

Extensions of the Poincaré group

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We construct an extension of the Poincaré group which involves a mixture of internal and space-time supersymmetries. The resulting group is an extension of the superPoincaré group with infinitely many generators which carry internal and space-time indices. It is a closed algebra since all Jacobi identities are satisfied and it has, therefore, explicit matrix representations. We investigate the massless case and construct the irreducible representations of the extended symmetry. They are divided into two sets, longitudinal and transversal representations. The transversal representations involve an infinite series of integer and half-integer helicities. Finally, we suggest an extension of the conformal group along the same line. © 2011 American Institute of Physics. [doi:10.1063/1.3607971]

I. INTRODUCTION

In an attempt to discuss higher spin gauge fields in a new setting, a generalization of the Poincaré algebra has been suggested.²¹ In it the Poincaré generators are enlarged by infinitely many new bosonic generators which carry internal and space-time indices. In this article we shall construct a supersymmetric extension of this algebra. The resulting algebra contains the ordinary superPoincaré generators together with infinitely many bosonic generators which form a current algebra between themselves. It is a closed algebra since all Jacobi identities are satisfied and it can, hence, have explicit matrix representations.

Let us first introduce the infinite set of translationally invariant generators which carry internal and space-time indices:

$$L_a^{\lambda_1 \dots \lambda_s}, \quad s = 0, 1, 2, \dots, \quad (1)$$

where $L_a(s=0)$ are the generators of the internal Lie algebra L_G and the generators $L_a^{\lambda_1 \dots \lambda_s}$ are totally symmetric with respect to the indices $\lambda_1 \dots \lambda_s$. These generators carry space-time and internal indices and transform under the operations of both groups. In a sense these generators remind us of gauge fields having both Lorentz and internal indices and, as we shall see, there are some properties inherent of gauge fields in them. The current algebra of these generators is defined as follows:²¹

$$[L_a^{\lambda_1 \dots \lambda_n}, L_b^{\lambda_{n+1} \dots \lambda_s}] = if_{abc} L_c^{\lambda_1 \dots \lambda_s}, \quad s = 0, 1, 2, \dots, \quad (2)$$

where at the basic level ($s=0$) it contains the internal algebra L_G with commutators $[L_a, L_b] = if_{abc} L_c$ ($a, b, c = 1, \dots, \dim L_G$). The current algebra (2) is not yet completely defined because it does not specify how the new generators $L_a^{\lambda_1 \dots \lambda_s}$ transform under space-time transformations. Assuming the generators $L_a^{\lambda_1 \dots \lambda_s}$ be translationally invariant tensors of rank s , the following extension of the Poincaré algebra was suggested:²¹

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$$\begin{aligned}
[P^\mu, P^\nu] &= 0, \\
[M^{\mu\nu}, P^\lambda] &= i(\eta^{\lambda\nu} P^\mu - \eta^{\lambda\mu} P^\nu), \\
[M^{\mu\nu}, M^{\lambda\rho}] &= i(\eta^{\mu\rho} M^{\nu\lambda} - \eta^{\mu\lambda} M^{\nu\rho} + \eta^{\nu\lambda} M^{\mu\rho} - \eta^{\nu\rho} M^{\mu\lambda}),
\end{aligned} \tag{3}$$

$$\begin{aligned}
[P^\mu, L_a^{\lambda_1 \dots \lambda_s}] &= 0, \\
[M^{\mu\nu}, L_a^{\lambda_1 \dots \lambda_s}] &= i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2 \dots \lambda_s} - \eta^{\lambda_1\mu} L_a^{\nu\lambda_2 \dots \lambda_s} + \dots + \eta^{\lambda_s\nu} L_a^{\lambda_1 \dots \lambda_{s-1}\mu} - \eta^{\lambda_s\mu} L_a^{\lambda_1 \dots \lambda_{s-1}\nu}),
\end{aligned} \tag{4}$$

$$[L_a^{\lambda_1 \dots \lambda_n}, L_b^{\lambda_{n+1} \dots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \dots \lambda_s} \quad (s = 0, 1, 2, \dots). \tag{5}$$

The first three commutators define the Poincaré algebra as its subalgebra. The next two commutators tell us that the generators $L_a^{\lambda_1 \dots \lambda_s}$ are translationally invariant tensors of rank s , and the last commutator defines the current subalgebra (2). One can check that all Jacobi identities are satisfied and we have an example of fully consistent algebra, which is called an *extended Poincaré algebra* $L_G(\mathcal{P})$ associated with a compact Lie group G . Thus, the algebra $L_G(\mathcal{P})$ incorporates the Poincaré algebra and an internal algebra L_G in a nontrivial way, which is different from the direct product. The generators $L_a^{\lambda_1 \dots \lambda_s}$ have a nonzero commutation relation with $M^{\mu\nu}$ and, therefore, carry higher spins.

II. SUPERSYMMETRIC EXTENSION OF THE $L_G(\mathcal{P})$ ALGEBRA

We are interested in constructing further extensions of the $L_G(\mathcal{P})$ algebra which should include anticommuting generators. *A priori*, it is not obvious that such an extension can be constructed. With this intention in mind, let us compare the above $L_G(\mathcal{P})$ extension of the Poincaré algebra with the superPoincaré algebra which is defined as follows:^{15-18,24,25}

$$\begin{aligned}
[P^\mu, P^\nu] &= 0, \\
[M^{\mu\nu}, P^\lambda] &= i(\eta^{\lambda\nu} P^\mu - \eta^{\lambda\mu} P^\nu), \\
[M^{\mu\nu}, M^{\lambda\rho}] &= i(\eta^{\mu\rho} M^{\nu\lambda} - \eta^{\mu\lambda} M^{\nu\rho} + \eta^{\nu\lambda} M^{\mu\rho} - \eta^{\nu\rho} M^{\mu\lambda}),
\end{aligned} \tag{6}$$

$$\begin{aligned}
[P^\mu, Q_\alpha^i] &= 0, \\
[M^{\mu\nu}, Q_\alpha^i] &= \frac{i}{2}(\gamma^{\mu\nu} Q_\alpha^i)_\alpha, \quad \gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu],
\end{aligned} \tag{7}$$

$$\{Q_\alpha^i, Q_\beta^j\} = -2\delta^{ij}(\gamma^\mu C)_{\alpha\beta} P_\mu, \quad i = 1, \dots, N, \tag{8}$$

where we allowed for an R -symmetry specified by the indices i and j . Q_α^i is a Majorana spinor. Both algebras contain Poincaré subalgebra as it is in (3) and (6). The next two commutators (4) and (7) express the fact that the extended generators Q_α^i and $L_a^{\lambda_1 \dots \lambda_s}$ are translationally invariant operators and carry a nonzero spin. The last commutators (5) and (8) are essentially different in both algebras: in super-Poincaré algebra the generators Q_α^i anticommute with the operator P^μ , while in our case $L_a^{\lambda_1 \dots \lambda_s}$ commute with themselves to form an infinite series of commutators of the current algebra (2) which cannot be truncated. Therefore, the index s runs from zero to infinity, providing an example of an infinitely dimensional current subalgebra.⁷

In the first attempt to combine the above algebras one can try to consider an infinite set of spinor-tensor generators $Q_{\alpha\lambda_1 \dots \lambda_s}^i$, but this does not work. Therefore, the natural suggestion is the following unification of these algebras (Our notational conventions follow the article in Ref. 6):

$$\begin{aligned}
[P^\mu, P^\nu] &= 0, \\
[M^{\mu\nu}, P^\lambda] &= i(\eta^{\lambda\nu} P^\mu - \eta^{\lambda\mu} P^\nu), \\
[M^{\mu\nu}, M^{\lambda\rho}] &= i(\eta^{\mu\rho} M^{\nu\lambda} - \eta^{\mu\lambda} M^{\nu\rho} + \eta^{\nu\lambda} M^{\mu\rho} - \eta^{\nu\rho} M^{\mu\lambda}),
\end{aligned} \tag{9}$$

$$\begin{aligned}
[P^\mu, L_a^{\lambda_1 \dots \lambda_s}] &= 0, \\
[P^\mu, Q_\alpha^i] &= 0, \\
[M^{\mu\nu}, L_a^{\lambda_1 \dots \lambda_s}] &= i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2 \dots \lambda_s} - \eta^{\lambda_1\mu} L_a^{\nu\lambda_2 \dots \lambda_s} + \dots + \eta^{\lambda_s\nu} L_a^{\lambda_1 \dots \lambda_{s-1}\mu} - \eta^{\lambda_s\mu} L_a^{\lambda_1 \dots \lambda_{s-1}\nu}), \\
[M^{\mu\nu}, Q_\alpha^i] &= \frac{i}{2}(\gamma^{\mu\nu} Q_\alpha^i)_\alpha, \quad \gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu], \\
[L_a^{\lambda_1 \dots \lambda_n}, L_b^{\lambda_{n+1} \dots \lambda_s}] &= i f_{abc} L_c^{\lambda_1 \dots \lambda_s}, \quad s = 0, 1, 2, \dots, \\
\{Q_\alpha^i, Q_\beta^j\} &= -2\delta^{ij} (\gamma^\mu C)_{\alpha\beta} P_\mu, \quad i = 1, \dots, N, \\
[L_a^{\lambda_1 \dots \lambda_s}, Q_\alpha^i] &= 0.
\end{aligned} \tag{10}$$

$$\begin{aligned}
\{Q_\alpha^i, Q_\beta^j\} &= -2\delta^{ij} (\gamma^\mu C)_{\alpha\beta} P_\mu, \quad i = 1, \dots, N, \\
[L_a^{\lambda_1 \dots \lambda_s}, Q_\alpha^i] &= 0.
\end{aligned} \tag{11}$$

Here, at $s = 0$, we have the relations

$$[P^\mu, L_a] = 0, \quad [M^{\mu\nu}, L_a] = 0;$$

therefore, the internal bosonic algebra L_G obeys the Coleman-Mandula theorem.⁵ Thus, we have the superPoincaré algebra $L_G(\mathcal{SP})$ associated with a compact Lie group G .

Let us now investigate the commutators between Q_α^i and the rest of the generators. First, we have to check the Jacobi identities which contain at least one anticommuting generator. They are

$$\begin{aligned}
[[L_a^{\lambda_1 \dots \lambda_s}, P^\mu] Q_\alpha^i] + Perm. &= 0, \\
[[L_a^{\lambda_1 \dots \lambda_s}, M^{\mu\nu}] Q_\alpha^i] + Perm. &= 0, \\
[[L_a^{\lambda_1 \dots \lambda_n}, L_b^{\lambda_{n+1} \dots \lambda_s}] Q_\alpha^i] + Perm. &= 0,
\end{aligned}$$

and as one can check, they are indeed identically true. The identities with two anticommuting generators have the form

$$\{[L_a^{\lambda_1 \dots \lambda_s}, Q_\alpha^i] Q_\beta^j\} + \{[L_a^{\lambda_1 \dots \lambda_s}, Q_\beta^j] Q_\alpha^i\} + \{Q_\alpha^i, Q_\beta^j\} L_a^{\lambda_1 \dots \lambda_s} = 0,$$

and they are also true. The rest of the identities are satisfied since they coincide with the identities of the known subalgebras (3)–(5) and (6)–(8).

III. GENERAL PROPERTIES OF THE EXTENDED ALGEBRA $L_G(\mathcal{SP})$

The algebra (9)–(11) is invariant with respect to the following “gauge” transformations:

$$\begin{aligned}
L_a^{\lambda_1 \dots \lambda_s} &\rightarrow L_a^{\lambda_1 \dots \lambda_s} + \sum_1 P^{\lambda_1} L_a^{\lambda_2 \dots \lambda_s} + \sum_2 P^{\lambda_1} P^{\lambda_2} L_a^{\lambda_3 \dots \lambda_s} + \dots + P^{\lambda_1} \dots P^{\lambda_s} L_a, \\
P^\lambda &\rightarrow P^\lambda, \\
M^{\mu\nu} &\rightarrow M^{\mu\nu}, \\
Q_\alpha^i &\rightarrow Q_\alpha^i,
\end{aligned} \tag{12}$$

where the sums \sum_1, \sum_2, \dots extend over all inequivalent index permutations. It is not an internal isomorphism since it cannot be represented as conjugations by elements U of the group itself: $L \rightarrow U^{-1} L U$. Similar to the case of $L_G(\mathcal{P})$ algebra the above transformations contain polynomials of the commuting momenta and are reminiscent of the gauge transformations for the gauge fields.²¹ They are “off-shell” transformations because the invariant operator P^2 can have any value. Note that the square mass operator P^2 , is a Casimir invariant for the above algebra, while the spin operator $W^\mu W_\mu$ (W^μ being the Pauli-Lubansky vector) is not. As a result, to any given representation of $L_a^{\lambda_1 \dots \lambda_s}$, $s = 1, 2, \dots$ of the extended algebra, one can add the longitudinal terms, as it follows from the transformation (12). Thus, all representations are defined modulo “gauge transformations” and we can identify these generators as “gauge generators.”

To any given representation of the gauge generators $L_a^{\lambda_1 \dots \lambda_s}$, $s = 1, 2, \dots$ of the extended algebra one can add longitudinal terms. All representations are therefore defined modulo longitudinal terms.

The second general property of the extended algebra is that each gauge generator $L_a^{\lambda_1 \dots \lambda_s}$ cannot be realized as an irreducible representation of the Poincaré subalgebra of a definite helicity, i.e., to be a symmetric and *traceless tensor*. The reason for this is that the commutator of two symmetric traceless generators in the current subalgebra (2) is not any more a traceless tensor. Therefore, the gauge generators should realize a reducible representation of the Poincaré subalgebra and each of them carries a sequence of helicities, which we shall find out in Secs. IV–V.

Finally, the extended algebra $L_G(\mathcal{SP})$ has a general reducible representation in terms of differential operators of the following form:

$$\begin{aligned} P^\mu &= k^\mu, \\ M^{\mu\nu} &= i(k^\mu \frac{\partial}{\partial k_\nu} - k^\nu \frac{\partial}{\partial k_\mu}) + i(\xi^\mu \frac{\partial}{\partial \xi_\nu} - \xi^\nu \frac{\partial}{\partial \xi_\mu}) - \frac{i}{2} \bar{\vartheta} \gamma^{\mu\nu} \frac{\partial}{\partial \vartheta}, \\ Q_\alpha &= -i \frac{\partial}{\partial \bar{\vartheta}_\alpha} + i(\gamma^\mu \vartheta)_\alpha k_\mu, \\ L_a^{\lambda_1 \dots \lambda_s} &= \xi^{\lambda_1} \dots \xi^{\lambda_s} \otimes L_a, \end{aligned} \quad (13)$$

where the vector superspace of complex-valued functions is parameterized in terms of momentum coordinates k^μ , translationally invariant vector variables ξ^μ , and anticommuting Grassmann variables ϑ_α :

$$\Psi(k^\mu, \xi^\nu, \vartheta_\alpha). \quad (14)$$

This representation allows us to further justify the interpretation of the transformation (12) as a gauge transformation and of the generators $L_a^{\lambda_1 \dots \lambda_s}$ as gauge generators if one considers how this transformation acts on the representation (13). Indeed, the transformation (12) induces a transformation for the vector variable ξ^μ of the form

$$\xi^\mu \rightarrow \xi^\mu + k^\mu, \quad (15)$$

reminiscent of a gauge transformation for the photon polarization vector. Furthermore, in order to obtain the irreducible representations from (13), we shall follow Wigner's prescription imposing invariant constraints on the vector space of functions, defined in (14), of the following form:^{10,27,28}

$$k^2 = 0, \quad k^\mu \xi_\mu = 0, \quad \xi^2 = -1. \quad (16)$$

These equations have a unique solution,

$$\xi^\mu = \xi k^\mu + e_1^\mu \cos \varphi + e_2^\mu \sin \varphi, \quad (17)$$

where $e_1^\mu = (0, 1, 0, 0)$, $e_2^\mu = (0, 0, 1, 0)$ when $k^\mu = k(1, 0, 0, 1)$; thus, justifying the interpretation of the vector variable ξ^μ as a polarization vector. In this article we shall consider only massless representations with $k^2 = 0$. The invariant subspace of functions is now reduced to the form $\Psi(k^\mu, \xi, \varphi, \vartheta_\alpha)$, where ξ and φ remain as independent variables on the cylinder $\varphi \in S^1$, $\xi \in R^1$.

There are important properties of the above representations (13), (16) and (17) which are worth mentioning:

(i) The gauge transformation (12) and (15) cannot trivialize the above representation by nullifying the generators $L_a^{\lambda_1 \dots \lambda_s}$, but what it can do is to change the parameter ξ in front of k^μ in (17), and

(ii) This representation is transversal in the sense that

$$k_{\lambda_1} L_a^{\lambda_1 \dots \lambda_s} = 0, \quad s = 1, 2, \dots \quad (18)$$

Having in hand this interpretation of the generators $L_a^{\lambda_1 \dots \lambda_s}$, we can divide the vector space of representations into *pure longitudinal* and *transversal* subsets.

IV. LONGITUDINAL REPRESENTATIONS

Let us consider an irreducible representation of the superPoincaré algebra (6), in which the generators $P^\mu, M^{\mu\nu}, Q_\alpha^i$ realize a matrix representation with maximal helicity h and the L_a realize an irreducible matrix representation of the internal algebra L_G . If one now takes the gauge generators in the trivial form $L_a^{\lambda_1 \dots \lambda_s} = 0, s = 1, 2, \dots$, it is easy to check that this set of generators fulfils all commutation relations of the algebra (9) and, therefore, forms a true representation of the extended algebra $L_G(\mathcal{SP})$. Applying the above theorem to the representation just described we find that it is isomorphic to the representation in which all generators remain in the same matrix form, except that the gauge generators $L_a^{\lambda_1 \dots \lambda_s}$ are now purely longitudinal. Thus, we have the following equivalence relation:

$$\begin{aligned} L_{\mathcal{SP}} : \quad & P^\mu, M^{\mu\nu}, \\ & L_a^{\lambda_1 \dots \lambda_s} = 0 \\ L_G : \quad & L_a \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} & Q_\alpha^i P^\mu, M^{\mu\nu}, Q_\alpha^i \\ & L_a^{||\lambda_1 \dots \lambda_s} = k^{\lambda_1} \dots k^{\lambda_s} \oplus L_a \\ & L_a, \end{aligned} \quad (19)$$

where $s = 1, 2, \dots$. It states that representations with trivial generators $L_a^{\lambda_1 \dots \lambda_s} = 0$ and representations with purely longitudinal generators $L_a^{||\lambda_1 \dots \lambda_s} = k^{\lambda_1} \dots k^{\lambda_s} \oplus L_a$ are isomorphic to each other. In other words, pure longitudinal representations factorize into super-Poincaré $L(\mathcal{SP})$ and internal L_G algebra multiplets. Or, if one reads this statement from right to left, it says that the representations with pure longitudinal generators $L_a^{||\lambda_1 \dots \lambda_s}$ carry no more helicities than the ones carried by the representation of the superPoincaré subgroup, since it is equivalent to a trivial representation of $L_a^{\lambda_1 \dots \lambda_s}$, namely, $L_a^{\lambda_1 \dots \lambda_s} = 0$ ($s = 1, 2, \dots$). The result of this section will be further illustrated below when we shall consider the properties of the corresponding little algebra.

V. TRANSVERSAL REPRESENTATIONS

As we have seen in Sec. IV, any representation of the algebra $L_G(\mathcal{SP})$, in which the generators of the superPoincaré subalgebra (6)–(8) realize a matrix representation of maximal helicity h , is always equivalent to a representation in which the gauge generators are purely longitudinal and are therefore trivial. It was realized in the case of the $L_G(\mathcal{P})$ algebra that in order to get a nontrivial representation for the gauge generators, one should consider infinite-dimensional representations of Poincaré subalgebra;²¹ therefore, it seems natural to think that in the given case as well one should also consider infinite-dimensional representations now of the superPoincaré subalgebra (6). Such representations for the superPoincaré algebra have been constructed in the article.⁴

The irreducible representations of the extended algebra can be found by the well-known method of induced representations.^{4,26} This method consists of finding a representation of the Wigner's little group L and boosting it up to a representation of the full group. The subgroup L is a group of transformations which leave a fixed momentum, in our case time-like momentum $k^\mu = k(1, 0, 0, 1)$, invariant. The Poincaré generators in L form the Euclidean algebra $E(2)$ (see Appendix for definitions):

$$[h, \pi'] = i\pi'', \quad [h, \pi''] = -i\pi', \quad [\pi', \pi''] = 0.$$

Notice that transformations generated by the gauge $L_a^{\lambda_1 \dots \lambda_s}$ and supercharge Q_α generators leave the manifold of states with fixed momentum invariant, since they all commute with P^μ ; therefore, all these generators should be included into the little algebra L , so that we have the following generators in L (in this section we shall use two component Weyl spinors):

$$h, \pi', \pi'', Q_\alpha, \bar{Q}_{\dot{\alpha}}, L_a^{\lambda_1 \dots \lambda_s}. \quad (20)$$

The full set of commutators of the L algebra is presented in the Appendix and have the following form (Not all of them are presented here in the main text.):

$$\begin{aligned}
[h, \pi'] &= +i\pi'', & [h, \pi''] &= -i\pi', & [\pi', \pi''] &= 0, \\
[h, \bar{Q}_1] &= -\frac{1}{2}\bar{Q}_1, & [h, Q_1] &= +\frac{1}{2}Q_1, & \{Q_1, \bar{Q}_1\} &= 4k \\
[\pi', Q_1] &= iQ_2, & [\pi'', Q_1] &= -Q_2, & [\pi', \bar{Q}_1] &= i\bar{Q}_2, & [\pi'', \bar{Q}_1] &= \bar{Q}_2.
\end{aligned} \tag{21}$$

The supercharges commute with the gauge generators:

$$[Q_1, L_a^\lambda] = 0, \quad [\bar{Q}_1, L_a^\lambda] = 0. \tag{22}$$

The commutators between the $E(2)$ and the $L_a^{\lambda_1}$ generators are

$$\begin{aligned}
[h, L_a^0] &= [h, L_a^3] = 0, & [\pi', L_a^0] &= -iL_a^1, & [\pi'', L_a^0] &= -iL_a^2, \\
[h, L_a^1] &= [h, L_a^2] = 0, & [\pi', L_a^1] &= -iL_a^0, & [\pi'', L_a^1] &= -iL_a^3, \\
[h, L_a^2] &= +iL_a^1, & [\pi', L_a^2] &= -i(L_a^0 - L_a^3), & [\pi'', L_a^2] &= 0, \\
[h, L_a^3] &= -iL_a^2, & [\pi', L_a^3] &= 0, & [\pi'', L_a^3] &= -i(L_a^0 - L_a^1),
\end{aligned} \tag{23}$$

and the higher rank generators $L_a^{\lambda_1 \dots \lambda_s}$ have similar structure of commutators (see details in the Appendix). The problem reduces to the construction of the unitary irreducible representations of the L algebra.

The key of understanding the representations of both algebras $L_G(\mathcal{P})$ and $L_G(\mathcal{SP})$ is hidden in the commutation relations (23) (see also (A3)). Indeed, if the π generators are realized trivially $\pi' = \pi'' = 0$, then from (23) it follows that

$$L_a^0 - L_a^3 = 0, \quad L_a^1 = L_a^2 = 0, \tag{24}$$

that is, the generator L_a^λ is purely longitudinal because $L_a^1 = L_a^2 = 0$ and $k_\lambda L_a^\lambda = 0$. Therefore, only in the case when π generators are realized nontrivially one can get transversal representation for the gauge generators $L_a^{\lambda_1 \dots \lambda_s}$.²¹ Here, in the supersymmetric case the situation is very similar to the non-supersymmetric case because the commutation relations (23) are the same in both algebras. We are able, therefore, to formulate the following result:

(α) *If the Poincaré (3) or superPoincaré (9) subalgebras are realized as the massless finite dimensional matrix representations of definite helicity, then the gauge generators $L_a^{\lambda_1 \dots \lambda_s}$, $s = 1, 2, \dots$ are purely longitudinal, (β) If Poincaré or superPoincaré subalgebras are realized as massless infinite dimensional representations, then the gauge generators are transversal and carry nonzero helicities.*

Let us explicitly construct the representations of the little algebra L by restricting the general representation (13) into the invariant subspace defined by the conditions (16) and solution (17) which has the following form:

$$\Psi(k^\mu, \xi^\nu, \vartheta_\alpha) \delta(k^2) \delta(k \cdot \xi) \delta(e^2 + 1) = \Phi(k^\mu, \varphi, \xi, \vartheta_\alpha). \tag{25}$$

Making use of the chain rule we may re-express (13) as a differential operator in the new variables, so that the generators of the L algebra reduce to the form

$$\begin{aligned}
h &= -i \frac{\partial}{\partial \varphi} - \frac{1}{2} (\vartheta^1 \frac{\partial}{\partial \vartheta^1} - \bar{\vartheta}^1 \frac{\partial}{\partial \bar{\vartheta}^1} - \vartheta^2 \frac{\partial}{\partial \vartheta^2} + \bar{\vartheta}^2 \frac{\partial}{\partial \bar{\vartheta}^2}), \\
\pi' &= \rho \cos \varphi - i \vartheta^1 \frac{\partial}{\partial \vartheta^2} - i \bar{\vartheta}^1 \frac{\partial}{\partial \bar{\vartheta}^2}, & \pi'' &= \rho \sin \varphi + \vartheta^1 \frac{\partial}{\partial \vartheta^2} - \bar{\vartheta}^1 \frac{\partial}{\partial \bar{\vartheta}^2}, & \rho &= -\frac{i}{k} \frac{\partial}{\partial \xi}, \\
Q_1 &= -i \frac{\partial}{\partial \vartheta^1} - 2ik \bar{\vartheta}^1, & Q_2 &= -i \frac{\partial}{\partial \vartheta^2}, \\
\bar{Q}_1 &= +i \frac{\partial}{\partial \bar{\vartheta}^1} + 2ik \vartheta^1, & \bar{Q}_2 &= +i \frac{\partial}{\partial \bar{\vartheta}^2},
\end{aligned} \tag{26}$$

and taking into account (17) the gauge generators become

$$L_a^{\lambda_1 \dots \lambda_s} = \prod_{i=1}^s (\xi k^{\mu_i} + e_1^{\mu_i} \cos \varphi + e_2^{\mu_i} \sin \varphi) \oplus L_a. \tag{27}$$

This is a purely transversal representation and, as we have already mentioned in Sec. III, it cannot be trivialized by the transformations (12) and (15). It is transversal in the sense that

$$k_{\lambda_1} L_a^{\perp \lambda_1 \dots \lambda_s} = 0, \quad s = 1, 2, \dots \quad (28)$$

What is important to notice is that the commutators between the generators of $E(2)$ and the $L_a^{\perp \lambda_1 \dots \lambda_s}$ generators of the little algebra L are fulfilled only if $\rho \neq 0$. An example of this is the commutator $[\pi', L_a^0] = -iL_a^1$ in (23).

Next, we are interested in knowing the helicity content of the transversal gauge generators just constructed. The supercharges Q_1, \bar{Q}_i carry helicities $h = (1/2, -1/2)$, as one can see from the commutators of the helicity operator with supercharges in (21). The Poincaré generators $\pi^\pm = \pi' \pm \pi''$ carry helicities $h = (1, -1)$. The fact that $L_a^\pm = L_a^1 \pm iL_a^2$ carry helicities $h = (1, -1)$ is seen from the commutators in the first column of (23):

$$[h, L_a^\pm] = \pm L_a^\pm. \quad (29)$$

The rank-2 generators $L_a^{++}, L_a^{+-}, L_a^{--}$ carry helicities $h = (2, 0, -2)$, where

$$L_a^{++} = L_a^{11} + 2iL_a^{12} - L_a^{22}, \quad L_a^{+-} = L_a^{11} + L_a^{22}, \quad L_a^{--} = L_a^{11} - 2iL_a^{12} - L_a^{22},$$

so that $[h, L_a^{\pm\pm}] = \pm 2L_a^{\pm\pm}, [h, L_a^{+-}] = 0$, and, in general, the rank- $s(L_a^{+\dots+}, \dots, L_a^{-\dots-})$ generators carry helicities in the following range:

$$h = (s, s-2, \dots, -s+2, -s) \quad (30)$$

in total $s+1$ states. (Remember that gauge generator $L_a^{\lambda_1 \dots \lambda_s}$ cannot be realized as an irreducible representation of the Poincaré subalgebra of a definite helicity.) This can be seen also from the explicit representation (27):

$$L_a^{\perp \mu_1 \dots \mu_s} = \prod_{n=1}^s (\xi k^{\mu_n} + e^{i\varphi} e_+^{\mu_n} + e^{-i\varphi} e_-^{\mu_n}) \oplus L_a, \quad (31)$$

where $e_\pm^\mu = (e_1^\mu \mp i e_2^\mu)/2$. The last formula also illustrates the realization of the transformation rule (12). Indeed, if we perform the multiplication in (31) and collect terms with a given power of momentum, we get the following expression:

$$\begin{aligned} L_a^{\perp \mu_1 \dots \mu_s} &= \prod_{n=1}^s (e^{i\varphi} e_+^{\mu_n} + e^{-i\varphi} e_-^{\mu_n}) \oplus L_a + \\ &+ \sum_1^{s-1} \xi k^{\lambda_1} \prod_{n=1}^{s-1} (e^{i\varphi} e_+^{\mu_n} + e^{-i\varphi} e_-^{\mu_n}) \oplus L_a + \dots + \xi k^{\lambda_1} \dots \xi k^{\lambda_s} \oplus L_a, \end{aligned} \quad (32)$$

where

$$\prod_{n=1}^s (e^{i\varphi} e_+^{\mu_n} + e^{-i\varphi} e_-^{\mu_n}) \oplus L_a \quad (33)$$

is the transversal part of the generator which we describe in terms of $(L_a^{+\dots+}, \dots, L_a^{-\dots-})$. The rest of the terms (corresponding to the terms with indices 0 or 3) are purely longitudinal, transforming under (12), and can be gauged away. The situation is analogous to the polarization tensor of the graviton $e^{\mu\nu}(k) = e_1^{\mu\nu} + e_2^{\mu\nu} + k^\mu \xi^\nu + k^\nu \xi^\mu$. The first two terms describe transversal polarizations, the last two terms describe the longitudinal part, and if one takes $\xi^\mu = e_1^\mu$, then there will be a spin one part, but still representing a pure gauge.

Let us first recapitulate the contraction of states in the case of the superPoincaré algebra with nontrivially realized π^\pm generators.⁴ The massless irreducible representation of $\mathcal{N} = 1$ supersymmetry comprises the two states with helicities λ and $\lambda - 1/2$:

$$\begin{pmatrix} |\lambda \rangle \\ \bar{Q}_i |\lambda \rangle \end{pmatrix}, \quad (34)$$

where $h|\lambda\rangle = \lambda|\lambda\rangle$ and $Q_1|\lambda\rangle = 0$. Because the operators π^\pm commute with the supercharges (21), they generate an infinite tower of high helicity states (This is because on the state $|\lambda\rangle$ the supercharges Q_2, \bar{Q}_2 are realized trivially $Q_2|\lambda\rangle = \bar{Q}_2|\lambda\rangle = 0$):

$$\begin{array}{cccccc}
 \dots & \pi^+|\lambda\rangle & |\lambda\rangle & \pi^-|\lambda\rangle & \dots & \\
 \dots & \pi^+\bar{Q}_1|\lambda\rangle & \bar{Q}_1|\lambda\rangle & \pi^-\bar{Q}_1|\lambda\rangle & \dots & \\
 \dots & \lambda + 1 & \lambda & \lambda - 1 & \dots & \\
 \dots & \lambda + 1/2 & \lambda - 1/2 & \lambda - 3/2 & \dots &
 \end{array} \tag{35}$$

From the above formulae it follows that the infinite multiplets built up by any integer λ are isomorphic to each other. The same is true for multiplets built up by any half-integer λ . The supersymmetry transforms simultaneously different pairs of states within the large multiplet, the vertical columns in (35). It does not transform nontrivially the whole multiplet, that is, the horizontal states in (35). These infinite representations are those found in Ref. 4, where the continuous spin representations of the superPoincaré group were derived.

Now let us consider the case when in addition to the above h, π^\pm and Q_1, \bar{Q}_1 generators we have the operators $(L_a^{++++}, \dots, L_a^{----})$ which are commuting with the supercharges (22) and with the π^\pm generators (23) and are similar to the creation and annihilation operators of the Kac-Moody algebra without central charges. This is exactly the case with the $L_G(\mathcal{P})$ algebra where the ‘‘vacuum’’ state $|\lambda\rangle_i$ belongs to the irreducible representation of the internal algebra L_G from which the states of high helicity are generated by applying the operators $(L_a^{++++}, \dots, L_a^{----})$.²¹ Now in the supersymmetric case $L_G(\mathcal{SP})$ instead of the single helicity state $|\lambda\rangle_i$, we have the $\mathcal{N} = 1$ vacuum supermultiplet,

$$\left(\begin{array}{c} |\lambda\rangle \\ |\lambda - 1/2\rangle \end{array} \right)_i, \quad \mathcal{N} = 1, \tag{36}$$

and one should apply to it the first level generators L_a^+ and L_a^- ,

$$\begin{array}{c}
 \left(\begin{array}{c} |\lambda + 1\rangle \\ |\lambda + 1/2\rangle \end{array} \right)_j \\
 \left(\begin{array}{c} |\lambda\rangle \\ |\lambda - 1/2\rangle \end{array} \right)_i \begin{array}{l} \nearrow \\ \searrow \end{array} \\
 \left(\begin{array}{c} |\lambda - 1\rangle \\ |\lambda - 3/2\rangle \end{array} \right)_k
 \end{array} \tag{37}$$

as a result we shall get two supermultiplets in which the helicities have increased and decreased by one unit of angular momentum quanta and isospins are changing in accordance with the applied generators. Applying to the vacuum multiplet the second level generators $(L_a^{++}, L_a^{+-}, L_a^{--})$, we shall get three supermultiplets in which helicities are changing by ± 2 and 0 units:

$$\begin{array}{c}
 \left(\begin{array}{c} |\lambda + 2\rangle \\ |\lambda + 3/2\rangle \end{array} \right)_l \\
 \left(\begin{array}{c} |\lambda\rangle \\ |\lambda - 1/2\rangle \end{array} \right)_i \begin{array}{l} \nearrow \\ \rightarrow \\ \searrow \end{array} \left(\begin{array}{c} |\lambda\rangle \\ |\lambda - 1/2\rangle \end{array} \right)_k \\
 \left(\begin{array}{c} |\lambda - 2\rangle \\ |\lambda - 5/2\rangle \end{array} \right)_j
 \end{array} \tag{38}$$

Drawing the supermultiplets as vertical columns and continuing this process we shall get a tree which grows to the right direction indefinitely ($s = 0, 1, 2, 3, \dots$) with increasing multiplicity of spins. Then, one should also apply to this tree the π^\pm generators which will make each of the above supermultiplet columns infinite as in Ref. 4.

This construction can be generalized to the expended supersymmetry with \mathcal{N} supercharges. In that case we shall have instead of the vacuum supermultiplet (36) the supermultiplet of dimension $2^{\mathcal{N}}$ (Ref. 6) and the rest of the construction will be the same.

VI. GENERALIZATION OF DE SITTER AND CONFORMAL GROUPS

We might ask if the extension above can be made for the de Sitter and the conformal groups too. Consider first the algebras $SO(4, 1)$ or $SO(3, 2)$,

$$[J^{AB}, J^{CD}] = i(g^{AD} J^{BC} - g^{AC} J^{BD} + g^{BC} J^{AD} - g^{BD} J^{AC}),$$

where $g^{AB} = (+ - - -)$ or $g^{AB} = (+ - - +)$ and $A, B = 0, 1, \dots, 4$. The Wigner-Inönü contraction $J^{A\mu} = RP^\mu$, $J^{\mu\nu} = M^{\mu\nu}$, where $\mu, \nu = 0, 1, 2, 3$ and $R \rightarrow \infty$, reduces it to the Poincaré algebra. In the previous analysis there were no restrictions on the dimension of space-time when we considered the bosonic part. We can then drop the translation generators and just consider the following sets of commutators:

$$\begin{aligned} [J^{AB}, J^{CD}] &= i(g^{AD} J^{BC} - g^{AC} J^{BD} + g^{BC} J^{AD} - g^{BD} J^{AC}), \\ [J^{AB}, L_a^{C_1 \dots C_s}] &= i(\eta^{C_1 B} L_a^{AC_2 \dots C_s} - \dots - \eta^{C_s A} L_a^{C_1 \dots C_{s-1} B}), \\ [L_a^{C_1 \dots C_n}, L_b^{C_{n+1} \dots C_s}] &= i f_{abc} L_c^{C_1 \dots C_s} (s = 0, 1, 2, \dots). \end{aligned} \quad (39)$$

This is an obvious generalization to the cases of the (anti)de Sitter groups.

Similarly, since the $SO(d, 2)$ algebra is isomorphic to the conformal algebra, the algebra $L_G(\mathcal{P})$ can be extended to the conformal group as well with the following well known identification:

$$J^{\mu\nu} = M^{\mu\nu}, \quad J^{\mu, d} = \frac{1}{2}(K^\mu - P^\mu), \quad J^{\mu(d+1)} = \frac{1}{2}(K^\mu + P^\mu), \quad J^{(d+1)d} = D, \quad (40)$$

where $g^{AB} = (+ - - - \dots - +)$ and $A, B = (0, \dots, d, d+1)$. Thus, we have the algebra $L_G(\mathcal{SO})$ of the following form:

$$\begin{aligned} \frac{1}{i} [J^{AB}, J^{CD}] &= g^{AD} J^{BC} - g^{AC} J^{BD} + g^{BC} J^{AD} - g^{BD} J^{AC}, \\ \frac{1}{i} [J^{AB}, L_a^{D_1 \dots D_s}] &= \eta^{D_1 B} L_a^{AD_2 \dots D_s} - \dots - \eta^{D_s A} L_a^{D_1 \dots D_{s-1} B}, \\ \frac{1}{i} [L_a^{D_1 \dots D_n}, L_b^{D_{n+1} \dots D_s}] &= f_{abc} L_c^{D_1 \dots D_s} (s = 0, 1, 2, \dots). \end{aligned} \quad (41)$$

We defer to future work the study of generalizations of these algebras to superalgebras.

VII. CONCLUSIONS

In this article we have studied infinite-component massless supermultiplets which arise from a new extension of the superPoincaré algebra $L_G(\mathcal{SP})$. We find that they combine the continuous spin representations of the superPoincaré algebra $L(\mathcal{SP})$ and of the extended Poincaré algebra $L_G(\mathcal{P})$. This provides us with a new framework to discuss such representations. Interesting extensions of the Poincaré algebra recently were considered in Refs. 3, 13, 14, and 23. There has been a struggle since the advent of the string theory to describe the zero-tension limit of such a theory, which should be a theory with massless particles of all possible spins.^{1,2,8,9,11,12,19,20,22} It is hence interesting to study methods to generate infinite-component massless supermultiplets. In this article we have only taken a first step to include half-integer spins in a recently proposed scheme.²¹ In future work we will extend this to higher-dimensional algebras and possibly further extension along the lines of this article.

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APPENDIX: LITTLE ALGEBRA

The irreducible representations of the extended algebra can be found by the well-known method of induced representations.^{4,26} This method consists of finding a representation of the Wigner's little group L and boosting it up to a representation of the full group. The subgroup L is a group of transformations which leave a fixed momentum, in our case the time-like momentum $k^\mu = k(1, 0, 0, 1)$, invariant. Under the Lorentz rotations the action of the element $U_\theta = \exp(\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu})$ creates an infinitesimal transformation $k^\mu \rightarrow \omega_\nu^\mu k^\nu + k^\mu$, and $k^\mu = k(1, 0, 0, 1)$ is left invariant provided the parameters obey the relations $\omega_{30} = 0$, $\omega_{10} + \omega_{13} = 0$, $\omega_{20} + \omega_{23} = 0$. Therefore, the little subalgebra L contains at least the following generators:

$$h = M_{12}, \quad \pi'/P^0 = M_{10} + M_{13}, \quad \pi''/P^0 = M_{20} + M_{23}.$$

The M_{12} represents the helicity operator h :

$$h = \frac{\vec{P}\vec{S}}{P^0} = \frac{\vec{P}\vec{J}}{P^0} = \frac{P_i\epsilon_{ijk}M_{jk}}{P^0} = M_{12},$$

where ($\vec{J} = \vec{R} \times \vec{P} + \vec{S}$). The super-Poincaré little algebra is

$$\begin{aligned} [h, \pi'] &= +i\pi'', & [h, \pi''] &= -i\pi', & [\pi', \pi''] &= 0, \\ [h, \bar{Q}_1] &= -\frac{1}{2}\bar{Q}_1, & [h, Q_1] &= +\frac{1}{2}Q_1, & \{Q_1, \bar{Q}_1\} &= 4k, \\ [h, \bar{Q}_2] &= +\frac{1}{2}\bar{Q}_2, & [h, Q_2] &= -\frac{1}{2}Q_2, & \{Q_2, \bar{Q}_2\} &= 0, \\ [\pi', Q_1] &= iQ_2, & [\pi'', Q_1] &= -Q_2, & [\pi', \bar{Q}_1] &= i\bar{Q}_2, & [\pi'', \bar{Q}_1] &= \bar{Q}_2, \\ [\pi', Q_2] &= 0, & [\pi'', Q_2] &= 0, & [\pi', \bar{Q}_2] &= 0, & [\pi'', \bar{Q}_2] &= 0, \end{aligned} \tag{A1}$$

and the rest of the anticommutators between the supercharges is $\{Q_\alpha, Q_\beta\} = \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0$. The first level commutation relations in (10) are

$$[M^{\mu\nu}, L_a^\lambda] = i(\eta^{\lambda\nu}L_a^\mu - \eta^{\lambda\mu}L_a^\nu), \quad [Q_\alpha, L_a^\lambda] = 0, \quad [\bar{Q}_\alpha, L_a^\lambda] = 0,$$

and, when written in components, the little subalgebra L takes the following form:

$$\begin{aligned} [h, L_a^0] &= [h, L_a^3] = 0, & [\pi', L_a^0] &= -iL_a^1, & [\pi'', L_a^0] &= -iL_a^2, \\ [h, L_a^1] &= +iL_a^2, & [\pi', L_a^1] &= -i(L_a^0 - L_a^3), & [\pi'', L_a^1] &= 0, \\ [h, L_a^2] &= -iL_a^1, & [\pi', L_a^2] &= 0, & [\pi'', L_a^2] &= -i(L_a^0 - L_a^3). \end{aligned}$$

The second level commutation relations are

$$[M^{\mu\nu}, L_a^{\lambda_1\lambda_2}] = i(\eta^{\lambda_1\nu}L_a^{\mu\lambda_2} - \eta^{\lambda_1\mu}L_a^{\nu\lambda_2} + \eta^{\lambda_2\nu}L_a^{\mu\lambda_1} - \eta^{\lambda_2\mu}L_a^{\nu\lambda_1}), \quad [Q_\alpha, L_a^{\lambda_1\lambda_2}] = [\bar{Q}_\alpha, L_a^{\lambda_1\lambda_2}] = 0,$$

and in components they have the following form:

$$\begin{aligned}
[h, L_a^{00}] &= [h, L_a^{03}] = [h, L_a^{33}] = 0, \\
[h, L_a^{01}] &= +iL_a^{02}, \\
[h, L_a^{02}] &= -iL_a^{01}, \\
[h, L_a^{13}] &= +iL_a^{23}, \\
[h, L_a^{23}] &= -iL_a^{13}, \\
[h, L_a^{11}] &= +2iL_a^{12}, \\
[h, L_a^{22}] &= -2iL_a^{12}, \\
[h, L_a^{12}] &= +i(L_a^{22} - L_a^{11}),
\end{aligned} \tag{A2}$$

and with translation operators π' and π'' :

$$\begin{aligned}
[\pi', L_a^{00}] &= -2iL_a^{01}, & [\pi'', L_a^{00}] &= -2iL_a^{02}, \\
[\pi', L_a^{01}] &= -iL_a^{00} + iL_a^{03} - iL_a^{11}, & [\pi'', L_a^{01}] &= -iL_a^{12}, \\
[\pi', L_a^{02}] &= -iL_a^{12}, & [\pi'', L_a^{02}] &= -iL_a^{00} + iL_a^{03} - iL_a^{22}, \\
[\pi', L_a^{03}] &= -iL_a^{01} - iL_a^{13}, & [\pi'', L_a^{03}] &= -iL_a^{02} - iL_a^{23}, \\
[\pi', L_a^{11}] &= -2iL_a^{01} + 2iL_a^{13}, & [\pi'', L_a^{11}] &= 2iL_a^{23} - 2iL_a^{02}, \\
[\pi', L_a^{12}] &= iL_a^{23} - iL_a^{02}, & [\pi'', L_a^{12}] &= iL_a^{13} - iL_a^{01}, \\
[\pi', L_a^{13}] &= -iL_a^{11} + iL_a^{33} - iL_a^{03}, & [\pi'', L_a^{13}] &= -iL_a^{12}, \\
[\pi', L_a^{22}] &= 0, & [\pi'', L_a^{22}] &= 0, \\
[\pi', L_a^{23}] &= -iL_a^{12}, & [\pi'', L_a^{23}] &= -iL_a^{22} + iL_a^{33} - iL_a^{03}, \\
[\pi', L_a^{33}] &= -2iL_a^{13}, & [\pi'', L_a^{33}] &= -2iL_a^{23}.
\end{aligned} \tag{A3}$$

The current subalgebra in (11) between generators has the following form:

$$\begin{aligned}
[L_a^0, L_b^0] &= if_{abc}L_c^{00}, & [L_a^0, L_b^1] &= if_{abc}L_c^{01}, & [L_a^0, L_b^2] &= if_{abc}L_c^{02}, & [L_a^0, L_b^3] &= if_{abc}L_c^{03}, \\
[L_a^1, L_b^1] &= if_{abc}, & L_c^{11}, [L_a^1, L_b^2] &= if_{abc}L_c^{12}, & [L_a^1, L_b^3] &= if_{abc}L_c^{13}, \\
[L_a^2, L_b^2] &= if_{abc}L_c^{22}, & [L_a^2, L_b^3] &= if_{abc}L_c^{23}, \\
[L_a^3, L_b^3] &= if_{abc}L_c^{33},
\end{aligned} \tag{A4}$$

and so on to the higher levels.

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