to changes in the permittivity. The Jacobian multiplied with the unknown epsilon values is equal to the differences between the measured and simulated data in the right hand side of the equation. The problem is now to solve this matrix equation, which is a well known problem with several solution strategies.

One important point is how the Jacobian matrix rows are arranged. Each row represents a single transmission (receiving) procedure. Thus one row can be added to the Jacobean for every transmitting antenna. If the *(i)th* transmitter sends a pulse the receivers will register the scattered wave from the object, therefore by using the (3.2.10.A) regarding first receiver the first row of the Jacobian matrix will be formed and in a similar way more rows can be added for each transmitting receiving antenna combination. The right hand side of the equation is calculated as the difference between the simulated and measured receiver data for each transmitting receiving antenna combination. The differences should be calculated and added to relate (in front of) same row number of that receiver in the Jacobian matrix. [1][2][23-28]

This equation is then used in an iterative solution strategy where the unknown permittivity values are stepwise improved and optimized with a Newton type of algorithm. The step length must then be determined, this is made according to the description below. In this part the Newton step for any iteration is adjusted with a feedback technique from the error function. The feedback calculation modeled in equation (3.2.12) if loop number is equal to (i).

$$\text{NewtonStep}=\text{InitialNewtonStep}*\left(1-\frac{\sum(simulated-measuement)^2(i+1)}{\sum(simulated-measuement)^2(i)} \right) \qquad (3.2.12)$$

In order to understand the above equation first the accumulated absolute error should be mentioned which is represented by the differences of simulation and measurement as $(simulated - measuement)^2(i)$ in the equation .Therefore dividing the absolute error of the next loop to its previous value, results the proportion of changing within each loop. The behavior is simple, that as much as the proportion is higher the next Newton step is lower. This is helpful near the convergence where the Newton steps should become smaller. In the results part the power of equation (3.2.12) will be shown and how it can be used to avoid diverging solutions.

In the case that the number of equations is smaller than the unknown parameters, it is not possible to uniquely solve the matrix equation. If so regularization has to be applied to the equation system. The reqularization is also transforming the ill-posed problem into a well-posed.

3.3. Transforming an ill-posed problem to a well-posed

Microwave imaging consists of two parts, one is the forward computation and other one is the inverse calculation. Nevertheless it is possible that the number of unknown variables is much larger than the number of equations. In this case the regularization can be used to stabilize the solution. But also for an over determined equation system the regularization is essential for transforming the ill-posed problem into a well-posed. It is thus important to specify what is a well-posed problem and how large amount of regularization that should be added. The following two sections attempts to enlightening the idea to some extent.

3.3.1. Changing the ill-posed inverse problem into a well-posed problem

The mathematical formulas in this part is not implemented in this project, in the other words the regularization factor value (which is described here) is selected by testing the program (and not be calculated by the formula in 3.3.8 for example).This part more or less is made to clarify the regularization idea.

From the mathematical aspects, it is possible to deal with forward problems, which is a case where causes are given and the quantities that should be found are results. For inverse problems the situation is opposite. The results are known and causes are unknown. In a forward method, the action is to find information that satisfies some given partial differential equation, some initial and boundary conditions. In inverse problems, in general model (initial conditions or boundary conditions) are not fully determined on the other hand, it is possible to gain or access to some information which by using them the specified model with differential equation can be solved and be determined. For example in microwave problem the object shape can be estimated as a connected contour before processing especially, if the imaging propose is about the breast tumor issues, which the shape and the properties of the tumor can be estimated based on previous experiments. Moreover in the microwave tomography the equation quantity regarding solution can be increased by using different reconstruction frequencies, which reduce the values of regularization as the system rank in solution is improved. [1][29]

The definition of a well posed problem according to J. Hadamard, [29]:

1) the solution of the problem does exist;
2) the solution is unique;
3) the solution continuously depends on input data"

Basic approaches for the approximate solution of an ill posed case is utilizing perturbation of the initial problem, this is what was happen is equation (3.2.1). These perturbations are leading to a linear multivariable solution for the model, for example in microwave imaging the Jacobian matrix elements have this task in equation (3.2.9).

In microwave imaging, an ill posed problem has been replaced with a well posed one by a linear differential model(derivation) of the variable then deducting a Jacobian matrix (or operator) to solve the problem. The Tikhonov regularization method is commonly used to enforce a smooth solution, however an appropriate value on the regularization parameter must be determined in each iteration.

Regarding Tikhonov the main technique is like many other algorithms, which is making a stable algorithm to overcome the problem with an acceptable estimation or as optimized as possible. The method is based on the linear first order derivation which lead to Jacobian matrix, then adding stabilizing element (in microwave imaging the regularization added in the (3.2.8) noted by γ) help to an estimation of the target answer.

$$Au\alpha = f\delta \qquad (3.3.1)$$

The backbone of constructing stable methods for solving ill posed problems is to implement an algorithm that utilizing some priori data about the input data inaccuracy. Many manipulations is existed in order to handle the uncertainty related to this problem, however in this work transforming from time domain to frequency domain and adding a regularization factor (3.2.8) has been selected, thus the selection of FDTD as forward computation become clear now, since FDTD is the time domain method that allow to reconstruct the image from many different frequencies (regarding bandwidth on Nyquist criteria) by one forward computation in time domain.

Assume $A\delta u\alpha = f\delta$, that operator $A\delta$ have improved properties compared to A by adding the additional information .Also based on [1], instead of solving above equation it is practical to minimize the norm of the difference $r = Av - f\delta$, or the *functional differences* $J0(v) = \|Av - f\delta\|^2$.There exists are many solutions to this model which satisfy equation (3.3.1) regarding differences δ. What is needed ,is to wisely consider all available information about the inaccuracy and ensure that it will constrained in to the equations . In the microwave imaging procedure A is the Jacobian matrix and (v) in Av is the dielectric value for each pixel. In the Tikhonov regularization method, they can be introduce according to (3.3.2).

$$J\alpha(v) = \|Av - f\delta\|^2 + \alpha\,(\,\|v\|\,)^2 \tag{3.3.2}$$

The approximate solution of the initial problem (3.3.1) is the optimum point of the equation (3.3 .3) in below.

$$J\alpha(u\alpha) = \min\alpha(v) \tag{3.3.3}$$

In (3.3.2), $\alpha > 0$ is the regularization parameter, whose value determined related to inaccuracy δ. For the general case of Tikhonov it is practical to use a stabilizing functional $\|v\|$ which help for better regularization in different inverse loops , however in this project of microwave imaging just a constant value of α have been used.

Due to the fact that the final goal of all actions is the convergence of the approximate solution to the exact solution. It is required to find under which situation the approximate solution of $u\alpha$ (found from (3.3.2) and (3.3.3)), converges to the exact solution of problem (3.3.1).Thus an important factor is showing up which is how the rate of convergence by this manipulations is changing .We represent the approximate solution in the operation (3.3.4) that is coming below. [29]

$$u\alpha = R(\alpha)\, f\delta \tag{3.3.4}$$

If the approximate solution converges to the exact solution with some settings, then inaccuracy tending to zero and it is practical to determine the operator $R(\alpha)$ as a regularizing operator.

Regarding this new labeling $R(\alpha)$ and its formation in (3.3 .4) ,the regularization parameter (α) defined as a function of δ , on the other hand below equation (3.3 .5) is what we used and discussed before in order to be solved .

$$\|Au\alpha - f\delta\| = \delta \tag{3.3 .5}$$

According to the reference [29] ”this choice of regularization parameter, or, in other words, the convergence of the approximate solution $u\alpha$ with $\alpha = \alpha(\delta)$ to the exact solution of (3.3.1) with $\delta \to 0$ was considered for many classes of problems. The difference between the approximation and the exact solution, or the discrepancy, somehow depends on α. ” Then using the $\varphi\ (\alpha) = \|Au\alpha - f\delta\|$ (φ determinated as new function by $\varphi\ (\alpha) = \delta$ notification)is proper based on this functional dependencies discussions .To find the regularization value related to (3.3.5), the equation $\varphi\ (\alpha) = \delta$ should be considered .According to [29] ”In almost any situation , the function $\varphi\ (\alpha)$ is a non-decreasing function, and equation $\varphi\ (\alpha) = \delta$ has a solution. From a mathematical point of view there are variety of solution techniques to deal with $\varphi\ (\alpha) = \delta$ For instance, we can use the succession as in (3.3 .7) showed.

$$\alpha k = \alpha 0 q^{k}\ ,\ q > 0 \tag{3.3 .7}$$

Here, the calculations are to be made starting from $k = 0$ and going to a certain $k = K$ at which equality in $\varphi\ (\alpha) = \delta$ becomes fulfilled to an acceptable accuracy. With so defined regularization parameter, we need $K + 1$ calculation of the differences (for the solutions of variation problems of type (3.3.2). In finding the approximate solution of $\varphi\ (\alpha) = \delta$, more rapidly converging iterative methods can also be used. It was found that the function $\psi(\beta) = \varphi\ (1/\beta)$ is a decreasing convex function. Hence, in solving the equation $\psi\ (\beta) = \delta$ one can use the Newton iterative method, in which noted by (3.3 .8).

$$\beta k{+}1 = \beta k - \frac{\psi(\beta k) - \delta}{\Delta\psi(\beta k)} \tag{3.3.8}$$

.

This method converges for any initial approximation $\beta 0 > 0$. To avoid the calculation of the derivative of $\psi(\beta)$, we can use the iterative secant method, more descriptive showed below (3.3.9).

$$\beta k{+}1 = \beta k - \frac{\beta k - \beta k{-}1}{\psi(\beta k) - \psi(\beta k{-}1)} * (\psi(\beta k) - \delta) \tag{3.3.9}$$

The use of such iterative procedures reduces the computational cost in the determination of α.”The above descriptions between quotations are not implemented in this project; nevertheless it is one way of optimization of the regularization .The information n the quotation is taken from [29].

3.3.2. Regularization parameter setting

In this project a simplified method of selecting the regularization parameters has been used. The important thing is in every iteration this parameters should be reduced because, in comparison with previous iteration the system is more converged, thus the solution can be stable even with lower regularization factor. On the other hand selecting the regularization factor equal to previous iteration cause the low filtering effects the resulted image, that is not acceptable in many cases. However too much decreasing the regularization factor makes the solution unstable and artifact will occur.

The simple regularization selection is based on (3.3.7), yet accepting some error and selecting the estimated regularization for first iteration it is possible to lead this parameters for next loops properly. Here the hard part is to estimate this parameter values for the first iteration by trial and error technique, which can be very time consuming in many cases particularly when the object is more complicated. Then after adjusting the regularization values for the first loop, it should be reduced in each integration by a constant factor (it is possible to used an adaptive values of reduction in each loops but this process is too much time consuming).

According to the description above the best formulation is, assume the constant reduction factor be equal to β then for nth iteration the initial regularization factor should be multiplied with β^{n} ,which cause the proper reduction occur.

So far the microwave imaging idea is fully determined from the mathematical views, on the other hand how to implement it in computer, needs some more descriptions that here is the best place to be disserted. In the next part the software implementation and the program parameters settings in order to simulate the forward and inverse part in described.

3.4. Implementing the microwave imaging simulation.

In this part the fundamental c++ program structure and settings the parameters will be discussed which coming in to two separated part. The first part is focusing on settings and parameters of the forward method and the second part is discussing the inversion setup parameters. It is important to delineate the simulation parameters, because in this project both forward and inverse problems are simulated and just a few parameters change to reconstruct the images in different experiments. Thus in this part the parameters used in simulation of the system are described.

3.4.1. Forward method implementation

It is also practical to review the previous discussion about microwave imaging from some other aspects. The image generation procedure is based on two different main parts where each part itself have some parameters that are controlling the quality of the convergence from different views like rate of convergence, artifacts, shape and position of object in the reconstructed image.

The first part in the image reconstruction algorithm is the forward method that includes transmitting of the wave into the object placed in the imaging area and storing the corresponding receiver data. In the forward method the electromagnetic field will be simulated and propagated into different locations. In principle any forward method can be used (FDTD , MOM, FMM,…). The Maxwell equation will be solved in the imaging area. Thus the electric and magnetic field related to dielectric properties of different pixels can be determined. The other action is simple, that is to create the matrix form of these data like in (3.2.11).
In this project the implementation has been done with VC++ based on an object oriented programming. The main.c manages the program from different parts that is setting up the FDTD and then the inverse method, will be constructed by matrix division regarding (3.2.11). There are many different parameters that have to be set before running the simulation program the fundamental settings are coming in the following:

1_Two integers values specifying the number of Spatial Grid (nX , nY)

2_The Spatial Step Length (ΔX , ΔY), in the other words the size of each pixel.

3_ Set the number of Pml layers,
   The computation domain is surrounded by layers of absorbing equal to this number

4_Simulation time

5_Source Center Frequency,(the central frequency of the propagated wave from each antenna)

6_Source Band Width, (transmitted wave band width)

7_Specify the locations of the transmitters, (geometrical specification of the imaging system)

8_Receiver Locations, (geometrical specification of the imaging system)

9_BackgroundPermittivity and Background Conductivity

10_Set object dielectric features (like width, location, permittivity and conductivity) for the measurement calculation propose.

3.4.2. Inverse method implementation

There are many inversion techniques that can iteratively generate a reconstructed image, however in this project the matrix division regarding (3.2.11) is used to obtain the permittivity values. Exactly like the forward computation the inverse part has some setup parameters that must be set before starting.

1_Set the specific area to be evaluated in inversion .It is possible to selected an specific pixels and try to solve the inverse equation just for them, because in the matrix form of (3.2.11) we face multivariable convergence problem .Therefore it is possible to discard some variable from (pixels) of the overall solution, this will decrease the matrix size and reduce the consumed time of calculation. Also from another point of view selecting just a specific area for inverse matrix ,can be regarded as prior data in some respect ,since discarding these pixels means we know that their values are equal to background and no need to be evaluated . This is a proper action in many problems for example in detecting the tumor in breast its location with high probability is not beneath the skin layer (no need of imaging in there).

2_The regularization value and how this values should be changed within different loops .Again it is important to emphasize on this fact that the task of regularization in microwave imaging is stabilizing the solution thus its effect on the image is like a low pass filter which has been implied on the image with exact solution to find the object ,then by increasing the regularization factor the lower frequencies amplitude in the reconstruction image will be increased (Therefore selecting a proper regularization value is very important) and can be done by trial and error too.

3_The Jacobian matrix elements are constructed regarding a reconstruction frequencies which should be determined before running the program, and its effect on reconstruction will be discussed in next part. In other words the equation (3.2.9) calculated with a selected reconstruction frequences in order to make the Jacobian matrix, moreover some time it is required to increase the Jacobian matrix’s size, then some new elements can be calculated by different frequencies and added as new row in the matrix. This technique let us to have as many as required equation (even as many as we like) which (regarding the previous discussion about the ill posed problem) can change the problem to well posed situation. It is why one problem can be ill posed in on space and well posed in other space .In other word simulation with FDTD in time domain let us to convert the situation to be well posed since Fourier transform let the changing space happened.

4_The number of inverse loops is very decisive because if the parameters set properly in each iteration the resulted image converging more to the exact object image, however usually with respect to the acceptable relative error the number of required loops can be controlled .

5_The FDTD center frequency in each inverse loop .Based on the fact that in each iteration it is required to run one FDTD to obtain the simulation data, it is important to select the center frequency in its proper range regarding the interaction with body tissue (1.0 to 3.0 GHz) .

Now it is possible to show the results of all theoretical parts that was coming before .In the other words in the previous part the fundamental mathematical construction and program structure was discussed in detail, therefore it is time to run the program and show the results regarding it.

Again it is practical to remember that the microwave imaging procedure can be implemented by different software like MATLAB or VC++ , also different forward techniques like FDTD , MOM and FMM can be used. Each method has some pros and cons, which has been discussed before. In this project all the implementations have been done by Microsoft visual studio (VC++) (because of its faster calculation in comparison with Matlab) and the images generated with Matlab.

It is highly important to declare the numerical constant setting in this experiment, because some parameters managed to be altered in different reconstructions, but some of them, that are constant all over the experiment, showed in table (3.4.2).

The Spatial Step Length	4.0e-3 4.0e-3
Number of pml layers	11
Simulation time	1.0e-8
Number of Spatial Grid Cells	30 , 30

Table (3.4.2), One example of the setting file

Summary of the Program steps :

Collecting the receiver values of measurement simulation section

Set the dielectric properties of the object, set the center frequency values
Run the fdtd and collect the receiver values
Make the dielectric and the field values equal to zero

Run the inverse part :

Add initial condition if it is needed
Run the FDTD , then save the receivers values and the electric filed values.
Calculate the error matrix $\varphi(\varepsilon) = \left\|RS_{ij} - RM_{ij}\right\|^2$, The receiver signals is in time domain ,thus processing on it is necessary to transform them in to the frequency domain then selecting one (or more if using multi frequency procedure) frequency sample (called Jacobian frequency sample or samples in multi frequency aspect) to calculate the error matrix .
Calculate the Jacobian matrix elements $\frac{d\varphi_r^t}{dk_l^2} = \frac{Rc<E^t,E^r>_l}{2j\omega V_t V_r}$, yet as above the signals are in time domain ,thus it is needed to transform them to frequency domain , then to calculate the Jacobian elements choose one (or more in multi frequency aspect) frequency to calculate the elements .
The multi frequency consideration cause the Jacobian and error signal calculated in some specific frequencies, which should be determined before running the program.

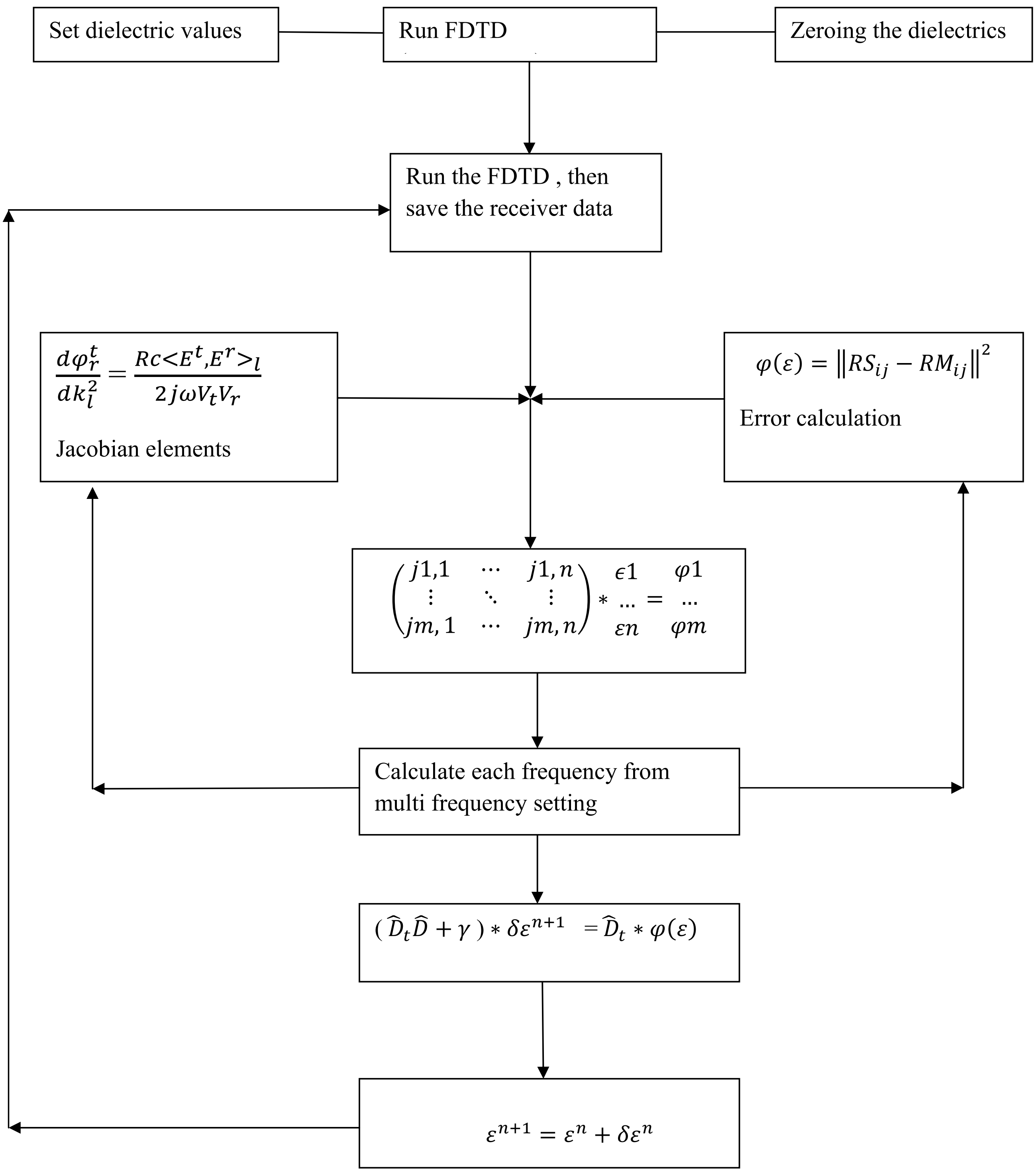
The Resulted equation is $\begin{pmatrix} j1,1 & \cdots & j1,n \\ \vdots & \ddots & \vdots \\ jm,1 & \cdots & jm,n \end{pmatrix} * \begin{matrix} \epsilon 1 \\ \ldots \\ \varepsilon n \end{matrix} = \begin{matrix} \varphi 1 \\ \ldots \\ \varphi m \end{matrix}$ with the

Jacobian size equal to (m=M*N^2) when M is the number of frequency samples (multi frequency) and the N is the number of antennas .Also the columns are equal to the number of image pixels (the variables) . The arrangement of the matrix is the first N^2 equations regarding the first Jacobian frequency posed in N^2 first rows and it is happened for remaining m-1 other frequencies . Now the equation should be solvable by some manipulation as bellow:

$$(\widehat{D}_t\widehat{D} + \gamma) * \delta\varepsilon^{n+1} = \widehat{D}_t * \varphi(\varepsilon)$$

After this step the last thing is matrix inversion, finding the $\delta\varepsilon$ and feed them to next equation $\varepsilon^{n+1} = \varepsilon^n + \delta\varepsilon^n$.

The further action is going to next iteration by zeroing the field and run a new FDTD to compute the new error and jacobian matrix.


Set dielectric values
Run FDTD
Zeroing the dielectrics
Run the FDTD , then save the receiver data
$\frac{d\varphi_r^t}{dk_l^2} = \frac{Rc<E^t,E^r>_l}{2j\omega V_t V_r}$
Jacobian elements
$\varphi(\varepsilon) = \left\|RS_{ij} - RM_{ij}\right\|^2$
Error calculation
$\begin{pmatrix} j1,1 & \cdots & j1,n \\ \vdots & \ddots & \vdots \\ jm,1 & \cdots & jm,n \end{pmatrix} * \begin{matrix} \epsilon 1 \\ \ldots \\ \varepsilon n \end{matrix} = \begin{matrix} \varphi 1 \\ \ldots \\ \varphi m \end{matrix}$
Calculate each frequency from multi frequency setting
$(\widehat{D}_t \widehat{D} + \gamma) * \delta\varepsilon^{n+1} = \widehat{D}_t * \varphi(\varepsilon)$
$\varepsilon^{n+1} = \varepsilon^n + \delta\varepsilon^n$

3.5 Algorithm Optimization

There is information in the matrix equation of microwave imaging which is redundant, like some details involved in high frequencies of the image Fourier transform, which consume many CPU operations without gaining any significant valuable things, from the object dielectric characteristics .Thus an optimization technique is essential to reduce the calculation by discarding the dissipating information to be processed. Succeeding two sections of the part (3.5) elucidate the new optimization technique.

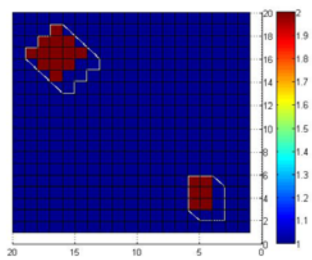


Figure (3.5.1) The object

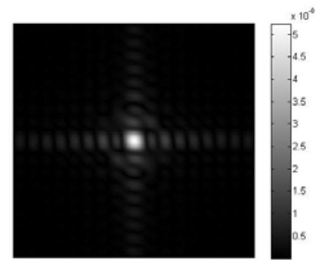


Figure (3.5.2) Frequency response of the figure (3.5.1) , the higher content of information is in the center set as low frequencies

3.5.1 Non pixel based reconstruction to optimize the computational time

In this work the reconstruction is implemented in frequency domain, then instead of finding the epsilon values in different location, the spectrum of it will be presented on the image screen which will yield the epsilon values after applying two dimensional IFFT on it.
The important point is that the reconstruction is made in the frequency domain, it is possible to discard the higher frequencies and solve the inverse matrix just by lower frequencies. Higher frequencies contain lower feasible information, then by this approximation the calculation cost will be very appropriate while the approximation error is remaining the same mostly.

In other words the goal is to make a method that do the reconstruction in a faster way .The first step which is inspired from FMM (forward method) is to expand the green function to become a summation of a linear functions ,then discard the coefficients that their energy (information) are lower than a threshold .In other words by using filtering ,an approximation of the Jacobian matrix is gained and its calculation is much faster in the inverse method (because higher frequencies are discarded and the new reduced Jacobian in frequency domain has lower size).

In order to answer that how much this method is faster, the matrix inversion operation should be considered. Assume the fastest methods like Winograd-Coppersmith O(n^2.376) [34].
If (M) member of the matrix size (from each row) is discarded (the higher frequencies that contain low energy) then, the inversion needs (O((N-M)^2.3) +4*O(N-M)log(N-M)), which additional (4(N-M)*log(N-M) in the algorithm implementation is coming from FFT and IFFT. Figures (3.5.1.1)showing how this method is faster regarding the number of N and M .In this figure M=N/10 and the green graph characterizing the effect of reduction in operation regarding discarding 10% of higher frequency samples.
It is important to remember that the Jacobian elements contain much more energy in lower frequencies, figure (3.5.1.2) showing the Jacobian matrix spectrum energy regarding one row.

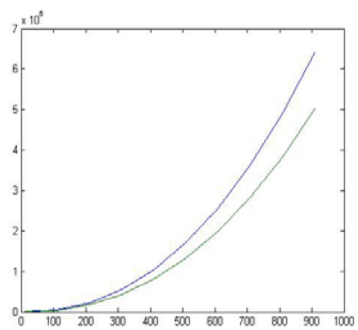


Figure 3.5.1.1 , horizontal direction showing the number of variables vertical direction is the inversion cost of normal (blue color) and the non pixel based method

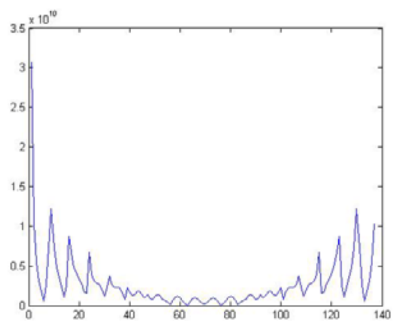


Figure 3.5.1.2,the jacobian frequency response showing the high frequency amplitude in very low (high frequencies are posed in the middle)

What is exactly happened in cost reduction is, firstly cut a selected range of higher frequencies. Also as here the FFT function is utilized from FFT DLL added to the program, the generated FFT results are symmetrical based on the fact that the time domain signals are real values.

Secondly that the higher frequencies coming in the middle of the spectrum signal, it is easy to cut the meddle part from a selected range and make a new matrix which is symmetrical again (but because of reduction, its size is lower). The next step is to feed the lower sized matrix, to the matrix inversion function.
It is practical to discuss it from the mathematical points of views on reconstructing the image. In the following J(r) means one row of the Jacobian matrix where (r) is the location and ε(r) means permittivity values in different locations.
Assume for one transmission rewrite the equation (3.2.8):

1) $\sum_r J(r).\,\varepsilon(r) = Error$

2) $\sum_r J(r) * \sum_{wn} \alpha(n)\varepsilon(wn)e^{+jwnr}$: ***using FFt to get ε (r) spectrum***

3) $\sum_{wn} \alpha(n) * \sum_r J(r).\,\varepsilon(wn)e^{+jwnr}$

4) $\sum_{wn} \alpha(n) * \varepsilon(wn) * \sum_r J(r).\,e^{+jwnr}$: ***using iFFt to get J(r) spectrum***

5) $\sum_{wn} \alpha(n) * \varepsilon(wn) * FFt(J(r))$

6) $\sum_{wn} \alpha(n) * \varepsilon(wn) * FFt(J(r))$

7) $\sum_{wn} \alpha(n) * \varepsilon(wn) * (J(wn))$

8) $\sum_{w(ux,uy)} \alpha(ux,uy) * \varepsilon(w(ux,uy) * (J(w(ux,uy))) = Error$
   ux ,uy are the spatial frequencies in 2D.

The important point is that this method cost two additional FFt but, as it let many higher frequency coefficients become discarded, the overall calculation tend to a very faster computation procedure.Grounding to the fact that the theoretical discussions are fulfilled by this part, it is practical to present the results in the following parts.

4. Introduction to the results of different reconstructions

Although the fundamental mathematical theory of microwave imaging has been described the capability of reconstruction can be best understood from different examples. Especially the new techniques like multi frequency algorithm and non pixel reduction should be tested in a variety of situations. Therefore in the following sections, a collection of examples are exposed from the simplest form, which lead to the hardest object for reconstruction that is the breast model.

4.1. Centered object results

The simplest object in this project to be imaged is showing in the figure (4.1), because this object is a wide objet in the center. The best reconstruction (respect to relative error from other experiments with this object that coming below) of this shown in figure (4.2) after eight inverse loop and reconstruction frequency 9.0e9.

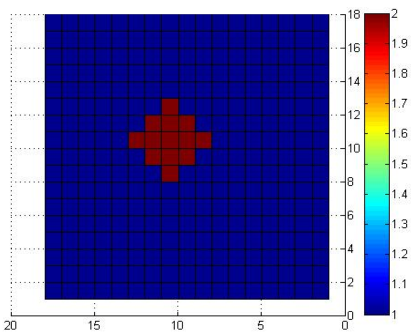


Figure 4.1, The object

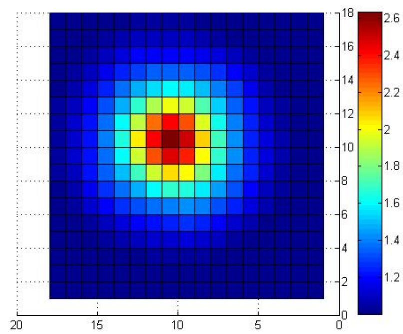


Figure 4.2, Reconstruction the center object.

In figure (4.3) the error (differences between the simulation and measurement data) regarding each inverse loop is depicted.

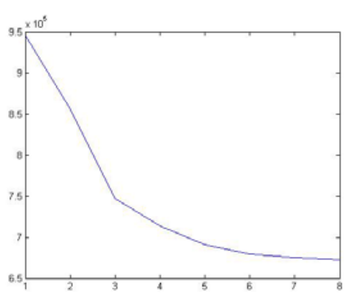


Figure 4.3 ,The error signal within eight loop of iteration .Each number in horizontal direction mean one iterated loop.

Figures from (4.4) to (4.8) are trying to show how the reconstruction quality changing with just altering the reconstruction frequency within five loop of iterations .Brief information of them is as below while the bandwidth is constant for all equal to 1.0e7 (RF=Reconstruction frequency):

figure (4.4)-> 5loop, RF=5.5e9, relative error =0.2212

figure (4.5)-> 5loop,RF=6.5e9, relative error =0.1578

figure(4.6)-> 5loop,RF=8.5e9, relative error = 0.0889 figure(4.7)-> 5loop,RF=9.5e9, relative error =0.0644

figure(4.8)-> 5loop,RF=9.7e9, relative error =0.1483

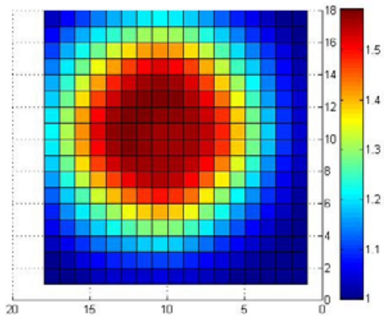


Figure 4.4, Reconstruction with 5.5GHz

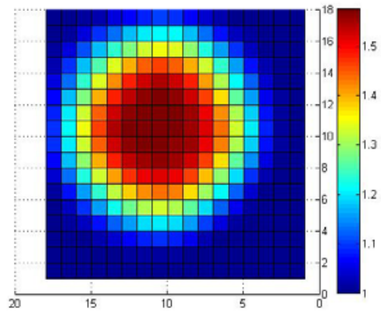


Figure 4.5, Reconstruction with 6.5GHz

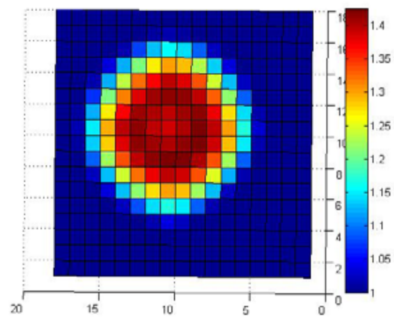


Figure 4.6, Reconstruction with 8.5GHz

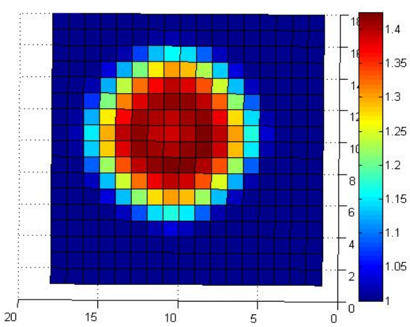


Figure 4.7, Reconstruction with 9.5 GHz

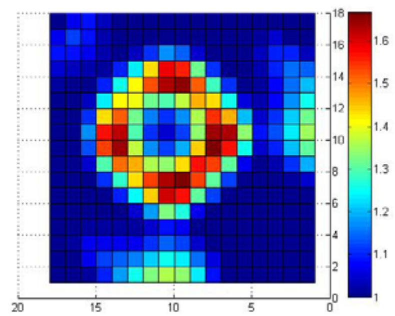


Figure 4.8, Reconstruction with 9.7 GHz

This result is figured in graph number (4.9), it is very important that according figure (4.9) the relative error is reduced by increasing the reconstruction frequency till reaching the 9.5e9 .Also it is clear that lower reconstruction frequency detecting the object with higher proportion of low frequencies characteristics like in figure (4.4) and more details appear in the figure 4.7 because the higher frequency is used to make the imaging equations.

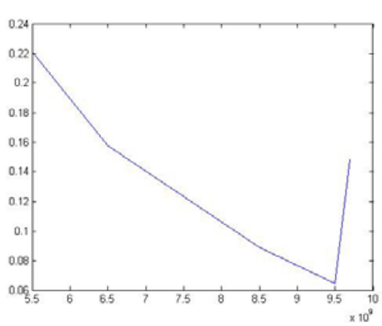


Figure 4.9,
The relative error in y-direction and the reconstructed frequency in x-direction .

Related to above discussion again reconstruction with one frequency is mentioned below, which for better understanding the relative error regarding each reconstruction is calculated in their following. Unlike the previous experiment where the effect of different reconstruction is tested, here the convergence quality with different frequencies is shown.

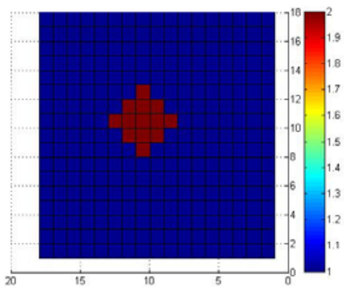


Figure 4.10 , the object

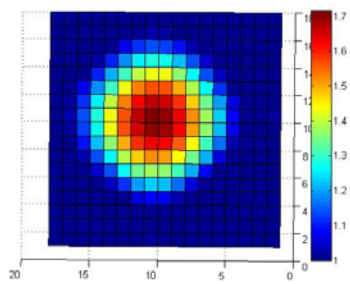


Figure 4.11
reconstruction frequency GHz=4.5

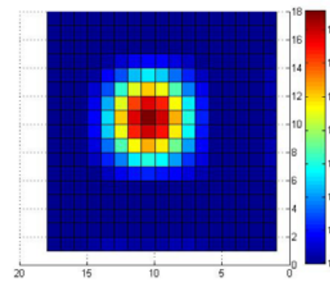


Figure 4.12
reconstruction frequency GHz=8.0

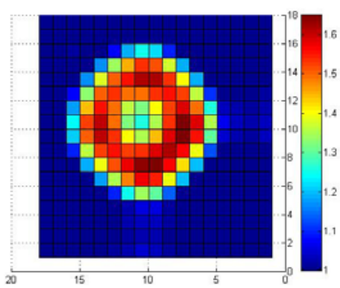


Figure 4.13
reconstruction frequency GHz=12.0

Regarding the object in figure (4.10), the goal in this part is showing the performance of different reconstruction frequencies.
The center frequency is set to 9.5e9.

figure(4.11),frequency GHz=4.5-relativeerror = 0.0942
figure(4.12), frequency GHz=8.0 -relative error = 0.0522
figure(4.13), frequency GHz=12.0-relativeerror =0.1271

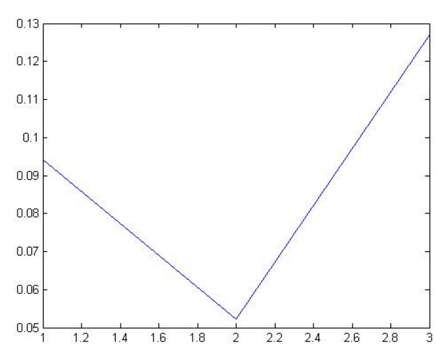


Figure 4.14 ,the relative error graph horizontal direction is show the frequencies {4.5,8.0.12.0 GHz} ,the vertical direction is the relative error.

The obvious conclusion from the relative error analyses is that the reconstruction frequency selection is important and based on these images if selected in low values , the reconstructed image contains more low frequencies amplitude .On the other hand high values of reconstruction frequency cause more focusing on object details . Nevertheless too high reconstruction frequencies like in figure 4.13 cause more artifacts showing up which tends the convergence relative error to be increased.

This is very important to remember that how the reconstruction frequency have effects on the results. Especially regarding the next section which illustrating the multi frequencies convergence.

4.2 Analyzing the reconstruction with one frequency

In Medical situations normally targets are more complicated than a single object. For example in breast tumor diagnostic problem, the tumors can be separated from each other with variation in shape and dielectric values. Then it is important to show the best convergence ability of this algorithm when confronting to this level of problems. It is important to say that in previous parts the algorithm behavior was described, however in this part one of the best possible results of convergence will be discussed in order to show the power of the algorithm and especially their application . As the first target with two objects, results come below which their locations and shape is detected as in figures (4.2.2) an (4.2.4).

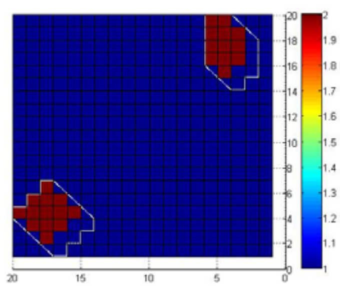


Figure 4.2.1, reconstruction frequency is 5.0 GHz

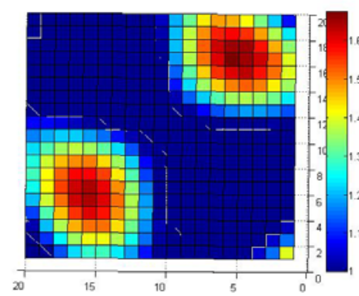


Figure 4.2.2, reconstruction frequency is 5.0 GHz

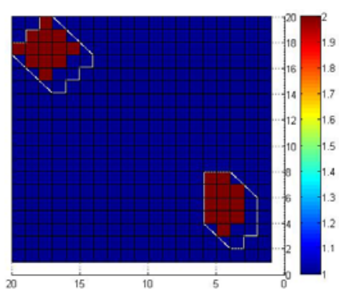


Figure 4.2.3, reconstruction frequency is 5.0 GHz

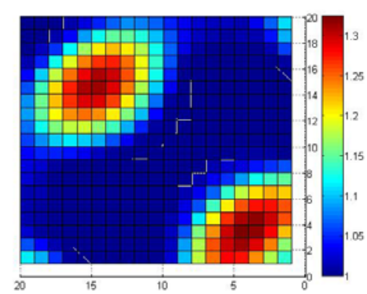


Figure 4.2.4, reconstruction frequency is 5.0 GHz

In above figures the shape and the locations detected properly, which resulted after many tests. The golden point for such a convergence is using, almost near full Jacobian matrix rank (the number of variables are near to be equal to the number of equations) where the number of equations are (20-1)*(20-1) (20 is the number of the antennas, minus one is because the sender antenna equations discarded) and the number of variable are 20*20 pixels.

Also in each inverse loop the regularization factor is reduced a bit in order to decrease the low pass filtering (its task is to make the solution stable). At last it is practical to determine the reconstruction frequency for this part, that is 5.0 GHz and both images resulted after 4 iterations. Moreover it is essential to try the algorithm for more complex targets. The results below are gained with the setting as reconstruction frequency equal to 4.0GHz, and the reason to repeating the experiment is to prepare the discussion for the breast model which is the ultimate goal.

It is well known that the higher the reconstruction frequency the better resolution. On the other hand higher frequency might cause artifacts due to increasing the nonlinearities in the reconstruction procedure. These are located with higher proportion in higher frequencies. In other words scattering is correlated with nonlinearities, i.e. when multiple objects are present, high dielectrics values and high reconstruction frequency will escalate the nonlinear behavior. Another example with different object is coming below, the results are gained by reconstruction in 4.0GHz , in figure (4.2.6) with an acceptable detection (the screen size set 16*16 showing the algorithm can be adjusted with different screen size).

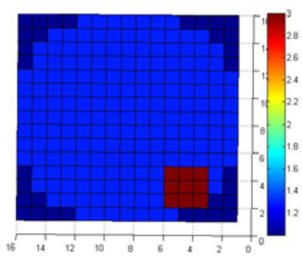


Figure 4.2.5, the object

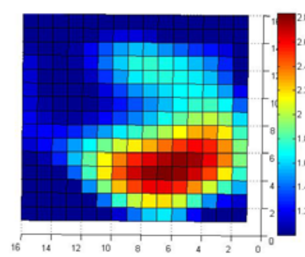


Figure 4.2.6 ,reconstruction with 4.0GHz frequency and smaller screen

Now it is time to increase the complexity of the object a little harder to be imaged , which is the breast shape object with tumor (4.2.7) and its imaging result in (4.2.8) with reconstruction frequency equal to 1.4 GHz .The relative error related to figure (4.2.8) is 0.558 , and the importance of this result is , its multi frequency reconstruction will come in last part with lower relative error (with same Jacobian rank) .

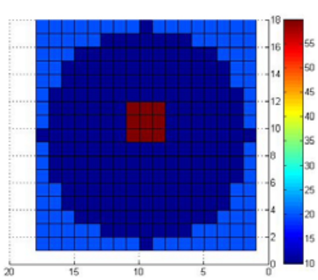


Figure 4.2.7 ,The object which

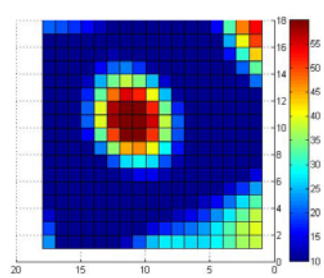


Figure 4.2.8 ,reconstruction with 1.4 GHz

As final words regarding convergence in this part, it should to noticed that the image can be improved or degraded by the different parameters settings, one of them is the regularization parameter. Remembering the previous discussion this factor helps to stabilizing the solution, yet if it selected too high, the reconstruction will fail. It is because regularization factor role is performing the low frequency filtration on the image, thus if adjusted higher than required it will cause the blurring effect on the reconstructed object .One simple possible example of high regularization setting is coming in the figure 4.2.10.

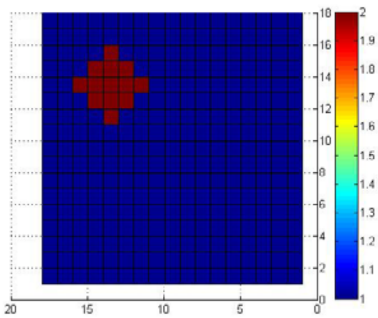


Figure 4.2.9, The object which posed in the

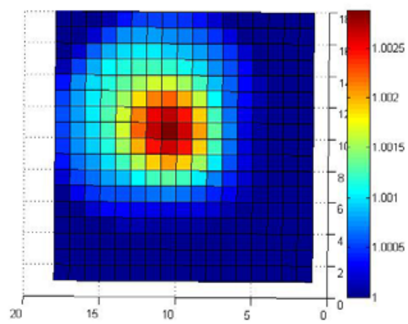


Figure 4.2.10, Reconstruction with very high regularization value

The regularization value is one of the decisive one that should be selected wisely and within many trial and error .In figure (4.2.9) the object is depicted while the regularization factor that selected much more higher than its proper value 4.2.10.

4.3. Time domain Jacobian

This part is based on the idea that instead of using constructed Jacobian matrix with one or multi frequency, we instead try to calculate the Jacobian matrix in time domain. One condition which let this be implemented is, when the transmitter sends only single frequency (not a pulse kind signal). Also based on the fact that, in this part just one frequency is used in constructing the Jacobian matrix in forward and inverse method, there is no need to use FFT function to make the Jacobian elements.

The mathematics behind is described in detail next. Assume that an antenna transmits a wave similar to $A\sin(\omega t + \emptyset)$, then by a time shift that cause $\pi/2$ phase differences, two independent equations will be generated. These can be solved in order to get the values of A (the wave amplitude) and $\emptyset$ (the wave phase) .

In figures (4.3.1) and (4.3.2) a left position object is tested in 5 loops and the corresponding error for each iteration is depicted in figure (4.3.3).

Important conclusion from the time domain Jacobian matrix calculation is, the object and its location has been found using fewer setup parameters, which is based on a strong reason that, by utilizing the time domain Jacobian there is no need to find the reconstruction frequency as the center frequency is the only frequency that has been utilized to propagate in the field. Also discarding the FFT function cause lower error in calculation regarding the window effect.

Another critical issue is that the calculation should be occurred after a transition time of the received wave, thus the first 600 (from 1072) time samples have been discarded. In other words when the wave transmitted from one antenna it needs to pass a temporary time to get to its stable situation on the object medium and either on the receivers, because of this the simulation of the wave in the transition time should be discarded.

Moreover about the results, the differences of measured and simulated signal are calculated by using the complex number of sinusoidal function with $\frac{\pi}{2}$ shift like in Jacobian part.

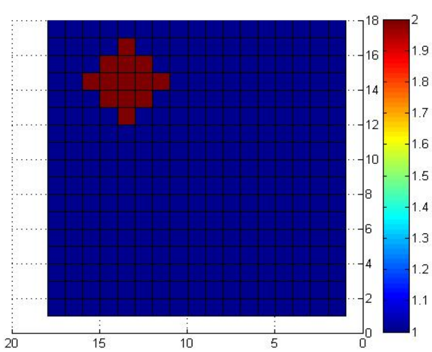


Figure 4.3.1 , Object position for following reconstruction

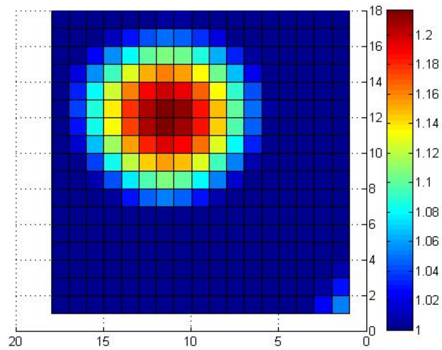


Figure 4.3.2, Reconstruction
reconstruction to time domain jcobian.

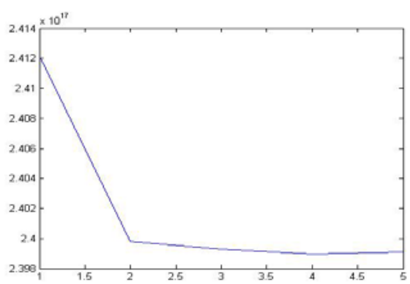


Figure 4.3.3, related error within iterations
related to figure 4.15

It is important to show the effect of phase shifting by a values around the $\frac{\pi}{2}$ (for example $\frac{\pi}{2}+\frac{\pi}{12}$), the result will be changed regarding this inaccuracy. Assume the reconstruction of (4.3.4) with $\emptyset=\frac{\pi}{2}+\frac{\pi}{12}$, the result of this reconstruction is depicted in (4.3.5). It is obvious that the object shape distorted (twisted to its corners), as the phase shift is not equal to $\frac{\pi}{2}$. In other words the reconstruction is not successful, because the sinusoidal functions used to build the Jacobian are not independent. On the other hand due to the digital computation and discerete calculation this inaccuracy in phase (not to be perfectly independent) always happened related to different settings.

.

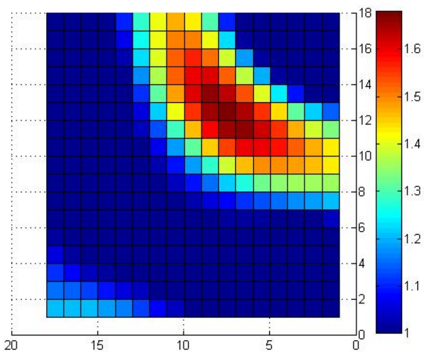
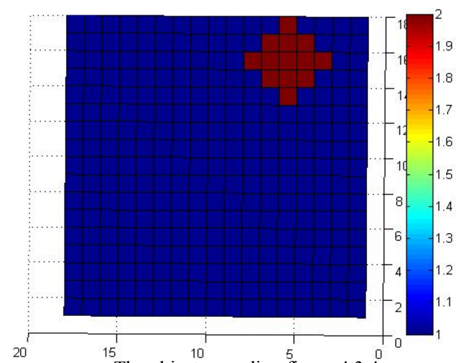


Figure 4.3.5, The reconstruction regarding object in figure 4.3.4 ,
the time domain jacobian elements $\frac{\pi}{2}$ phase difference did not satisfied .



The object regarding figure 4.3.4

Figure (4.3.6) and (4.3.7) are the objects in 20*20 screen size, in different positions with Jacobian calculated in time domain .It is clear that the wider screen resulted in the same manner like above.

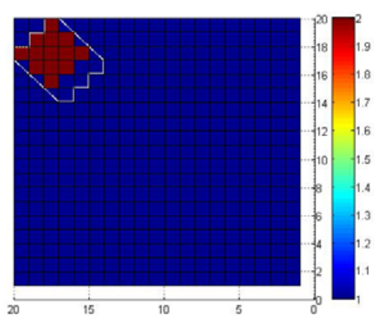


Figure 4.3.6 , the object
which posed in the left in the 20*20 screen

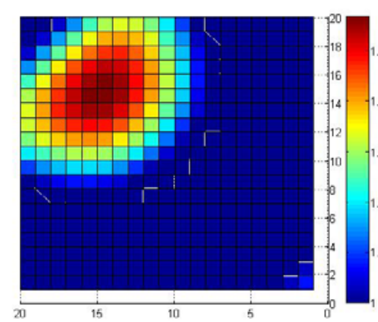
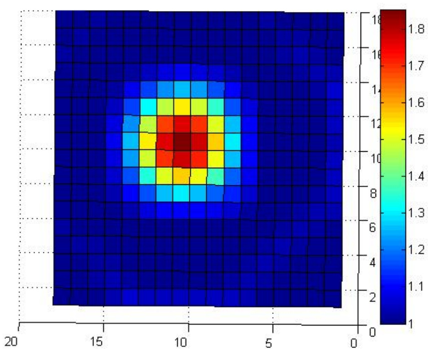


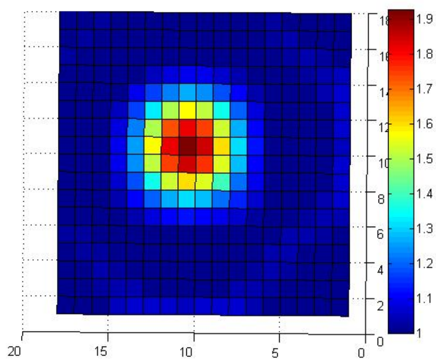
Figure 4.3.7 , Reconstruction from the object in the figure 4.3.6, with time domain jacobian

4.4 Multi frequencies aspect

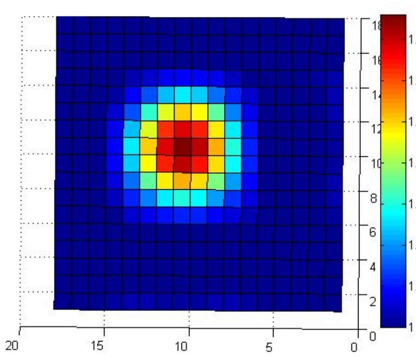
In this section the multi frequencies are used to make the Jacobian matrix, the related values and respected plots are coming below with the center object like in figure (4.10). In other words it is possible to increase the Jacobian matrix row's size (the column are equal to epsilon values) by adding the new elements calculated with different frequencies. This action cause the equations quantity increased in comparison with the variables which means improving the ill-posed situation by adding more information to the matrix solution system in order to make it more stable in convergence. Also the frequencies are selected from the descrete frequency samples which can be called the frequency number according to the DFT (discrete Fourier transform) definition. It should be mentioned that the purpose of using the multi frequency is to increase the number of equations in comparison with the variables with one FDTD calculation (without adding more antennas).



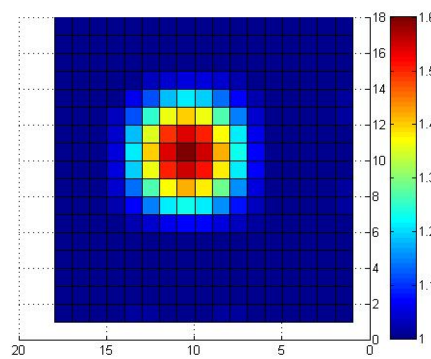
Figure(4.4.1) recontruction of figure 4.10 with frequencies 8.0 and 8.5 GHz



Figure(4.4.2) reconstruction of figure 4.10 with frequencies 8.0 and 8.3 GHz

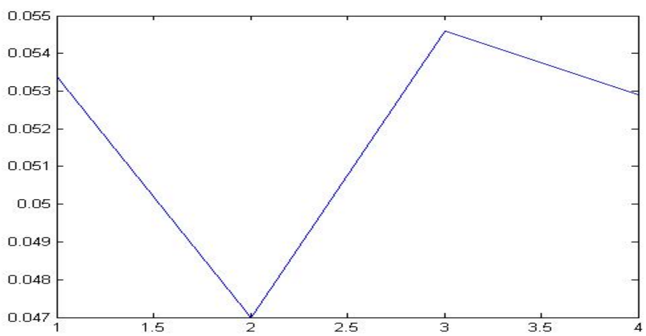


Figure(4.4.3) reconstruction of figure 4.10 with frequencies 8.0 and 9.0 GHz



Figure(4.4.4) reconstruction of figure 4.10 with frequencies 8.0 and 12.0 GHz

figure(4.4.1),frequency GHz=8.0 and 8.5 - relative error = 0.0470
figure(4.4.2), frequency GHz=8.0 and 8.3 - relative error = 0.0534
figure(4.4.3), frequency GHz=8.0 and 9.0 - relative error =0.0546
figure(4.4.4), frequency GHz=8.0 and 12.0 -relative error = 0.0529



Graph (4.G2) showing the multi frequency relative error in a graph and signifying that increasing the two frequency to make Jacobian matrix is very sensitive since the result seems not to be a linear behavior .
The x-direction is (x1=80,83), (x2=80,85), (x3=80,90) and (x4=80,120) .

As simple conclusion from this strategy is that by adding more equations formed by different frequencies, injects higher amount of information about the object into the solution in the inverse part, that not only lower the degree of freedom (the ill-posed situation is improved), but also the selected frequencies managing how much the reconstruction tends to the object details . Following examples clarifying the ideas regarding different objects and frequency range.

It is practical to describe the meaning of multi frequencies in relation with the center frequency of the simulation and how the sampling in time domain can change it .The description about it coming in the section **4.4.1 ,** also the deeper analysis of multi frequency idea is enlightened in section **4.5.**

4.4.1 Reconstructing frequency's relation with center frequency (frequency number in the array of the frequency coefficients)

Before any discussion the FFT function which has been used in this program should be defined especially it features in the application. The FFT function here is a dll file that is linked to the program and called everywhere that DFT is need. This FFTW package was developed at MIT by Matteo Frigo and Steven G. Johnson and according to them "FFTW is a C subroutine library for computing the discrete Fourier transform (DFT) in one or more dimensions, of arbitrary input size, and of both real and complex data". [32]

In order to describe the frequency number meaning, it is practical to assume a determined situation as: Simulation Time = 5.0e-9. Also according to the program simulation settings, there are 1071 time samples regarding this time frame, and then it can be written as:

Tsampling= 5.0e-9/1071 = 4.6685e-012

also $fs = \frac{1}{Tsampling}$ then fs = 2.1420e+011 ,

On the other hand the FFT DLL arranges the results collected (according to its documentation) from DC component to highest component from first to N/2, then as the input values are real the function copy them to the second side to make a symmetrical results . Now it is clear that reconstruction frequency is related to sampling frequency by Rf=f(n)*(fs/N),where f(n) means nth sample in the DFT . Also regarding this the maximum amplitude dedicated where the reconstruction frequency is equal to the center frequency .For example if the simulation time be equal to 1.0e-8 and 1071 samples are taken in this frame then, fs=1.0710e+011and then grounding to above equations the 80^{th} frequency number (sample) means 8.0e9 GHz, while highest amplitude is gained if the center frequency set as 8.0e9 GHz.

4.5 Analyzing the Multi frequencies behavior

Utilizing FDTD has one important benefit that, it makes it possible to increase the solution rank by mutli frequency information. In other words the number of variables (pixels) are much higher than the number of equations in the first step (in this project for example the image screen is 20*20 and when there are ten antennas the number of equation become equal to 10*10), however since the FDTD is a time domain method then we can transfer it to frequency domain which provide a Jacobian matrix in each frequency sample. These equations are independent (as they are constructed in different frequency) it is possible to use them all in one Jacobian matrix till the number of equation and variables become almost equal. It is practical to say that an over determined system in this solution (the number of equations become greater than variables) is not providing the lowest relative error because of the nonlinearities and discretization error is involved into information.

Following images are presented to show how different selection of the reconstruction frequencies (or how the combination of reconstruction frequencies) effecting the results especially in primate iteration loops. In other words in this part the behavior of multi frequency aspect is analyzed, therefore instead of full convergence the results selected in their early iterations of convergence (mostly the first four or five iterations), that helps the differences (of the variation in selected frequencies) be presented with higher contrast in comparison with each other. Regarding the first example in figure 4.5.1, and its reconstruction settings are coming in table 4.T1.

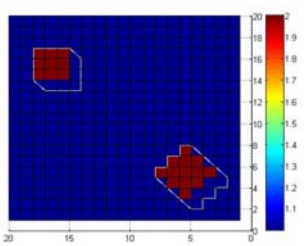


Figure 4.5.1, The objects

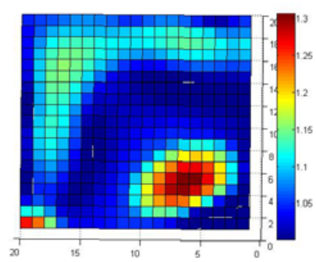


Figure 4.5.2, reconstruction with lower jacobian rank

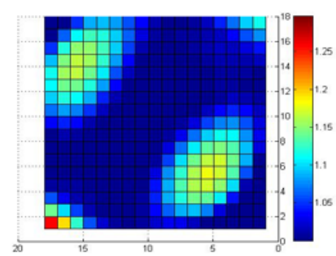


Figure 4.5.3, reconstruction with higher jacobian rank

Figure	4.5.2	4.5.3
Multi frequency GHz	6.5 5.1 5.6	6.5 5.1 5.6

4.T1, The table describing the reconstruction frequency regarding images 4.5.2,and 4.5.3 .

One conclusion is, that the image 4.5.2, reconstructed with the same settings as in 4.5.3, however just the number of variables are different in two images for reconstruction. Thus the results showing how much higher Jacobian rank (ration of the equation quantity to the variables) is vital for better convergence.

Following images are reconstructing a harder object, because the objects have, not only different sizes, but also there are three objects posed here (just 4 loop of iterations is performed here, in order to show how reconstruction frequencies behave in first sequences).

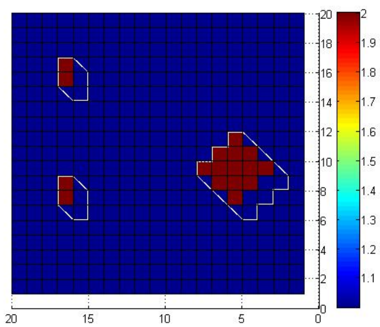


Figure 4.5.4, the object

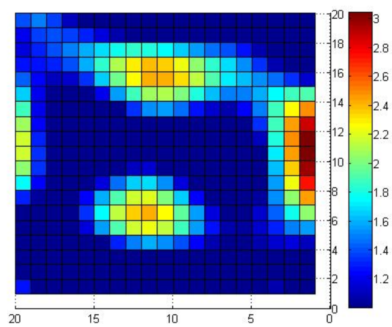


Figure4.5.5,reconstruction with three frequency {6.0,6.5,5.5}

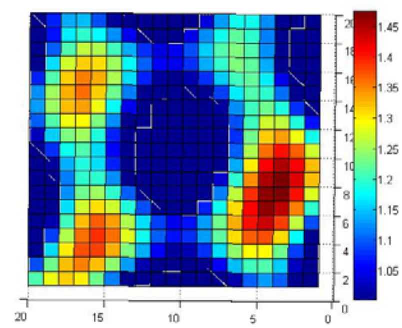


Figure 4.5.6, reconstruction with three frequency {5.1,5.3,5.5}

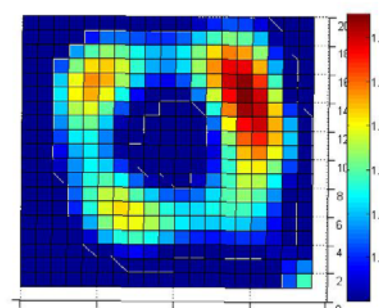


Figure 4.5.7, reconstruction with three frequencies {6.5, 6.0, 5.0}

figure	4.5.4	4.5.5	4.5.6	4.5.7	
Multi frequency GHz	object	6.0 6.5 5.5	5.1 5.3 5.5	5.0 6.0 6.5	

Table 4.T2, the reconstruction frequencies for above images with three objects.

In order to show how the system responding to detect the shape of an object a new object in triangular form used in figure 4.5.8 and the related table 4.5.T3.

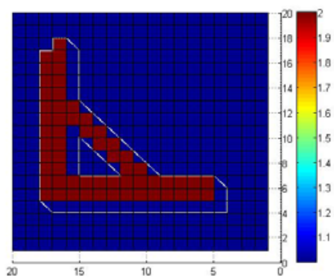


Figure 4.5.8, The object

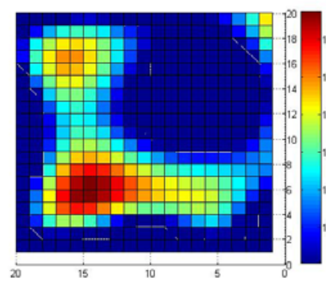


figure 4.5.9, reconstruction by three frequencies {2.5,3.0,2.0}

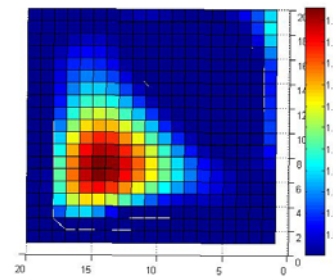


figure 4.5.10,reconstruction by three frequencies {2.0,3.0,2.6}

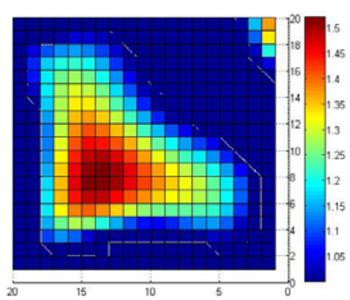


Figure4.5.11 ,reconstruction three frequencies {2.1,3.5,2.6}

figure	(4.5.9)	(4.5.10)	(4.5.11)
Multi frequency GHz	2.0 2.5 3.0	2.0 2.3 2.6	2.1 2.6 3.5
Center frequency GHz	2.0	2.0	2.0

Table 4.5.T3 , The reconstruction regarding above images

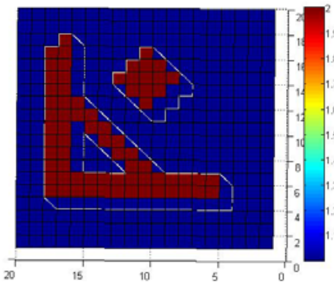


Figure 4.5.12, the object

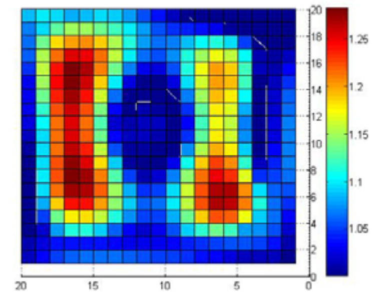
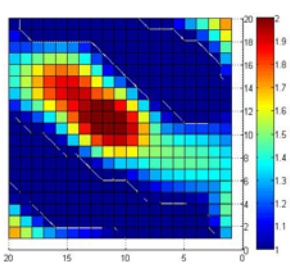


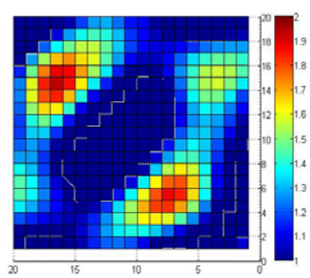
Figure 4.5.13,The reconstruction
by frequencies {2.0,2.5.3.0} GHz

The object in the figure 4.5.12, reconstructed in 4.5.13, by setting as multi frequencies equal to {2.0,2.5,3.0} in GHz . The problem is with a complicatd object ,finding a proper solution in reconstruction is much more harder .Especially the regularization factor setting ,in iterating loops is very critical for a stable solution ,on the other hand its lowpass filtering feature casuse low frequencies content increasing in the image .

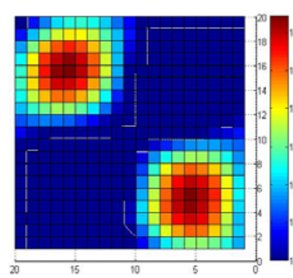
After surveying these discussions and examples it can be understandable that he multi frequency algorithm is vital because the added new information (new equations) by each reconstruction frequency in to the Jacobian, lead the ill-posed situation to become the well posed situation to some extent .One strong reason for this statement is in the above examples (with acceptable detection of the object which is the goal in diagnostic cases), the results are yielded within even few iterations with acceptable detection. Thus multiple frequencies not only vital in convergence but also reduce the computational time, because with higher proportion of information the convergence will be occurred in smaller quantity of inverse loops (as the degree of freedom (matrix rank) is lower, below figures try to show this idea in order to clarify the rank roles in reconstruction.



Reconstruction by 5.5 GHz
from the object in figure 4.6.2
equation quantity =100
The object is figure 4.6.2



Reconstruction by 6.5 GHz
from the object in figure 4.6.2
equation quantity =100
The object is figure 4.6.2



Reconstruction by {5.5,6.5} GHz
from the object in figure 4.6.2
equation quantity =200
The object is figure 4.6.2

To complete this idea another optimization technique will be introduced in the next part which helps the redundant frequencies be deleted from the inverse part. This discarding helps the multi frequency algorithm consume less elapsed time because the variables size will be reduced effectively .Section (4.6) is dedicated to this new optimization method.

4.6 Reconstruction based on non pixel based reduction

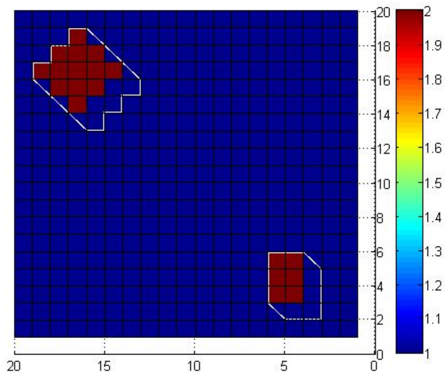


Figure 4.6.2 the object

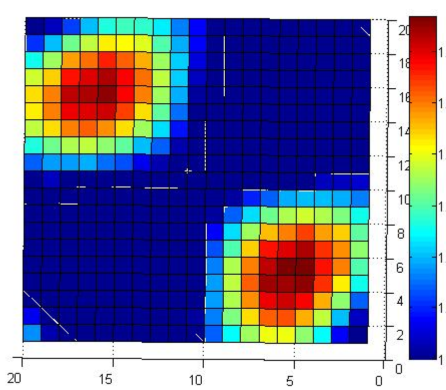


Figure 4.6.3 ,image
reduction quantity is 0

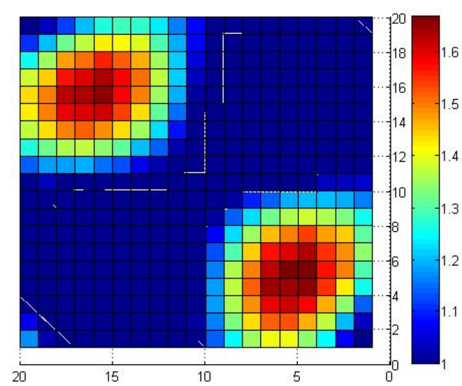


Figure4.6.4, image
reduction quantity is 40

The images are reported larger , in order to make the detail more clear by different reduction settings.

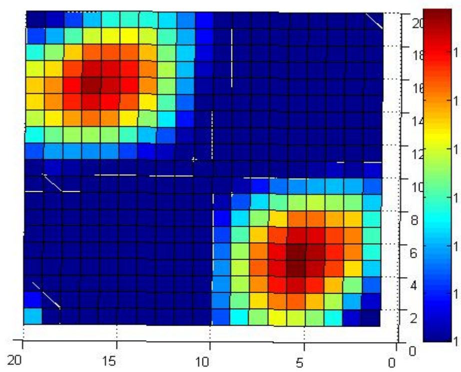


Figure 4.6.5 , image
reduction quantity is 120

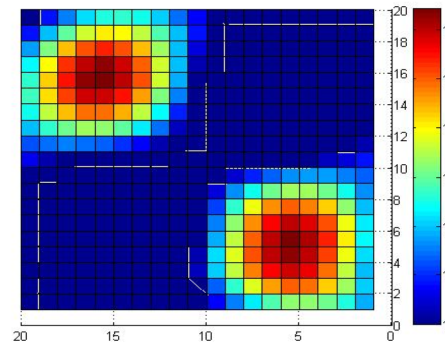


Figure 4.6.6, image
reduction quantity is 350

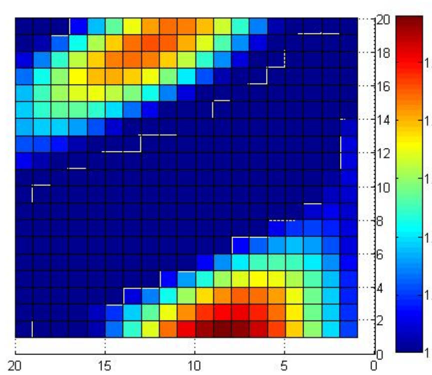


Figure 4.6.7, image
reduction quantity is 360

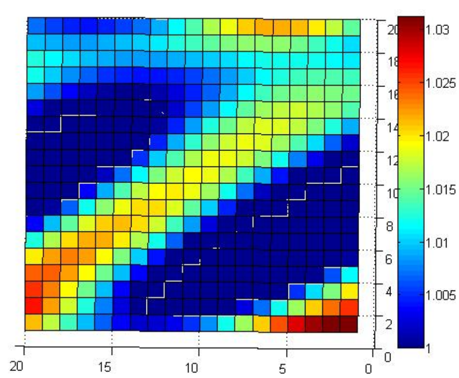


Figure 4.6.8, image
reduction quantity is 362

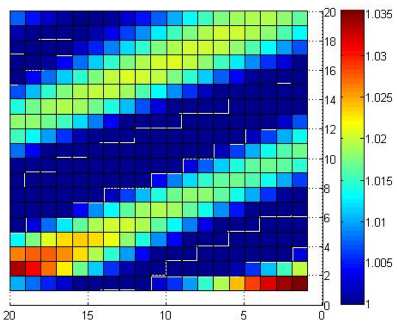


Figure 4.6.9, image
reduction quantity is 363

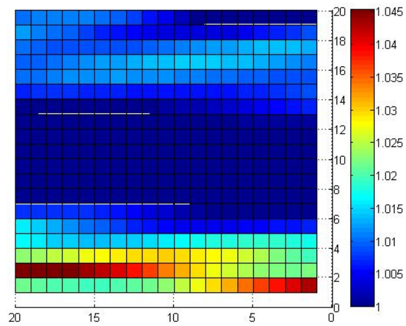


Figure 4.6.10, image
reduction quantity is 370

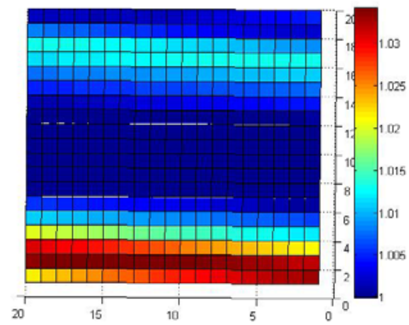


Figure 4.6.11, image
reduction quantity is 390

Figure	4.6.3	4.6.4	4.6.5	4.6.6	4.6.7	4.6.8	4.6.9	4.6.10	4.6.11
Discarded elements	0	40	120	350	360	362	363	370	390

Table 4.6 , The reduction quantity regarding each images

According to above table and the related images the reduction effect generates some error which is acceptable around 350 elements discarding. This means that , for an application with 400 variable and reduction of 350 the overall procedure is O(9.6619e+005) faster according to the algorithm changing operation cost equation ((N-M)^2.3) + 4*(N-M)*log(N-M) where N is the variables quantity and M is the reduced quantity .

It is important to give an time estimation about this reduction, at first experience assume for each iteration with no reduction in reconstruction (with 400 variables), and the second experiment, the settings change to again one loop iteration but 350 reduced elements. The time for first experiment is around two minute and seven seconds and for next setting it is one minute and thirteen second.

The golden point in using this method is, by solving the reconstruction in frequency domain and discarding the higher frequencies, one important feature also improved which is the matrix rank. In the other words, the variable quantity is reduced in this method, therefore the Jacobian matrix have same quantity of equations with fewer variables. Moreover as the discarded frequencies are at the higher frequency end where the nonlinear behavior is more pronounced, deleting or filtering these frequency components is very beneficial for the convergence.

4.7. Reconstruction based on non pixel based reduction, with more complex objects.

In this section we analyze this algorithm in to deeper level by comparing it with other similar methods and consider the behavior of convergence for more complex objects like when the target constructed from three separated parts which the size of one of them is much larger, the object is coming in figure (4.7.1).

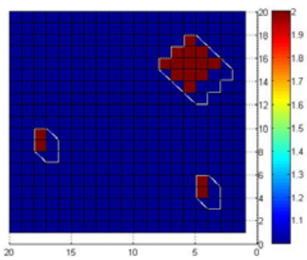


Figure 4.7.1, the three separated object

The results according to the object with selecting the proper parameters, is coming in next figures and the table (4.7.T).

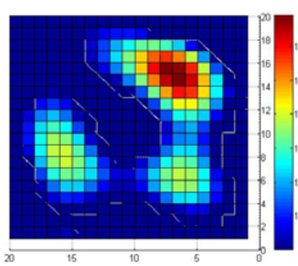
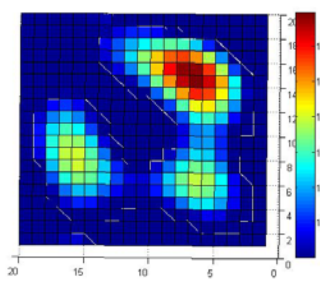


Figure (4.7.2),reduction is 0%



Figure(4.7.3) reduction is 25%

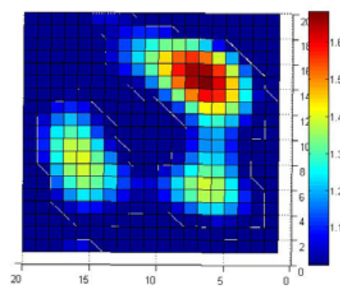


Figure4.7.4 reduction is 50%

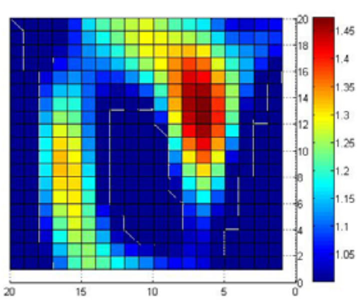


Figure4.7.5, reduction is 75%

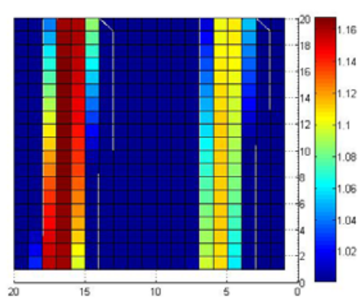


Figure 4.7.6, reduction 85%

figure	(4.7.2)	(4.7.3)	(4.7.4)	(4.7.5)	(4.7.6)
Multi frequencies	5.5 6.5 7.3	5.5 6.5 7.3	5.5 6.5 7.3	5.5 6.5 7.3	5.5 6.5 7.3
Discarded percentage	0	25%	50%	75%	85%
Relative error	0.078	0.076	0.081	-	-

Table 4.7.T, The relation of the images and their reduction.

One important conclusion about this experiment is that by discarding the 50% of the variables, not only the convergence is acceptable also the solution is twice faster. In other words the results gathered in table 4.7 suggest that 50% reduction results in a very efficient reconstruction with respect to time and image quality. Also the relative error is stable up to as much as 50% reduction in problem size.

It is interesting to compare this approach with similar methods which working more or less like the non pixel based method. One of the previously successful method is multi scale, that use the low frequency content of the reconstructed image (like the non pixel based method), yet it is working on the spatial domain. In multi scale method the resolution of the convergence is changing during the inverse part, in other words the sampling scale or (sampling rate) is just changed as the scaling is in spatial domain. In order to describe how the sampling rate effects the solution assume one dimensional signals below in frequency domain.

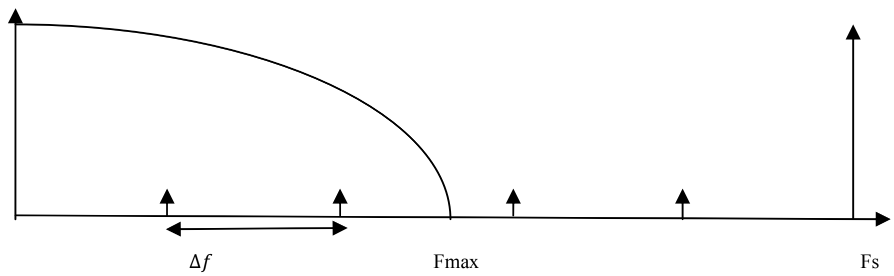


Figure 4.7.7, the sampling in frequency domain regarding the sampling frequency and the object maximum included frequency.

The Δf depends on the sampling rate, as Fs/N=Δf and Fs=1/Ts where Ts is the sampling rate in time domain. However if the signal in time domain contains a larger proportion of the low frequencies (in imaging it occur as the objects usually have very small amount of high frequency content which is visible to the human eye). In an image it is possible to find an upper frequency, Fmax, where the detailes in higher frequency content is not seen and could be discarded. In the multi scale method by changing the sampling rate the Δf will be changed correspondingly, which can reduce the frequency samples after Fmax (but never discard all of them). On the other hand in multi scale method the reduction quantity is not flexible, for example if each two pixels in X direction merged together (minimum reduction), it cause 50% reduction and if each two pixels in X and Y directions merges together, then there is 75% reduction. [21][30]

Nevertheless in non pixel base method

1) The reduction rate is flexible for example it is possible to remove about 20% of the higher frequencies.

2) All of the higher frequencies will be discarded after Fmax , which is more optimum in comparison with multi scale method, that always some higher frequencies will remain according to above figure.

There is also some drawbacks in non pixels based method which are:

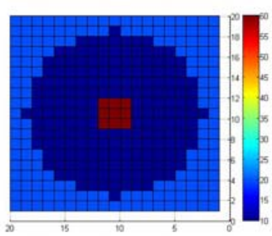
1) Two more FFt function cost related to the variables quantity should be added (which are ignorable in comparison with inverse part cost , FFT -->N*log(N) inverse part ->(N^3)

2) For small size images the widow effect, cause some small artifact.

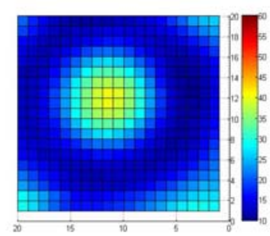
3) If the object does not contain a large portion of low frequencies. In other words the Fmax's posed near to Fs then the multi scale method is working the same as non pixel base method (no priority in discarding).

4.8 Different situation of convergence on the breast model

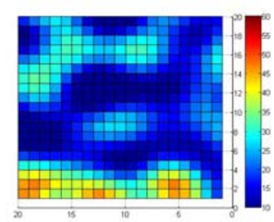
Reconstructing the breast with tumor is one of the important goals of this project, especially utilizing the two different algorithms; the non pixel based and the multiple frequency method. We analyze this optimization problem using the relative error for the reconstruction as a measure of imaging quality. This part is focusing on the breast model and the last part (4.8.2) showing the results based on equation (3.2.12). In this equation the Newton step is calculated in each iteration of the reconstruction procedure using the error signal in each loop as feedback.



Figure(4.8) the breast model object



Reconstruction with (1.2,1.4) GHz
Relative Error 0.241



Reconstruction with (2.2.2.4)
Higher reconstruction frequency is not proper due to high amount of scattering of the breast model.

4.8.1. Convergence to the breast model.

Finding the breast tumor is a hard procedure because the dielectric contrasts are high, therefore the nonlinear effects will be large. In order to solve the matrix equations for the solution properly it is very practical to use some prior information like the dielectric properties of the air and the breast which are the same in every experiment. Following results are yield with this idea.

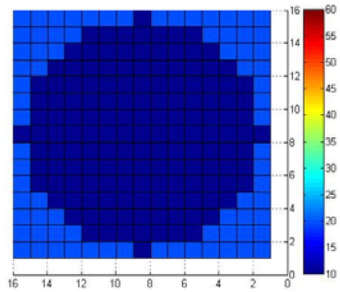
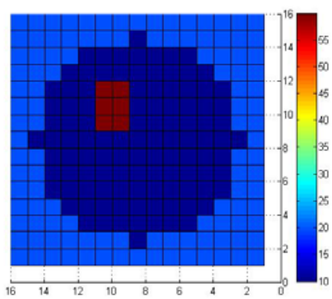
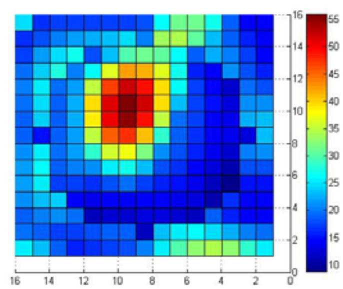


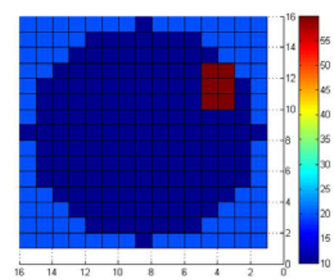
Figure (4.8.1.0) the prior information



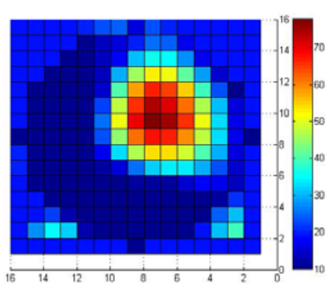
figure(4.8.1.1),the object



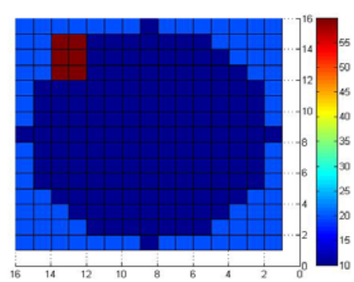
figure(4.8.1.2),reconstruction frequencies {1.6,1.2}



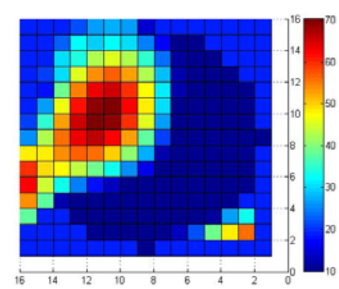
figure(4.8.1.3),the obejct



figure(4.8.1.4) reconstruction frequencies {1.7,2.0}



figure(4.8.1.5) the object



figure(4.8.1.6) reconstruction frequencies {1.8,2.2}

The simple conclusion is the tumor location and dielectric values detected acceptably, especially in comparison with the next part which no prior information in used. In other words with no prior information the detection is much more harder and needs more precise selection of the parameters , yet the reconstructed tumor in the next part is blurred even with the best selection after many trials .

Reconstruction related to a center position tumor in the breast model, but without the prior information . The background and the breast dielectric is reconstructed as well as the tumor, in the figure 4.8.1.8 which its error graph depicted in figure 4.8.1.9 with relative error equal to 0.204 .

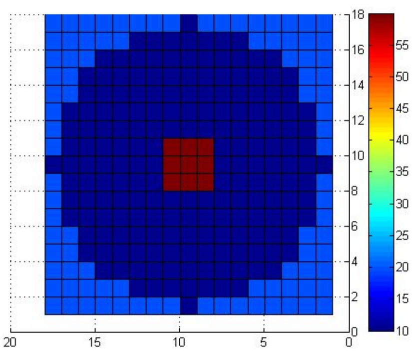


Figure 4.8.1.7, The breast model

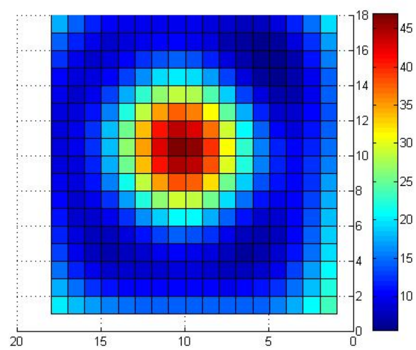


Figure 4.8.1.8 , reconstruction with 1.2 and 1.4 frequencies.

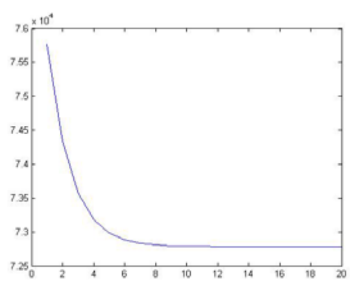


Figure 4.8.1.9, the error graph within twenty iteration (the horizontal direction showing each iterated loop)

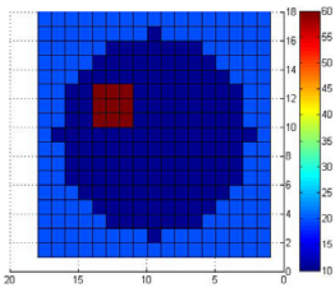
figure	4.8.1.2	(4.8.1.3)	(4.8.1.4)	(4.8.1.6)	(4.8.1.8)
Reconstruction frequencies GHz	1.6 and 2.0	object	1.7 and 2.0	1.8 and 2.2	1.2 and 1.4

Table 4.8.1 the reconstruction description according the above images

It is possible to conclude that in Figure 4.8.1.8 the reconstruction is successful as the object and background is clearly detectable ,also figure 4.8.1.9 is the absolute error regarding each iteration ,that after the 12^{th} loop remains the same as the reconstruction resulted image converged to the object .

4.8.1.2 The breast model reconstruction with non pixel based idea

In this part the non pixel based method with multi frequency idea are added together, which calculates their regarded relative errors to demonstrate a deeper level of its capability.



The object

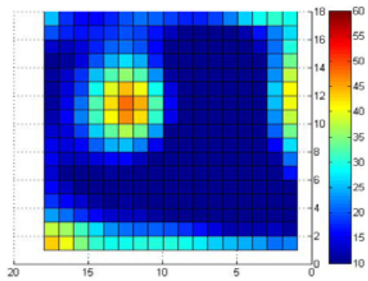


Fig4.8.1.2.1 reconstruction with 0% variables discarding

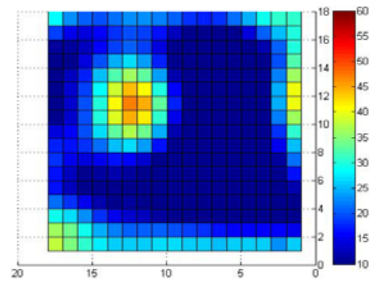


Fig4.8.1.2.2 reconstruction with 33% variables discarding

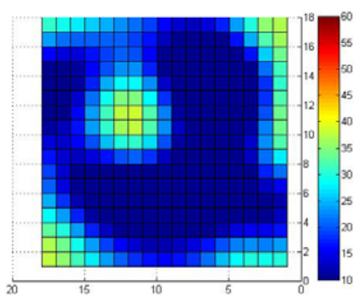


Fig4.8.1.2.3 reconstruction with 61% variables discarding

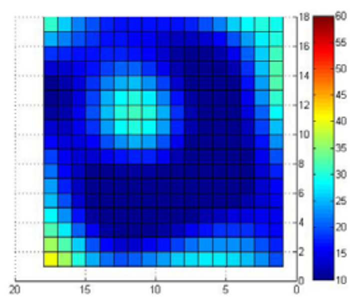


Fig4.8.1.2.4 reconstruction with 77% variables discarding

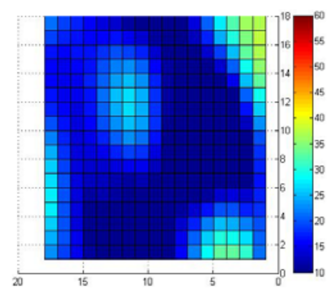


Fig4.8.1.2.5 reconstruction with 84% variables discarding

Fig. number	4.8.1.2.1	4.8.1.2.2	4.8.1.2.3	4.8.1.2.4	4.8.1.2.5
Reconstruction frequencies GHz	1.0 1.6	1.0 1.6	1.0 1.6	1.0 1.6	1.0 1.6
Discarding variables quantity	0 (0%)	100 (33%)	200 (61%)	250 (77%)	270 (84%)
Relative error	0.339	0.336	0.334	0.341	0.382

As a conclusion the non pixel based method show very acceptable results in convergence, especially it helps to reduce the relative error for images (4.8.1.2.2)and (4.8.1.2.3) because of lower quantity of variables and discarding higher frequencies, that means decreasing some of the nonlinear information . Also the numbers of iterations are the same for all images from, 4.8.1.2.1 to 4.8.1.2.5, in order to better understanding the role of discarding effects.

4.8.2. Controlling the convergence steps with error signal feedback in each iteration

Convergence analysis with error steps in each iteration is helpful to control the $\Delta\varepsilon$ values in each iteration .There are examples to describe this role according to the figure(4.8.2.2) and its related absolute error graph(4.8.2.4),also the figure (4.8.2.7) and (4.8.2.8) that is a harder type of object to be reconstructed, yet due to the fact that the equation (3.2.12) defining a manageable Newton stepping with the derivational feedback (depicted in figure 4.8.2.5) which help the convergence not too be diverged due to too high updating values , the procedure lead to better quality of reconstruction.

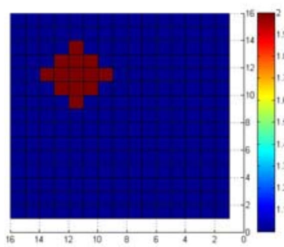


Figure 4.8.2.1,the object

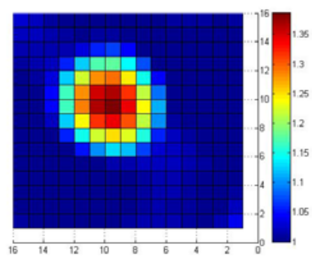


Figure 4.8.2.2, Reconstruction after ten iteration

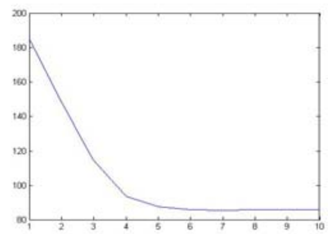


Figure 4.8.2.4 , The absolute error signal

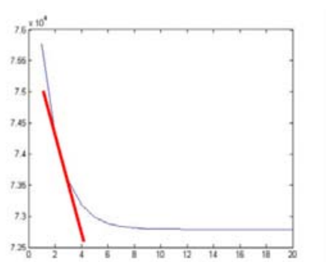


Figure 4.8.2.5, the curve derivation specification of the error signal , to make a feedback to manage the Newton steps

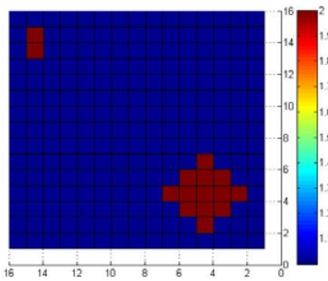


figure 4.8.2.6,the object
with two object with different size

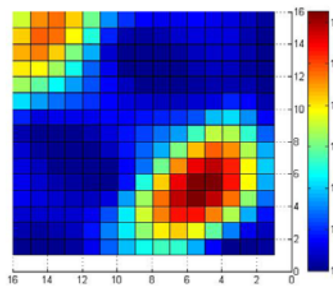


figure 4.8.2.7 ,reconstruction
with two different sized object

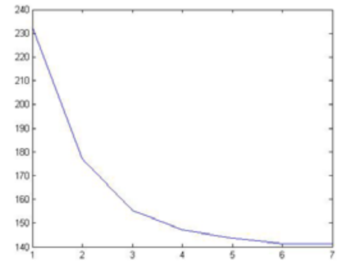


figure 4.8.2.8 ,absolute error
signal

On the other hand using several loops of inversion and how the error is manageable within this quantity of loops is considered in figures (4.8.2.9) , (4.8.2.10) and (4.8.2.11) .

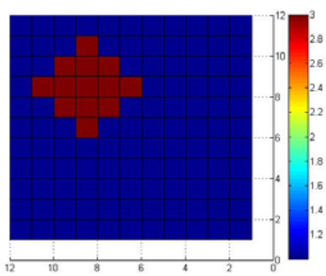


figure 4.8.2.9 ,the object

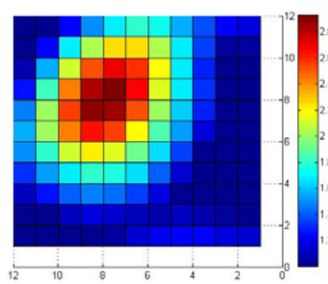


figure 4.8.2.10 , left position
related to 4.8.2.18

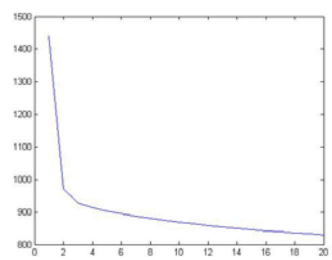


figure 4.8.2.11,left position
absolute Error graph

In the above examples the equation (3.2.12) is a helpful strategy to reduce the Newton steps meaningfully regarding subsequent iterations.

In the figure (4.8.2.13) the breast model in respect to the tumor position is figure (4.8.2.12) is reconstructed, which utilized this method to manage the Newton steps. The reconstruction frequencies are 1.0 and 1.6 GHz that generated the relative error equal to 0.426 for (4.8.2.13) and 0.431 regarding (4.8.2.14) with 77 % discarding of higher frequencies .

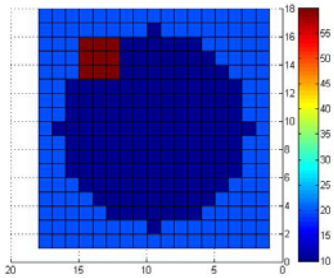


Figure 4.8.2.12
The tumor in red and breast model

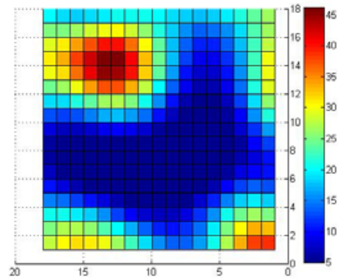


Figure 4.8.2.13
reconstruction with 1.0 and 1.6 GHz

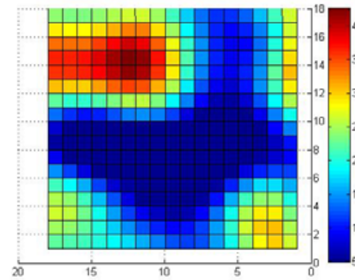


Figure 4.8.2.14
reconstruction with 1.0 and 1.6 GHz , also 250 of higher frequencies variables are discarded

This reconstruction according to 4.8.2.13 is acceptable with lower relative error in comparison with previous part (which means, using the equation (3.2.5) is feasible), thus managing the Newton steps with this adaptive consideration promoting the algorithm.

4.8.3. The MRI breast model reconstruction

In this part two types of the breast models are utilized for the reconstruction, and their properties are described in detail. ''Numerical Breast Phantom Repository contains a number of anatomically-realistic MRI-derived numerical breast phantoms for breast cancer detection and treatment applications. The breast tissues in these phantoms have the realistic ultrawideband dielectric properties''.[35]

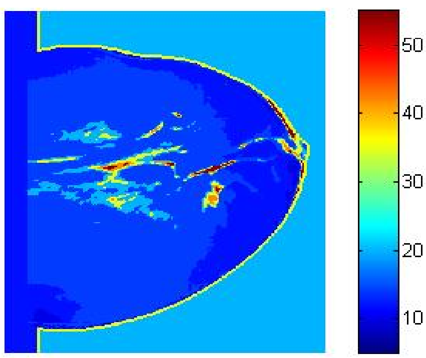


Figure (4.8.3.1)

The first breast model

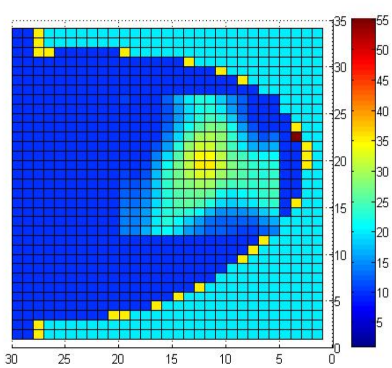


Figure (4.8.3.2)
Reconstruction frequencies are 1.6 and 2.0 GHz .
The relative Error is 0.52

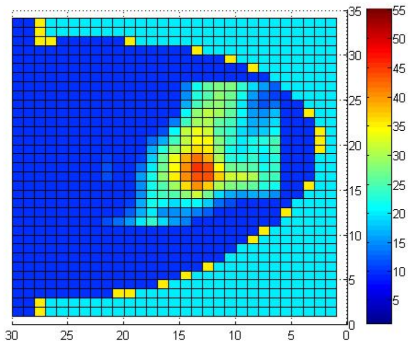


Figure (4.8.3.3)
Reconstruction frequencies are1.4 and 1.8 GHz
The relative Error is 0.58

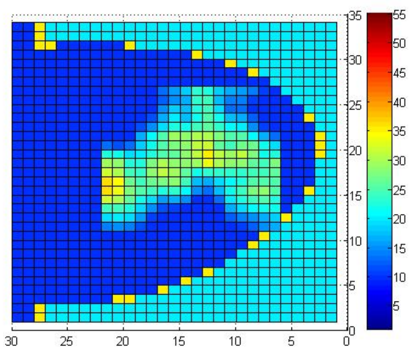


Figure (4.8.3.4)
Reconstruction by 1.6 and 2.2 GHz
The relative error is 0.466

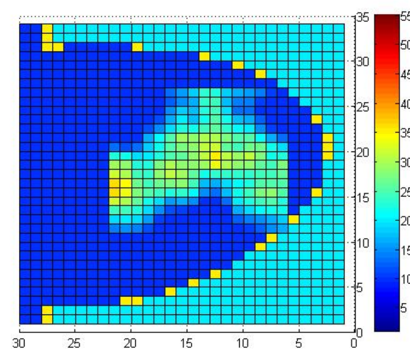


Figure (4.8.3.5)
Reconstruction by 1.6 and 2.2 GHz, with 25% discarding of the variables quantity . The relative error is 0.43

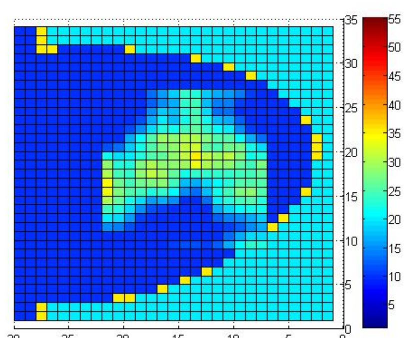


Figure (4.8.3.5)
Reconstruction by 1.6 and 2.2 GHz,

with 50% discarding of the variables.
The relative error is 0.47

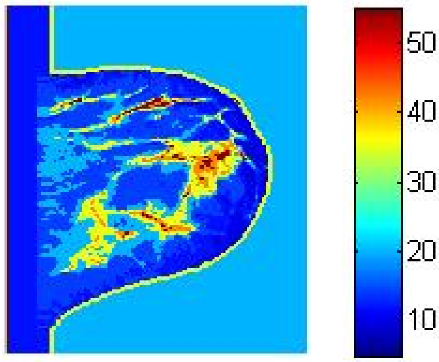


Figure (4.8.3.6)

The breast model

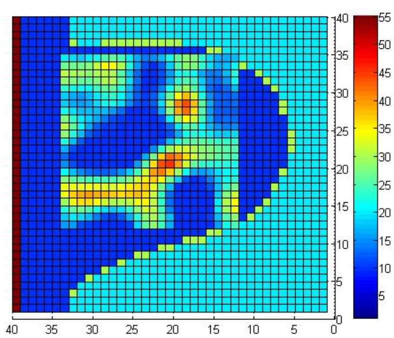


Figure (4.8.3.7)
Reconstruction frequencies
1.8 and 1.2 GHz.
Relative error 0.46

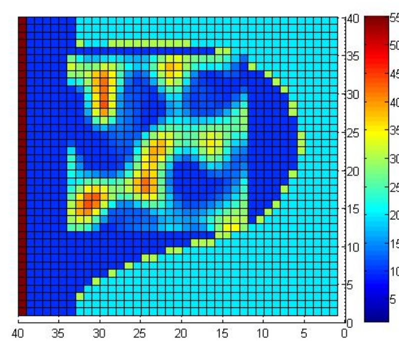


Figure (4.8.3.8)
Reconstruction frequencies
1.6 and 1.8 GHz
Relative error 0.62

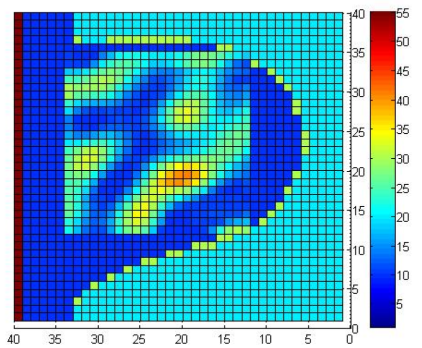


Figure (4.8.3.9)
Reconstruction frequencies
1.0 and 1.6 GHz
Relative error = 0.53

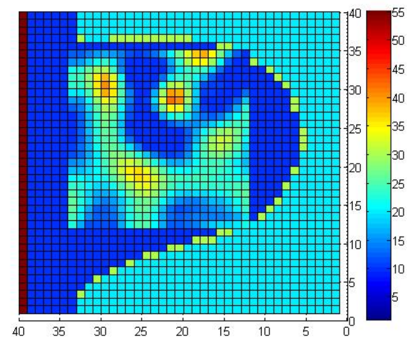


Figure (4.8.3.10)
Reconstruction frequencies
1.8 and 1.4 GHz
Relative error = 0.39

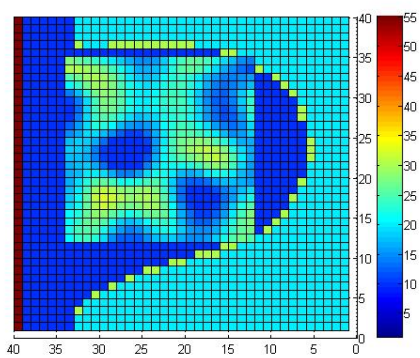


Figure (4.8.3.11)
Reconstruction frequencies
1.8 and 1.4 GHz , reduction is 50%
Relative error = 0.41

The reconstruction is successful about both models, especially the shape and locations of the tumors are detected with acceptable relative error. Also comparing figures (4.8.3.1) to (4.8.3.11), is showing that, how the selection of the reconstruction frequencies is characterizing the detection of the details.

Conclusion

In this project microwave imaging was tested and evaluated in many different situations, using a variety of test objects, different number of unknown and equations, a range of reconstruction frequency and time optimized algorithm, which according to the results the following conclusion has been proposed.

Regarding the results from this project the number of antennas have been selected between10 to 20 because the screen grids are adjusted between 14*14 up to 20*20. This selection was made because with less than 10 antennas the Jacobian becomes very small and we end up with a low rank problem. With more than 20 antennas the time consumption of the reconstruction procedure is very large. With the number of equations around 70% of the unknown variables the reconstruction is on its best optimized state regarding the computational time and relative error. Moreover the frequency range selected from 1.0GHz to 9.0GHz is not only proper for medical objects like body tissue, but also it has been shown that higher frequencies cause more sensitive results according to variation of the dialectic in the area of the object, which increase the scattering and the nonlinear behavior of the propagated wave. Also in order to increase the equation quantity, using multi frequency and increasing the antennas has been used. However the multi frequency idea shown much more beneficial because of its wide range of different frequencies that can be selected just with one numerical filed computation by FDTD method.

The reconstruction algorithm is constructed by iterative Newton method in order to find the permittivity values. On the other hand the Jacobian requires (for the inverse solution) to find the electric field in different locations of the imaging area, which obtained by utilizing the FDTD method, and it is important to know that the FDTD was used in the simulation part in this project too. However the Jacobean can be calculated by different ways like in frequency domain by using FFT function or by $\frac{\pi}{2}$ phase shifting. Different types of Jacobians have been implemented and a Jacobian combined with multi frequency concluded more beneficial because of its flexibility of choosing a variety of equation quantity with different reconstruction frequencies.

One essential problem was the Jacobian matrix rank, since the quantity of the equations to be solved is often smaller than the number of the pixels of the images (the variables). This tangle is tackled by using reciprocal feature of microwave modeling by Maxwell and self-adjoint response, which let respecting every receiver as a source in one iteration. This technique increase the number of equation by N^2 if N equal to the number of the antennas. Nevertheless calculating the Jacobian matrix in different frequencies (what is allowed by FDTD regarding Nyquist criteria) let the number of equation increased up to N^2, thus the rank of the Jacobian can be increased to be full .

In order to describe the performance of it, different kind of objects in variety of locations was tested with the successful results. The simplest object is a circular one in the center of the area, then in next step its location was changed. It is practical to know that the imaging algorithm detect one object in every position with acceptable relative error ,therefore single object is the easiest thing to be detected properly .

The convergence becomes harder when more than a single object placed in imaging area. Especially the two objects inside each other is very important model because it can be regarded as a breast cancer model in medical diagnostic cases. The results of these objects is more complicated as the scattered wave can encounter with other object (when multiple objects used) and then again become scattered. This problem increases the nonlinear behavior of the system which destroy the quality or convergence of the image . This nonlinear behavior can be handled according to the results in this project by using higher Jacobian matrix rank (even full rank). In other words using multiple frequencies let increasing the number of equation near to number of the variables (full rank) then the problems tend to change from ill-pose to a well pose problem . By these techniques imaging from more than single object will be resulted with acceptable relative error and convergence.

One important behavior of the convergence is, the regularization factor should be change (decreased) in each inverse iteration, because in each subsequent inverse loop in reconstruction the correlation of the reconstructed epsilon values with object epsilon values will be higher (convergence cause less error) , thus the permittivity values are more converging in the next inverse iteration and this cause, escalating the stability of the solution which lead to reduce the regularization factor. In other words regularization factor do the low filtering task on the image, thus should be lowered in each subsequent inverse iterations in order to improve the convergence, especially for complex objects. Also selecting the reconstruction frequencies is very critical as the higher frequency cause some instability in the solution yet bring more details if selected propely. On the other hand lower reconstruction frequency shows the resulted image with higher low frequency content (that discards the details), for example it might cause two separated objects to stick together.

Moreover utilizing the multi frequency in order to improve the matrix equation rank is one of the most improving technique which based on the results in this report, its vital application is proven. Again it is good to mention that by using each new frequency one set of N^2equation added to the Jacobian matrix, and the selection of the frequencies should not be very near because it tend the solutions of the matrix to a singularity causes by lower (or similar)quantity of equation in comparison with variables .

The final part in conclusion is about the novel technique which can regard as state of art in this project, which is reconstruction in the frequency domain of the epsilon values. That means instead of generating the image in time domain, try to solve the Jacobian of the image spectrum. This method let the higher frequency of epsilon values be discarded (as medical image have higher information in their low frequency coefficients) .In other words the number of variable will be reduced while the image after implying inverse FFt function will be presented even in better quality . The better quality is because lower variables quantities have been used which let the solution implemented in much more stable situation.

This method converged to the acceptable result much faster and with lower memory consumption. Especially the non-pixel base method has been compared with multi scale method and as results the non pixel based method shown more flexible and optimized regarding this similar method. From the imaging cases we have tried we conclude that around 60% reduction of the variables is the proper choice related to diagnostic application.

The breast model is the most complicated object which has been tested with and without prior information in the reconstruction procedure for improving the convergence. The results show that the prior information (the breast location and the background) allows for easier convergence. On the other hand solving the breast model without prior information yield acceptable results too, yet the tumor location and shape detected is not consisted with same determination quality .More over it is important to be mentioned that regarding high dielectric values of the breast model the reconstruction frequencies selected in the range of 1.0 GHz to 2.0 GHz otherwise the scattering (non linear behavior)is too high for a stable trajectory of solution within iterations, and it is not possible to selected lower than 1.0GHz because the interaction of propagated wave (lower from 1.0 GHz) with body tissue are less than what is needed for imaging procedure .

As last sentences it is feasible to summarize the mathematical procedure that, microwave imaging technique is like a system with a nonlinear differential equation which with some proper parameter selection converged to the acceptable results, or the convergence is an equilibrium that by changing the reconstruction frequency the pass of convergence is changed to reach the answer.

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