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The effective stability parameter for two-component galactic discs: is $Q^{-1} \approx Q_{\rm stars}^{-1} + Q_{\rm gas}^{-1}$?

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ABSTRACT

The Wang–Silk approximation, $Q^{-1} \approx Q_{\rm stars}^{-1} + Q_{\rm gas}^{-1}$, is frequently used for estimating the effective Q parameter in two-component discs of stars and gas. Here we analyse this approximation in detail, and show how its accuracy depends on the radial velocity dispersions and Toomre parameters of the two components. We then propose a much more accurate but still simple approximation for the effective Q parameter, which further takes into account the stabilizing effect of disc thickness. Our effective Q parameter is a natural generalization of Toomre's Q, and as such can be used in a wide variety of contexts, e.g. for predicting star formation thresholds in galaxies or for measuring the stability level of galactic discs at low and high redshifts.

Key words: instabilities – stars: kinematics and dynamics – ISM: kinematics and dynamics – galaxies: ISM – galaxies: kinematics and dynamics – galaxies: star formation.

1 INTRODUCTION

It is well known that both stars and cold interstellar gas play an important role in the gravitational instability of galactic discs (e.g. Lin & Shu 1966; Jog & Solomon 1984a,b; Bertin & Romeo 1988, and references therein). The local stability criterion for two-component discs of stars and gas can be expressed in the same form as the Toomre (1964) stability criterion, $Q \geq 1$, provided that Q is redefined appropriately. Bertin & Romeo (1988), Elmegreen (1995), Jog (1996), Rafikov (2001) and Shen & Lou (2003) calculated the effective Q parameter as a function of the radial velocity dispersions and surface mass densities of the two components. Shu (1968), Romeo (1990, 1992, 1994) and Wiegert (2010) evaluated the stabilizing effect of disc thickness, which is usually neglected but significant.

As galactic discs contain both stars and gas, the effective Q parameter is clearly more accurate and useful than Toomre's Q. Bertin et al. (1989a,b) and Lowe et al. (1994) showed that the radial profile of this parameter has a large impact on the dynamics and evolution of spiral structure in galaxies. The results of those comprehensive analyses are discussed further in the books by Bertin & Lin (1996) and Bertin (2000). The effective Q parameter is also a useful diagnostic for exploring the link between disc instability and star formation in galaxies (e.g. Hunter, Elmegreen & Baker 1998; Li, Mac Low & Klessen 2005, 2006; Yang et al. 2007; Leroy et al. 2008). More applications and references are given below.

Wang & Silk (1994) proposed a remarkably simple recipe for computing the effective Q parameter in the case of infinitesimally thin discs: $Q^{-1} \approx Q_\star^{-1} + Q_{\rm g}^{-1}$, where $Q_\star$ and $Q_{\rm g}$ are the stellar and gaseous Toomre parameters. Bertin (private communication) points out that such an approximation was already used by him, before the 1990s, for illustrating the efficiency of a small amount of cold gas to destabilize a disc (see also Bertin 1996; Bertin & Lin 1996). Jog (1996) pointed out that the Wang–Silk approximation is invalid since it results from an incorrect analysis. In spite of that, the Wang–Silk approximation has been used in several important contexts: star formation (e.g. Martin & Kennicutt 2001; Boissier et al. 2003; Corbelli 2003; Wong 2009), galaxy formation and evolution (e.g. Immeli et al. 2004; Naab & Ostriker 2006; Kampakoglou & Silk 2007; Stringer & Benson 2007; Wetzstein, Naab & Burkert 2007; Foyle, Courteau & Thacker 2008; Benson 2010), gravitational instability of clumpy discs at low and high redshifts (e.g. Bournaud & Elmegreen 2009; Burkert et al. 2010; Puech 2010) and others (e.g. Hitschfeld et al. 2009; Wong et al. 2009).

In spite of such a burst of applications, there has been no attempt to assess how good the Wang–Silk approximation is. In this paper, we evaluate its accuracy by performing a rigorous comparative analysis (see Section 2.1). Besides, we introduce a new approximation for the effective Q parameter: simple, accurate and applicable to realistically thick discs (see Sections 2.2 and 2.3). We also show how to use our effective Q parameter for measuring the stability level of galactic discs, and why such a diagnostic is more predictive than the classical Toomre parameter (see Section 2.4). The conclusions of our paper are drawn in Section 3.

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2 APPROXIMATING THE EFFECTIVE Q

2.1 Disc instability and the Wang–Silk approximation

Wang & Silk (1994) investigated the link between star formation and disc instability in galaxies. They reconsidered the two-fluid dispersion relation of Jog & Solomon (1984a), which is valid for infinitesimally thin discs of stars and gas, and found that the effective Q parameter can be approximated as follows:

$$\frac{1}{Q_{\mathrm{WS}}} = \frac{1}{Q_\star} + \frac{1}{Q_{\mathrm{g}}}, \qquad (1)$$

where $Q_\star = \kappa\sigma_\star/\pi G\Sigma_\star$ and $Q_{\mathrm{g}} = \kappa\sigma_{\mathrm{g}}/\pi G\Sigma_{\mathrm{g}}$ are the stellar and gaseous Toomre parameters. This approximation is appealing because it is as simple as the formula for the total resistance in a parallel circuit. To evaluate the accuracy of equation (1), we rewrite it as $Q_{\mathrm{WS}} = Q_\star/\overline{Q}_{\mathrm{WS}}$, where

$$\overline{Q}_{\mathrm{WS}} = 1 + \frac{Q_\star}{Q_{\mathrm{g}}}. \qquad (2)$$

The local stability criterion, $Q_{\mathrm{WS}} \geq 1$, translates into $Q_\star \geq \overline{Q}_{\mathrm{WS}}$. This is of the same form as the local stability criterion found by Bertin & Romeo (1988); see also Romeo (1985). Bertin & Romeo (1988) determined the stability threshold $\overline{Q}_{\mathrm{BR}}$ numerically, starting from the same dispersion relation as Wang & Silk (1994) but without introducing further approximations. In contrast to $\overline{Q}_{\mathrm{WS}}$, $\overline{Q}_{\mathrm{BR}}$ depends on two parameters: $\sigma_{\mathrm{g}}/\sigma_\star$ and $\Sigma_{\mathrm{g}}/\Sigma_\star$. Since $\Sigma_{\mathrm{g}}/\Sigma_\star = \sigma_{\mathrm{g}}Q_\star/\sigma_\star Q_{\mathrm{g}}$, we can easily express $\overline{Q}_{\mathrm{BR}}$ in terms of

$$s \equiv \frac{\sigma_{\mathrm{g}}}{\sigma_\star}, \quad q \equiv \frac{Q_{\mathrm{g}}}{Q_\star}. \qquad (3)$$

We can then compare $\overline{Q}_{\mathrm{WS}}(q)$ with $\overline{Q}_{\mathrm{BR}}(s, q)$ and evaluate the accuracy of the Wang–Silk approximation as a function of s and q.

Let us first see how spiral galaxies populate the (s, q) plane. We use the 12 nearby star-forming spirals analysed by Leroy et al. (2008), namely NGC 628, 2841, 3184, 3198, 3351, 3521, 3627, 4736, 5055, 5194, 6946 and 7331. These are galaxies with sensitive and spatially resolved measurements of kinematics, gas surface density and stellar surface density across the entire optical disc. For each galaxy of this sample, we compute the radial profiles $s = s(R)$ and $q = q(R)$, and hence the track left by the galaxy in the (s, q) plane. The result for the whole sample is shown in Fig. 1. The data span a range of two orders of magnitude in s and q, so we show them using a log–log plot. The typical value of s can be robustly estimated by computing the median of the data points along s, which is $s_{\mathrm{med}} \simeq 0.27$. This value is comparable to that found in the solar neighbourhood ($s \approx 0.2$; see Binney & Tremaine 2008, p. 497), but is much smaller than that expected in high-redshift star-forming galaxies ($s \sim 1$; e.g. Burkert et al. 2010; Krumholz & Burkert 2010). The median value of q, $q_{\mathrm{med}} \simeq 1.5$, is close to unity. This suggests that, on average, stars and gas contribute equally to the gravitational instability of the disc. Similar values of q are found in the solar neighbourhood ($q \approx 0.6$; see Binney & Tremaine 2008, p. 497), and are also expected at high z ($q \sim 1$; e.g. Burkert et al. 2010; Krumholz & Burkert 2010; Tacconi et al. 2010).[1] Last but not least,

[1] Hereafter we will use s_{med} and q_{med}, i.e. the median values of s and q computed from the galaxy data of Leroy et al. (2008), for estimating the typical accuracy of the Wang–Silk approximation and of our approximation. This is meant to be a complement to the detailed error maps shown and discussed throughout the paper. We do not 'hint that the stability properties can be characterized by a median value of an effective Q parameter'.

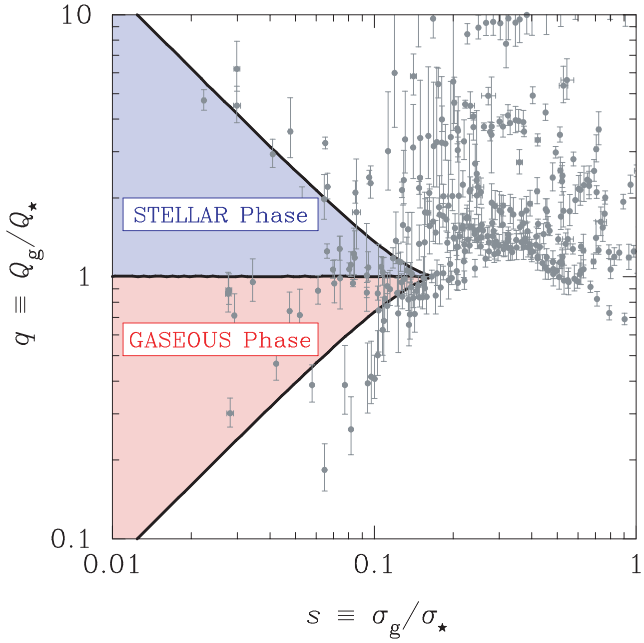

Figure 1. The parameter plane populated by nearby star-forming spirals. The galaxy data are from Leroy et al. (2008), $Q_\star$ and Q_{g} are the stellar and gaseous Toomre parameters, $\sigma_\star$ and σ_{g} are the radial velocity dispersions of the two components. The shaded part of the (s, q) plane is the two-phase region discussed in the text. The dispersion relation $\omega^2(k)$ has two minima inside this region, and one minimum outside it. The transition between the gaseous and stellar stability phases occurs for $q = 1$. This line intersects the boundaries of the two-phase region at $(s, q) \simeq (0.17, 1)$, where the stability threshold is $\overline{Q}_{\mathrm{BR}} \simeq 1.4$.

note that 20 per cent of the data fall within the shaded part of the (s, q) plane. This is the 'two-phase region' of Bertin & Romeo (1988), here shown using our parametrization and logarithmic scaling. In this region, the contributions of stars and gas to the gravitational instability of the disc peak at two different wavelengths. If $q < 1$, then the gaseous peak is higher than the stellar one and gas will dominate the onset of gravitational instability. Vice versa, if $q > 1$, then stars will dominate. These are the gaseous and stellar stability 'phases' shown in Fig. 1. In the rest of the parameter plane, the dynamical responses of the two components are strongly coupled and peak at a single wavelength. More information is given in section 3.2.2 of Romeo (1994).

Fig. 2 shows the contour maps of $\overline{Q}_{\mathrm{BR}}$ and $\overline{Q}_{\mathrm{WS}}$ (left-hand panel), and the error map of Q_{WS} (right-hand panel). Remember that $\overline{Q}$ denotes the stability threshold, i.e. the value of $Q_\star$ above which the two-component disc is locally stable, while $Q = Q_\star/\overline{Q}$ denotes the effective Q parameter. Both stability thresholds are above unity and converge to $1 + q^{-1}$ as $s \to 1$. The first property means that a disc of stars and gas can be gravitationally unstable even when both components are separately stable, as is well known (e.g. Lin & Shu 1966). The second property simply means that stars and gas act as a single component when they have the same radial velocity dispersion, so that $Q = \kappa\sigma/\pi G(\Sigma_\star + \Sigma_{\mathrm{g}})$. Apart from satisfying those properties, the two stability thresholds are clearly different. Look in particular at the contour levels 1.1–1.4 of $\overline{Q}_{\mathrm{BR}}$. Their slope changes abruptly across the line $q = 1$, revealing the existence of two stability phases. As discussed in the previous paragraph, this is an important characteristic of two-component discs, which $\overline{Q}_{\mathrm{WS}}$ fails to reproduce. From a quantitative point of view, the error

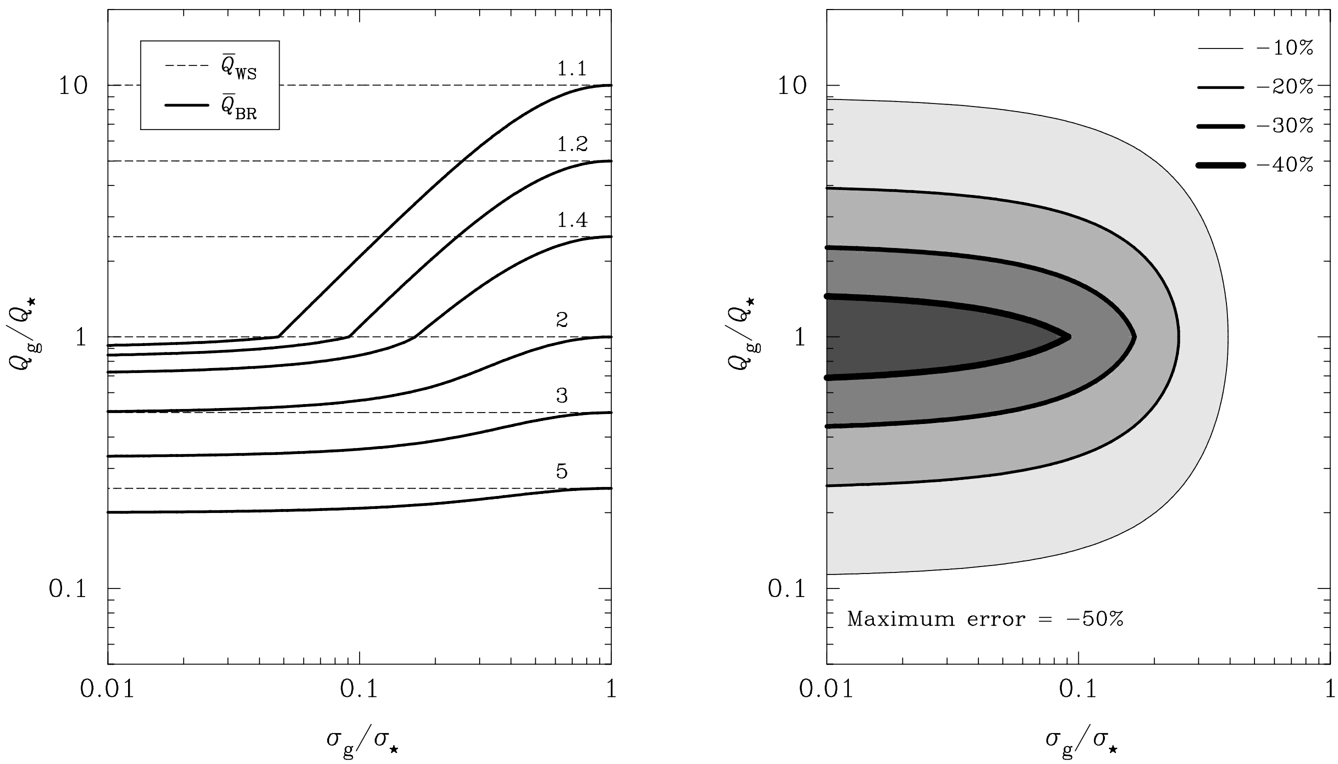

Figure 2. Accuracy of the Wang–Silk approximation. The curves shown are the contour lines of $\overline{Q}_{\rm WS}$ and $\overline{Q}_{\rm BR}$ (left-hand panel), and the contour lines of the relative error $(Q_{\rm WS} - Q_{\rm BR})/Q_{\rm BR}$ (right-hand panel). Q and $\overline{Q}$ denote the effective Q parameter and the stability threshold, the subscripts WS and BR refer to Wang & Silk (1994) and Bertin & Romeo (1988). In addition, $Q_\star$ and $Q_{\rm g}$ are the stellar and gaseous Toomre parameters, $\sigma_\star$ and $\sigma_{\rm g}$ are the radial velocity dispersions of the two components. Note how far the Wang–Silk approximation is from the correct stability threshold, except for $\sigma_{\rm g} \sim \sigma_\star$ (see left-hand panel).

that affects $Q_{\rm WS}$ is significant but below 50 per cent (see now the right-hand panel of Fig. 2). Using the median values of s and q computed from the galaxy data of Leroy et al. (2008), $s_{\rm med} \simeq 0.27$ and $q_{\rm med} \simeq 1.5$, one finds that the typical error is about 20 per cent. Remember, however, that a significant fraction of the data populate the two-phase region, where the error can be more than 40 per cent. Note also that the error is negative, which means that the Wang–Silk approximation underestimates the effective Q parameter systematically.

2.2 Our approximation

Let us now illustrate how to find a better approximation for Q. The first ingredient is to determine the asymptotic behaviour of $\overline{Q}$ as $s \to 0$ and $s \to 1$. These are in fact the natural bounds of s. A rigorous analysis was performed by Romeo (1985). His results can be summarized as follows.

(i) For $s \ll 1$ and $q \leq 1$, i.e. in gas-dominated stability regimes, $\overline{Q} \approx q^{-1} + 2s$.

(ii) For $s \ll 1$ and $q \geq 1$, i.e. in star-dominated stability regimes, $\overline{Q} \approx 1 + 2s\,q^{-1}$.

(iii) For $s \approx 1$, i.e. in the limiting case of a one-component disc, $\overline{Q} \approx 1 + q^{-1}$.

Note that $\overline{Q}$ behaves asymptotically as a weighted sum of two terms: 1 and q^{-1}. Note also that the weight factors change symmetrically as we move from case (i) to case (iii): $(2s, 1) \to (1, 2s) \to (1, 1)$. Such symmetry suggests that we should search for an approximation of the form

$$\overline{Q} = \begin{cases} W(s) + q^{-1} & \text{if } q \leq 1, \\ 1 + W(s)\,q^{-1} & \text{otherwise,} \end{cases} \tag{4}$$

where $W(s) \approx 2s$ as $s \to 0$ and $W(s) \approx 1$ as $s \to 1$. A further constraint on $W(s)$ follows from the fact that the original system of fluid and Poisson equations remains unaltered if we interchange the stellar and gaseous components. Equation (4) must then be invariant under the transformation $s \mapsto s^{-1}$, $q \mapsto q^{-1}$ and hence $\overline{Q} \mapsto q\overline{Q}$, where $q\overline{Q}$ is the value of $Q_{\rm g}$ above which the two-component disc is locally stable. Invariance requires that $W(s^{-1}) = W(s)$. A simple function that satisfies this requirement and matches the asymptotic behaviour above is

$$W(s) = \frac{2s}{1+s^2}\,. \tag{5}$$

Since $Q = Q_\star/\overline{Q}$, equations (4) and (5) lead us to the following approximation for the effective Q parameter:

$$\frac{1}{Q} = \begin{cases} \dfrac{W}{Q_\star} + \dfrac{1}{Q_{\rm g}} & \text{if } Q_\star \geq Q_{\rm g}\,, \\[2ex] \dfrac{1}{Q_\star} + \dfrac{W}{Q_{\rm g}} & \text{if } Q_{\rm g} \geq Q_\star\,; \end{cases} \tag{6}$$

$$W = \frac{2\sigma_\star\sigma_{\rm g}}{\sigma_\star^2 + \sigma_{\rm g}^2}\,. \tag{7}$$

Our approximation is almost as simple as the Wang–Silk approximation (equation 1), but differs from that in one important respect: it gives less weight to the component with larger Q. The weight

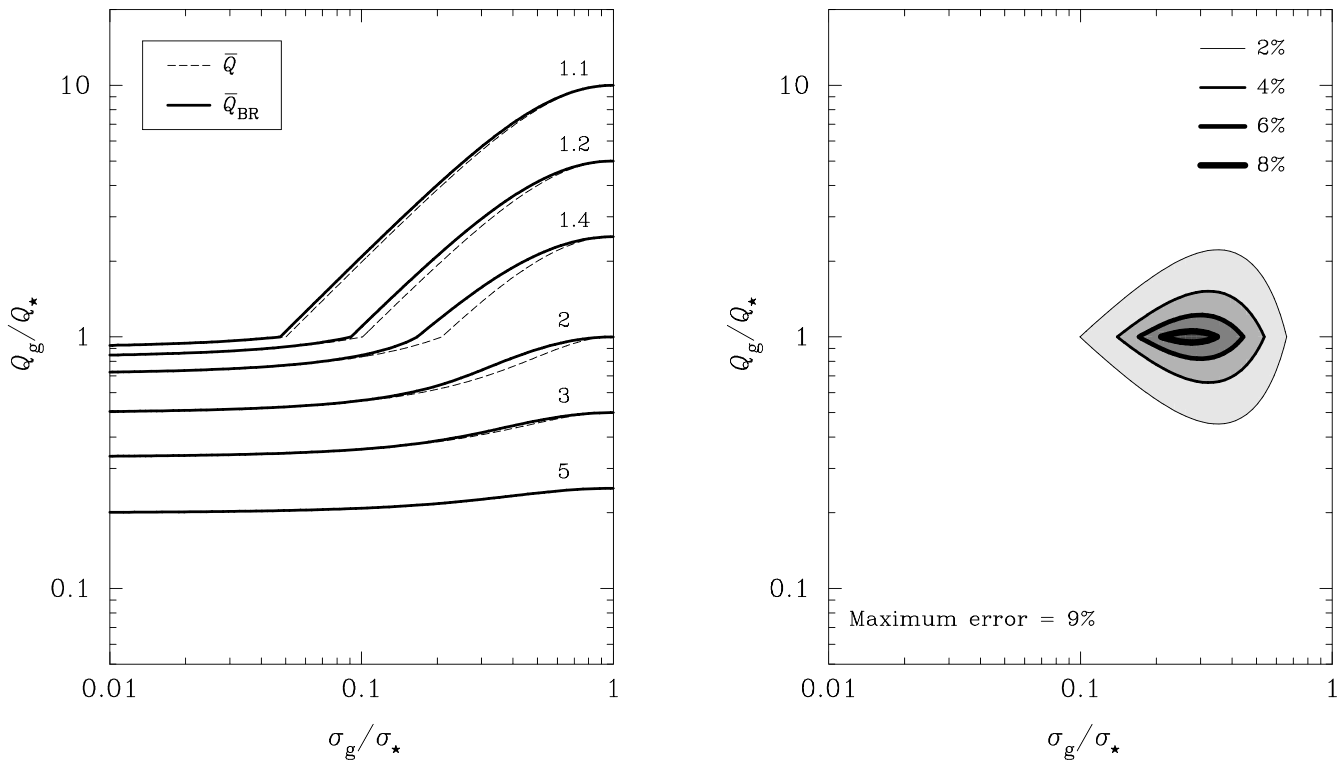

Figure 3. Accuracy of our approximation. The curves shown are the same as in Fig. 2, but for our effective Q parameter and stability threshold. These quantities are denoted by Q and $\overline{Q}$, without subscripts. Note how close our approximation is to the correct stability threshold, especially for $\overline{Q} \lesssim 1.2$ and $\overline{Q} \gtrsim 2$ (see left-hand panel).

factor W depends symmetrically on the radial velocity dispersions of the two components, and is generally small.

To evaluate the accuracy of our approximation, we compare $\overline{Q}(s,q)$ with $\overline{Q}_{\rm BR}(s,q)$ and compute the relative error $(Q - Q_{\rm BR})/Q_{\rm BR}$ as a function of s and q, as we did for the Wang–Silk approximation. Fig. 3 shows that $\overline{Q}$ works well in the whole parameter space (see left-hand panel). Note in particular how successfully our approximation reproduces the gaseous and stellar stability phases for $\overline{Q} \lesssim 1.2$. Fig. 3 also shows that Q overestimates the effective stability parameter, but the error is well below 10 per cent even inside the two-phase region (see right-hand panel). The error can be reduced further by fine-tuning the weight factor, but the approximation will no longer be consistent with the asymptotic behaviour of the stability threshold.

2.3 How to apply our approximation to realistically thick discs

As pointed out in Section 1, the stabilizing effect of disc thickness is usually neglected but significant. In this section, we show that our approximation can easily be modified so as to take this effect into account.

Romeo (1992) investigated the gravitational instability of galactic discs taking rigorously into account two factors: (i) their vertical structure at equilibrium; (ii) the coupling between scaleheight, h, and vertical velocity dispersion, σ_z, in the stellar and gaseous layers. He calculated the effective Q parameter both as a function of $h_\star$ and $h_{\rm g}$, and as a function of $\sigma_{z\star}$ and $\sigma_{z{\rm g}}$. He also discussed the advantages of using $\sigma_{z\star}$ and $\sigma_{z{\rm g}}$ as input quantities. This effective Q parameter has been studied further by Wiegert (2010). Hereafter we will denote it with $\mathcal{Q}_{\rm WR}$.

Let us now illustrate how to find a simple and accurate approximation to $\mathcal{Q}_{\rm WR}$. In the infinitesimally thin case (see equation 6), the local stability level of the disc is dominated by the component with smaller Q. The contribution of the other component is weakened by the W factor, which is generally small. This suggests that the effect of thickness can be estimated reasonably well by considering each component separately. Romeo (1994) analysed this case in detail. The effect of thickness is to increase the stability parameter of each component by a factor T, which depends on the ratio of vertical to radial velocity dispersion:

$$T \approx 0.8 + 0.7 \left(\frac{\sigma_z}{\sigma_R}\right) . \tag{8}$$

Equation (8) can be inferred from fig. 3 (top) of Romeo (1994) and applies for $0.5 \lesssim \sigma_z/\sigma_R \lesssim 1$, which is the usual range of velocity anisotropy. To approximate $\mathcal{Q}_{\rm WR}$, use then equation (6) with $Q_\star$ and $Q_{\rm g}$ replaced by $T_\star Q_\star$ and $T_{\rm g} Q_{\rm g}$:

$$\frac{1}{\mathcal{Q}} = \begin{cases} \dfrac{W}{T_\star Q_\star} + \dfrac{1}{T_{\rm g} Q_{\rm g}} & \text{if } T_\star Q_\star \geq T_{\rm g} Q_{\rm g}, \\ \dfrac{1}{T_\star Q_\star} + \dfrac{W}{T_{\rm g} Q_{\rm g}} & \text{if } T_{\rm g} Q_{\rm g} \geq T_\star Q_\star, \end{cases} \tag{9}$$

where $\mathcal{Q}$ is our effective Q parameter for realistically thick discs, W is given by equation (7) and $T_\star$ and $T_{\rm g}$ are given by equation (8). Equation (9) tells us that the local stability level of the disc is now dominated by the component with smaller TQ. The contribution of the other component is still suppressed by the W factor.

The left-hand panel of Fig. 4 shows the error map of $\mathcal{Q}$ for a galactic disc with $(\sigma_z/\sigma_R)_\star = 0.5$ and $(\sigma_z/\sigma_R)_{\rm g} = 1$, and the corresponding

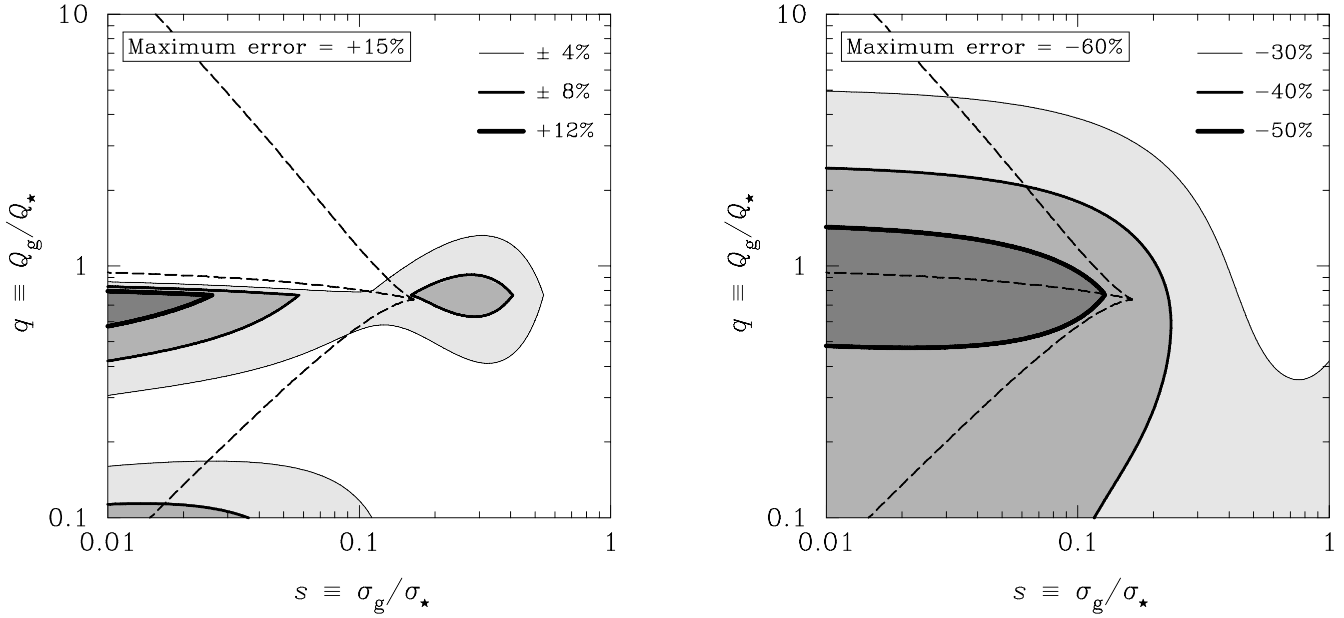

Figure 4. Accuracy of our approximation (left-hand panel) versus accuracy of the Wang–Silk approximation (right-hand panel) for realistically thick discs. The curves shown are the contour lines of the relative errors $(\mathcal{Q} - \mathcal{Q}_{\rm WR})/\mathcal{Q}_{\rm WR}$ (see left-hand panel) and $(Q_{\rm WS} - \mathcal{Q}_{\rm WR})/\mathcal{Q}_{\rm WR}$ (see right-hand panel) for $(\sigma_z/\sigma_R)_\star = 0.5$ and $(\sigma_z/\sigma_R)_{\rm g} = 1$. Here $\mathcal{Q}$ is our effective Q parameter, $\mathcal{Q}_{\rm WR}$ is the effective Q parameter of Romeo (1992) and Wiegert (2010), $Q_{\rm WS}$ is the effective Q parameter of Wang & Silk (1994) and σ_z/σ_R is the ratio of vertical to radial velocity dispersion. The rest of the notations are the same as in Figs 1–3. Also shown is the corresponding two-phase region (dashed lines). The boundaries of this region and the transition line intersect at $(s, q) \simeq (0.16, 0.74)$, where the stability threshold is $\overline{\mathcal{Q}}_{\rm WR} \simeq 1.3$.

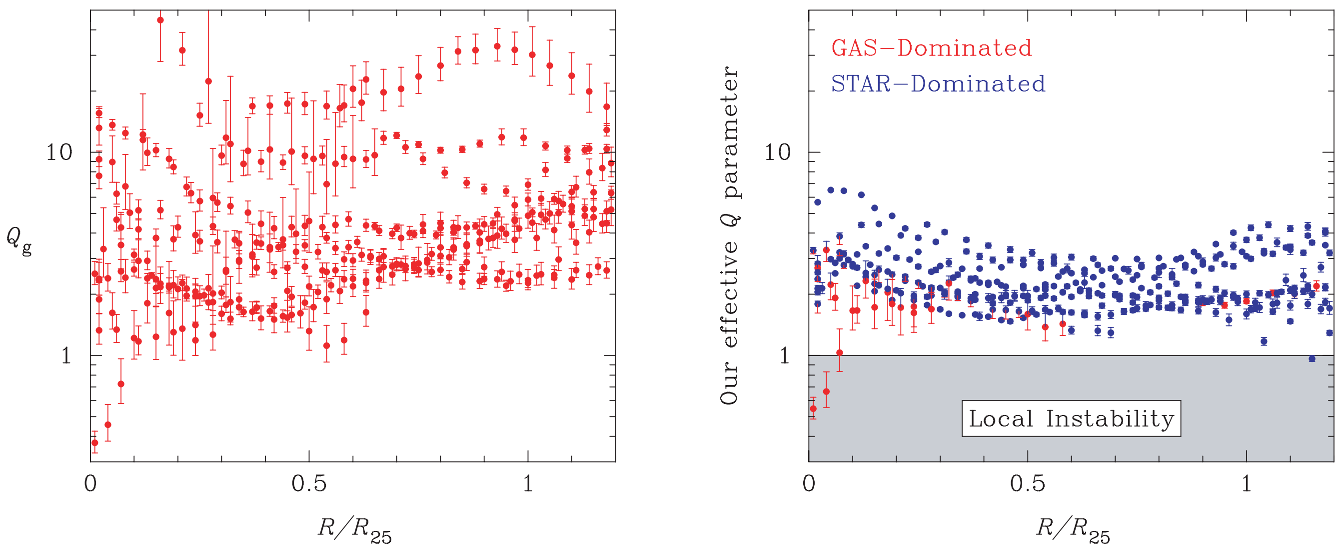

Figure 5. The stability level of nearby star-forming spirals, as measured by two diagnostics: the gaseous Toomre parameter, $Q_{\rm g}$, and our effective Q parameter, $\mathcal{Q}$ (see equation 9). The galaxy data are from Leroy et al. (2008), R is the galactocentric distance and R_{25} is the optical radius. In the right-hand panel, the data are colour-coded so as to show whether the stability level is gas dominated or star dominated, as predicted by equation (9). The two data points that lie well below the critical stability level tell us that the nuclear region of NGC 6946 is subject to strong gas-dominated instabilities. This is consistent with the fact that NGC 6946 hosts a nuclear starburst (e.g. Engelbracht et al. 1996).

two-phase region (Wiegert 2010). Note that the error is below 15 per cent even inside this region, which confirms the high accuracy of our approximation in this more realistic context. What about the accuracy of the Wang–Silk approximation? The right-hand panel of Fig. 4 shows that the relative error $(Q_{\rm WS} - \mathcal{Q}_{\rm WR})/\mathcal{Q}_{\rm WR}$ is much larger than ours, and can be well above 50 per cent inside the two-phase region.

2.4 Application to nearby star-forming spirals

In this section, we show how to use our effective Q parameter for measuring the stability level of galactic discs, and why such a diagnostic is more predictive than the classical Toomre parameter.

We consider the same sample of spiral galaxies as in Section 2.1, and refer to Leroy et al. (2008) for a detailed description of the data and their translation into physical quantities. For each galaxy, we compute the radial profile of our effective Q parameter, $\mathcal{Q}$, using equation (9). We adopt $(\sigma_z/\sigma_R)_\star = 0.6$, as was assumed by Leroy et al. (2008), and $(\sigma_z/\sigma_R)_\mathrm{g} = 1$, as is natural for a collisional component. We also compute the radial profile of the gaseous Toomre parameter, Q_g, which is the traditional diagnostic used for predicting star formation thresholds in galaxies (e.g. Quirk 1972; Kennicutt 1989; Martin & Kennicutt 2001; Schaye 2008; Elmegreen 2011).

Fig. 5 shows $Q_\mathrm{g}(R)$ and $\mathcal{Q}(R)$ for the whole galaxy sample. Note that Q_g spans a much wider range of values than $\mathcal{Q}$ at any given R. This is true even at distances as large as the optical radius, R_{25}, where Q_g is supposed to be a reliable diagnostic. A similar fact was noted by Leroy et al. (2008), using an effective Q parameter that neglects the stabilizing effect of disc thickness (Jog 1996; Rafikov 2001). Why are Q_g and $\mathcal{Q}$ so weakly correlated across the entire optical disc? Equation (9) helps us to clarify this point. It tells us that the value of $\mathcal{Q}$ is dominated by the gaseous component if $T_\mathrm{g}Q_\mathrm{g} < T_\star Q_\star$, and by the stellar component if $T_\star Q_\star < T_\mathrm{g}Q_\mathrm{g}$. In the right-hand panel of Fig. 5, we have colour-coded the data so as to show whether $T_\mathrm{g}Q_\mathrm{g} < T_\star Q_\star$ or vice versa. It turns out that in 92 per cent of the cases the value of $\mathcal{Q}$ is dominated by the stellar component. Gas dominates the stability level only in 8 per cent of the cases. This is why Q_g and $\mathcal{Q}$ are so weakly correlated. This result illustrates (i) how important it is to consider both gas and stars when measuring the stability level of galactic discs and (ii) the strong advantage of using our effective Q parameter as a stability diagnostic.

3 CONCLUSIONS

(i) The approximation of Wang & Silk (1994) (equation 1) underestimates the effective Q parameter. The error is typically 20 per cent, but can be as large as 40 per cent or more if $\sigma_\mathrm{g} \lesssim 0.2\sigma_\star$ and $Q_\mathrm{g} \sim Q_\star$. In this case, the gaseous and stellar components should contribute separately to the gravitational instability of the disc (Bertin & Romeo 1988). But such dynamical decoupling is difficult to approximate because it involves two stability regimes, one dominated by the gas and the other dominated by the stars, and because there is a sharp transition between the two ‘phases’. So it is not strange that the Wang–Silk approximation becomes less accurate when $\sigma_\mathrm{g} \lesssim 0.2\sigma_\star$ and $Q_\mathrm{g} \sim Q_\star$.

(ii) Our approximation (equation 6) overestimates the effective Q parameter, but the error is less than 9 per cent and typically as small as 4 per cent. The accuracy and simplicity of our approximation result from a rigorous analysis, which takes into account the stability characteristics of the disc as well as the symmetries of the problem.

(iii) We provide a simple recipe for applying our approximation to realistically thick discs (see equation 9). The ratio of vertical to radial velocity dispersion is usually 0.5 for the stars and 1 for the gas. In this case, our approximation is in error by less than 15 per cent, whereas the Wang–Silk approximation can be in error by more than 50 per cent. Note also that the effective Q parameter is 20–50 per cent larger than in the infinitesimally thin case. Thus the effect of thickness is important and should be taken into account when analysing the stability of galactic discs.

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