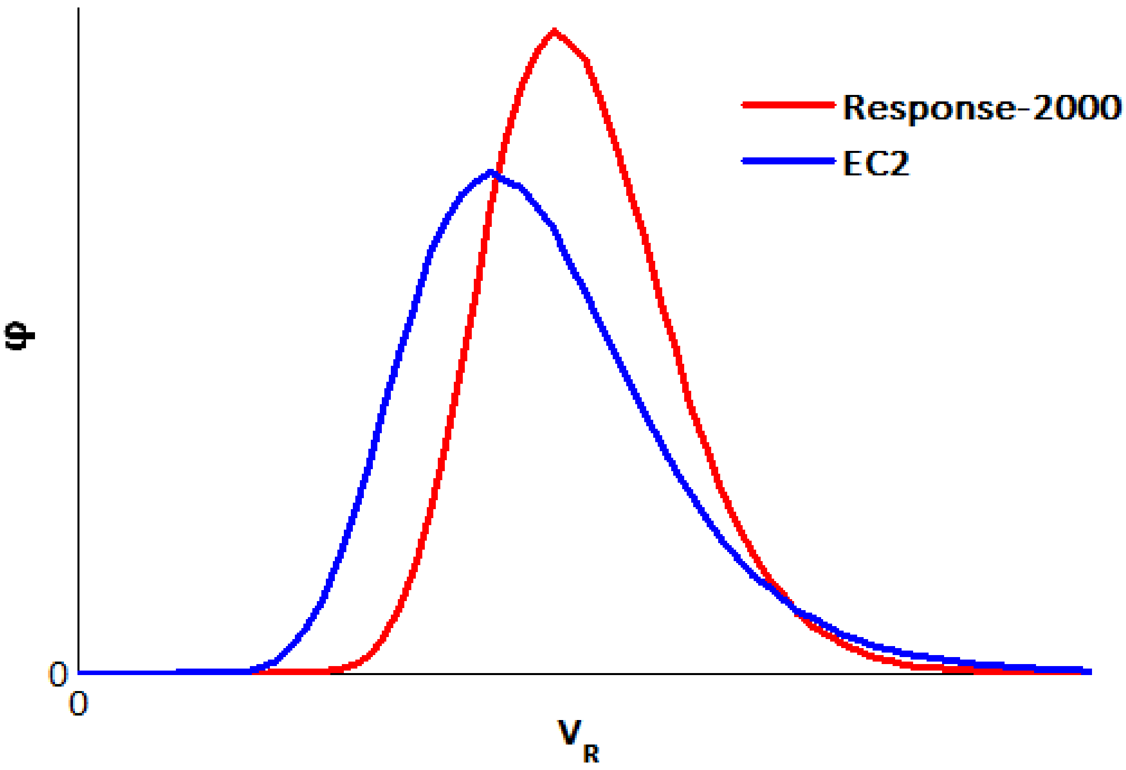


# Safety evaluation of shear capacity of reinforced concrete bridges

*Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design*

THOMAS ÅHNBERG

Department of Civil and Environmental Engineering
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Cover:
Average predictions of shear resistance distribution for reinforced concrete beams ($\rho_l$ = 1.2 %, $\rho_w$ = 0.1 %), using Response-2000 and the EC2 shear model, evaluated based on the JCSS Probabilistic model code (JCSS, 2001) and the EC2 Basis for structural design (CEN, 2002)

Chalmers Reproservice / Department of Civil and Environmental Engineering
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ABSTRACT

Predicting the load carrying capacity of concrete bridges often calls for a conservative approach which leads to high costs, especially in the maintenance of existing structures. The need for conservativeness arises not only from natural variations but also from inconsistency of available calculation models and safety formats. This master thesis presents a probabilistic evaluation of analysis methods and safety formats used to establish design shear capacity of reinforced concrete bridge girders. An assessment is made of the relative favorableness of using either the shear analysis procedure described in the European construction code (CEN, 2004) or the sectional analysis tool Response-2000 (Bentz, 2000). The accuracy of the EC2 partial safety factor format is compared to a safety format proposed by Schlune *et al.* (2010). The performed evaluations are founded on a parametric study of standard beam cross sections and the probabilistic model used in the evaluations is prepared in accordance with the JCSS Probabilistic model code (2001) along with relevant guidance given by the CEN (2002).

Key words: Concrete, reinforced, bridges, girder, shear, capacity, probabilistic, Monte Carlo

Säkerhetsutvärdering av skjuvkapacitet för armerade betongbroar

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SAMMANFATTNING

Det finns idag ca 11000 betongbroar i Sverige och det satsas i runda tal en halv miljard kronor per år på att underhålla dessa. Samtidigt som broarna blir allt äldre öker trafiklasterna och det ställs därmed allt högre krav på konstruktionerna. Konsekvensen blir ofta att broar behöver förstärkas eller byggas om, vilket innebär stora kostnader. Detta innebär att det finns ett stort ekonomiskt intresse av att finna metoder för att bättre kunna precisera bärförmågan hos broarna.

Sedan årsskiftet 2010/2011 sker all dimensionering och bärighetsbestämning av broar i Sverige enligt den Europeiska normen, EC2 (CEN, 2004). Med fokus på bestämning av skjuvkapacitet har det i detta examensarbete undersökts om säkerhetsformatet som presenteras i normen skulle kunna tänkas vara onödigt konservativt samt om det på sikt skulle vara intressanta att introducera en alternativ skjuvmodell. En serie probabilistiska analyser har utförts i syfte att dels jämföra hur mycket större bärförmåga som skulle kunna påvisas med beräkningsverktyget Response-2000 (Bentz, 2000), dels utvärdera och jämföra säkerhetsformatet i EC2 med ett alternativt säkerhetsformat som presenterats av Schlune *et al.* (2010).

Utvärderingarna har gjorts genom parameterstudier på typiska balktvärsnitt. Upprättandet av den probabilistiska modellen har i sin tur huvudsakligen gjorts enligt den vägledning som ges i JCSS Probabilistic model code (2001) samt tillämpliga riktlinjer som ges i EC2. En slutsats som dragits från användandet av de tillgängliga regelverken är att det fortfarande återstår arbete med att färdigställa allmänna riktlinjer för utförandet av probabilistisk analys. Detta gäller särskilt anvisningar för bestämning av modellosäkerhet. De kvantitativa resultat som presenteras i denna rapport bör därför användas med viss försiktighet men är fortfarande användbara i jämförelsesyfte.

Jämförelsen mellan beräkningsmodeller gav en indikation på att högre bärförmåga ofta påvisas med hjälp av beräkningsmodellen i Response-2000 (Collins & Mitchell, 1991) . Det gällde särskilt för balkar med små mängder skjuvarmering och stora mängder böjarmering. Utifrån de använda uppskattningarna av modellosäkerhet visades också att den dimensionerande bärförmågan som beräknas med skjuvmodellen och säkerhetsformatet i EC2 inte är konservativa utan tvärtom leder till inkonsekventa säkerhetsindex som underskrider det uppsatta riktvärdet i EC2. Det samma visades gälla när samma säkerhetsformat användes för skjuvkapacitetsberäkning i Response-2000. En jämförelse visade sedan att det säkerhetsformat som presenterats av Schlune *et al.* ledde till en något högre och jämnare nivå på säkerhetsindex. Utvärderingen visade dock att det tycks finnas en del att vinna på en vidare kalibrering av detta format.

Nyckelord: Betongbroar, armerade, balkar, skjuvkapacitet, probabilistisk, Monte Carlo

# Contents

# Preface

This Master's thesis project is the final part of a Master of Science in Structural Engineering and Building Performance Design at Chalmers University of Technology in Gothenburg Sweden. It has been carried out in collaboration with the concrete structures research group at Chalmers.

The project was supervised by Hendrik Schlune and examined by Mario Plos. The author would like to thank them both for their valuable comments and especially Hendrik for his support and encouragement throughout the work.

Göteborg Juni 2011

Thomas Åhnberg

# Notation

**Roman upper case letters**

| | |
|---|---|
| $A_c$ | Cross sectional area of concrete |
| $A_s$ | Cross sectional area of reinforcement |
| $A_{s,min}$ | Minimum cross sectional area of reinforcement |
| $A_{sw}$ | Cross sectional area of shear reinforcement |
| D | Diameter |
| E | Load effect |
| $E_d$ | Design value of load effect |
| LN | Lognormal distribution |
| M | Margin (safety margin) |
| M | Bending moment |
| $M_{Ed}$ | Design value of the applied bending moment |
| $M_{Rd}$ | Design value of moment resistance |
| N | Normal distribution |
| N | Number of simulations |
| P | Probability |
| $P_f$ | Probability of failure |
| $P_s$ | Measurement of reliability |
| R | Resistance |
| $R_d$ | Design value of resistance |
| $R_\mu$ | Resistance based on mean values |
| $R_{\Delta fc}$ | Resistance based on low concrete compressive strength |
| $R_{\Delta fct}$ | Resistance based on low concrete tensile strength |
| $R_{\Delta fy}$ | Resistance based on low steel strength |
| V | Coefficient of variation |
| V | Shear force/resistance |
| $V_{Ed}$ | Design value of applied shear force |
| $V_{Rd}$ | Design value of shear resistance |
| $V_c$ | Concrete contribution to shear transfer |
| $V_f$ | Coefficient of variation representing physical uncertainty |
| $V_g$ | Coefficient of variation representing geometrical uncertainty |
| $V_m$ | Coefficient of variation representing model uncertainty |
| $V_n$ | Nominal shear resistance |

| | |
|---|---|
| $V_p$ | Vertical component of prestressing force |
| $V_s$ | Shear reinforcement contribution to shear transfer |
| X | Stochastic variable |
| **X** | Set of stochastic variable |

**Roman lower case letters**

| | |
|---|---|
| a | Geometrical data |
| $a_{nom}$ | Nominal geometrical data |
| $b_w$ | Width of the beam web |
| d | Effective depth of cross-section |
| $d_g$ | Largest nominal maximum aggregate size |
| $f_c$ | Compressive strength of concrete |
| $f_{cd}$ | Design value of concrete cylinder compressive strength |
| $f_{ck}$ | Characteristic concrete cylinder compressive strength |
| $f_{cm}$ | Mean value of concrete cylinder compressive strength |
| $f_{ctk}$ | Characteristic axial tensile strength of concrete |
| $f_{ctm}$ | Mean value of axial tensile strength of concrete |
| $f_y$ | Yield strength of reinforcement |
| $f_{yk}$ | Characteristic yield strength of reinforcement |
| $f_{yd}$ | Design yield strength of reinforcement |
| $f_{ywd}$ | Design yield strength of shear reinforcement |
| g | Limit state function |
| $g_u$ | Limit state function represented in a stochastic variable space |
| h | Overall depth of a cross-section |
| i | Index vector |
| k | Coefficient; Factor |
| $q_{ud}$ | Maximum value of combination reached in non linear analysis |
| s | Spacing of shear reinforcement stirrups |
| t | Thickness |
| t | Time being considered |
| $t_0$ | The age of concrete at the time of loading |
| u | Normalized stochastic variable |
| **u** | Set of normalized stochastic variables |
| x | Basic variable |
| **x** | Set of basic variables |
| z | Lever arm of internal forces |

**Greek upper case letters**

$\Phi$ Cumulative standard normal distribution

$\Phi$ Diameter

$\Phi_M$ Model bias with regard to moment resistance

$\Phi_V$ Model bias with regard to shear resistance

**Greek lower case letters**

$\alpha_E$ Sensitivity factor with regard to uncertainty of load effect

$\alpha_R$ Sensitivity factor with regard to uncertainty of resistance

$\alpha_{cc}$ Coefficient taking account of long term effects on compressive strength and of unfavourable effects resulting from the way load is applied

$\alpha_{cw}$ Coefficient taking account of state of stress in compression chord

$\beta$ Reliability index

$\gamma$ Partial factor

$\gamma_c$ Partial factor for concrete strength

$\gamma_s$ Partial factor for steel strength

$\gamma_{Rd}$ Partial factor for design resistance

$\theta$ Angle

$\mu$ Mean value

$\mu_x$ Mean value of variable x

$\nu$ Average shear stress

$\nu_1$ Strength reduction factor for concrete cracked in shear

$\rho_l$ Longitudinal reinforcement ratio

$\rho_w$ Transversal reinforcement ratio

$\sigma$ Standard deviation

$\sigma_x$ Standard deviation of variable x

# 1 Introduction

## 1.1 Background

There are today approximately 11000 concrete bridges in Sweden and roughly half a billion SEK is every year spent on their maintenance (Vägverket, 2001). At the same time as the bridge population is getting older traffic loads are ever increasing, resulting in demand for setting harder requirements for many bridges. Hence, there is large economical interest in finding out what the capacities of the bridges actually are.

Both design and structural assessment of reinforced concrete bridges is today conducted following the set of guidelines presented in the European construction code, EC2 (CEN, 2004). Regarding shear capacity it has in several cases been found that application of the code seems to imply a higher requirement for the amount of shear reinforcement compared to when using previous construction codes. As a result many existing bridges are deemed not to have sufficient shear capacity and need to be strengthened or replaced before they can be allowed to accommodate stipulated traffic loads. Since these are rather costly measures the problem has spurred the interest in finding alternative more accurate calculation models which allow showing higher shear capacities.

Within a research project carried out by the division of structural engineering at Chalmers University of Technology researchers are looking at possible advantages from using a combination of probabilistic analysis and an alternative shear strength model implemented in Response-2000 (Bentz, 2000), based on the modified compression field theory (Collins & Mitchell, 1991) and developed at University of Toronto. The model itself has shown to produce results which correlate better with test results than does the Eurocode model (Bohigas, 2002) and together with the full probabilistic analysis, which is a more accurate way of guaranteeing acceptable failure probability, it is expected to show a significantly higher shear capacity for many bridges.

## 1.2 Aim and purpose

The overall purpose of this Master's project is to contribute to the work on improving the methods for structural assessment of bridges, by evaluating, and thereby facilitating, the use of more accurate models and tools for analysis.

The principal aim of the project is to create an overview of for which types of concrete beams the highest improvement of capacity utilization can be gained, both from using the Toronto model instead of the EC2 shear model and from using a more accurate reliability method, than the partial safety factor method presented in EC2. In the latter respect the intention is to incorporate a continued evaluation of an alternative safety format for non-linear analysis recently presented by Schlune *et al.* (2010), as well as employing a full probabilistic approach for determining appropriate design resistances. The resulting overview is meant to be used as guidance on if and when it could be worth using an alternative safety format and or another structural model for assessment of shear strength.

Ensuing questions to which answers are also sought include:

- What can be said about the performance of the current EC2 safety format when used for linear and non-linear analysis of shear strength based on a full probabilistic analysis?
- What can be said about the calibration of model uncertainties intended to indicate the accuracy of the alternative shear strength models?

## 1.3 Specifications and delimitations

The target group for this thesis is mainly people working with management or assessment of concrete bridges. However, the thesis does include an introductory description of the underlying theories on which the presented study is based, partly to make the thesis interesting for a broader range of possible readers.

The opening chapters of the thesis are intended to serve as an introduction to the theoretical background of the models and procedures that are dealt with in the main study. They are not aimed at giving a comprehensive overview of e.g. other competing shear models or safety formats.

The conducted study concerns methods of carrying out cross section analyses of critical bridge cross sections based on given values of moment and shear forces, it does not deal with full analysis of entire bridge structures. Furthermore, only beams with shear reinforcement have been studied and only considering shear resistance with regard to uni-axial shear, not shear from torsion. The effects of interacting moment have been looked at but normal forces from prestressing are not considered.

Due to the restricted scope of the study, it also only involves a limited number of bridge types. The focus has therefore been on assorted conventional bridge types for which shear capacity in some cases has proven critical.

## 1.4 Method

In order to meet the objectives for this project both the Toronto model and the shear strength model presented in the Eurocode, together with full probabilistic analysis, was applied on 144 different reinforced concrete sections. The obtained results were then compared and the evaluation presented in a set of tables and charts. These are meant to show both the influence of the choice of shear strength model and the effect of including the full probabilistic analysis.

The evaluation was preceded and supported by a continuous literature study. Apart from dealing with the shear behavior of reinforced concrete and how it is modeled, this state-of-the-art review was especially aimed at aiding the understanding of how probabilistic theory has been implemented into safety formats for the design and assessment of reinforced concrete structures. Particular focus was paid to the principles behind and derivation of methods to perform full probabilistic analysis as part of shear strength assessments.

# 2 Underlying theory

According to Ryall (2001) there are generally three reasons for carrying out a strength assessment of a bridge: First it might have been decided that the bridge should facilitate heavier traffic loads, second the structure can have suffered from serious damage or deterioration, third there might have been a change in design codes, setting higher demands on the bridge.

The goal of the appraisal is to determine the safe loading capacity of the bridge. In principle the task is much similar to that of initially designing the structure, but unlike when still on the drawing table it is often no longer possible to idealize and therefore usually poses a greater engineering challenge. For example the material strengths that in design were considered as constant throughout the various members can now have changed to the worse or better and need to be sampled and estimated. It can also be hard to conceptualize and model structural forms that in their design are very different from today's norms.

Many of the codes used in construction today are specifically meant to be used when dealing with contemporary materials and design. They are therefore often not directly applicable on older constructions, not even when they have been constructed using today's methods. New codes can be used but then they must first be properly modified, often by going back to the principles from which they were derived. One can also, as a start, use the old codes to determine how the structures meet the original requirements.

As outlined by Hille *et al.* (2005) another aspect of using design codes for the assessment of existing structures is that it is usually tied to a fair amount of conservatism. Due to the fact that the codes should cover a wide range of structures, with subsequent high variability of material quality and construction practice, safety margins are normally set high. This does not usually affect the cost of construction significantly but can induce much unjustified cost for improvements if setting the same requirements in a later assessment.

Furthermore, the simple rule that generally applies is that; the simpler the assessment methods, the higher the tolerance levels needed. Some simple analyses do not always do the bridges justice, and the result of this can be that serviceable bridges are closed for lengthy periods in wait for further assessment and repair. Luckily the tools of today include computer modeling programs that in combination with refined analysis methods can help making a more fair evaluation.

Structural assessment is today conducted following a step level procedure in which simple calculations, based on readily available data, are gradually complemented by increasingly sophisticated methods, given both that the simpler analyses fail to show sufficient capacity and that it is plausible to gain benefit from higher accuracy. As can be read in the guideline from Sustainable Bridges (2007), the steps by which accuracy can be improved involve: using models that allow showing redistribution of stresses due to non-linear material behavior, reducing idealizations in models through increased detailing and use of FEM applications, and finally, making increasing use of a probabilistic approach both for the enhancement of resistance safety formats and better modeling of loads. Out of these different research areas this thesis will, as explained in the introduction, mainly deal with topics regarding probabilistic modeling and reduced idealization of structural models.

## 2.1 Shear response of reinforced concrete

When a structural member is subjected to transverse loading, as is typical for the main members of a bridge superstructure, this will result in not only bending moments but also considerable shear forces in some sections of the bridge. When enough load is applied the shear forces in the concrete beams will give rise to diagonal cracking of the concrete and can, if not taken proper account for, lead to premature failure of the structure. Therefore, in order to avoid such failures, both the cross section geometries and reinforcement of the bridge must be designed with careful consideration to shear resistance.

### 2.1.1 Shear cracking

Initiation of cracking in concrete takes place when principal tensile stresses at some point reach the tensile strength of the material. In the case of beams subjected to a combination of shear and moment it will typically occur either at the centre line of the beam, where the shear stresses are the greatest, or in the bottom or top layer, where the tensile stresses due to moment are dominant (Collins & Mitchell, 1991). The cracks which form under the influence of shear are diagonal, resulting from the inclination of principal tensile stresses as shown in Figure 1.

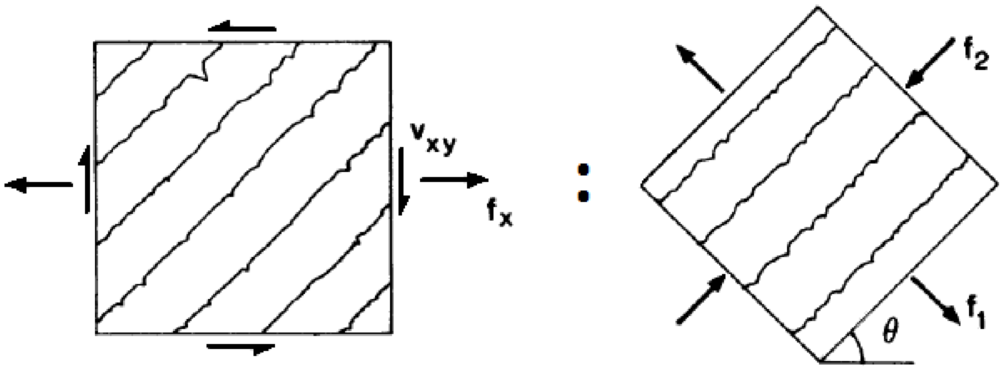


*Figure 1: Conversion of element shear stresses to principal stresses*

Concrete cracks will form perpendicular to the tensile stresses and if starting as flexural cracks at the bottom or top of the beam they will be almost vertical. Then, if in a shear region, they will curve off horizontally as they progress inwards. At what loading the shear cracks will form depends on the tensile strength of concrete, as well as the thickness and internal lever arm of the cross section. The influence of non-prestressed reinforcement is yet of negligible importance.

### 2.1.2 Behaviour after cracking

The formation of cracks will drastically reduce the ability of the concrete to transfer shear through principal tensile stresses. Consequently there will be a first sudden but then also continuous change of equilibrium conditions in the cracked region, given that there, as illustrated at a certain stage in Figure 2, is sufficient reinforcement to prevent immediate collapse. It will also become more difficult to predict the exact response of the concrete section due to a number of uncertainties surrounding the remaining shear transfer mechanisms.

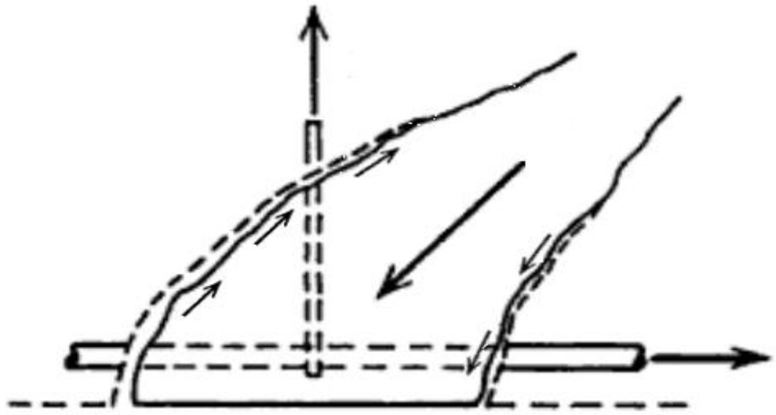

*Figure 2: Equilibrium forces acting on a console of concrete in a cracked region*

A way of treating some of these uncertainties is by saying that shear will only be transferred through the uncracked compression zone of the beam and, if such has been applied, by the shear reinforcement. Since the compression zone in relatively slender beams is quite small and the corresponding shear transfer has little importance this approach can be even further simplified into truss models consisting of exclusively compressive concrete struts and ties of reinforcement. These models, an illustration of which is given in Figure 4 in the following chapter, have traditionally provided a reliable basis for design of beams with shear reinforcement, e.g. in the shear design procedures recommended in EC2.

The problem with the above approach is however that the diagonal concrete cracks are basically considered as consisting of flat frictionless surfaces with no interaction, which, as can be seen in Figure 3, is hardly realistic. Instead as e.g. explained by Jung *et al.* (2008), a combination of aggregate interlocking and other mechanisms constitute a significant transfer of shear between the crack surfaces. In addition to some remaining tensile stresses due to tensile softening in the crack interface, this consequently results in transverse tensile stresses in the cracked concrete which contribute to the balancing of vertical shear forces. These principal tensile stresses in the concrete will gradually decrease as the crack opens up under increasing load but a considerable portion will generally still occur at a point of failure.

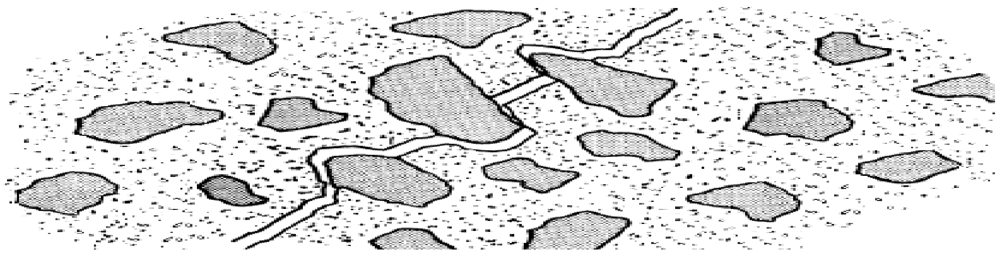

*Figure 3: Principle close-up view of a concrete crack*

The fact that some amount of shear is transferred along the shear cracks has for a long time been rather undisputed and is also, if not always directly regarded, part of the reason why concrete beams can be designed without shear reinforcement. Instead the question has been to what extent the so called concrete contribution can be considered as well established enough to be taken full account of in the assessment of shear capacity, especially when shear reinforcement is provided. Some extensive research on the subject has resulted in a number of shear models which, more or less accurately, take the transfer of shear across cracks into account. Perhaps one of the most notable of these models is based on the so called Modified compression field theory developed by Collins &Vecchio (1986).

### 2.1.3 Failure modes and parameters influencing shear strength

As a concluding summary, and much in analogy with the strut and tie models for shear transfer discussed earlier in this chapter; shear failure of concrete members is generally subdivided into to two types. The first one is called sliding shear failure and occurs when the tying and shearing capabilities of the concrete section are exceeded; for members with shear reinforcement this happens after yielding of the reinforcement steel. The second type of failure occurs when the compressive strength of the concrete held up by the above forces is reached and is accordingly called crushing failure.

There are many parameters influencing the shear strength of a reinforced concrete member, but the main ones are: concrete compressive and tensile strength (as well as aggregate size for relatively low strength concrete), amount of longitudinal and vertical reinforcement, the simultaneous axial forces acting on the member, possible arch effects depending on the span to depth ratio and size effects governed by the cross section depth. The meaning of the term shear strength in this context is the ultimate average shear stress which a member can carry, which as a rule decreases with increasing size, as exemplified by Shioya *et al*. (1989).

When assessing the capacity of a bridge cross section it is often important to note that moment and shear are hardly ever isolated occurrences. This means that the resistance with regard to shear is always, to a varying degree, dependent on the moment capacity and vice versa. Consequently, although somewhat distinctly separate failure modes can be distinguished observationally, clear definitions of corresponding resistances are difficult to make.

## 2.2 Shear strength modeling

To some extent distinguished by the relative importance they place on different shear transfer mechanisms there are as noted earlier various types of structural models used to model shear strength. All of these models will not be discussed in this chapter; instead focus will be on the models which are part of the study presented in this report.

The presented shear models are both used in sectional analysis which is performed to determine the capacity of critical sections of a structural member. In order to find these critical sections, an analysis of the overall structural behavior must first be done to determine not only the shear forces acting along the bridge but also the ratios between shear and moment. This preceding, linear or non-linear, analysis can be done analytically or numerically using finite element methods.

### 2.2.1 Response-2000

Following extensive testing and calibration with empirical data, it has been shown that, as presented by Vecchio & Collins (1986), the use of a shear model based on the so called modified compression field theory leads to analysis results that consistently correspond well with reality. The model has also been developed into easy to use computer programs, such as Response-2000, which can be used to evaluate concrete cross sections with a wide range of geometries and are applicable both for members with and without shear reinforcement (Bentz, 2000). They furthermore incorporate the estimation of diagonal tensile stresses in the cracked concrete, from the transfer of shear across cracks as well as in the uncracked compression zone. However, they do not take into account arch effects near supports and are therefore not suitable for analysis of beams with span to depth ratios less than four.

When having added the shear strength provided by diagonal tensile stresses to the components which make up for the total shear resistance these can, according to the model, be summed up by the following expression:

$$V_n = V_c + V_s + V_p \qquad (2.1)$$

, where $V_n$ is the nominal shear resistance of a section, $V_c$ is the so called concrete contribution, $V_s$ is the vertical force from the shear reinforcement and $V_p$ is the vertical stress component potentially provided by prestressing tendons.

In order to accurately predict the response of the reinforced concrete section subjected to shear there are a number of unknowns that need to be solved for. This requires not only the setting up of equilibrium equations but also the establishing of the involved strain compatibility and constitutive relations, the latter of which have to some extent been drawn up based on test measurements of strains in different parts of the cracked region.

The transfer of shear across a crack is as mentioned in the previous chapter dependent on the crack width and the mean aggregate size. The crack width is then in turn dependent on the average resulting tensile strain as well as the average crack spacing, resulting from the crack control characteristics of both the vertical and longitudinal reinforcement. As is also a topic for discussion concerning the truss models presented in the previous chapter, another of the parameters which cannot be determined directly is the inclination of the compression field.

As e.g. thoroughly described by Collins and Mitchell (1991), the values of all the involved variables discussed above are instead found through an iterative process combining the different constitutive, stress-strain relations. The capacity of a section is then found by performing the combination of interdependent equations for a range of strain values until a maximum value of shear is obtained. In design the obtained stress and strain distribution at the maximum value can also be used to determine what modifications are needed.

### 2.2.2 Eurocode Shear model

Following the provisions in EC2 concerning concrete structures the assessment of shear strength for beams without shear reinforcement is based on using a shear resistance equation of the form: $V_{Rd} = v * b_w d$. In this expression $b_w d$ is the area within the internal lever arm of the section and υ is the average shear strength, of which the influencing parameters are derived directly from empirical studies. In other words, there is no real distinguishing between different components of the shear transfer.

If it is determined that shear reinforcement is needed, contribution from vertical tensile stresses in the concrete will be discarded and the full vertical shear force shall be taken up by the added vertical reinforcement. The inclination of the compressive struts is then chosen within a stipulated accepted range and after having checked that the concrete can withstand the resulting compressive forces the spacing of the reinforcement is estimated in accordance with the principle model shown in Figure 4, via the number of needed stirrups within the range of the inclined crack.

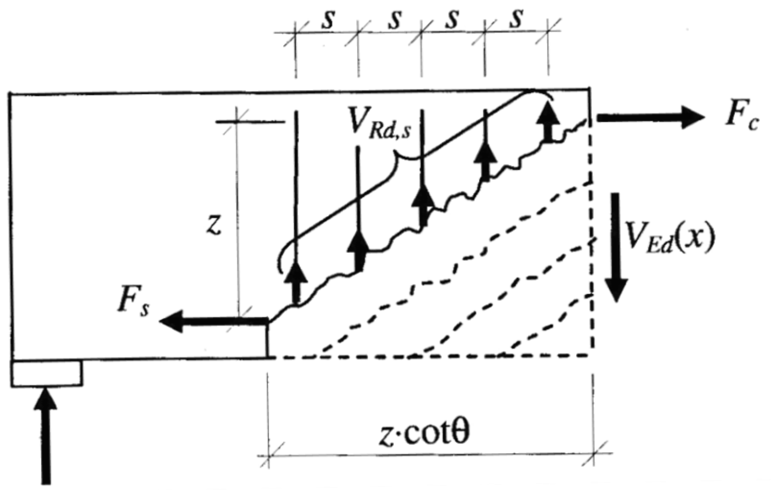


*Figure 4: Model for determining the shear resistance provided by added shear reinforcement according to EC2 (Al-Emrani* et al.*, 2008).*

In order for the equilibrium equations describing the limits of the shear resistance to hold it is furthermore required that sufficient longitudinal reinforcement is provided to restrict crack widths. Provisional procedures also include e.g. reducing design shear forces in regions influenced by arch effects near supports and checking of maximum spacing of shear reinforcement to ensure adequate spreading of the tying forces.

## 2.3 Structural reliability

The reliability of a structure can essentially be explained as the probability that the structure will fulfill its purposes throughout its design lifespan. When designing structures the goal is to make certain that this probability is adequately high. The fulfillment of purpose is in this context expressed as the structure being in a so called admissible state. In most standard design procedures this is equivalent to saying that the resistances, R, of all components of the structure are greater than the respective load effects E. The structural reliability can accordingly be written as the probability:

$$P\,(R - \mathrm{E} > 0) \tag{2.2}$$

A more general way of expressing the condition of the structure is by saying that there are at any given time one possible set of input values which corresponds to failure denoted $\mathbf{w}_f$ and one complementary set of variables $\mathbf{w}_s$ which constitutes a safe state. The two sets, or variable spaces, are divided by a failure surface which can be described by the equation $g(\mathbf{x}) = 0$, in which $g(\mathbf{x})$ is the limit state function of a set of random variables $\mathbf{x} = (x_1, ..., x_n)$.

When regarding design of structures the term reliability can refer to different limit states, or types of failure. The ones that constitute the collapse of a structure are either called ultimate or conditional limit states, depending on whether the stresses that lead to failure originate from the intended load to be carried or from accidental loads.

Serviceability limit state is the second type of failure which does not denote a collapse but yet a failure to serve the intended purpose, caused by unacceptable performance under normal use.

With the intention of reaching normalized, widely accepted levels of safety, target reliabilities for structures are set according to agreed conventions. These optimal design reliabilities should be decided based on economic decision theory, i.e. on the ratio between the risk of failure (cost times probability) and cost of reconstruction. Generally target reliability can be used both, as in this context, as a set lower limit and more freely in decision making based on cost benefit estimations for maximum utility (Sorensen, 2004).

For main bridge structures, which are ranked as class 3 structures in the European construction standards (CEN, 2002), the target minimum reliability is 1 failure per million during a reference period of 1 year, see marked box in Table 1.**Error! Reference source not found.** Reliability in this case is the reliability of design performed according to best practice, which is decided by a board of recognized experts. Decisions regarding new best practices, e.g. use of new probabilistic methods, are typically made as far as possible on the basis of existing codes.

### 2.3.1 Uncertainties

The reason why the performance of a structure cannot be decisively predicted is that it is ruled by many uncertainties. These originate from the stochastic nature of the related variables, but also from uncertainties of how they affect the state of the structure. The various uncertainties which all add up to a joint total uncertainty of resistance are often divided into three main categories:

**Physical uncertainty** mainly regards the natural randomness of material strengths, and is therefore often also referred to as *material uncertainty*.

**Measurement uncertainty** (or *geometrical uncertainty*) is a term that considers the imperfectness of measurements of quantities, e.g. geometrical forms and dimensions.

**Model uncertainty** refers to the lacking knowledge of how to model the behavior of the structure, simplifications made in that process, as well as the uncertainty regarding the probability distributions of the involved stochastic variables.

Due to the fact that models are never perfect, reliability can also never be an absolute number. Instead, the degree to which the models can be relied upon is mainly defined by the amount of information available from previous experience. Although neither exact nor completely accurate the models should always be aimed at reflecting correct mean values and it is therefore valuable when models can be continuously updated e.g. with the help of Bayesian statistics (Sorensen, 2004).

An additional uncertainty which is often not discussed but nevertheless can have critical impact on structural reliability is gross human error. This factor is very hard to appreciate quantitatively and also, just as e.g. risk of terrorist actions, not reasonable to design for. It is therefore not part of what is nominally referred to as reliability and this measure is consequently not necessarily indicative of the actual frequency of structural failure but rather only a quantification used for comparison between structures. Gross errors are however treated in quality assurance, the cost of which in turn influences the initial choice of appropriate reliability classes.

Finally, due to the inevitable deterioration of structures, it is important to note that the parameters that determine the structural resistance are not only variable as such, but also time dependent, thus constituting a time dependent reliability (JCSS, 2001). This aspect is always central when carrying out assessment and maintenance planning. It can, as e.g. proposed in Sustainable Bridges (2007) be considered by adding a factor which includes the influence of the actual condition of structural members when calculating resistance. Also, for the sake of whole life management it can, as discussed by Capriani *et al.* (2007), be elaborated on in different ways, to quantify the time dependent reliability, e.g. for calculation of lifetime probabilities of failure.

### 2.3.2 Reliability methods and safety formats

Accounting for the many uncertainties and ensuring an intended structural safety can be done in a number of ways, however as a principle it is done by making sure that there is a large enough safety margin between the expected mean resistances and mean load effects. How large the safety margin has to be is then essentially depending both on how large and on how well known the variability of the loading and performance is.

One way of expressing this so called safety margin is by replacing the basic variables **x** in the failure function g(**x**) with stochastic variables **X**, i.e. by saying that the margin M = g(**X**). This leads to the following expression for the probability of failure:

$$P_f = P(M \leq 0) = P(g(\boldsymbol{X}) \leq 0) = \int_{w_f} f_x(\boldsymbol{x})dx \qquad (2.3)$$

, where $f_x$ is the density function for the variable set **x.** The safety margin can then in turn also be used to define the currently adopted standard measure of reliability called the reliability index β.

In the fundamental case in which the failure function, and thereby the margin is linearly dependent on two independent and normally distributed stochastic variables, namely the resistance R and the load effect E, the margin will also be normally distributed and β can by definition be expressed as:

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}} \qquad (2.4)$$

However, the failure function is generally not a linear representation of parameters and the safety margin is not necessarily normally distributed. Instead, a more universal way of defining the reliability index is by the geometric interpretation that β can be seen as the shortest distance from the failure surface $g_u(\boldsymbol{u}) = g\big(\mu_{X_i} + \sigma_{X_i} u_1, \ldots, \mu_{X_n} + \sigma_n u_n\big) = 0$ to origo in a normalized stochastic variable space, formed by variables $u_i = \frac{x_i - \mu_{X_i}}{\sigma_{X_i}}$ $i = 1, 2, .., n$. A two dimensional illustration of this is given in Figure 5.

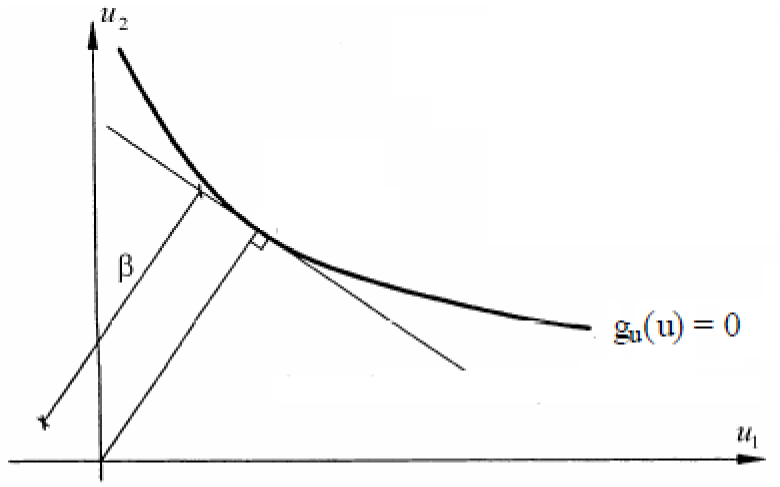


*Figure 5: Geometrical illustration of the reliability index β*

According to the above definitions the reliability index has the relation $\beta = -\Phi^{-1}(P_f) \Leftrightarrow P_f = \Phi(-\beta)$ to the probability of failure, where Φ is the cumulative standard normal distribution, see Table 1**Error! Reference source not found.** below.

*Table 1: Provisional target reliability indices β (and related target failure rates) related to one year reference period and ultimate limit states (modified from JCSS, 2001).*

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| Relative cost of safety measure | Minor consequences of failure | Moderate consequences of failure | Large consequences of failure |
| Large (A) | $\beta$=3.1 ($p_F \approx 10^{-3}$) | $\beta$=3.3 ($p_F \approx 5\ 10^{-4}$) | $\beta$=3.7 ($p_F \approx 10^{-4}$) |
| Normal (B) | $\beta$=3.7 ($p_F \approx 10^{-4}$) | $\beta$=4.2 ($p_F \approx 10^{-5}$) | $\beta$=4.4 ($p_F \approx 5\ 10^{-6}$) |
| Small (C) | $\beta$=4.2 ($p_F \approx 10^{-5}$) | $\beta$=4.4 ($p_F \approx 5\ 10^{-6}$) | $\beta$=4.7 ($p_F \approx 10^{-6}$) |

As a result, the goal of analyses regarding the reliability of structures is often either to assure the reliability defined by a reliability index or conversely to determine the reliability index. Depending on whether the failure functions are treated as being almost linearly dependent on the stochastic variables or if quadratic representations are used these analyses can be either denoted first or second order reliability methods, FORM or SORM (Sorensen, 2004). In either case, finding the closest point on the failure surface can be expressed as the optimization problem:

$$\beta = \min_{g_u(\boldsymbol{u})=0} \sqrt{\sum_{i=1}^{n} u_i^2} \qquad (2.5)$$

Reliability methods, i.e. procedures of measuring and dealing with structural reliability, can be taken to different levels of complexity and scope. Accordingly, the methods are conventionally divided into the following categories as described by Sorensen (2004) and in CEN (2002):

**Level I**: At this lowest level, uncertain parameters are modeled using one characteristic value; no attempt is made to calculate the actual probability of failure, only that it is within the accepted limits. The way of expressing these bounds is typically via the use of partial safety factors, as discussed in the following chapter.

**Level II**: In this category, parameters are represented by their mean values and standard deviations, as well as the correlation coefficients between the stochastic values. These are all implicitly assumed to be normally or log-normally distributed. This is the lower level methods which can be made use of when implementing a so called FORM or SORM.

**Level III:** At this level a complete analysis is made of the reliability problem. Uncertain quantities are modeled by their joint distribution functions and, the estimated probability of failure is used directly to quantify reliability. In doing so a FORM or SORM can be employed, as well as different simulation techniques for full probabilistic analysis.

**Level IV:** In principle the same methods are used as in level II and III, but cost benefit analyses are also incorporated for economy based comparison of different design alternatives. These methods are principally used for structures of major economic importance.

The higher levels of evaluation are essentially used to calibrate the results from the lower levels. As reminded in Sustainable Bridges (2007), the higher levels of enhanced evaluation are only used when a bridge fails the intermediate assessments and the cost of repair and strengthening is significant. This is also in line with the general approach in structural assessment; as noted by Happold, *et al.* (1996), it is important to keep in mind that the activities involved in the appraisal of a structure should never be taken further than is necessary for a conclusion to be reached.

#### 2.3.2.1 EC2 safety format - The partial safety factor method (Level 1)

As prescribed in the standard safety format of EC2 (CEN, 2002), an acceptable reliability is achieved via the deriving of design values for all basic variables. The resulting design load effect should be smaller than the design resistance:

$$E_d < R_d \tag{2.6}$$

Maximum and minimum values for the design load effect and resistance respectively are according to the Eurocode procedures treated as separate limit states. The corresponding separate target reliability indexes for resistance and load effect are obtained from using fixed weight factors $\alpha_R$ and $\alpha_E$, also called sensitivity factors, which are supposed to reflect the ratio between the respective variability.

The weight factors are defined in such way that the overall target reliability index is achieved given that the probability that the actual effect is less than the design load effect is $P(E > E_d) = \Phi(\alpha_E \beta)$ and likewise that $P(R > R_d) = \Phi(\alpha_R \beta)$ The significance of the weight factors can also be visualized as in Figure 6, in which $\alpha_E \beta$ and -$\alpha_R \beta$ form the coordinates of the design point P, i.e. the closest point in the design limit state to the point of mean resistance and load effect. The recommended values for the sensitivity factors are $\alpha_R = 0.8$ and $\alpha_S = -0.7$.

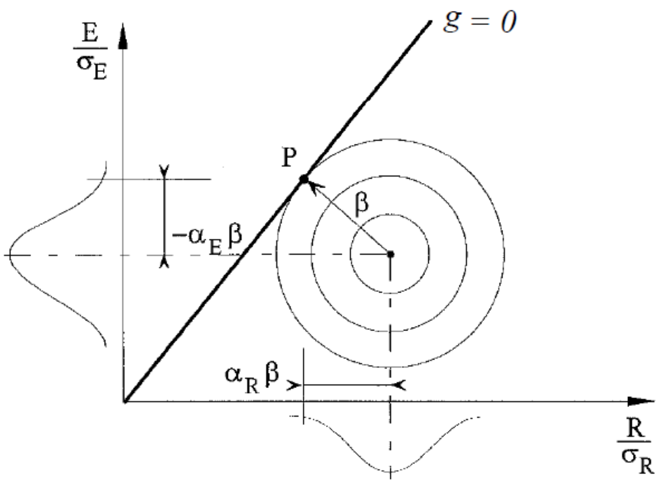


*Figure 6: Design point and reliability index $\beta$ for normally distributed uncorrelated variables (CEN, 2002)*

The design values of load effect and resistance are generated through the application of partial safety factors which are used to first adjust characteristic values of input parameters and then also to amplify or reduce the respective resulting values from calculations to account for model uncertainty. The design resistance can be written as:

$$R_d = R\left\{\frac{f_{ck}}{\gamma_c}, \frac{f_{yk}}{\gamma_s}, a_d\right\} /\gamma_{Rd} \tag{2.7}$$

With regards to resistance the derivation of partial factors is based upon the assumption that the resistance R can be calculated as a product of the nominal resistance and factors expressing the involved material, geometrical and model uncertainties. The uncertainties can then be measured in terms of coefficients of variation denoted $V_f$, $V_g$ and $V_m$ respectively, which in turn can be used to express the coefficient of variation of the resistance:

$$V_R = \sqrt{V_f + V_g + V_m} \tag{2.8}$$

From this reasoning the partial safety factor for steel $\gamma_s = 1.15$ and the partial factor for concrete $\gamma_c = 1.5$ have been derived using the following expressions:

$$\gamma_s = \exp(\alpha_R \beta V_f - 1.64V_f)\ , \gamma_c = 1.15\exp(\alpha_R \beta V_f - 1.64V_f)$$

The set value of β in the above expression is, 3.8, is in accordance with earlier mentioned target reliability (reference period 50 years) and the number 1.64 is the index of the 95$^{th}$ percentile, used for determining characteristic values. The additional factor 1.15 in the expression for concrete has been introduced to account for long-term observations of concrete strength in real structures compared to values from initial testing (CEN, 2004).

Estimating the values of the coefficients of variation, which in current formulations are meant to cover a wide range of circumstances, is quite difficult and requires a combination of considerable testing, experience and engineering judgment. It is therefore also an open topic for discussion. As an example it was estimated by Schlune *et al.* (2010) that in the case of non-linear analysis of shear type failures $V_m$ was in the range of 10-40 %, whereas in the Eurocode recommended values of $V_m$ are given in the range of 2.5-5% depending on the considered material, see Table 2. Material uncertainty can for simple structures be determined directly as the COV of the input material parameters influencing the failure load. Otherwise the COV of resistance depending linearly on a number of parameters can be assessed from a sensibility study in which the covariance and partial derivative with respect to different, steel and concrete, parameters are established. $V_g$ is as shown in table 2 specified as approximately 5%, e.g. by JCSS (2001).

*Table 2: Coefficients of variation for the determining of Eurocode partial safety factors (Schlune* et al. *2010)*

| | **Steel** | **Concrete** |
|---|---|---|
| $\mathbf{V_m}$ | 2.5% | 5% |
| $\mathbf{V_f}$ | 4% | 15% |
| $\mathbf{V_g}$ | 5% | 5% |

In part 2 of EC2 (CEN, 2004) a modification of the safety format has been made to make it better suited for nonlinear analysis. When using this modified format the yield strength of steel and compressive strength of concrete used in the analysis are initially adjusted to be close to mean values and thereby constituting a more realistic behavior of the structure. The same safety factor as in the original format is then obtained by dividing the calculated resistance with an additional overall reduction factor proportional in effect to the initial increase of material strengths:

$$R_d = \frac{R(\tilde{f}_y, \tilde{f}_c, a_{nom})}{\gamma_0 \gamma_{Rd}} \tag{2.9}$$

For non-linear modeling one alternative method discussed has been to employ a semi probabilistic analysis in which global safety factors are estimated from the resulting resistance when using mean and characteristic values of input parameters. The working procedures based on this approach have however not quite been established, and to the extent such have been proposed they tend to be unsuited for analysis of shear capacity (Broo *et al.*, 2008).

#### 2.3.2.2 Schlune *et al.* safety format

For non-linear analysis a somewhat different safety format from that in EC2 was proposed by Schlune *et al.* (2010). According to this format an initial crude value of the resistance is calculated using mean yield strength of reinforcement steel, $f_{ym}$, mean *in situ* compressive strength of concrete, $f_{cm,is}$, and nominal values of geometrical parameters, $a_{nom}$. Similar to EC2, the design resistance is then obtained by dividing the initial crude value with a resistance safety factor $\gamma_R$:

$$R_d = \frac{R(f_{ym}, f_{cm,is}, a_{nom})}{\gamma_R} \tag{2.10}$$

Assuming a lognormal distributed resistance the safety factor according to Schlune et al. can be written as:

$$\gamma_R = \exp\,(\alpha_R \beta V_R) \tag{2.11}$$

As before, the overall coefficient of variation of the resistance, $V_R$, is the square root of the sum of COV for geometrical, model and material uncertainty, see previous chapter. Geometrical and model uncertainty are as in EN1990 (CEN, 2002) both given tabulated values based on earlier testing and estimations.

What is new is that the material uncertainty is estimated by first measuring the influence of the involved input parameters. This is done by performing a number of nonlinear analyses, with different combinations of mean and reduced values of the parameters and using the resulting resistances as weight factors for the respective variances. If the main factors limiting the resistance are assumed to be steel yield strength and concrete compressive and tensile strength the derivation of $V_f$ can be written:

$$V_f \approx \frac{1}{R_\mu}\sqrt{\left(\frac{R_\mu - R_{\Delta fc}}{\Delta_{fc}}\right)\sigma_{fc}^2 + \left(\frac{R_\mu - R_{\Delta fct}}{\Delta_{fct}}\right)\sigma_{fct}^2 + \left(\frac{R_\mu - R_{\Delta fy}}{\Delta_{fy}}\right)\sigma_{fy}^2} \tag{2.12}$$

, where $R_\mu$ is the resistance obtained from using mean values of all parameters; $R_{\Delta fc}$, $R_{\Delta fct}$ and $R_{\Delta fy}$, are the resistances calculated with, in sequence, reduced values of concrete compressive and tensile strength and steel yield strength. $\Delta_{fc}$, $\Delta_{fct}$, $\Delta_{fy}$ are the respective reductions of material strengths (Schlune *et al.*, 2010).

#### 2.3.2.3 First and second order reliability methods (Level II and III)

In the higher level reliability analyses a first order approximation of the closest point on a failure surface can be made through an iterative process following the procedure of:

$$\boldsymbol{u}^{i+1} = \nabla g(\boldsymbol{u}^i)\frac{\nabla g(\boldsymbol{u}^i)^T \boldsymbol{u}^i - g(\boldsymbol{u}^i)}{\nabla g(\boldsymbol{u}^i)^T \nabla g(\boldsymbol{u}^i)} \tag{2.13}$$

in which $\mathbf{u}^0$ is an assumed coordinate in the normalized stochastic variable space (Sorensen, 2004). Improved assumptions of the design point and corresponding β are made until convergence in β is reached i.e. until $|\beta^{i+1} - \beta^i|$ is less than a certain value. When a satisfactory approximation is found and if the stochastic variables are non-correlated the corresponding elements of the unit normal vector to the failure surface can also be used as a measure of the importance of the different uncertainties.

In the case of correlated and non-normal stochastic variables the procedure is the same, although a transformation of the variables into regular U-type variables is needed. The transformation of correlated variables, e.g. denoted Y, is done with the help of a transformation matrix T=Y\U derived from the correlation coefficients and in the case of failure functions of non-normally distributed variables, the variables X are substituted more or less directly via the relation between the distributions according to:

$$\Phi(U_i) = F_{X_i}(X_i) \Rightarrow g(F_{X_1}(\Phi(u_1)), \dots, F_{X_n}(\Phi(u_n))) \tag{2.14}$$

If a second order approximation of the failure function is used to obtain the reliability estimate, the procedure becomes more difficult. Although the failure function of certain values in the variable space can be expressed in analogy with the first order procedure, the iterative process is no longer applicable. Instead a value of the second order failure probability is determined via a number of orthogonal transformations of the variable space and solving of eigenvectors through Jacobi-iteration, see Ditlevsen & Madsen (1986).

If the failure surface is far from linear the non-linear estimate can be expected to be much more accurate. But also, the smaller the probability of failure is the smaller is the difference between the first and second order approximation and when $\beta \rightarrow \infty$ both the first and second order approximations converge to the exact one.

#### 2.3.2.4 Probabilistic analysis (Level III)

In general terms probabilistic modeling can be explained as any process that employs deduced probability distributions as input to calculate the probability distribution of a certain output. The most widely used technique for executing these models is called Monte Carlo simulation; reference to the famous casino comes from the random sampling ingredient of the method. Utilizing this approach comprises running a prescribed deterministic model a large number of times for different random input values. Specialized software is used for this purpose, which make sure that the random sampling is consistent with assumed relations and registers the produced results, to be presented e.g. as histograms showing the probable distribution.

As mentioned earlier, full probabilistic modeling can be used for determining structural reliability. The reliability is then expressed as the probability

$$P_s = 1 - P_f \tag{2.15}$$

, where $P_f$ stands for the adverse portion of the sampled outcomes. The first step of such a probabilistic analysis is to identify the basic variables and then to develop an appropriate model based on the uncertainties that are tied to the specific limit states at hand (CSSN, 2001).

Suitable probability distribution types and parameters for the physical variables, i.e. the material properties and geometrical parameters should preferably be taken from direct measurements presented in published large sample studies. If such are not available the distribution can be chosen from an experience based set of possible distributions, by sampling and evaluating using e.g. the method of maximum likelihood. If two distributions fit equally well the one resulting in the lowest reliability should be chosen.

The second step of setting up the model is to deal with the overlaying model uncertainties, which arise not only from the physical model of structural capacity but also from statistical uncertainty of e.g. moderately sized samples in the additional gathering of data mentioned above. This is generally done by adding a stochastic variable $X_m$ to represent the model uncertainty. The statistical properties of this, typically normally distributed variable are evaluated by comparing the results from the used model with results from either physical experiment or other more detailed models, such as FE models, the latter of which can ensure consistency of input.

In a full probabilistic analysis computation of the failure probability is then, as described above, done via the use of simulation methods. The problem when using crude Monte Carlo simulation techniques for structural reliability purposes is that the needed number of simulations, N, increases with decreasing probability of the modeled outcome. Since the failure probability is very small the consequence is that a very large number of simulations is needed to attain an adequate confidence level, as can be seen from the expression for standard error of the failure probability estimate:

$$s = \sqrt{\frac{\hat{P}_f\left(1-\hat{P}_f\right)}{N}} \tag{2.16}$$

In order to reduce the computational effort there are two categories of methods which can be used, namely indicator function methods and conditional expectation methods (JCSS, 2001). The aim of these methods is to reduce the variance of the failure probability estimate, without having to increase the sample size. Of the former methods the perhaps most relevant is importance sampling, see Figure 7a, in which an initial FORM or SORM analysis is used to obtain a likely failure point (design point, generally expressed for all involved stochastic variables, see previous chapter) and then concentrate the sampling points to the vicinity of this point. Similarly, it is also often possible to, as illustrated in Figure 7b, deduct a large portion of the sampling space where the probability of failure is known to be zero and therefore no calculation of the failure function is needed. However, in some cases it can be difficult to define a single important sampling region. Then direction sampling, which belongs to the second of the above mentioned categories, is another commonly used technique, which uses so called conditional expectation in the normalized stochastic variable space and generates precise information on the position of the failure limit.

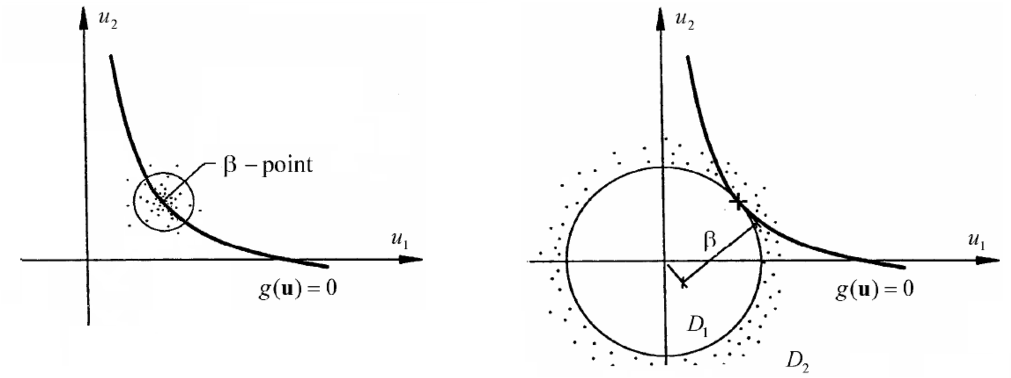


*Figure 7: Principle illustrations of alternative sample reduction techniques (modified from Sorensen, 2004)*

As stated by Sorensen (2004), it is important that probabilistic models are as far as possible not influenced by subjectivity and therefore must use a formalized reference, otherwise results do not reflect the actual reliability of the structure but can merely be used for rough comparison. It is further noted in CEN (2002) that full probabilistic methods give in principle correct answers to reliability problems, but Level III methods are seldom used in the calibration of design codes because of the frequent lack of data.

#### 2.3.2.5 System reliability analysis

The reliability methods discussed in the preceding chapters are used to determine the reliability of individual components in a structure whereas as also hinted earlier, overall reliability is usually governed by a system of several more or less interdependent components with their own failure modes. Consequently, in order to perform a full evaluation of whether a structure lives up to the set target reliability or not, an additional system reliability analysis is needed. This aspect of structural reliability is outside the scope of the study presented in this report, but will still be briefly summarized.

The aim of the system reliability analysis is to convert various component reliability indices into one representative value of the system failure probability. The manner by which this should be done depends on to what degree the system should be regarded as a series or parallel system. Another way of expressing this is by saying that the structure can be designed to have a certain redundancy. An ideally redundant structure, constituting a parallel system, would keep fulfilling its purpose as long as there is at least one component that had not yet failed, whereas a system with no redundancy would fail capitally if not all involved components are intact. Consequently, the parallel and series systems have the opposite relations both to the number and the possible correlation of components involved in the system.

Finally, structural redundancy is also to some extent intertwined with the concept of robustness, i.e. what possible wide spread effect local accidental actions can have on a structure. Starossek, (2006) states that except when designing very long bridges robustness of structures is often neglected and in the cases where recent recommendations have been given concerning the issue these often lack general applicability. By and large, risks of low probability-high consequence types of actions are often left uncared for.

### 2.3.3 The Eurocode program and the JCSS model code

In 1975, The European Commission decided on an action program with the aim of eliminating obstacles to trade through harmonization of technical specifications in the field of construction. In a first stage, an initial set of uniform rules would serve as an alternative to national standards, before ultimately replacing them. A first generation of codes was established during the following decade, but it is only recently that the codes have started to become fully implemented.

The European construction code comprises standard regulations on the execution of design and assessment work within a number of sub disciplines of structural engineering. The most particular purpose is to facilitate compliance with essential requirements on mechanical resistance and stability, as well as safety in case of fire. The code does not however distinguish values with regards to levels of safety used in individual countries. These matters are included in the informative annexes of the code and left to be decided on by the responsible national regulatory authorities.

According to note 3.5.4 and 3.5.5 in the Eurocode basis for structural design (CEN, 2002), structural reliability models used in limit state design shall be formulated either according to the provided partial safety factor method, see chapter 2.3.2.1, or directly based on probabilistic methods, if agreed on by the relevant authority. In case the latter approach is used, the only governing rule is that the thereby determined design values should, as stated in note 6.1 (5), correspond to at least the same degree of reliability as intended by the use of partial factors given in the code.

The safety formats used in EC2, partly based on a probabilistic background, partly calibrated for design procedures of the past, have been designed to allow for easy application in design and assessment practice. However, as a result of the intended general applicability, the drawback of the safety formats is that they cannot always be entirely accurate (Vrouwenvelder, 1997).

A more consistent reliability should naturally result from using a more direct, full probabilistic approach, which is also sometimes implemented in design and assessment of particularly important structures. In order for such assessments, or evaluation of the present safety formats, to be conclusive it is required that accessible probabilistic data, on which the models should be based, is complete. As an aid to fulfill this requirement, with the aim of reducing the need for pragmatic and subjective decisions in the above processes, the Joint committee on structural safety, JCSS, has produced a comprehensive set of guidelines on how to construct stochastic models for structural analysis.

# 3 Analysis

To meet the objectives of this thesis a parametric study was conducted in which the shear assessment methods and safety formats described in the previous chapter were evaluated and compared. This chapter is aimed at describing the methodology used in preparation for this evaluation.

## 3.1 Choice of typical cross sections

The intention of the parametric study was to conduct evaluations of concrete cross sections which could represent conventional types of bridge girders. Considering that there is quite a range of possible designs this can seem as a rather major task. However, in the light of what was meant to be investigated, narrowing down the number of cross section configurations would perhaps not only be justified by limitations, but also did not have to significantly affect the generality of results.

It was concluded that if focusing on the design of simple horizontal concrete bridge superstructures there are in principle only two main alternatives; the structure can either be built as a solid slab or it can be shaped into a number of separate girders. The girders can then in turn be formed as simple T-shaped deck girders or joined at the bottom to form so called box girders. There are many ways by which these basic designs can be tapered and modified, but the question was whether such changes in configuration would have any effect on the reliability of capacity estimates and if such differences could be modeled.

As stated in CEN (2002): “Unusual forms of construction or design conditions are not specifically covered and additional expert consideration will be required by the designer in such cases.” Apart from the obvious extra effort needed, it was also found that since there was e.g. no guidance available on how this would affect variability of geometry and concreting quality there was neither any real point in trying to evaluate more intricately shaped beams. For that reason it was decided that the geometries used in the analysis should be kept simple and such that standard formulas given in the code applied.

It was assumed that in order to enable showing possible variations due to geometry in the sensitivity to variation of different parameters it would be sufficient to model a cross section with a distinct difference in width between top and bottom and then alternate which of the two ends was to be in compression or tension. Accordingly, in addition to a solid rectangular section, a regular T-section with the same web dimensions was defined to be loaded with alternatively positive and negative moment.

## 3.2 Choice of parameters

As shown in Table 3, the quantities to which primary focus was given in the parametric study of specific cross sections were the concrete strength and the amount of transverse and longitudinal reinforcement, reinforcement ratios $\rho_w$ and $\rho_l$. Different load cases were also defined by setting a varying ratio between moment and shear force.

*Table 3: Tested parameter values*

| | | | |
|---|---|---|---|
| **Concrete class** | C25 | | C45 |
| **Shear reinforcement ratio, $\rho_w$** | 0.1 % | → | 0.9%<br>1.3 % |
| **Longitudinal reinforcement ratio, $\rho_l$** | 0.4 % | → | 2.0 % |
| **Moment/shear ratio** | 0.5 | | 2.0 |

The ranges of parameter values were chosen with the aim of producing a general view of the variation of capacities and corresponding reliability indices for beams with common configurations. In some aspects however, they were also to some degree made in order to allow for focusing more specifically on parameters influencing shear capacity, i.e. configurations and loading cases for which variability of longitudinal steel parameters is of relatively little or, according to the deterministic EC2 shear model, no importance.

The amounts of shear reinforcement used in the parametric study were set so that the reinforcement ratio ranges from little more than the minimum allowed (eq. 9.5N in EN 1992, 2004) to slightly less than what is still effective with regard to full use of concrete strength. Both the minimum and maximum value depends on the concrete strength, characteristic and in-situ design value respectively, and the ranges of reinforcement ratios used were therefore varied depending on the applied concrete strength class. Also, the area per leg was altered for the different concrete strengths in order to keep the stirrup spacing within reasonable ranges.

Longitudinal reinforcement ratios were chosen so that they should fulfill requirements regarding ductility as well as minimum amounts. They were also chosen with respect to what seemed to be practical in terms of detailing.

## 3.3 Defining cross sections

All input parameters that were not mentioned in the previous section were given fixed values. As shown in Table 4, this includes the outer dimensions of the concrete cross section as well as the steel strength class.

*Table 4: Fixed input parameters*

| | |
|---|---|
| **Height** | 1.0 m |
| **Web thickness** | 0.5 m |
| **Flange thickness** | 0.25 m |
| **Effective flange width** | 2.5 m |
| **Steel type** | B500 |
| **Maximum aggregate size** | 16 mm |
| **Concrete cover** | 40 mm |

In order to define realistic cross sections consideration was given to the placing of reinforcement bars. Different reinforcement ratios require different disposition of the reinforcement bars resulting in a shifting ratio between the height of the section and the internal lever arm. Consequently, these two parameters could not be defined independently, instead the option stood between defining the size of the section in terms of either of the two. It was decided to use fixed concrete dimensions and let the lever arm result from, in this regard, optimal steel distributions. These were based on the most suitable longitudinal reinforcement bar dimensions within a range of 16 to 32 mm, a minimum spacing of the bars based on an assumed maximum aggregate size of 32 mm, as stipulated in the Swedish national construction code (Vägverket, 2004), an assumed minimum nominal concrete cover of 40 mm and a stirrup dimension of 16 mm.

In the Response-2000 non-linear analysis, stress-strain relations for concrete in compression was modeled by the standard equation given by Collins & Mitchell (1991) and the so called tension stiffening factor was set to the Response default value of 1.0. Softening as a result of transverse tensile strains was modeled according to Vecchio and Collins (1986). Finally, based on the custom equation in Response-2000, the stress-strain behavior of the reinforcing steel was modeled by a linear relation until yielding, followed initially by a constant stress and then, at a default strain of $7 \cdot 10^{-3}$, a quadratic strain hardening relation until the maximum stress was reached.

## 3.4 Calculation of resistance according to EC 2

As previously mentioned, in accordance with the shear model presented in EN1992-2 (CEN, 2004) the capacity of a concrete beam section is governed by separate limit state formulations with regard to shear and bending capacity. Concerning shear there are principally two equations describing the capacity. Assuming that vertical shear reinforcement stirrups have been provided these can be written as:

$$V_{Ed} \leq V_{Rd} = min \begin{cases} V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta \\ V_{Rd,max} = \alpha_{cw} b_w z\, v_1 f_{cd} / \cot\theta \end{cases} \tag{3.1}$$

The shear resistance is however connected to the moment resistance via the condition that enough bending reinforcement should be applied to provide not only the tensile force needed for moment resistance but also an additional force $\Delta F_{td}$ needed for shear transfer. The infliction of this requirement on the shear capacity can via substitution be described as:

$$\Delta F_{td} \leq \frac{M_{Rd}-M_{Ed}}{z} \;\; , \;\; \Delta F_{td} = 0.5\, V_{Ed} cot\theta \;\; \xrightarrow{yields} \;\; V_{Ed} \leq 2\, \frac{M_{Rd}-M_{Ed}}{z cot\theta} \tag{3.2}$$

Normally, the need for sufficient anchored bending reinforcement would be dealt with separately and then would not be directly relevant in the shear strength assessment. However in order to allow for comparison with results obtained from Response-2000 the two assessment procedures had to be made consistent. In other words, the moment and shear analyses of EN 1992 had to be combined.

### 3.4.1 Internal lever arm

According to CEN (2004) the internal lever arm z, governing the shear response of a concrete cross section, can be approximated as 0.9d. There is however no specific guidance on how to determine the value of d. For calculation of moment resistance d is usually seen as the weighted lever arm of the bending reinforcement, but if this approach is applied for shear resistance calculations it would mean that adding longitudinal reinforcement would result in a reduction of the shear capacity. This did not seem reasonable and therefore, as long as the moment resistance was not a governing factor, the value of d was set as the height from the compressed edge of the beam to the outer layer of tensile reinforcement, as in the case of minimum longitudinal reinforcement. For evaluation of whether the longitudinal reinforcement can provide the additional tensile forces needed for shear transfer, the value of d was still treated as based on the gravity center of all the bars in tension.

### 3.4.2 Design resistance

Calculation of resistance according to the EC2 shear model for component check is a deterministic, and at least calculation-wise linear, analysis. Therefore the EC safety format for linear analysis was used. As described in chapter 2.3.2.1, the alternative safety format for non-linear analysis could also have been used; however by definition it should have given the same result. Design moment resistances of the cross sections were also calculated using the basic partial safety factor format given in EN 1992-1 (CEN, 2004a). No strain hardening of the steel was allowed for and a parabola shaped ultimate state stress distribution in the concrete compression zone was assumed.

Design in-situ concrete strengths were taken in accordance with EN 1992-2 (CEN, 2004), in which it is stated that for all strength classes in-situ strengths are derived by multiplying the concrete cylinder strength with a correction factor $\alpha_{cc}$, taking into account time dependent effects and unfavorable effects from specific loading conditions. Following the EN recommendations concerning design of buildings the value of $\alpha_{cc}$ should be taken as 1.0, whereas for bridges the same value is 0.85. The correction factor $\alpha_{cc}$ is however a nationally determined parameter and it is also treated in various ways in different countries of the EU. In the Swedish annex to EN 1992-2 the recommended value of $\alpha_{cc}$ is 1.0, but since JCSS and EC2 should have the same considerations in mind when they set their recommendations the value of $\alpha_{cc}$ was still chosen as 0.85, which also to a higher degree enables testing the consistency of recommendations given by the two functions, in principle both intended for the same purpose by the European Commission.

## 3.5 Calculation of resistance with Response-2000

The assessment of resistances using Response-2000 was done as prescribed by Bentz (2000). As described previously, when assessing design shear resistances using Response-2000 the acting moments and shear forces are combined in an evaluation of equilibrium. Separate failure modes can then be distinguished from interpretation of resulting strains, but there is no distinction between moment and shear capacity. For comparison of design resistances the ratio between acting moment and shear was set in accordance with the EC2 resistance analyses.

### 3.5.1 Defining cross sections

Sample input in text file format was created as prescribed by Bentz (2000) using a preprocessor, see Schlune *et al.* (2010)

### 3.5.2 Design resistances

The assessment of design resistance was done using the EC2 safety format for non-linear analysis, described in chapter 2.3.2.1 and the safety format proposed by Schlune *et al.* (2010). The latter was used both as previously prescribed and with a modification, see equation (3.3), whereby the COV representing geometrical uncertainty was increased to compensate for the large variability of stirrup spacing reported by Turan *et al.* (2007).

$$V_g = \sqrt{V_g^2 + \frac{\sigma_s^2}{R_\mu}\left(\frac{R_\mu - R_{\Delta s}}{\Delta s}\right)} \tag{3.3}$$

## 3.6 Probabilistic analysis

A full probabilistic evaluation of design resistances of the various cross section configurations was conducted using crude and bounded Monte Carlo simulations, based on the previously described estimates of the variance of involved parameters. The resulting reliability indices when using stipulated safety formats were determined, as well as the design resistance corresponding to the target resistance reliability index $\beta = 3.04$.

### 3.6.1 Modeling of uncertainties

All the definitions concerning the variability and correlation of statistical parameters involved in the probabilistic analyses were made in accordance with the JCSS model code (JCSS, 2001). The resulting distributions can be seen in Table 5 and some explanations are then given in the following subsections.

*Table 5: Stochastic model*

| | | Distribution | Mean | COV |
|---|---|---|---|---|
| Materials | | | | |
| C25 | Compressive strength | | 26.9 MPa | 16.2 % |
| | Tensile strength | | 2.7 MPa | 32.0 % |
| C45 | Compressive strength | | 36.7 MPa | 10.6 % |
| | Tensile strength | | 3.3 MPa | 30.9 % |
| B500 | Yield strength | N | 560 MPa | 5.4 % |
| | Ultimate strength | N | 605 MPa | 6.6 % |

| | | | | |
|---|---|---|---|---|
| Geometry | | | | |
| | Height | LN | 1000 mm | 1.0 % |
| | Width of web | LN | 500 mm | 1.4 % |
| | Concrete cover to bottom reinforcement | LN | 40 mm | 25.0 % |
| | Shear reinforcement spacing | N | - | 10 % |
| | Area of reinforcement cross sections | N | - | 2 % |
| Model uncertainty | | | | |
| EC2 | JCSS - Shear capacity | LN | 1.4 | 25.0 % |
| | JCSS - Moment capacity | LN | 1.2 | 15.0 % |
| Response | Bentz | LN | 1.05 | 12.0 % |
| | Bohigas | LN | 1.07 | 17.4 % |

#### 3.6.1.1 Concrete parameters

The concrete strength distributions used in the analysis are described by sets of parameters listed in the model code, which take into account both the variability derived from direct strength testing as well as the spatial variations in situ, e.g. in terms of curing and hardening conditions. Similar to when deriving design values according to CEN (2004), time dependent in-situ effects are also accounted for via a correction factor $\alpha(t,\tau)$, here with the approximated value of 0.8, whereas effects of creep are disregarded in order to obtain an equally general applicability.

#### 3.6.1.2 Steel parameters

Ultimate and yield strength of steel are said to be obtained as the sum of three stochastic variables representing differences between individual mills and melts as well as variations within melts. COVs of the mean steel parameters as well as mean reinforcement area are provided; variance in these properties along bars is assumed to be negligible. The mean yield strength is taken as the characteristic value plus 2 standard deviations and the mean ultimate strength is calculated in accordance with EC2 (CEN, 2004). Full correlation of steel parameters is assumed between bars.

#### 3.6.1.3 Geometrical uncertainty

The variance of concrete dimensions and cover to reinforcement were also given in agreement with JCSS model code. In the latter respect, effective depth variations of the bottom reinforcement are the ones considered.

Although variability of the effective flange width could possibly be seen as a geometrical uncertainty it is rather an effect of loading conditions and therefore not part of the probabilistic resistance model. Furthermore, it has previously been seen that neither variations of effective width nor thickness of the flanges are of great importance (Turan *et al.*, 2008), hence the latter of the two is also not modeled.

A normally distributed spacing of shear reinforcement stirrups with a coefficient of variation 0.1 was assumed based on the study by Turan *et al*. (2008), in which 1,600 measurements were taken on 8 in situ, mid 20-th century, reinforced concrete deck girder bridges. The COV corresponds to the variability of mean spacing within the horizontal length of a traditionally idealized 45 degree shear crack, expectedly representative in high shear regions.

As for all other uncertainties, the geometrical dimensions are represented by a prior probabilistic model, i.e. a model based on the assumption that nominal dimensions originate from a survey of construction documents, not from control measurements in-situ. Naturally, if such measurements were available the uncertainty should be significantly reduced.

#### 3.6.1.4 Model uncertainty

Random variables describing the uncertainty of the Response-2000 resistance model were derived both from the study on shear design methods for high strength concrete conducted by Bohigas (2002) and from observations made by Bentz (2000). Factors of 1.07 and 1.05 respectively were given as representative for the average conservative bias of models and the COV of these factors were said to be 0.1739 and 0.12. The first of the two estimates was made on the basis of 123 strength tests on shear reinforced beams failing in shear and the other on a collection of 534 corresponding test results. Bentz however does not distinguish between concrete with or without transverse reinforcement and according to Bohigas the discrepancy between these two sets is significant, wherefore more focus has been put on the first set of parameters.

For the shear resistance model in EC2 (CEN, 2004) JCSS (2001) provides a mean bias of 1.4 and a COV of 0.25. This is said to hold for reinforced concrete section were no additional normal force is applied (Braml *et al.*2009)

In the EC2 analyses the constriction of shear resistance due to need for sufficient longitudinal reinforcement is a factor dictated by moment resistance. Correspondingly, the limitation of shear capacity set by available surplus tensile capacity in the longitudinal steel bars is treated as governed solely by the model uncertainty of moment capacity. Statistical parameters used for this model uncertainty, bias = 1.2 and COV = 0.15, are also derived from the JCSS model code (JCSS, 2001) and the resulting equations used to determine stochastic values of the capacity are shown in the following expression:

$$V_R = min\begin{cases} \Phi_V \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta \\ \Phi_V \alpha_{cw} b_w z\, v_1 f_{cd} / \cot\theta \\ 2\, \frac{\Phi_M M_R - M_E}{z cot\theta} \end{cases} \qquad (3.3)$$

, where $\Phi_V$ and $\Phi_M$ denote the model bias with regard to moment and shear capacity respectively.

### 3.6.2 Monte Carlo Simulation

Reliability indices and probabilistic design resistances were computed as described by Schlune *et al.* (2010a, 2010b). For the deterministic EC2 shear analyses the computer software Matlab was used while the probabilistic analyses with Response-2000 were done using a pre- and postprocessor written in Python. The random samples of input variables for the Response-2000 resistance evaluations were produced using randomizing functions in the mathematical toolboxes Numpy and Scipy.

For the computationally heavy analyses using Response-2000 the number of required function evaluations was reduced by the use of bounds, setting the limits of a safe variable space. Since the limit function of the shear resistance is monotonous or inverse monotonous with respect to all the involved variables, bounds could be created using the approach of Rajabalinejad (2009). Accordingly, a certain number of initial function evaluations were used to determine the combinations of minimum or maximum values for which the function yields a safe result. In accordance with Schlune *et al.* (2010b) the procedure of deriving the bounds was then enhanced through shifting of the random variable distribution to center around the values constituting the limit states, i.e. the before determined design resistances.

The number of simulations needed to achieve an acceptable confidence level was estimated following the guidance given by Waarts (2000). When conducting the parametric study of typical cross sections Monte Carlo simulation of random input variables was run 200000 times, according to the formula (Waarts, 2000):

$$N > \frac{1}{V\left(P_f\right)^2}\left(\frac{1}{P_f} - 1\right) \tag{3.4}$$

This corresponds to a COV of the failure probability $V(P_f) = 6.5\%$ and a standard deviation of the beta value[1] $\sigma(\beta) = 0.02$ at $\beta = 3.04$. Bounds were created from 3000 initial analyses, which approximately reduced the number of analyses in the subsequent Monte Carlo simulation by a factor of 20.

The design resistance corresponding to target reliability was found via iteration around an estimated value based on earlier determined indices obtained from using the two safety formats.

## 3.7 Additional comments on the choice and modelling of parameters

In this section a number of supplementary notes have been added to elaborate on some considerations made when setting up the analysis. These are aimed at explaining some personal choices and approximations as well as details on how available guidance in literature has been interpreted.

### 3.7.1 Design of cross sections

When choosing which parameters were interesting to study the influence of, and which could be treated as fixed, the focus was on enabling showing trends in the fluctuations of obtained reliability indices and comparative strength estimates for

[1] Waarts (2000) recommended value of $V(\beta)$ for FEA reliability calculations is 0.05, i.e. $\sigma(\beta) = 0.152$

different beams, without having to change that many parameters. It was assumed that as long as the COV of parameters remain constant it is not so much changes in size of parameters that result in differences, but rather changes in ratio between parameters.

#### 3.7.1.1 Geometry and material parameters

Regarding the outer dimensions of the beam cross sections it was deduced that it is mainly the relation between concrete and steel area, i.e. the reinforcement ratio, that is important, not so much the size and shape of the concrete area. The exception from this rule is the size of the flanges which in effect changes the ratio between steel and concrete area even though it does not affect the nominal reinforcement ratios $\rho_l$ and $\rho_w$.

The relation between steel and concrete strength was also of prominent importance, wherefore two different concrete strengths were used. An even larger span in the relation between steel and concrete strength would have been achieved if also the steel strength had been varied, but this was deemed superfluous.

Another reason for changing material strength parameters is that to some extent the COV of both concrete and steel strength parameters change when the strengths change. For concrete this fluctuation was shown to mainly regard compressive strength, for which the COV, as prescribed by JCSS (2001) was 50 % higher for C25 concrete than for C45, see Table 5. Also according to JCSS, $\sigma$ of steel should be a fixed value. This would obviously result in larger strength variability for low strength steel and would perhaps form a reason for including this variable in the parametric study. It is however debatable whether it is the $\sigma$ or COV that should be treated as fixed and there have been studies presented which point in either direction (JCSS, 2001).

#### 3.7.1.2 Amounts of reinforcement

The amounts of reinforcement used in the parametric study were set with quite large spans within the maximum allowed ranges set by EC2 detailing rules. Although some consideration has been given to practicality it is therefore still possible that some of the combinations of values may not be very likely to be used in real cases.

Regarding the total amount of longitudinal reinforcement, there are recommendations that, at the bottom side of bridge girders, 25 % of the reinforcement needed to resist the maximum moment should be continued over intermediate supports (Vägverket, 2004). This is however, even within a certain distance from the supports, usually more than is apparently needed with regard to moment capacity. It was therefore decided that this recommended value makes for a good assumption of approximate ratio between reinforcement amounts on compression and tension side in ordinary high shear regions.

In the estimation of resulting internal lever arms when using certain amounts of steel, bar diameters were assumed to be less than 32 mm. One reason for this choice was that for larger sizes of bars certain additional requirements apply regarding prevention against loss of bond through spalling of concrete cover (CEN, 2004a).

### 3.7.2 Ratio between moment and shear

To get an idea of what range of ratios between designing moment and shear force was reasonable to use in the parametric study a simple analysis was made in which a live point load was applied over the length of a three span bridge. Resulting moments and shear forces were evaluated for each loading case and maximum moments and shear forces in each section was gathered together with the concurrent ratios between acting moment and shear force.

The dead weight of the bridge cross section was approximated based on the cross section dimensions chosen for the parametric study, a concrete density of 2500 kg/m$^3$ and 100 mm paving of density 2000 kg/m$^3$. The relation between the lengths of the inner and outer spans of the bridge, 17 and 14 m respectively, were chosen so that maximum resulting field moments should be fairly equal and the live load was then selected so that it should give rise to a maximum reaction moment approximately corresponding to the EC2 design moment obtained when either using C25 concrete and longitudinal reinforcement ratio = 0.5 % or C25, $\rho_l$ = 2.0 %. To further vary the ratio between live and dead load the total width of the section was also varied from 1-7 times the width of the web, $b_w$.

From the results shown in Figure 8, it can be seen that the ratios between moment and shear corresponding to maximum sectional load effects are rather wide spread and ranges from 0 to 11 over the full length of the bridge. However, when looking at maximum sectional shear forces it is also evident that these are sometimes dominant over moment in the so called high shear regions near the supports. Since the focus of this thesis is mainly meant to be on sectional shear capacity it was decided that the more extreme low values were to be assumed, as can be seen in Table 3.

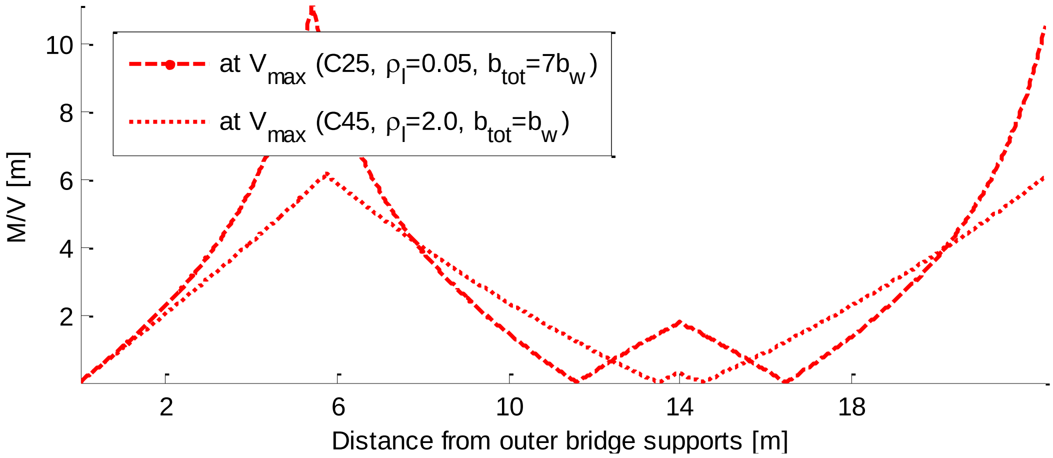


*Figure 8: Absolute value of Moment/shear-ratio (M/V) at maximum sectional shear forces along the bridge (only half of the bridge model is shown due to symmetry)*

### 3.7.3 Correlation of steel parameters

The coefficients of variation of steel strength parameters recommended by JCSS are based on studies of individual bars. Although it is reasonable to assume that some correlation exists between the variability of the bars used in one cross section the extent and type of this correlation is debatable. Most conservative would be to assume that the relative deviation of area is the same for all reinforcement bars and that all steel comes from the same batch, i.e. the same part of the same steel melt, but this would not be very realistic.

Apart from the question of whether correlation of bars exists within bundles, it could also be argued that full correlation between transverse and longitudinal reinforcement would be unlikely. The question of whether this assumption could be considered feasible would then e.g. depend on whether or not the same dimensions of steel bars have been used for transverse and longitudinal reinforcement and in the first case if the stirrups have been bought as separate bundles or formed from straight bars at the construction site.

When considering the EC2 shear model the choice of treating the longitudinal and stirrup reinforcement parameters as correlated or uncorrelated will affect the estimated reliability index of cross sections for which the two are of equal or almost equal importance, i.e. where the optimal inclination of the compressive struts is a function of both the vertical and longitudinal steel yield strengths. Consequently, at least when modeling shear capacity according to EC2, the types of beams for which this modeling choice is important are those for which moment capacity is limiting and since this study is aimed at investigating the reliability of beams failing due to shear it is probably not of great importance. When modeling according to the modified compression field theory however, making the distinction of concerned beams and loading cases is not so easy. This is due to the fact that this model takes into account other factors such as cracking behavior, largely affected by longitudinal reinforcement configuration.

In the general recommendation by JCSS (2001) it is appreciated that the correlation coefficient between the yield forces of individual bars with equal diameter can be set to 0.9 and for bars of different diameter the same coefficient can be set to 0.4. As an example, for cooperating bars of equal diameter this would, according to Bienaymé formula, theoretically mean that the COV of the average strength and area should be reduced with 2 to 5 %, see Figure 9.

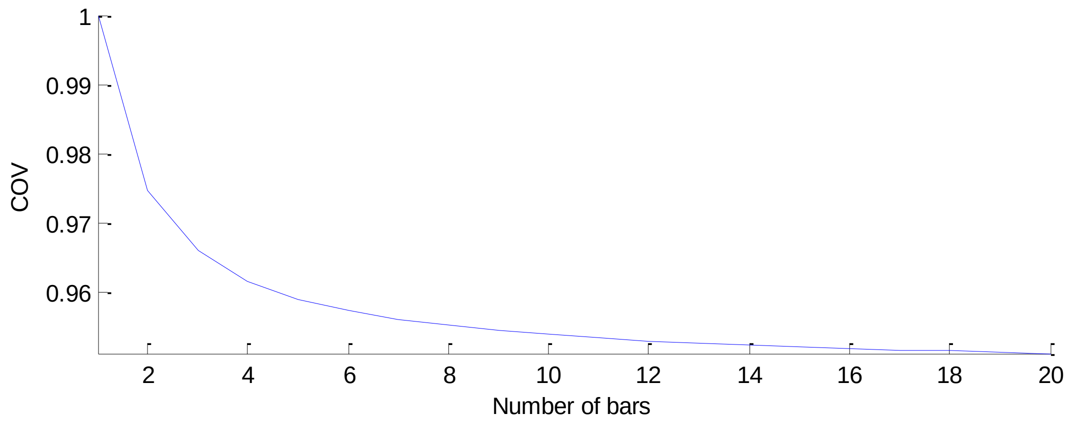


*Figure 9: Relative coefficient of variation of the yield force of bundles of 90 % correlated steel bars*

The steel yield force is however not the only stochastic steel parameter and enough information to model this correctly is hence not given. In mathematical terms another way of expressing this is by saying that it would not be possible to produce positive definite correlation matrices for the parameters of multiple bars. Random modeling of multiple bars separately would also be largely impractical and if the number of dimensions is increased this especially limits the gain of producing bounds as described in section 3.5.2. Since no conclusive recommendations have been given on how to treat the above considerations, and for the sake of computational efficiency, it was decided that the more conservative alternative would be chosen, i.e. to say that all bars are fully correlated.

### 3.7.4 Variability of stirrup spacing

As illustrated in Figure 10, the variability of the mean size of a population of uncorrelated variables decreases with the number of included variables. For that reason, it could be expected that in high shear regions, where shear reinforcement stirrups should be fairly tightly spaced, variability of mean spacing should also be relatively low if the length over which the mean was evaluated was kept constant. The study conducted by Turan *et al.* (2008) however showed that this difference was not very significant. In the regions within one fourth of a spans length away from supports the COV of mean stirrup spacing was only 10 % lower than for other regions of the beams, 0.09 instead of 0.1. It was therefore assumed in this study that the more conservative of the two variability estimates should hold for a general application, at least as long as the spacing was roughly within the same range as for the beams in the study by Turan *et al.*, s ≈ 0.2 d – 0.4 d.

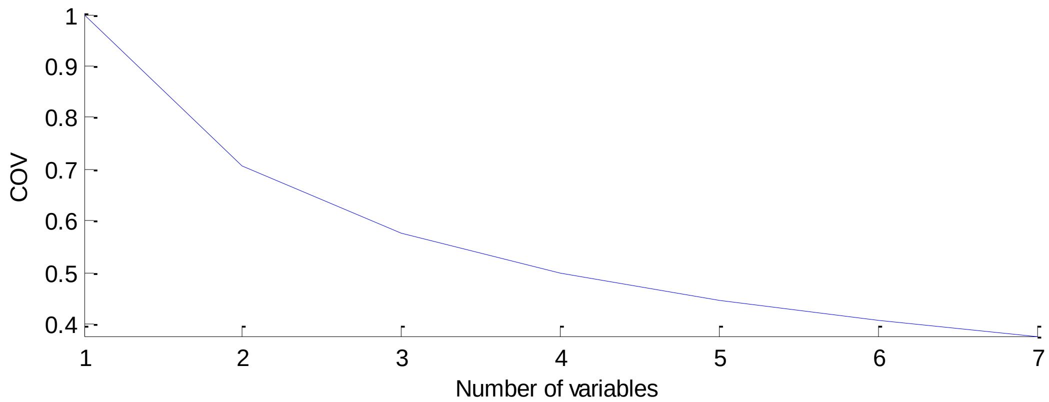


*Figure 10: Relative coefficient of variation of the mean value of uncorrelated variables (Bienaymé formula).*

Variability of stirrup spacing was derived from a study of American bridges built in the 60's. It could of course be questioned whether these values are directly applicable to bridges built in other countries, during more recent years and possibly with other degrees of quality control. The most objective way of treating the figures is however probably to assume that no such differences exist. In either case the study by Turan *et al.* was conducted because other sources of information was, and still is, not available.

### 3.7.5 Model uncertainty

As noted by Schlune *et al.* (2010) data to quantify model uncertainty of non-linear analyses are generally scarce and the scatter of results from different studies is large. According to a study of statistical parameters for EC2 model uncertainty when dealing with shear reinforced beams failing in shear by Braml *et al.* (2009), the estimated biases found in literature vary between 0.91 and 1.454 and the coefficients of variation range between 10 and 27 %. In other words, there is a large uncertainty surrounding the appropriate choice of model uncertainty.

Also, as mentioned earlier, it is essential to note that the reaction moment and shear force arising in a cross section come as result of the same stress distribution, they are not separate functions, and they should and cannot be treated independently. It is therefore not straightforward to define model uncertainties as being exclusively representative for either moment or shear capacity. One example of potential problems related to this issue is for instance given in the report by Bohigas (2002) in which it is stated that all the beams used to establish the estimate of model uncertainty failed in shear. Exactly where the line was drawn between moment and shear failure was however not defined.

It seems fairly clear that the model uncertainty is the largest factor governing the reliability of strength estimates. Therefore, it is also reasonable that the largest portion of efforts should be placed on defining and estimating this variable. It does not however make it less important to establish the other parts of the probabilistic model. In this perspective, Faber (2007) makes the fundamental point: ”Only if reliability assessments are performed on a standardized basis is it possible to compare reliability analysis results. Furthermore, only in this case is it possible to compare results with given requirements to the minimum acceptable reliability.”

# 4 Results and discussion

As described in the previous chapter, a series of probabilistic analyses were conducted based on the probabilistic framework set up by the JCSS (2001). The analyses were part of a parametric study with two main focuses, of which the first was to evaluate two different methods of performing shear analysis and the second was to assess the feasibility of two alternative safety formats which could be used for such analyses. The results from the performed evaluations together with some elaborating comments are presented in this chapter. A more complete but unprocessed account of the analysis results is found in Appendix A.

## 4.1 Comparison of alternative shear models

To assess the favourableness of either using the shear resistance model in EN1992 (CEN, 2004) or the one forming basis for Response-2000, the respective probabilistic design resistances corresponding to the target resistance reliability index of 3.04 were determined and compared. The comparison was meant to answer the question whether the initial hypothesis regarding the shear models could be verified; i.e. that the more accurate model used in Response-2000, with consequently smaller model uncertainty, would result in higher design resistances.

As described earlier, the intention was to compare beams of three different combinations of cross section shape and direction of moment. For some reason however, Response-2000 could not handle T-sections in which the flanges were loaded in tension. What remained was therefore to try the models on cross sections with varying width of the compression zone. The overall results from these comparisons are presented in Figure 11.

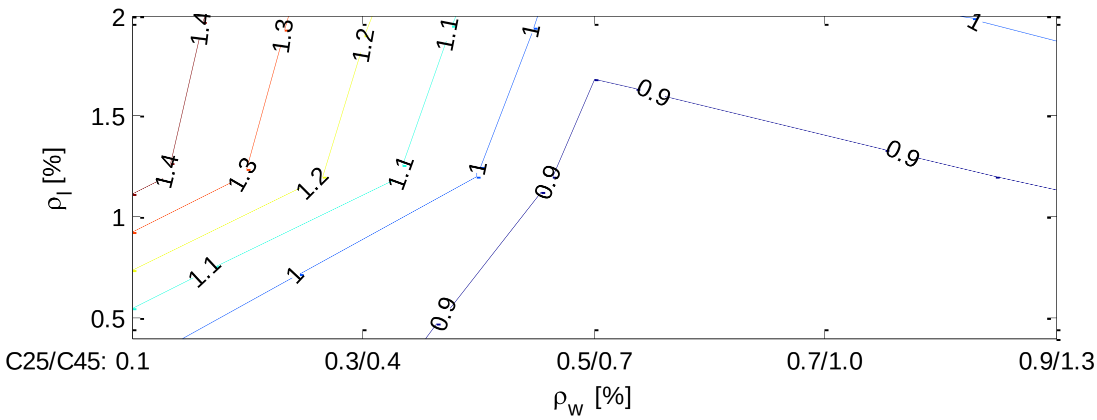


*Figure 11: Average ratio between probabilistic design resistances when using Response-2000 and EC2 shear model, $R_{d,\ Response\text{-}2000}/\ R_{d,EC2}$.*

The general observation from looking at the relation between obtained design resistances when using the two calculation models is first of all that in average the difference between the results is not that large, but also that the ratio varies depending on the amount of shear and longitudinal reinforcement. For a considerable portion of the beams it can be observed that the Response-2000 design resistances were in fact larger than ones obtained with the EC2 model, but for more than half of the range this was not the case.

In order to assess the reasonableness of the obtained relation between the two models a continued comparison was then made. This was partly aimed at investigating how the resistances change relatively with different altering of parameters, but also, to a large part, at finding convincing reasons for the distribution of the general results. Logically, the latter would have to do either with differences between the estimates of nominal resistances or with the corresponding variability of the modelled resistances.

### 4.1.1 Influence of shape, material strength and loading conditions

Apart from the differences incurred by varying configuration of reinforcement it was expected that some variation would also result from shifting the geometry and strength of the concrete section, as well as from changing the ratio between moment and shear. The effects of the last two of these factors are shown in Table 6. As can be seen, there were quite small incurred variations, but still a significant 5 % difference due to both of the changes.

*Table 6: Average ratios between probabilistic design resistances when using Response-2000 and EC2 shear model, $R_{d,Response-2000}/ R_{d,EC2}$, at different M/V-ratios and concrete strengths.*

| | **C25** | **C45** |
|---|---|---|
| **M/V = 0.5** | 1.03 | 1.08 |
| **M/V = 2** | 0.98 | 1.04 |

There was also an average 5% variation of the ratio depending on whether the section was rectangular or had an effective flange width equal to five times the width of the beam web. The largest difference, 10.5 %, was obtained for beams with little shear reinforcement and especially when loaded principally in shear, which would perhaps be explained by increasing uptake of dowel action accounted for in the Response-2000 analyses. However, none of the parameters looked at in this section had large influence on the average results.

### 4.1.2 Relative calibration of model uncertainties

The source of the model uncertainty is that in same way as the capacity of the described reinforced concrete beams, the accuracy of a model depends on both the specific design of the beam cross sections and the combination of load effects. The larger the range of situations the model is meant to cover the larger the variability of the ratio between real and estimated capacity. Also depending on the specific case, different model uncertainties represent more or less conservative estimates of real resistances.

As described earlier model uncertainties are generally specified in terms of two parameters; mean bias and COV of the relation between real and estimated capacity. Although it is not possible to determine which model is the most correct in specific cases, it can be concluded is that if the model uncertainty parameters for all models were calibrated based on the same empirical data the average mean value of expected resistances should also be the same irrespective of chosen calculation model. This is however only true if the range of cases on which the models are being tried is the same as in the calibration of specified uncertainties.

To investigate what possible differences there were in how the choice of parameters had influenced the conservativeness of estimated model uncertainties for the two models in the study, a comparison was made between the obtained estimates of mean resistances from using Response-2000 and the EC2 resistance model respectively. As presented in Figure 12, the comparison clearly showed that although the ratio varies within the range of chosen beams and loading ratios the estimate of mean bias of the EC2 shear model is distinctively less conservative than the one used for Response-2000, 12.4 % in average. This implies that the estimates of design resistance made with Response-2000 might be underestimations in comparison with the ones resulting from using the EC2 resistance model.

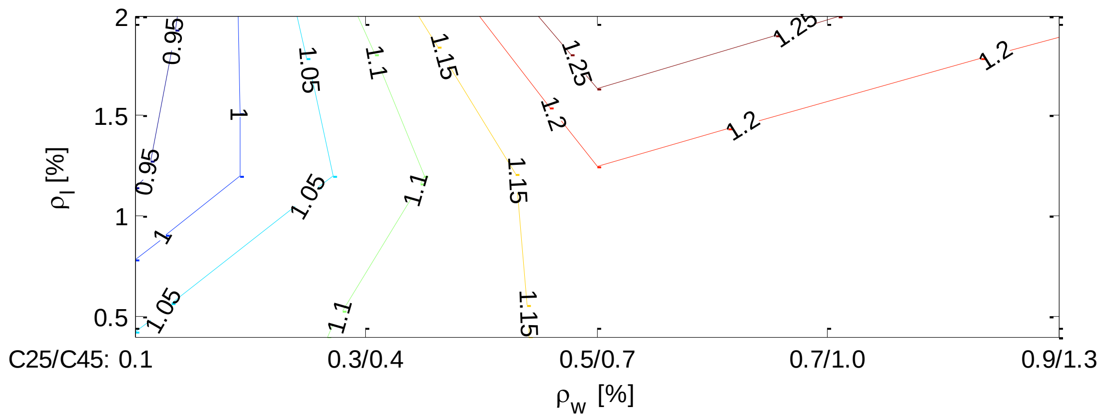


*Figure 12: Ratio between estimated mean values of resistances when using Response-2000 and EC2 shear model, $R_{\mu,EC2}/R_{\mu,Response\text{-}2000}$.*

As could have been expected, and also shown in Figure 12, the smallest ratio between expected mean shear resistances is obtained for beams with little shear reinforcement. One of the most obvious reasons for this is the larger influence of the additional shear transfer mechanisms accounted for when applying the modified compression field theory. Some increases of these observed ratios could also be expected both from using a larger maximum aggregate sizes and smaller difference between the amounts of added reinforcement on the compression and tension sides. Furthermore, another contributing factor is the smaller expected conservative bias of the EC2 model specified for the resistance of beams failing due to lack of moment capacity.

Another notable trend of the average ratios between estimated mean values of resistances when using Response-2000 and EC2 shear model, $R_{EC2}/R_{Response\text{-}2000}$ is more clearly shown in Table 7.Table 7: Average ratios between estimated mean values of resistances when using Response-2000 and EC2 shear model, $R_{EC2}/R_{Response\text{-}2000}$ The two shear models yield in average much more similar estimates of capacity when the moment to shear ratio is increased, i.e. when the moment resistance becomes more important. The difference was also slightly lower, 1.1 instead of 1.15, for beams with the higher strength concrete, but no conclusions could be drawn from this relation. However, this would go well in hand with the fact that the study by Bohigas (2002) was done on predominantly higher strength concrete beams.

*Table 7: Average ratios between estimated mean values of resistances when using Response-2000 and EC2 shear model, $R_{EC2}/R_{Response\text{-}2000}$, at different M/V-ratios and concrete strengths.*

| | **C25** | **C45** |
|---|---|---|
| **M/V = 0.5** | 1.22 | 1.14 |
| **M/V = 2** | 1.08 | 1.05 |

Given that the interpretation of how the biases of the EC2 resistance model should be applied was correct, the perhaps most important observation that can be made from looking at the results in Figure 12 is that the estimates of actual mean resistance match quite well in the area representing beams with small amounts of shear reinforcement. It was first thought that if not entirely conclusively this coincidence might possibly pose an indicative answer to the question of which beams and loading conditions the estimated model bias by Bohigas (2002) is applicable for. However, when looking more closely at what amounts of reinforcement were considered in the study made by Bohigas it was seen that the line in Figure 12 which represents full agreement between the two models does not fall within close proximity of the mean of those amounts. It was seen that although the shear reinforcement ratios as expected were much lower for the collection of tested beams, average 0.18 %, the average amounts of longitudinal reinforcement, 2.75 %, was well outside the spectrum used in this study. Also, the estimate of mean bias of the EC2 shear model presented here was much higher than the one given by JCSS, 1.8 instead of 1.4.

What instead appeared to cause the good correlation between predictions of mean bias observed in parts of Figure 12 was introduced after another parametrical analysis done by Bohigas (2002). Apart from verifying that the tested moment to shear ratios, 0.75 to 2.0 were within almost exactly the same ranges as used in this thesis, see Table 3 and Figure 8, the study by Bohigas showed first that the EC2 underestimation of shear resistance was significantly smaller for lower than for higher strength concrete, whereas for AASHTO resistances (quite similar to Response-2000) the bias was more or less the same, and second that the relation between the two models was the same with respect to how their biases were influenced by increasing longitudinal reinforcement amounts. Since the concrete strengths used in this study was found in the lower part of the spectrum in Bohigas's study, both of these relations indicated that the mean expected bias stated by the JCSS may be applicable in the here made comparison of models.

Collectively the tendencies indicate that both the estimated bias and model uncertainty for Response may also be applicable. The reasonableness of such an assumption is strengthened by the general trend which can be seen in the results presented by Bohigas, see Figure 13, together with the observation made by Bohigas: "For beams with the same geometric amount of transverse reinforcement, the higher their compressive strength, the higher their failure shear strength."

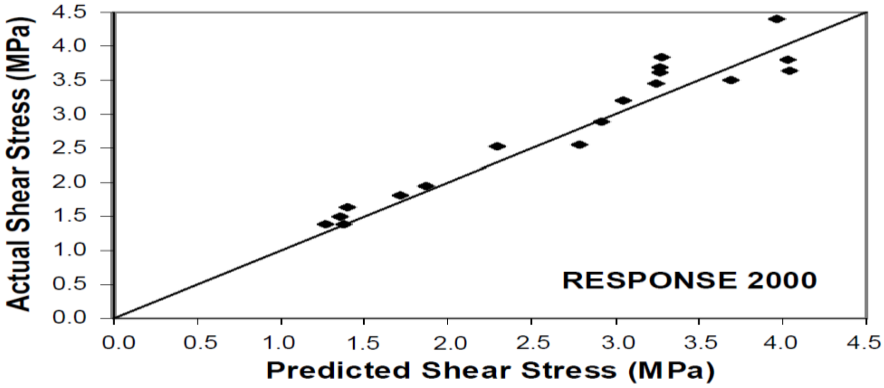


*Figure 13: Summary of measured shear resistance and predicted values when using Response-2000 (Bohigas, 2002).*

It could still be argued that in the comparison the specified model biases are only applicable for those beams for which loading capacity is dominantly restricted by shear resistance, which constitutes those beams reinforced with a large amount of longitudinal reinforcement and a close to minimum amount of shear reinforcement

### 4.1.3 Required safety margins

After the evaluation of dictated model biases, the next step was to look at the other factor influencing the design resistances, namely the safety margins between mean and design values. These would depend on the relative sensitivity of the calculation models to the natural variability of input parameters, as well as the difference between estimated scatter of model biases.

Bearing in mind that the implemented variation of model bias of the Response-2000 shear analysis was fixed for all input, it was not unexpected that the corresponding variation of the safety margin was much smaller than the one resulting from the EC2 analyses. Also, as shown in Table 8: Mean value and standard deviation of the required resistance safety factor $R_\mu / R_d$ for alternative shear strength models.Table 8, the results well reflected the overall relation between the respective COV.

*Table 8: Mean value and standard deviation of the required resistance safety factor $R_\mu / R_d$ for alternative shear strength models.*

| | **μ** | **COV** |
|---|---|---|
| **EC2** | 2.01 | 0.17 |
| **Response-2000** | 1.77 | 0.04 |

The comparison furthermore verified, see *Figure 14,* that the required safety margin between mean and design value was consistently higher for the design according to the EC2 shear model than it was when using Response-2000. The exception is when dealing with beams with very small amounts of longitudinal- and large amounts of shear reinforcement, failing predominantly due to moment, see Figure 15. This reflects that the model uncertainty in those cases is even defined as being smaller for the EC2 resistance model than for Response-2000, see Table 5.

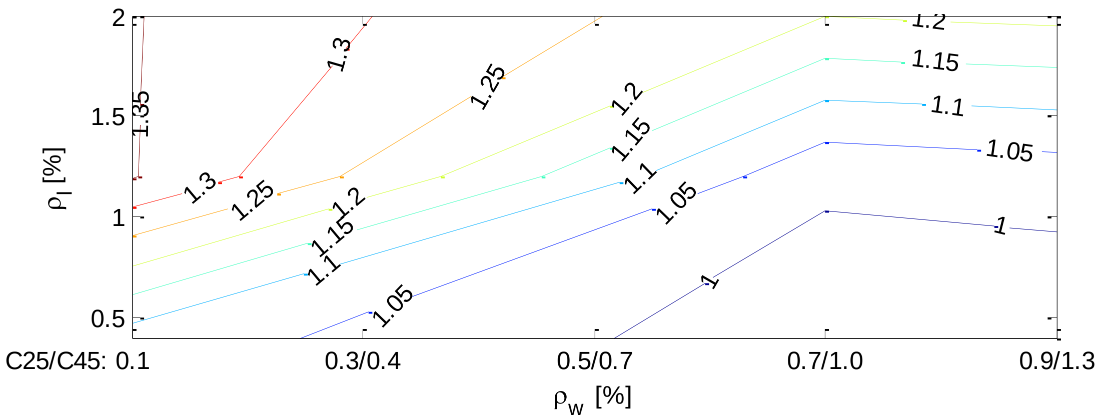


*Figure 14: Ratio between the required resistance safety factors when using Response-2000 and the EC2 shear model, $\gamma_{EC2}/\gamma_{Response\text{-}2000}$.*

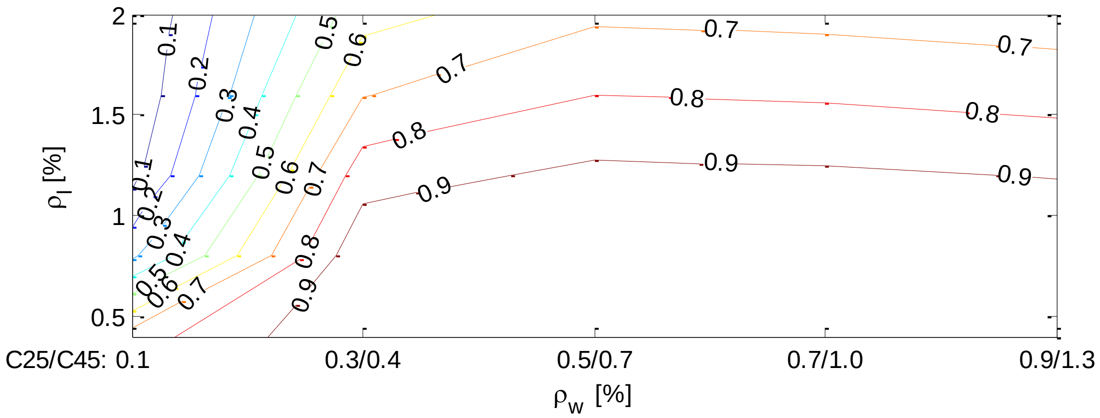


*Figure 15: Frequency of beams for which moment resistance is one of the limiting factors according to EC2 resistance model vs. nominal reinforcement ratios.*

Regarding the beams for which the difference is the largest, predominantly with little applied shear reinforcement, the discrepancy between the two models can also be credited to the fact that in the Response-2000 analysis the shear capacity is assured by means of more than one (uncorrelated) mechanism, which leads to a smaller number of extreme low values of the resistance.

Concerning the scatter of model bias of shear prediction, it is perhaps also worth mentioning that the predicted COV of the model bias based on the study made by Bohigas (2002) was 60 % higher than the one presented by JCSS (2001). Following the same reasoning as in the explanation to Figure 12 in section 4.1.2 this might not be a disagreement assuming that the number and variation of beams in the latter study was significantly smaller.

As when looking at the mean bias estimations in section 4.1.2, there was once more some variation of values depending on the loading and concrete strength. From studying Table 9 it can be seen that the rather small variation shown earlier in Table 6 can be explained by the fact that for beams loaded more dominantly with moment, the larger bias in the estimation of mean actual resistances is compensated by a smaller difference between the required resistance safety factors, $\gamma_R = R_\mu/R_d$ and vice versa. Yet again this reflects the relation between the estimated bias scatter of the EC2 resistance model when either representing beams failing mainly due to applied shear force or beams failing principally due to moment.

*Table 9: Average ratios between the required resistance safety factors when using Response-2000 and the EC2 shear model, $\gamma_{R,EC2}/\gamma_{R,Response\text{-}2000}$, at different M/V-ratios and concrete strengths.*

| | **C25** | **C45** |
|---|---|---|
| **M/V = 0.5** | 1.23 | 1.16 |
| **M/V = 2** | 1.06 | 1.08 |

### 4.1.4 The effect of reducing variability of stirrup spacing

Since stirrup spacing was expected to be the geometrical factor of most importance, it was decided to make an estimate of how much the reliability, or appropriate design resistance, could be increased by making in-situ measurements of the spacing. Beams were therefore modeled both with and without variability of stirrup spacing.

When looking at the difference in obtained design resistances, shown in Figure 16, it was however seen that the effect of this variation was quite small. In other words, very little increase of probabilistic design resistance would result from eliminating variability of stirrup spacing. The only noteworthy exception was for the beams with the least amount of reinforcement and dominantly loaded in shear. For these beams the probabilistic design resistance, according to the EC2 shear model, was increased with in average 3.7%.

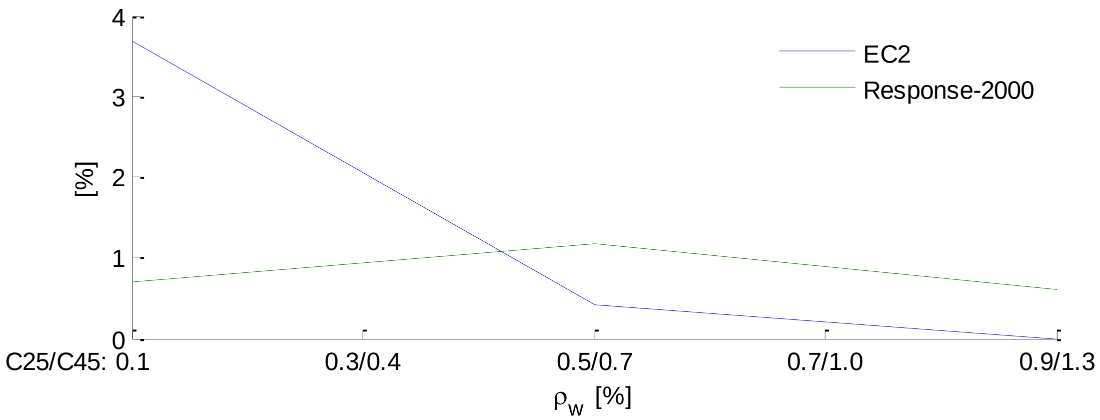


*Figure 16: Increase of probabilistic design resistance resulting from eliminating variability of stirrup spacing*

Even less distinct effects were shown when using Response-2000, which also demonstrates one of the differences between the models. Here the results show both that little influence can be expected both when there is an excessive amount of shear reinforcement and from changing the spacing when there is little steel and shear consequently is transferred by other actions.

### 4.1.5 Shortcomings of Response-2000 and potential effects on results

As mentioned in the beginning of this chapter, some problems occurred in the execution of Response-2000 when dealing with certain beam and loading configurations. It was however not investigated what caused the convergence problems, i.e. what made the analyses stop half ways for certain values of the concrete tensile strength. This raises questions whether this was an isolated problem or if there might be some less obvious and frequent errors also when running the other analyses. Due to the vast number of repeated analyses it was not practically possible to check whether some of the failures resulted from program errors. Consequently, this possibility cannot be completely discarded. Nevertheless the results were checked for any distinct and unexplainable deviations from the general trends and no noticeable differences were found.

## 4.2 Evaluation of safety formats

In this section the suitability of the two safety formats presented in EC2 (CEN, 2004) and by Schlune *et al.* (2010) was evaluated for use in shear strength analysis of reinforced concrete cross sections. This was done by determining and comparing the achieved level and consistency of reliability indices when using either of the two.

### 4.2.1 Evaluation of the EC2 safety format when used for EC2 shear analysis

One main part in reaching the aim of this thesis was to evaluate the performance of the EC2 partial safety factor format when used for shear analyses. Although not part of the comparison of safety formats, it was decided that this evaluation should include the performance of the safety format when used for EC2 shear analysis. This additional evaluation was motivated both by the fact that it could be expected that variability of stirrup spacing had not been used in the original calibration of the safety format and at the use of the two combined analyses and corresponding model uncertainties. It also did not involve any great extra effort since appreciation of the reliability indices for was a natural part of the following comparison between the two alternative shear models.

#### 4.2.1.1 General observations regarding the calibration of partial safety factors

The average reliability indices obtained for beams with given geometrical configurations and ranges of reinforcement amounts are shown in Figure 17. The figure shows the mean values of results obtained for the described T-section loaded with either positive or negative moments. Overall, when estimated on the basis of the JCSS probabilistic model code (JCSS, 2001) the reliability index is shown to be significantly lower than the one intended.

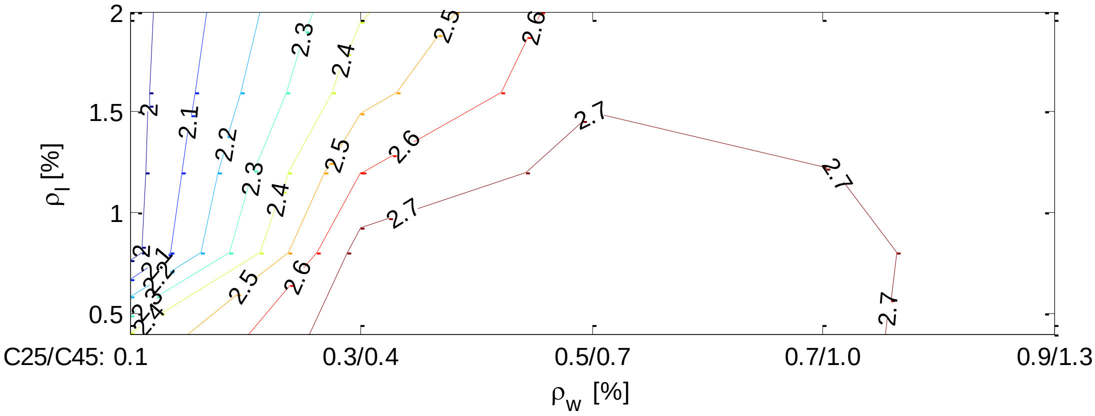


*Figure 17: Average obtained resistance reliability indices β from using the EC2 partial factor safety format for determining of design resistances*

Due to the great difference between estimated model uncertainties, made for the original calibration of the partial safety factor, see Table 2, or as presented by JCSS (2001), the general results are expected. The lowest indices are obtained for beams for which the ruling model uncertainty is the one representative for shear failure according to JCSS, as described in 4.1.3, see Figure 15. However, it was also observed that if only the first two limit functions were considered, see (3.1), there would still be an almost as large difference between beams with small and large amounts of shear reinforcement, which indicates that especially the partial safety factor for steel is not that well calibrated for determining design shear capacity.

The estimated levels of reliability were obtained with the most conservative choice of $\alpha_{cc}$ recommended in the main document of EN1992-2 (CEN, 2004). If instead the Swedish national recommendation had been used, the obtained β-index would have been even lower.

#### 4.2.1.2 Influence of shape, material strength and loading conditions

When looking at the average reliability obtained at varying loading condition and concrete strength it was seen that although they resulted in quite different graphs, see Figure 18, these factors barely made any difference when expressed irrespective of reinforcement ratios. It was also observed that if only the first two limit functions were considered, see (3.1), the higher strength concrete beams yielded comparatively lower reliability indices even though the COV of concrete strength is lower for high- than for low strength concrete. This probably comes as a result of the fact that for these beams there is a significantly larger discrepancy between the characteristic strength stated in EC2 and the resulting mean value from using JCSS strength parameters.

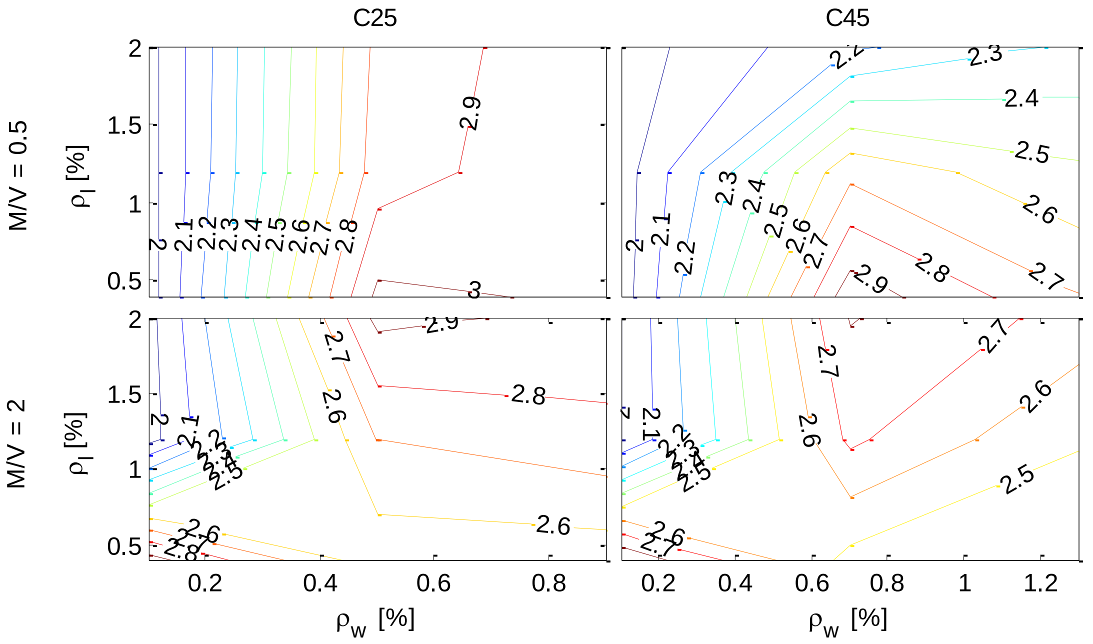


*Figure 18: Obtained resistance reliability indices β from using the EC2 partial factor safety format, at varying M/V-ratios and concrete strength.*

Regarding the cross section shape, it was observed that when the moment to shear ratio is low there is practically no importance of cross section shape, except for the beams with the least amount of longitudinal reinforcement. When the moment is increased, the effect of varying configuration will start playing a role in the sense that for T-beam cross sections which are loaded with moment in the more favourable direction, the corresponding reliability indices will be lower than for the beams with moment applied in the opposite direction.

### 4.2.2 Comparison of safety formats for Response-2000 shear analysis

The main part of the evaluation of safety formats was aimed at determining which of the two formats, EC2 (CEN, 2002) or Schlune *et al.* (2010), was most suitable to use when conducting Response-2000 shear analyses. From this comparison, see Table 10, it was found that using the Schlune *et al.* safety format generally led to more conservative results, and apparently also to a more uneven distribution of the reliability index. It should however be noted that, as described in the previous chapter (see eq. 3.4), the scatter of reliability indices is larger when the number of failures decreases. In other words, the coefficients of variation of the indices obtained when using either of the models may not allow a fair comparison. Another way of comparing the accuracy of the models could be to look at how the scatter of obtained β-values relates to the corresponding natural variation of β from running 200000 Monte Carlo simulations, at respective mean values: σ = 0.0146 and 0.0275. This instead showed that the relative variability obtained with the Schlune *et al.* safety formats was 22 % smaller than for the EC2 format

*Table 10: Mean value and standard deviation of the obtained resistance reliability indices β when using alternative safety formats.*

| | μ | COV |
|---|---|---|
| **EC2** | 2.84 | 0.07 |
| **Schlune *et al.*** | 3.28 | 0.09 |

#### 4.2.2.1 Separate evaluation of Schlune et al. safety format

When looking separately at the results from the evaluation of the Schlune Safety format, shown in Figure 19, it can be seen that the obtained reliability indices are not the same for the whole spectrum of reinforcement ratios. Also, if considering the average values from both M/V-ratios and concrete strengths, there is a clear trend of rising conservativeness with increasing reinforcement ratios. There are then, as in all previous evaluations, some minor differences depending on the above parameters, but more or less the same tendency is observed.

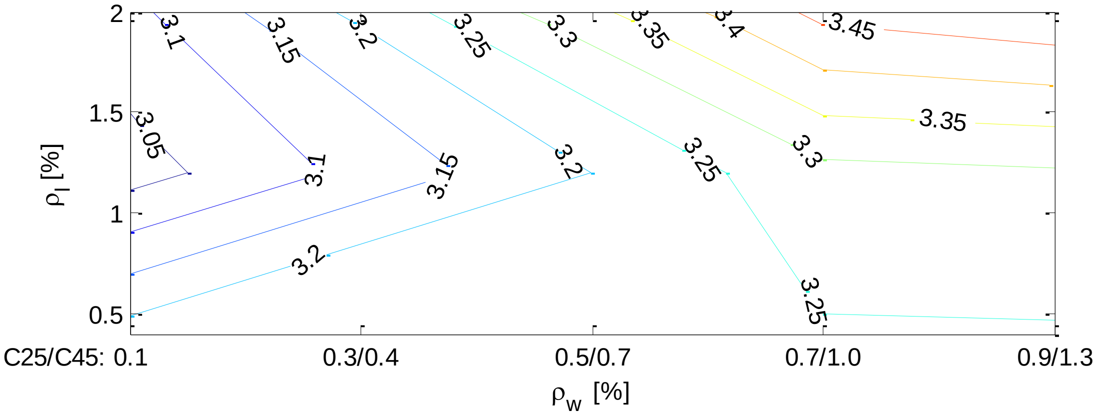


*Figure 19: Average obtained resistance reliability indices β from using the Schlune et al. (2010) safety format for determining of Response-2000 design resistances*

More significant differences due to loading conditions were shown in the mean levels of reliability, see Table 11: Average obtained resistance reliability indices β from using the Schlune et al. (2010) safety format for determining of Response-2000 design resistances, at varying M/V-ratios and concrete strength.Table 11. These were considerably higher when the applied shear was large in comparison to the moment.

*Table 11: Average obtained resistance reliability indices β from using the Schlune et al. (2010) safety format for determining of Response-2000 design resistances, at varying M/V-ratios and concrete strength.*

| | C25 | C45 |
|---|---|---|
| **M/V = 0.5** | 3.54 | 3.42 |
| **M/V = 2** | 3.00 | 3.13 |

Both of the observations made regarding the performance of the safety format indicate that there may be some calibrations needed to account for certain systematic variations. No attempts were made to investigate exactly what it was that caused the deviations. However, the estimated material uncertainty $V_f$ for all beams is gathered in Appendix A as aid for further development.

#### 4.2.2.2 Separate evaluation of the EC2 partial safety factor format

In the evaluation of the EC2 safety format there were, as shown in Figure 20, hardly any conclusive trends in the overall results. There also seemed to be little effects of different amounts of shear reinforcement.

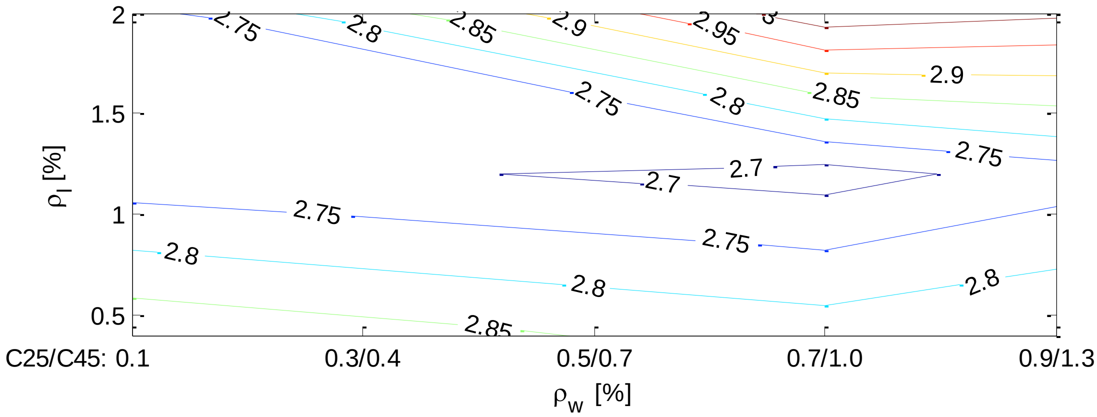


*Figure 20: Average obtained resistance reliability indices β from using the EC2 partial factor safety format for determining of Response-2000 design resistances*

Regarding the influence of loading condition and concrete strength, the reliability indices increased with increased moment to shear ratio, see Table 12. The most notable trend was that for the higher strength concrete beams.

The concrete cross section shape did not have any influence on the resulting average reliability index. It did affect the trends corresponding to those in Figure 20, but just as in the overall results these were difficult to define.

*Table 12: Average obtained resistance reliability indices β from using the EC2 partial factor safety format for determining of Response-2000 design resistances, at varying M/V-ratios and concrete strength.*

| | **C25** | **C45** |
|---|---|---|
| **M/V = 0.5** | 2.91 | 2.64 |
| **M/V = 2** | 2.92 | 2.84 |

## 4.3 Additional comments on the results

A general reflection regarding the results was that they in many ways demonstrate that, in some aspects, and to different extent, the model parameters on which the results are based are still incomplete. In particular what is referred to as the model uncertainty is rather vaguely defined.

As e.g. described by Bohigas (2002), there has been quite a range of tests made on reinforced concrete sections through the years. In his report he also illustrates how these empirical results can be used to show the approximate relation between real and estimated values of resistance based on certain geometrical-, material-, and loading parameters. If such information is available for a wide range of beams and varying loading conditions, this would form a relatively firm basis on which the model uncertainty can be estimated. In other words, the model uncertainty could be treated as a function of parameters, describing the sensitivity of the model bias to variation of input, which should be coupled with the statistical uncertainty of the predicted model biases originating from the limited number of tests used to establish the bias function.

Though the convenience of the above approach can be debated, it is the writer's opinion that it is inconsistent that, as it is at present the factor which is specified as model uncertainty can increase with the number and diversity of tests carried out to approximate it. Instead, it is reasonable to assume that the variability of model bias should be appreciated as higher when the model is applied for a wider range of input, given that no explicit information is available on how it varies.

In conclusion, it is important to note that when comparing design resistances obtained with different models, the extent of both the test sample basis to determine model uncertainty and the range of beams compared will affect the resulting estimations.

# 5 Conclusions

In this chapter the conclusions from this master thesis are presented followed by a couple of suggestions for further work to be done within the research area. It also contains a discussion on how the objectives of the thesis have been met and some thoughts regarding the potential use of results.

## 5.1 Conclusions from the application of codes

In order to facilitate consistency, both in general construction practice and in the implementation of probabilistic design methods, it is in some aspects, e.g. concerning the determining of in-situ strength parameters, still to be desired to reach greater uniformity in the European construction code recommendations. There is also reason to further clarify some parts of the JCSS (2001) Probabilistic model code, for instance regarding correlation of reinforcing steel properties.

The range of conditions at which estimates of model bias is applicable could be specified more clearly than has been done previously; the appreciation of model uncertainty is meant to be used in the prediction of failure, not in the examination of reasons why failures have occurred. Therefore it would be helpful if it was clarified for what types of beams and under what loading conditions the estimates are applicable, instead of at which resulting failure modes.

## 5.2 Conclusions from the comparison of resistance models

The more detailed shear model used in Response-2000 leads to a lower model uncertainty than does the EC2 shear model and it therefore enables using higher capacities under the same reliability criterion. This is especially true for beams with little shear reinforcement, predominantly failing due to lack of shear resistance. Response-2000 however still exhibits some convergence problems which have to be rectified before the program can become a reliable commercial design tool.

Despite the large variability of stirrup spacing shown in previous studies (Turan *et al.*, 2008) fairly little improvement of reliability is expected to be gained from verifying actual in-situ spacing.

## 5.3 Conclusions from the evaluation of safety formats

Judging from that there are bridges that do not pass assessment it was initially assumed that, at least in some cases and certain aspects, the current design code is more conservative than the ones used when many of the today existing constructions were built. More importantly, it was also suggested that in some aspects the models recommended in the code might lead to more conservative assessments of structural capacity than what has been decided regarding target levels of reliability. In the examination made in this thesis, based on the provisions by JCSS (2001), the second of these hypotheses was thoroughly contradicted. Instead it was shown that with respect to stipulated reliability requirement the partial safety factors provided for design according to EN1992 (CEN, 2002) are non-conservative when used for calculation of shear capacity, both when applied on the EC2 shear model and on Response-2000 resistance analysis.

The Schlune *et al.* (2010) safety format is slightly more accurate and consistent than the EC2 safety format when used for shear analysis with Response-2000. It also leads to more conservative estimates of the design resistance.

## 5.4 Meeting the objective

The principal aim of this thesis was to create an overview of for which reinforced concrete beams the highest improvement of capacity utilization can be gained from using the Response-2000 sectional analysis program instead of the EC2 shear model, and from using a more accurate reliability method than the partial safety factor method presented in EC2. In the latter respect the intention was also to incorporate a continued evaluation of the safety format presented by Schlune *et al.* (2010), as well as employing a full probabilistic approach for determining appropriate design resistances.

Both resistance models and safety formats have been evaluated and compared based on a series of probabilistic analyses which has been set up following standardized guidelines presented by the JCSS (2001) and CEN (2002). The evaluations have been done for a range of cross section configurations and loading conditions and the results have been presented and discussed. Although the probabilistic basis for the study allows for questioning of quantitative estimations, the presented results still form an overview which provides indicative answers to the questions posed in the beginning of this thesis. The author therefore believes that the objectives of the thesis have been met.

## 5.5 Further work

Given that Response-2000 allows for it, the work done in this project could be elaborated by looking at other cross section types than the ones already studied. The evaluations could furthermore be done over the full capacity envelope of chosen beams, i.e. for a larger range of moment-shear combinations and with the addition of normal forces from prestressing. Also, as mentioned in the previous chapter, the presented results indicate that, although relatively accurate and widely applicable, some further work can still be done on improving the Schlune *et al.* (2010) safety format

In a more general respect, in accordance with the comments already made in 5.1, further work that needs to be done is primarily a continuation of the development of the guidelines for probabilistic design. When this has been addressed, it should be noted that the here presented study only illustrates the benefits of probabilistic methodology in sectional analysis. This is just one of many possible applications. As stated in Sustainable Bridges (2007), non-linear FE analysis is perhaps the structural analysis method that has the highest potential for showing the load capacity of bridges. It is therefore natural that further work should be done to explore the possibility of combining probabilistic analyses with complete non-linear FE analyses.

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# 7 Appendix A – Tables of results

**Rectangular section (Slab)**

**Cross section properties**

| M/V | 0.5 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f_{ck}$ | 25 | | | | | | | | | 45 | | | | | | | | |
| $\rho_l$ | 0.40% | | | 1.20% | | | 2.00% | | | 0.40% | | | 1.20% | | | 2.00% | | |
| $\rho_w$ | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% |
| **Mean reistances from EC2 and Response-2000 shear analysis, corrected for model bias** | | | | | | | | | | | | | | | | | | |
| $V_{R,\mu,EC2\ (JCSS)}$ [1] | 794 | 1401 | 1639 | 819 | 2769 | 3397 | 819 | 3353 | 4047 | 799 | 1551 | 1795 | 819 | 3167 | 3869 | 819 | 4226 | 5178 |
| $V_{R,\mu,Response\ (Bohigas)}$ | 645 | 1192 | 1270 | 763 | 1932 | 2821 | 860 | 2032 | 3016 | 800 | 1252 | 1323 | 931 | 2584 | 3272 | 1010 | 2730 | 4106 |
| **Probabilistic design resistances from EC2 and Response-2000 shear analysis (β=3.04)** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2\ (JCSS)}$ | 339 | 867 | 1014 | 339 | 1361 | 1573 | 339 | 1348 | 1576 | 339 | 960 | 1075 | 339 | 1818 | 2149 | 339 | 1841 | 2147 |
| $V_{Rd,Response\ (Bohigas)}$ | 363 | 674 | 719 | 441 | 1070 | 1527 | 465 | 1138 | 1620 | 387 | 723 | 768 | 512 | 1452 | 1887 | 536 | 1536 | 2281 |
| **Design resistances and corresponding β-values from EC2 shear analysis, based on EC2 safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 457 | 857 | 1010 | 457 | 1442 | 1598 | 457 | 1442 | 1599 | 457 | 960 | 1120 | 457 | 1937 | 2384 | 457 | 2248 | 2609 |
| $\beta_{,EC2\ (JCSS)}$ | 1.95 | 3.11 | 3.08 | 1.96 | 2.84 | 3 | 1.96 | 2.82 | 2.99 | 1.95 | 3.05 | 2.8 | 1.95 | 2.74 | 2.59 | 1.95 | 2.3 | 2.33 |
| **Design resistances and corresponding β-values from Response-2000 shear analysis, based on EC2 and Schlune *et al.* (2010) safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 383 | 719 | 718 | 438 | 1118 | 1563 | 448 | 1180 | 1631 | 380 | 762 | 813 | 551 | 1624 | 2076 | 585 | 1721 | 2498 |
| $\beta_{,EC2\ (Bohigas)}$ | 2.8 | 2.68 | 3.05 | 3.07 | 2.83 | 2.92 | 3.22 | 2.86 | 3.01 | 3.07 | 2.77 | 2.76 | 2.66 | 2.46 | 2.54 | 2.6 | 2.49 | 2.57 |
| $V_{Rd,Schlune}$ | 369 | 618 | 676 | 439 | 1020 | 1350 | 435 | 1086 | 1341 | 280 | 685 | 725 | 526 | 1384 | 1797 | 521 | 1473 | 2129 |
| $\beta_{,Schlune\ (Bohigas)}$ | 2.95 | 3.53 | 3.4 | 3.05 | 3.24 | 3.69 | 3.37 | 3.25 | 3.92 | 4.5 | 3.34 | 3.28 | 2.88 | 3.25 | 3.33 | 3.17 | 3.23 | 3.42 |
| $V_{f,Schlune}$ [2] [%] | 3.11 | 11.8 | 10.1 | 1.99 | 10.7 | 16.1 | 13.3 | 9.85 | 19.6 | 29.5 | 8.14 | 6.69 | 4.88 | 9.7 | 7.81 | 12.1 | 9.2 | 11.8 |

| M/V | 2 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f_{ck}$ | 25 | | | | | | | | | 45 | | | | | | | | |
| $\rho_l$ | 0.40% | | | 1.20% | | | 2.00% | | | 0.40% | | | 1.20% | | | 2.00% | | |
| $\rho_w$ | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% |
| **Mean reistances from EC2 and Response-2000 shear analysis, corrected for model bias** | | | | | | | | | | | | | | | | | | |
| $V_{R,\mu,EC2\ (JCSS)}$ [1] | 463 | 554 | 554 | 819 | 1405 | 1496 | 819 | 2009 | 2178 | 467 | 560 | 560 | 819 | 1490 | 1560 | 819 | 2187 | 2344 |
| $V_{R,\mu,Response\ (Bohigas)}$ | 402 | 488 | 496 | 782 | 1255 | 1293 | 789 | 1863 | 1897 | 419 | 507 | 518 | 874 | 1316 | 1434 | 881 | 1996 | 2063 |
| **Probabilistic design resistances from EC2 and Response-2000 shear analysis (β=3.04)** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2\ (JCSS)}$ | 287 | 332 | 332 | 338 | 865 | 908 | 338 | 1223 | 1321 | 290 | 335 | 335 | 338 | 912 | 936 | 338 | 1360 | 1433 |
| $V_{Rd,Response\ (Bohigas)}$ | 233 | 281 | 285 | 438 | 711 | 747 | 446 | 1035 | 1170 | 239 | 290 | 301 | 484 | 749 | 788 | 501 | 1470 | 1458 |
| **Design resistances and corresponding β-values from EC2 shear analysis, based on EC2 safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 288 | 253 | 353 | 454 | 861 | 921 | 454 | 1166 | 1268 | 293 | 361 | 361 | 454 | 945 | 1002 | 454 | 1370 | 1480 |
| $\beta_{,EC2\ (JCSS)}$ | 3.04 | 2.65 | 2.66 | 1.96 | 3.07 | 2.96 | 1.96 | 3.32 | 3.28 | 3 | 2.6 | 2.6 | 1.96 | 2.84 | 2.62 | 1.96 | 3 | 2.84 |
| **Design resistances and corresponding β-values from Response-2000 shear analysis, based on EC2 and Schlune *et al.* (2010) safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 245 | 286 | 313 | 445 | 751 | 771 | 453 | 1024 | 1134 | 251 | 303 | 324 | 535 | 799 | 816 | 598 | 1224 | 1264 |
| $\beta_{,EC2\ (Bohigas)}$ | 2.78 | 2.96 | 2.55 | 2.95 | 2.75 | 2.85 | 2.95 | 3.08 | 3.15 | 2.78 | 2.81 | 2.65 | 2.52 | 2.71 | 2.84 | 2.15 | 4.17 | 3.62 |
| $V_{Rd,Schlune}$ | 231 | 280 | 285 | 447 | 720 | 742 | 451 | 1054 | 1051 | 240 | 291 | 297 | 500 | 755 | 815 | 505 | 1145 | 1184 |
| $\beta_{,Schlune\ (Bohigas)}$ | 3.08 | 3.08 | 3.05 | 2.93 | 2.96 | 3.08 | 2.98 | 2.95 | 3.41 | 3.01 | 3.03 | 3.13 | 2.86 | 3.01 | 2.84 | 3 | 4.42 | 3.9 |
| $V_{f,Schlune}$ [2] [%] | 2.05 | 2.73 | 2.67 | 3.28 | 2.67 | 2.58 | 3.2 | 4.84 | 7.04 | 2.95 | 2.75 | 1.59 | 3.23 | 2.56 | 4.22 | 2.72 | 2.56 | 2.49 |

[1] References for model bias estimates are given within paranthesis

[2] Schlune et al. Estimate of Material uncertainty

## T-section, compression in flange

**Cross section properties**

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **M/V** | 0.5 | | | | | | | | | | | | | | | | | |
| $f_{ck}$ | 25 | | | | | | | | | 45 | | | | | | | | |
| $\rho_l$ | 0.40% | | | 1.20% | | | 2.00% | | | 0.40% | | | 1.20% | | | 2.00% | | |
| $\rho_w$ | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% |
| **Mean reistances from EC2 and Response-2000 shear analysis, corrected for model bias** | | | | | | | | | | | | | | | | | | |
| $V_{R,\mu,EC2\ (JCSS)}$ [1] | 820 | 1456 | 1709 | 820 | 2908 | 3580 | 820 | 3353 | 4047 | 824 | 1617 | 1877 | 824 | 3307 | 4053 | 824 | 4353 | 5178 |
| $V_{R,\mu,Response\ (Bohigas)}$ | 716 | 1201 | 1331 | 885 | 2327 | 3084 | 874 | 2335 | 3180 | 823 | 1288 | 1443 | 1028 | 2918 | 3497 | 1034 | 3145 | 4316 |
| **Probabilistic design resistances from EC2 and Response-2000 shear analysis (β=3.04)** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2\ (JCSS)}$ | 339 | 864 | 1014 | 339 | 1376 | 1572 | 340 | 1366 | 1555 | 340 | 961 | 1080 | 340 | 1826 | 2147 | 341 | 1841 | 2147 |
| $V_{Rd,Response\ (Bohigas)}$ | 420 | 687 | 749 | 504 | 1265 | 1658 | 508 | 1291 | 1705 | 442 | 738 | 813 | 573 | 1612 | 1981 | 578 | 1771 | 2420 |
| **Design resistances and corresponding β-values from EC2 shear analysis, based on EC2 safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 457 | 891 | 1054 | 457 | 1442 | 1598 | 457 | 1442 | 1599 | 457 | 1002 | 1174 | 457 | 2021 | 2494 | 457 | 2248 | 2609 |
| $\beta_{,EC2\ (JCSS)}$ | 1.94 | 2.85 | 2.79 | 1.95 | 2.86 | 2.99 | 1.95 | 2.84 | 2.97 | 1.94 | 2.77 | 2.51 | 1.94 | 2.55 | 2.39 | 1.94 | 1.95 | 2.3 |
| **Design resistances and corresponding β-values from Response-2000 shear analysis, based on EC2 and Schlune *et al.* (2010) safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 456 | 718 | 782 | 512 | 1313 | 1684 | 510 | 1330 | 1700 | 444 | 785 | 855 | 653 | 1812 | 2176 | 606 | 1958 | 2715 |
| $\beta_{,EC2\ (Bohigas)}$ | 2.6 | 2.81 | 2.82 | 2.97 | 2.84 | 2.98 | 3.01 | 2.89 | 3.05 | 3.02 | 2.74 | 2.81 | 2.39 | 2.41 | 2.54 | 2.8 | 2.49 | 2.44 |
| $V_{Rd,Schlune}$ | 353 | 631 | 666 | 467 | 953 | 1291 | 499 | 1131 | 1393 | 397 | 701 | 719 | 534 | 1416 | 1784 | 566 | 1582 | 2247 |
| $\beta_{,Schlune\ (Bohigas)}$ | 3.94 | 3.47 | 3.64 | 3.45 | 4.17 | 3.94 | 3.11 | 3.78 | 3.81 | 3.65 | 3.3 | 3.47 | 3.39 | 3.68 | 3.59 | 3.13 | 3.57 | 3.43 |
| $V_{f,Schlune}$ [2] [%] | 14.6 | 10.9 | 13.9 | 10.7 | 23.1 | 22.2 | 3.47 | 15.5 | 20.2 | 8.11 | 13.6 | 11.6 | 15.8 | 8.63 | 14.1 | 11.8 | 15.4 | 12.8 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **M/V** | 2 | | | | | | | | | | | | | | | | | |
| $f_{ck}$ | 25 | | | | | | | | | 45 | | | | | | | | |
| $\rho_l$ | 0.40% | | | 1.20% | | | 2.00% | | | 0.40% | | | 1.20% | | | 2.00% | | |
| $\rho_w$ | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% |
| **Mean reistances from EC2 and Response-2000 shear analysis, corrected for model bias** | | | | | | | | | | | | | | | | | | |
| $V_{R,\mu,EC2\ (JCSS)}$ [1] | 484 | 585 | 585 | 819 | 1503 | 1607 | 819 | 2224 | 2426 | 489 | 592 | 592 | 819 | 1581 | 1668 | 819 | 2369 | 2551 |
| $V_{R,\mu,Response\ (Bohigas)}$ | 542 | 542 | 574 | 831 | 1343 | 1340 | 846 | 2032 | 2181 | 550 | 523 | 548 | 921 | 1394 | 1549 | 927 | 2168 | 2152 |
| **Probabilistic design resistances from EC2 and Response-2000 shear analysis (β=3.04)** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2\ (JCSS)}$ | 287 | 332 | 330 | 338 | 867 | 909 | 338 | 1225 | 1323 | 290 | 336 | 336 | 338 | 913 | 937 | 337 | 1362 | 1436 |
| $V_{Rd,Response\ (Bohigas)}$ | 251 | 304 | 309 | 459 | 754 | 792 | 472 | 1190 | 1327 | 262 | 312 | 320 | 507 | 803 | 843 | 526 | 1342 | 1321 |
| **Design resistances and corresponding β-values from EC2 shear analysis, based on EC2 safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 301 | 373 | 374 | 454 | 941 | 1012 | 454 | 1382 | 1519 | 307 | 383 | 383 | 454 | 1002 | 1065 | 454 | 1493 | 1620 |
| $\beta_{,EC2\ (JCSS)}$ | 2.75 | 2.32 | 2.31 | 1.96 | 1.96 | 2.38 | 1.96 | 2.27 | 2.17 | 2.68 | 2.23 | 2.23 | 1.96 | 2.48 | 2.26 | 1.96 | 2.42 | 2.28 |
| **Design resistances and corresponding β-values from Response-2000 shear analysis, based on EC2 and Schlune *et al.* (2010) safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 256 | 296 | 306 | 482 | 811 | 834 | 485 | 1160 | 1316 | 259 | 333 | 333 | 572 | 844 | 866 | 626 | 1317 | 1364 |
| $\beta_{,EC2\ (Bohigas)}$ | 2.97 | 3.15 | 3.08 | 2.77 | 2.67 | 2.8 | 2.89 | 3.15 | 3.08 | 3.09 | 2.7 | 2.84 | 2.44 | 2.78 | 2.9 | 2.14 | 3.11 | 2.89 |
| $V_{Rd,Schlune}$ | 297 | 308 | 321 | 476 | 770 | 772 | 484 | 1159 | 1251 | 299 | 300 | 313 | 529 | 804 | 881 | 531 | 1243 | 1237 |
| $\beta_{,Schlune\ (Bohigas)}$ | 2.23 | 2.96 | 2.87 | 2.84 | 2.94 | 3.21 | 2.92 | 3.15 | 3.34 | 2.42 | 3.25 | 3.15 | 2.83 | 3.04 | 2.81 | 3.01 | 3.36 | 3.29 |
| $V_{f,Schlune}$ [2] [%] | 7.99 | 4.5 | 5.89 | 2.92 | 2.63 | 0.76 | 3.22 | 3.74 | 2.54 | 8.54 | 2.6 | 3.02 | 2.52 | 0.81 | 4.2 | 2.75 | 2.72 | 2.02 |

[1] References for model bias estimates are given within paranthesis

[2] Schlune et al. Estimate of Material uncertainty

# T-section, compression in web

| Cross section properties | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| M/V | 0.5 | | | | | | | | | | | | | | | | | |
| $f_{ck}$ | 25 | | | | | | | | | 45 | | | | | | | | |
| $\rho_l$ | 0.40% | | | 1.20% | | | 2.00% | | | 0.40% | | | 1.20% | | | 2.00% | | |
| $\rho_w$ | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% |
| **Mean reistances from EC2 and Response-2000 shear analysis, corrected for model bias** | | | | | | | | | | | | | | | | | | |
| $V_{R\mu,EC2\ (JCSS)}$ [1] | 794 | 1402 | 1641 | 825 | 2805 | 3448 | 825 | 3369 | 4066 | 800 | 1552 | 1797 | 825 | 3209 | 3930 | 825 | 4317 | 5202 |
| $V_{R\mu,Response\ (Bohigas)}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Probabilistic design resistances from EC2 and Response-2000 shear analysis (β=3.04)** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2\ (JCSS)}$ | 340 | 867 | 1015 | 340 | 1371 | 1586 | 339 | 1354 | 1583 | 340 | 961 | 1076 | 340 | 1836 | 2169 | 340 | 1851 | 2158 |
| $V_{Rd,Response\ (Bohigas)}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Design resistances and corresponding β-values from EC2 shear analysis, based on EC2 safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 457 | 858 | 1011 | 457 | 1453 | 1611 | 457 | 1449 | 1607 | 458 | 960 | 1122 | 458 | 1962 | 2419 | 458 | 2259 | 2622 |
| $\beta_{,EC2\ (JCSS)}$ | 1.95 | 3.11 | 3.08 | 1.96 | 2.84 | 3 | 1.96 | 2.82 | 2.99 | 1.94 | 3.06 | 2.8 | 1.96 | 2.73 | 2.57 | 1.96 | 2.3 | 2.33 |

| Cross section properties | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| M/V | 2 | | | | | | | | | | | | | | | | | |
| $f_{ck}$ | 25 | | | | | | | | | 45 | | | | | | | | |
| $\rho_l$ | 0.40% | | | 1.20% | | | 2.00% | | | 0.40% | | | 1.20% | | | 2.00% | | |
| $\rho_w$ | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.5% | 0.9% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% | 0.1% | 0.7% | 1.3% |
| **Mean reistances from EC2 and Response-2000 shear analysis, corrected for model bias** | | | | | | | | | | | | | | | | | | |
| $V_{R\mu,EC2\ (JCSS)}$ [1] | 464 | 554 | 554 | 825 | 1440 | 1537 | 825 | 2105 | 2296 | 468 | 560 | 560 | 825 | 1528 | 1605 | 825 | 2295 | 2474 |
| $V_{R\mu,Response\ (Bohigas)}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Probabilistic design resistances from EC2 and Response-2000 shear analysis (β=3.04)** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2\ (JCSS)}$ | 288 | 330 | 330 | 340 | 888 | 932 | 339 | 1276 | 1394 | 290 | 336 | 336 | 340 | 936 | 963 | 339 | 1426 | 1517 |
| $V_{Rd,Response\ (Bohigas)}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Design resistances and corresponding β-values from EC2 shear analysis, based on EC2 safety formats** | | | | | | | | | | | | | | | | | | |
| $V_{Rd,EC2}$ | 288 | 354 | 354 | 458 | 882 | 947 | 458 | 1244 | 1363 | 293 | 361 | 361 | 458 | 969 | 1029 | 458 | 1436 | 1559 |
| $\beta_{,EC2\ (JCSS)}$ | 3.04 | 2.65 | 2.66 | 1.96 | 3.07 | 2.97 | 1.96 | 3.19 | 3.16 | 2.98 | 2.58 | 2.59 | 1.96 | 2.84 | 2.63 | 1.96 | 3 | 2.86 |

[1] References for model bias estimates are given within paranthesis