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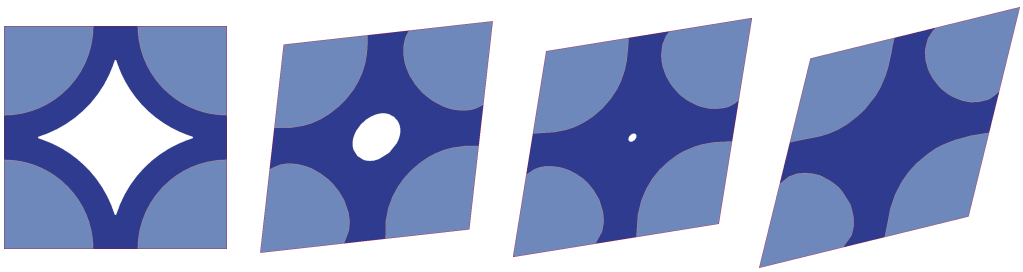

Computational Modeling of Liquid-Phase Sintering based on Homogenization

MIKAEL ÖHMAN

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2011

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING IN SOLID AND STRUCTURAL MECHANICS

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Initial state for a coupled FE^2 simulation with inhomogeneous density distribution.

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Abstract

Liquid-phase sintering is the process where a pre-compacted powder, "green body", is heated to the point where (a part of) the solid material melts, and the specimen shrinks while keeping (almost) net shape. In the case of hardmetal, the microstructure is defined by WC-Co-particles with large pores, whereby molten Co represents the liquid phase. In the ideal case, a fully dense material is achieved when the sintering is completed.

In this thesis, the sintering process is modeled within an idealized Representative Volume Element (RVE), which is evaluated at the Gaussian integration points in the macroscale FE-mesh. The "driving force" of the sintering procedure is surface tension along the free surfaces, i.e. Co-pore interfaces. In this thesis, the intrinsic deformation of both the solid phase and the melt phase is modeled as the creeping flow of the Stokes' type, whereby elastic deformation is ignored. The macroscopic properties are found via computational homogenization of the RVE's. Although the RVE is highly idealized, it shows important properties not easily captured with traditional macroscopic constitutive models.

The finite element mesh of the RVE becomes heavily deformed as the surface tension pulls the particles closer; hence, it was necessary to develop a versatile surface tracking method with remeshing. As an element in the mesh reaches a certain deformed state, defined by the condition number of the Jacobian, a new mesh is created.

The FE^2 algorithm has been implemented in the open source FE-code OOFEM (written in C++). In particular, the code is parallelized w.r.t. the Gauss points in the macroscale mesh.

The most straightforward RVE-problem is that with Dirichlet b.c. on the subscale velocity field, whereby the macroscopic rate-of-deformation is the control variable. In order to deal with eventual macroscopic incompressibility of the RVE (as the porosity vanishes) a new macroscopic format is introduced which allows for mixed control in terms of the deviatoric part of the macroscopic rate-of-deformation and the macroscopic pressure. In addition, this format allows for a seamless transition between compressible and incompressible RVE's. Numerical examples are shown for different loading scenarios, where the macroscopic behavior is studied.

Keywords: Liquid phase sintering, Computational homogenization, FE^2

Preface

The work presented in this thesis was carried out at the division of Material and Computational Mechanics at Chalmers University of Technology during 2009–2011. It was funded by the Swedish Research Council.

I would like to thank my supervisor Professor Kenneth Runesson and my co-supervisor Associate Professor Fredrik Larsson for their valuable guidance and help with material modeling, continuum mechanics and academic writing. My fellow Ph.D. students have also spent a lot of time discussing and helping me when problems arose. I could not hope for better friends and coworkers.

Göteborg in December, 2011
Mikael Öhman

Thesis

This thesis consists of an extended summary and the following appended papers:

The appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for the major progress of the work in preparing the papers, i.e. took part in planning the papers, took part in developing the theory, developed the numerical implementation and carried out the numerical simulations.

Contents

Part I
Extended Summary

1 Background

1.1 The process of sintering of hardmetal

Manufacturing of PM products is based on the "welding" (sintering) of particles due to heating or combined heating and mechanical loading (uniaxial or isotropic pressing). To model and simulate the sintering of hard metals, typically WC with Co as the binder metal in the case of a WC-Co-Ti-C system, is particularly challenging in view of the fact that the sintering process involves both solid and melt states of the constituents. In brief, the manufacturing process can be split into three different sequential stages: (I) Compaction of powder compound, with initially around 70 % porosity, into a "green body" with roughly 20 % to 40 % porosity. (II) Heating in oven up well above the melting temperature for Co (ca 1500 °C). (III) Cooling to room temperature.

The purpose is to achieve net-shape already of the green body, whereby the subsequent sintering would result mainly in change of volume (without distortion, i.e. shear deformation). Since uniaxial pressing is normally used for the initial compaction of hard metal, it is likely that wall friction leads to inhomogeneous distribution of residual stresses and bulk density in the green body (at least to some extent). This is an unwanted effect that obscures the goal of net-shape.

During the heating phase, thermal expansion is combined with a certain amount of solid state sintering before melting of the binder, which is necessary in order to achieve "liquid-phase" sintering. During the hold-time at the given temperature, significant compaction (densification, consolidation) takes place due to a combination of solid deformation, diffusion and liquid motion, which brings about reduced porosity. The so-called "sintering stress", which is a macro-scale manifestation of the surface tension between the constituents and the pores, is the "driving force" for compaction; hence, sintering can take place under zero external load of the specimen (known as "free sintering"). However, the residual stresses after compaction will effect the process.

Remark: It is possible to achieve a sintered product below or above the melting temperature of the Co-binder. Up to 80 % of the densification can be achieved in pure solid state sintering, although a completely dense product can only be achieved with liquid phase. These two cases are often denoted solid and liquid state sintering, respectively, in the literature.

Clearly, the aim of the process is that the final product is completely dense, i.e. there is no rest-porosity, with no net-shape distortion. This may be hard to accomplish if the green body bulk density is severely inhomogeneous and/or the liquid phase sintering is inefficient (in terms of insufficient amount of binder phase and/or incomplete melting).

1.2 Modeling and simulation efforts — A brief review

A wealth of literature deals with the modeling and simulation of the sintering process. Primarily, this relates to the constitutive modeling of (1) the powder material response for green body (pre)compaction and (2) the high temperature response and sintering mechanisms pertinent to both solid and liquid phase sintering. Modeling efforts can be classified by two major paradigms: *a priori macroscale modeling* and *micromechanics modeling and computational homogenization*.

1.2.1 Macroscale modeling

A vast majority of the existing literature on *a priori* macro-scale modeling is devoted to the compaction stage, and it is noted that very few (if any) attempts have been made to develop a unifying macroscopic constitutive model for the compaction and sintering stages. As to the compaction process, material rate-independence (elasto-plasticity) is a common (and valid) assumption. Such plasticity models are often taken, at least conceptually, from soil mechanics. The major focus is on the evolution of the yield surface due to changing porosity as the predominant hardening mechanism, e.g. Fleck et al. [4], Oliver et al. [19], Brandt and Nilsson [2], Redanz [21], Kraft [10].

For the solid phase sintering, the major feature is the strong rate-dependence close to, and above, the melting temperature of the binder. Models based on viscoelasticity and viscoplasticity have, therefore, been proposed to simulate the creep behavior of the highly deformable (and even partly melt) binder, e.g. Shinigawa [26], Brandt and Nilsson [2]. Clearly, it is of utmost importance to model the high sensitivity of yield stress to temperature, e.g. Mähler et al. [14]. The task of providing a rational thermodynamic definition of the sintering stress was addressed by, e.g. Reid and Oakberg [22], Mähler and Runesson [15].

As to liquid phase sintering, the text-book by German [5] is still an authority in the field. Examples of the rich literature are Svoboda et al. [29], Xu and Mehrabahdi [31], Lu et al. [12], who used a single-phase approach. To date, it seems that no attempts have been made in the literature to use mixture theory, which explicitly acknowledges the coexistence of a solid skeleton of the WC-particles and partly solidified and liquid Co.

1.2.2 Micromechanics modeling and computational homogenization

Most micromechanically based models consider idealized geometrical arrangement, such as a regular array of (in many cases only two) spheres, within a Representative Volume Element (RVE). One common approach is to consider grain boundary diffusion, e.g. Helle, et al. [6], McMeeking and Kuhn [16], Riedel and Svoboda [23] and Shinigawa [26] and particle bridging via diffusion, e.g. Svoboda and Riedel [28], Binet [1], Luque et al. [13], as the principal mechanisms for densification. Some attempts have been made too include more realistic microstructural arrangements and carry out computational homogenization to obtain response functions, e.g. Mähler and Runesson [15].

Early attempts to numerically simulate the surface-tension driven reshaping of contacting particles are by Jagota and Dawson [8, 7] and van de Vorst [30]. In a series of papers, Zhou and Derby [33, 32] emphasize efficient finite element algorithms to trace the complex 3-dimensional flow of multi-particle interaction. The main challenges are the complex subscale geometry and the moving free boundary giving rise to very large deformations and severe topology changes. Recent developments of free-boundary tracing FE-strategies for large deformations (without severe topological changes) are discussed by Peric and coworkers, [3], [25], [24]. All the mentioned work consider surface tension effects in fluids. A recent extension to include surface tension in the context of solid modeling, where anisotropic surface energy may be present, is due to Javili and Steinmann [9].

Micro-mechanical analysis must be accompanied by computational homogenization in order to obtain a predictive model for a component. One possibility is socalled upscaling, i. e. to use the subscale modeling to calibrate a macroscopic model. A more appealing, but theoretically and computationally more challenging, possibility is to carry out full-fledged simultaneous coupling between the micro- and macro-scales, which is known as Computational Multiscale Modeling (CMM), or the FE^2 strategy. This is certainly the current international trend in material modeling for engineering purposes; however, to our knowledge no work on the fully coupled FE^2 applied to the sintering problem has been published (at least not in main-stream journals).

2 Aim and Scope of Research

The thesis concerns the development of a predictive tool for the computational modeling of sintering of hardmetal, that involves a liquid (melt binder) phase. Virtually all modeling in the literature aiming for quantitative predictions on the engineering scale is based on *a priori* homogenized macroscopic material models. In this Ph.D. project (of which the present licentiate thesis represents the first part), the relevant equations are obtained via micromechanical modeling and computational homogenization.

The main purpose and specific aims of the Ph.D. project are two-fold:

- To establish a computational model for the prediction of final shape (distortion), residual stresses and remaining porosity in sintered hardmetal products. Particular emphasis is placed on the role of liquid phase sintering, e.g. with respect to the amount and composition of the (melt) binder metal, size distribution of particles, etc.

- To calibrate and validate the constitutive models using data from the literature and from present and past collaborators. Experimental data from component tests are available from a previous joint research project with Swedish partners.

In order to achieve this goal, the following tasks are identified in the present licentiate thesis:

- Develop the micro-mechanical relations for an ideal setting of viscous particles in contact that sinter due to surface tension.

- Use computational homogenization and a fully coupled FE^2 strategy to obtain a predictive model.
- Develop FE-software, in particular for efficient implementation of parallel algorithms.

3 Modeling Features

3.1 Constitutive modeling of constituents

Elastic deformation is neglected in the subscale constituents, which is a reasonable assumption since the (visco)plastic deformation is totally dominating the process. With plastic incompressibility, this leads to an isotropic incompressible Stokes flow within the particles. The chosen constitutive model is potentially nonlinear and the stress deviator $\boldsymbol{\sigma}_{\mathrm{dev}}$ is proportional the deviatoric strain rate $\boldsymbol{d}_{\mathrm{dev}}$:

$$\boldsymbol{\sigma}_{\mathrm{dev}}(\boldsymbol{d}) = 2\tilde{\mu}\boldsymbol{d}_{\mathrm{dev}}, \quad \tilde{\mu} \stackrel{\mathrm{def}}{=} \frac{\sigma_{\mathrm{e}}}{3d_{\mathrm{e}}} \tag{1}$$

Thermal expansion is not yet considered, which means that the process is assumed to take place after initial heating.

It is worth noting that, even though the subscale constituents are incompressible, the macroscopic change is compressible due to shrinking pores within the RVE's. The macroscopic response of a given specimen will eventually be partially incompressible at the later stages of sintering. In Paper B, the problem of incompressible RVE's is dealt with.

In the absence of acceleration, the balance equations for the quasi-static motion of the viscoplastic particles can be established in the spatial setting as follows:

$$-\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} = \boldsymbol{0} \tag{2a}$$

$$\boldsymbol{v} \cdot \boldsymbol{\nabla} = 0 \tag{2b}$$

Note that these equations are intrinsic to the particles.

3.2 Surface tension

A vital part of the simulation is the modeling of surface tension which acts as the "driving force" for liquid-phase sintering. An extensive theory on boundary energy potentials has been developed by Steinmann [27], which has served as the basis for the surface tension modeling in this work. In short; equilibrium across a surface Γ represented by a thin shell bounded by the curve $\mathcal{C}$, as in Figure 3.1 can be stated as:

$$\boldsymbol{t}^+ + \boldsymbol{t}^- + \boldsymbol{t}_{\mathrm{s}} = \boldsymbol{0} \quad \text{on } \Gamma \quad \text{with } \boldsymbol{t}_{\mathrm{s}} \stackrel{\mathrm{def}}{=} \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\nabla}} \tag{3}$$

$$\sum_i \hat{\boldsymbol{t}}_i = \boldsymbol{0} \quad \text{on } \mathcal{C} \tag{4}$$

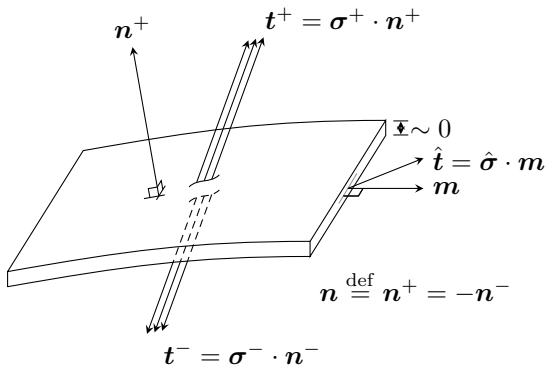

Figure 3.1: *Thin shell representing a surface with in-plane forces due to "surface tension".*

where $\hat{\boldsymbol{\nabla}} \stackrel{\text{def}}{=} \boldsymbol{\nabla} - (\boldsymbol{\nabla} \cdot \boldsymbol{n})\boldsymbol{n}$. For an isotropic surface potential one obtains, $\hat{\boldsymbol{\sigma}} = \gamma_{\text{s}}(\boldsymbol{I} - \boldsymbol{n} \otimes \boldsymbol{n})$, and the common expression for the surface traction representing surface tension is obtained:

$$\boldsymbol{t}_{\text{s}} = -\kappa \gamma_{\text{s}} \boldsymbol{n}, \quad \text{where} \quad \kappa \stackrel{\text{def}}{=} \boldsymbol{n} \cdot \hat{\boldsymbol{\nabla}} \tag{5}$$

where κ is the curvature.

3.3 RVE-problem

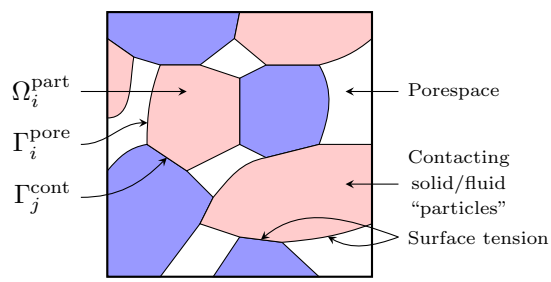

Figure 3.2: *Microstructure of porous particulate material with sintering particles in contact.*

The fundamental microconstituents of the subscale are defined in Figure 3.2. Surface tension could be considered on all boundaries; Γ_i^{pore} and Γ_j^{cont}; however, it is assumed to be zero on Γ_j^{cont} in all the numerical examples in the appended papers.

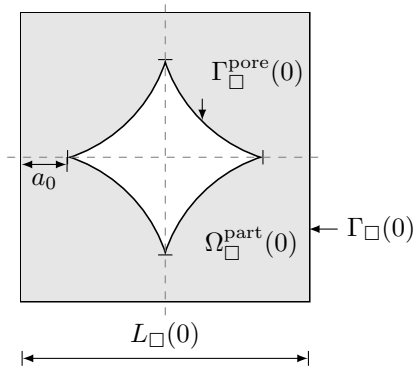

Figure 3.3: *Idealized initial configuration of the RVE in 2D consisting of circular particles in a square lattice.*

In the simplest form, the RVE is idealized to a unit cell with four contacting circular particles. The size of the initial contact surface after compaction is represented by the length a_0.

During the FE-simulation of the RVE, the mesh becomes very distorted, in particular when the sharp contact is rapidly rounded off and when the pores vanish. A remeshing strategy was developed in Paper A to deal with this issue.

4 Summary of Appended Papers

- **Paper A: Computational Mesoscale Modeling and Homogenization of Liquid-Phase sintering of Particle Agglomerates**. Liquid phase sintering of particle agglomerates is simulated as the viscous deformation of particle-particle contact, whereby the single driving force is the surface tension on the particle/pore interface. Particles are modeled as purely viscous fluids (with no elasticity). Computational homogenization is adopted for the RVE with Dirichlet boundary conditions. A surface motion algorithm was developed that requires complete remeshing of the FE-mesh based on a "maximum deformation" criterion. Since the particles are intrinsically incompressible, the macroscopic compressibility is determined from shrinking porosity in the substructure. The numerical examples include free sintering of an RVE and a fully coupled FE^2-simulation of a specimen with inhomogeneous initial distribution of porosity.

- **Paper B: FE^2 for Liquid-Phase Sintering with Seamless Transition from Macroscopic Compressibility to Incompressibility**. Liquid phase sintering of particle agglomerates is modeled on the mesoscale as the viscous deformation of particle-particle contact, whereby the single driving force is the surface tension on the particle/pore interface. On the macroscale, a quasistatic equilibrium problem allows for the prediction of the shrinkage of the sintering body. The present paper presents a novel FE^2 formulation of the two-scale sintering problem allowing

for the transition to zero porosity, implying macroscale incompressibility. The seamless transition from compressibility to incompressibility on the macroscale is accomplished by introducing a mixed variational format. This has consequences also for the formulation of the mesoscale problem, that is complemented with an extra constraint equation regarding the prolongation of the volumetric part of the macroscopic rate-of-deformation. The numerical example shows the sintering of a single representative volume element (RVE), which is sheared beyond the point where the porosity vanishes while subjected to zero macroscopic pressure.

5 Conclusions and Outlook

In this thesis a novel approach to simulate the sintering process as a problem of computational homogenization is presented. Examples of the response of a single RVE subjected to different macroscopic conditions showed that the results converged quite rapidly with increasing RVE-size for the adopted Dirichlet boundary conditions. The final FE^2-analysis for an inhomogeneous initial distribution of the macroscopic porosity was carried out using a code parallelization with respect to the macroscale integration points.

By introducing the mixed variational $(\bar{\boldsymbol{v}}, \bar{p})$-format of the macroscale problem, we can ensure a "seamless" transition from the macroscopically compressible to the incompressible response. Hence, there is no need to change the computational algorithm when the simulation is taken beyond the state of a fully dense macroscopic response (in some spatial point in the macro-domain). Without such a mixed formulation, the macroscopic ATS-tensor would become unbounded at the transition state.

The FE^2 algorithm has been implemented in the open source code OOFEM (cf. [20]). All parts have been implemented with a modular approach, implying that they can be used individually and with other problems. These modules will be available in the future official releases of OOFEM.

As an outlook to future developments, the new mixed RVE-format will be adopted in conjunction with micro-periodic and Neumann boundary conditions. It would also be of interest to extend the classical strong format of micro-periodicity to a weak variational setting, cf. Larsson et al. [11]. A major advantage is that the subscale FE-mesh does not need to be periodic, which is particularly beneficial in a context of adaptive mesh (re)generation.

The microstructural properties of the "green body", i.e. before the sintering process starts, should be represented in a more realistic way than is presently the case. For example, the RVE should be generated from a given statistical distribution of particle size and shape. Parameter variations should be carried out of the various geometrical and constitutive properties. In order to make (inverse) parameter identification meaningful, it is necessary to extend the description of the subscale geometry to three dimensions in the future, although this may represent a major increase in complexity and computational demand.

References

[1] C. Bient, K. L. Lencoski, D. F. Heaney, and R. M. German. "Modeling of distortion after densification during liquid-phase sintering". *Metallurgical and Materials Transactions A* 35.12 (Dec. 2004), 3833–3841. DOI: 10.1007/s11661-004-0289-z.

[2] J. Brandt and L.-G. Nilsson. "FE-simulation of compaction and solid state sintering of cemented carbides". *Mechanics of Cohesive-Frictional Materials* 3 (1998), 181–205.

[3] W. G. Dettmer and D. Perić. "A computational framework for free surface fluid flows accounting for surface tension". *Computer Methods in Applied Mechanics and Engineering* 195.23-24 (2006), 3038–3071. DOI: 10.1016/j.cma.2004.07.057.

[4] N. A. Fleck, L. T. Kuhn, and R. M. McMeeking. "Yielding of metal powder bonded by isolated contacts". *Journal of the Mechanics and Physics of Solids* 40.5 (July 1992), 1139–1162. DOI: 10.1016/0022-5096(92)90064-9.

[5] R. M. German. *Liquid phase sintering*. New York: Springer, 1985.

[6] A. S. Helle, K. E. Easterling, and M. F. Ashby. "Hot-isostatic pressing diagrams: New developments". *Acta Metallurgica* 33.12 (Dec. 1985), 2163–2174. DOI: 10.1016/0001-6160(85)90177-4.

[7] A. Jagota and P. R. Dawson. "Micromechanical modeling of powder compacts — II. Truss formulation of discrete packings". *Acta Metallurgica* 36.9 (Sept. 1988), 2563–2573. DOI: 10.1016/0001-6160(88)90201-5.

[8] A. Jagota and P. R. Dawson. "Micromechanical modeling of powder compacts — I. Unit problems for sintering and traction induced deformation". *Acta Metallurgica* 36.9 (Sept. 1988), 2551–2561. DOI: 10.1016/0001-6160(88)90200-3.

[9] A. Javili and P. Steinmann. "A finite element framework for continua with boundary energies. Part II: The three-dimensional case". *Computer Methods in Applied Mechanics and Engineering* 199.9-12 (Jan. 2010), 755–765. DOI: 10.1016/j.cma.2009.11.003.

[10] T. Kraft. "Optimizing press tool shapes by numerical simulation of compaction and sintering application to a hard metal cutting insert". *Modelling and Simulation in Materials Science and Engineering* 11.3 (May 2003), 381–400. DOI: 10.1088/0965-0393/11/3/310.

[11] F. Larsson, K. Runesson, S. Saroukhani, and R. Vafadari. "Computational homogenization based on a weak format of micro-periodicity for RVE-problems". *Computer Methods in Applied Mechanics and Engineering* 200.1-4 (Jan. 2011), 11–26. DOI: 10.1016/j.cma.2010.06.023.

[12] P. Lu, X. Xu, W. Yi, and R. M. German. "Porosity effect on densification and shape distortion in liquid phase sintering". *Materials Science and Engineering* A318 (2001), 111–121.

[13] A. Luque, J. Aldazabal, A. Martín-Meizoso, J. M. Martínez-Esnaola, J. G. Sevillano, and R. Farr. "Simulation of the microstructural evolution during liquid phase sintering using a geometrical Monte Carlo model". *Modelling and Simulation in Materials Science and Engineering* 13.7 (Oct. 2005), 1057–1070. DOI: 10.1088/0965-0393/13/7/004.

[14] L. Mähler, M. Ekh, and K. Runesson. "A Class of Thermo-Hyperelastic-Viscoplastic Models for Porous Materials: Theory and Numerics". *International Journal of Plasticity* 17 (2001), 943–969.

[15] L. Mähler and K. Runesson. "Modelling of solid-phase sintering of hardmetal using a mesomechanics approach". *Mechanics of Cohesive-frictional Materials* 5.8 (Nov. 2000), 653–671. DOI: 10.1002/1099-1484(200011)5:8<653::AID-CFM111>3.0.CO;2-A.

[16] R. M. McMeeking and L. T. Kuhn. "A diffusional creep law for powder compacts". *Acta Metallurgica et Materialia* 40.5 (May 1992), 961–969. DOI: 10.1016/0956-7151(92)90073-N.

[17] M. Öhman, K. Runesson, and F. Larsson. "Computational Mesoscale Modeling and Homogenization of Liquid-Phase sintering of Particle Agglomerates". *Technische Mechanik* (2011). Submitted.

[18] M. Öhman, K. Runesson, and F. Larsson. "FE^2 for Liquid-Phase Sintering with Seamless Transition from Macroscopic Compressibility to Incompressibility" (2011). In preparation.

[19] J. Oliver, S. Oller, and J. C. Cante. "A plasticity model for simulation of industrial powder compaction processes". *International Journal of Solids and Structures* 33.20-22 (Aug. 1996), 3161–3178. DOI: 10.1016/0020-7683(95)00249-9.

[20] B. Patzák. *OOFEM project home page*. 2000.

[21] P. Redanz. "Numerical modelling of cold compaction of metal powder". *International Journal of Mechanical Sciences* 40.11 (Nov. 1998), 1175–1189. DOI: 10.1016/S0020-7403(98)00021-6.

[22] C. R. Reid and R. G. Oakberg. "A continuum theory for the mechanical response of materials to the thermodynamic stress of sintering". *Mechanics of Materials* 10.3 (1990), 203–213. DOI: 10.1016/0167-6636(90)90043-F.

[23] H. Riedel and J. Svoboda. "A theoretical study of grain growth in poeous solids during sintering". *Acta Metallurgica et Materialia* 41 (1993), 1929–1936.

[24] P. H. Saksono and D. Perić. "On finite element modelling of surface tension: Variational formulation and applications – Part II: Dynamic problems". *Computational Mechanics* 38.3 (Nov. 2006), 251–263. DOI: 10.1007/s00466-005-0745-7.

[25] P. H. Saksono and D. Perić. "On finite element modelling of surface tension: Variational formulation and applications – Part I: Quasistatic problems". *Computational Mechanics* 38.3 (Nov. 2006), 265–281. DOI: 10.1007/s00466-005-0747-5.

[26] K. Shinigawa. "Finite element simulation of sintering process". *JSME International Journal, Series A: Mechanics and Materials Engineering* 39 (1996), 565–572.

[27] P. Steinmann. "On boundary potential energies in deformational and configurational mechanics". *Journal of the Mechanics and Physics of Solids* 56.3 (2008), 772–800. DOI: 10.1016/j.jmps.2007.07.001.

[28] J. Svoboda and H. Riedel. "New solution describing the formation of inter-particle necks in solid-state sintering". *Acta Metallurgica et Materialia* 43 (1995), 1–10.

[29] J. Svoboda, H. Riedel, and R. Gaebel. "Model for liquid phase sintering". *Acta Materialia* 44 (1996), 3215–3226.

[30] G. A. L. van de Vorst. "Integral method for a two-dimensional Stokes flow with shrinking holes applied to viscous sintering". *Journal of Fluid Mechanics* 257 (Apr. 1993), 667–689. DOI: 10.1017/S002211209300326X.

[31] K. Xu and M. M. Mehrabadi. "A micromechanical model for the initial rearrangement stage of liquid phase sintering". *Mechanics of Materials* 25.2 (1997), 157–137. DOI: 10.1016/S0167-6636(96)00048-8.

[32] H. Zhou and J. J. Derby. "An assessment of a parallel, finite element method for three-dimensional, moving-boundary flows driven by capillarity for simulation of viscous sintering". *International Journal for Numerical Methods in Fluids* 36.7 (Aug. 2001), 841–865. DOI: 10.1002/fld.159.

[33] H. Zhou and J. J. Derby. "Three-Dimensional Finite-Element Analysis of Viscous Sintering". *Journal of the American Ceramic Society* 81.3 (Jan. 1998), 533–540. DOI: 10.1111/j.1151-2916.1998.tb02371.x.