# Uncertainties in estimating ergodic MIMO capacity and diversity gain of multiport antenna systems with different port weights

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# Uncertainties in Estimating Ergodic MIMO Capacity and Diversity Gain of Multiport Antenna Systems with Different Port Weights

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***Abstract*— The performance of multiport antenna systems located in multipath environment is characterized by their ergodic capacity and diversity gain. The impact on convergence and statistical uncertainties involved in the estimation and evaluation of these performance metrics is studied for different port weights (i.e. different embedded element efficiencies) and correlations. The study is performed by using an upgraded version of the multipath simulation tool Rayleigh Lab which emphasizes the presentation of the statistical characteristics on the ports rather than the electromagnetic modelling. Therefore, the Rayleigh fading channels are generated in a very simple way by using arrays of randomly distributed complex numbers, and the user can input efficiencies and correlations rather than far-field functions and mutual couplings.**

## I. INTRODUCTION

During the last decade, multiple-input multiple-output (MIMO) technology has been very popular and favorite among researchers and communication engineers to exploit its performance capabilities in terms of increased capacity and diversity gain of the system. In the meantime, antenna engineers developed MIMO antenna systems' measurement facilities and measurement procedures to quantify the performance in terms of embedded element efficiency, ergodic capacity, diversity gain, etc. The characterization for isotropic environment can readily be done in reverberation chamber [1][2], but it is also possible to measure embedded far-field functions and efficiencies in reverberation chamber and compute capacity and diversity gain from them. This provides equivalent results [3]. Still, the reverberation chamber provides an environment with time delay spread and coherent bandwidth similar to real-life environments [4], and can therefore also be used to characterize receiver sensitivity of active terminals [5] and through-put of complete MIMO systems.

The measurement accuracy of diversity gain and capacity still remains a matter of concern, which is studied in [6]. This study is extended in the current paper to multiple and weighted antenna ports. Recently, MIMO technology has gained much more interest after being introduced in laptops, Long Term Evolution (LTE) mobile terminals, etc. This has motivated us to develop numerical simulation tools Rayleigh-lab [7] and ViRM-lab [8]. These tools are developed to study the uncertainty of MIMO capacity and diversity gain results in Rayleigh fading environments, and their order of convergence towards an ergodic value.

There exist statistical uncertainties in the estimation of embedded element efficiency, correlation, ergodic MIMO capacity, and diversity gain of multiport antenna systems, due to the statistical nature of the fading channel. Analysis of convergence of these quantities and statistical uncertainties involved in their estimation is presented in [6] and [7] where the study was limited to 2×2 MIMO systems under ideal assumptions of maximum embedded element efficiency and no correlation. Currently, LTE technology is considering up to 4×4 MIMO communication systems which eventually provides better Signal-to-Noise Ratio (SNR) to enhance MIMO channel capacity. Therefore, in the present paper, we extend our previous analysis to study the impact on the convergence and uncertainty for multiport antenna systems with more than 2 ports, and we also study the effect of using different port weights, i.e. different embedded element efficiencies and different values of correlation among different embedded elements.

Standard deviation among several different realizations of the same quantity is quite evident and it can be utilized to quantify statistical uncertainties and order of convergence. Mathematically, standard deviation $s$ can be expressed as:

$$s = \left( \frac{1}{n-1} \times \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{1/2}$$

where

$$\bar{x} = \frac{1}{n} \times \sum_{i=1}^{n} x_i$$

and $s_{dB}$ can be expressed as [6]:

$$s_{dB} = \frac{1}{2} \times \left(10\, log_{10}(1+s) - 10\, log_{10}(1-s)\right)$$

$$s_{dB} = 5\, log_{10}\left(\frac{1+s}{1-s}\right)$$

A graph of standard deviation plotted against increasing number of samples of any specific quantity like diversity gain, channel capacity, etc. provides a good picture to compute the magnitude of statistical uncertainties and the order of convergence towards an ergodic value. The magnitude of the standard deviation shows the magnitude of the statistical uncertainties present after gathering specific number of samples. The slope of the standard deviation determines the order of convergence of the quantity towards its ergodic value. Certainly the standard deviation will decrease as the number of samples increase, but the magnitude and slope of standard deviation can be quite different or similar for different quantities. For example, standard deviation of channel capacity is much lower compared to that of diversity gain at 1% CDF level, but both have similar order of convergence. Therefore, a plot of standard deviation can tell us about: (i) the range of possible uncertainty in our measurement results and (ii) the number of samples (proportional to measurement time) required to compute diversity gain, channel capacity, etc. with a desired accuracy. Ultimately, apart from cost and size, measurement uncertainty and measurement time will also assist in distinguishing among different measurement facilities developed for multiport antenna measurements.

## II. DESCRIPTION OF RAYLEIGH-LAB

This study is based on the results provided by the numerical simulation tool called ‘Rayleigh-lab’. This tool is developed and designed in MATLAB with a Graphical User Interface (GUI). Rayleigh-lab simulates the response of Rayleigh fading environments at each antenna port in a multiport antenna system. This is accomplished by using matrices of Gaussian distributed complex numbers representing voltage samples in terms of magnitude and phase received at each antenna port. The values for embedded element efficiencies, correlation matrix, signal-to-noise ratio (SNR), number of samples, and number of realizations are user-defined values. The values for these user-defined variables are provided through the graphical user interface of Rayleigh-lab. Using this smart technique, Rayleigh-lab is computationally faster compared to those simulation tools where embedded element radiation patterns are loaded and embedded efficiencies and correlation values are estimated using these patterns. This tool can compute MIMO capacity using Shannon’s channel capacity formula; and diversity gain using Cumulative Distribution Function (CDF) curves typically at 1% level. Selection Combining (SC) and Maximal Ratio Combining (MRC) diversity schemes can be used to render diversity gain. The key terms like samples, realizations, embedded element efficiency, ergodic MIMO capacity, and diversity gain, are defined and explained in [6]. Previously, Rayleigh-lab was limited to 2×2 MIMO antenna systems but recently it has been upgraded to compute diversity gain and channel capacity of any higher order MIMO antenna systems. This helps to compare accuracy of measurement results of higher order MIMO systems as well.

## III. ESTIMATION OF DIVERSITY GAIN AND MIMO CAPACITY

The simulated voltage samples at each port of the antenna are represented by Gaussian distributed complex numbers. These voltage samples from each antenna port of a MIMO system are stored in arrays of complex numbers known as channels. The channel includes the effect of correlation and the embedded element efficiency of the antenna on which it is received in terms of voltage samples. These received channels are then processed to estimate the diversity gain and MIMO capacity of the multiport antenna system. The estimation starts from the very first sample for embedded element efficiency, correlation, and MIMO capacity and it improves as we increase the number of samples. For diversity gain, the estimation starting sample number depends on the CDF level used to compute diversity gain which is typically set to 0.01 i.e. 1%.This simply means that we accumulate 1 sample to estimate diversity gain after every 100 samples. Therefore, the estimation of diversity gain starts from sample number 100 when 1% CDF level is used. After receiving 10,000 voltage samples from each antenna port of a MIMO system, it can be easily observed that the estimated values of embedded element efficiencies, correlation, and MIMO capacity have already converged to their ergodic value. However, the convergence of diversity gain computed at 1% CDF level is quite poor in comparison to other quantities. This is due to the reason that there are only 100 samples for estimation (i.e. 1% of 10,000) after receiving total 10,000 samples at each antenna port of a MIMO system. Apart from using 1% CDF level, there is another method where diversity gain can be computed as fast as MIMO capacity and converges to its ergodic value soon after a few hundred samples. This method is devised in [9] where estimated embedded element efficiencies and estimated correlation are used to compute diversity gain from the very first sample received at each antenna port. A comparison of both methods is shown in [6] in terms of statistical uncertainties and order of convergence.

Statistical uncertainties in the estimation of any quantity like diversity gain and MIMO capacity towards an ergodic value depends on the number of samples accumulated to estimate the performance. Greater the number of samples, the better it is for the estimation of multiport antenna performance and vice versa. Instantaneous values of diversity gain and MIMO capacity are completely random, but their average value over large number of samples approaches towards the ergodic value. Therefore, we need increased number of samples for accurate measurement of diversity gain and MIMO capacity. In general, diversity gain and channel capacity of $M_t \times M_r$ MIMO system can be estimated by using

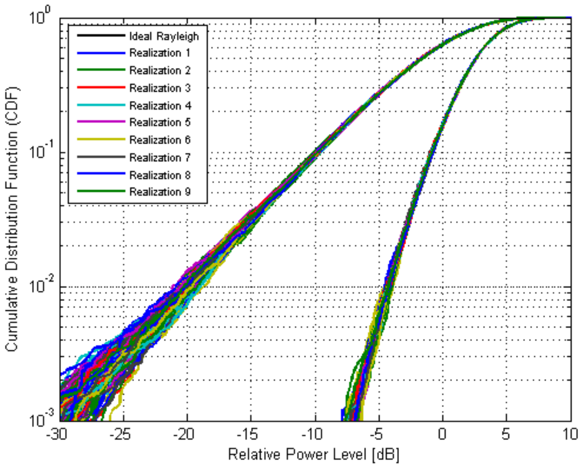

Fig. 1 CDFs of channel samples of a 4×4 MIMO system when all antenna branches have user-defined embedded element efficiency of 0 dB, and user-defined correlations of 0, i.e. the correlation matrix is an identity matrix.

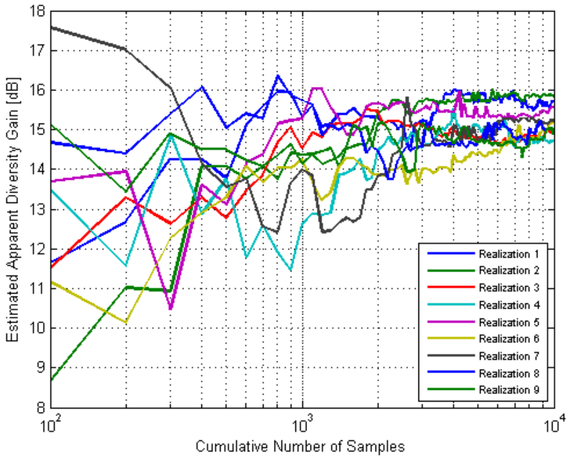

Fig. 2 Apparent diversity gain using 1% CDF level following the port weights in Fig. 1, for 9 different realizations, i.e. measurement sequences.

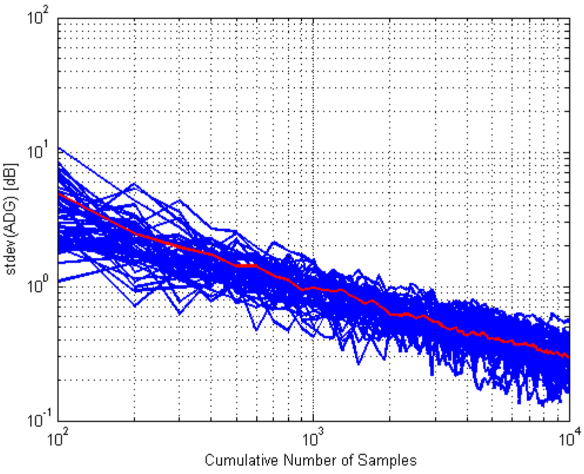

Fig. 3 Each of 50 blue curves shows standard deviation among 9 realizations of apparent diversity gain at 1% CDF level following the port weights in Fig. 1. The red curve is "true" standard deviation computed from 100 realizations

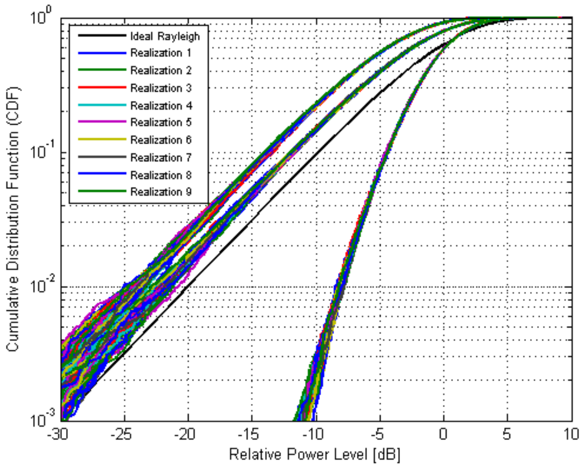

Fig. 4 CDFs of channel samples of a 4×4 MIMO system when two antenna branches have user-defined embedded element efficiency of -2 dB and the other two have -4 dB and user-defined correlation matrix.

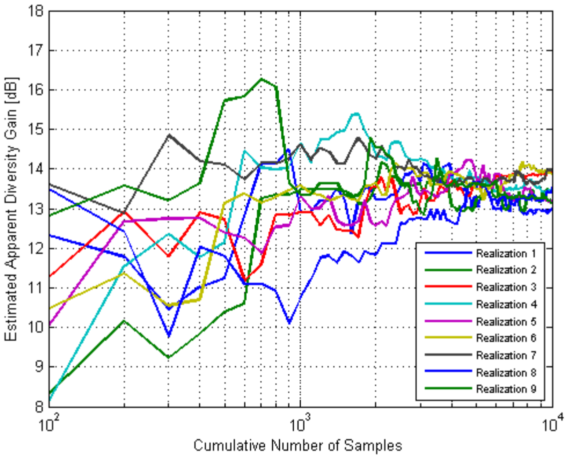

Fig. 5 Apparent diversity gain using 1% CDF level following the port weights in Fig. 4, for 9 different realizations, i.e. measurement sequences.

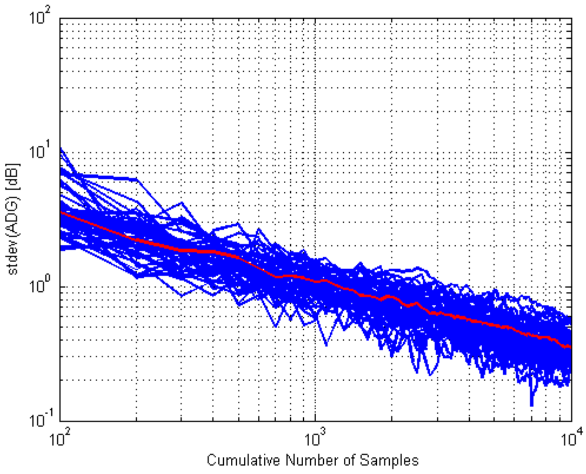

Fig. 6 Each of 50 blue curves shows standard deviation among 9 realizations of apparent diversity gain at 1% CDF level following the port weights in Fig. 4. The red curve is "true" standard deviation computed from 100 realizations

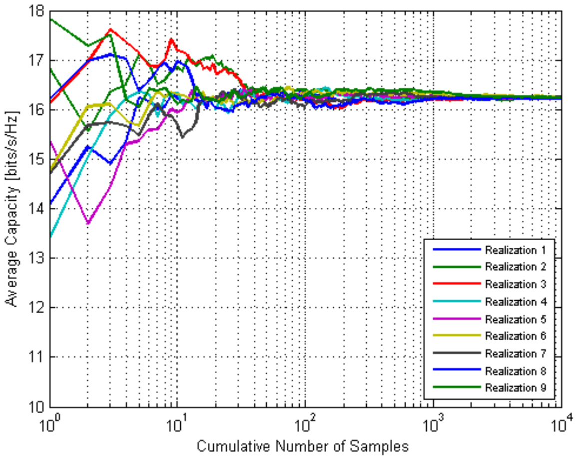

Fig. 7 A 4×4 MIMO channel capacity using 15 dB SNR level when all antenna branches have user-defined embedded element efficiency of 0 dB and user-defined correlations of 0, i.e. the correlation matrix is an identity matrix.

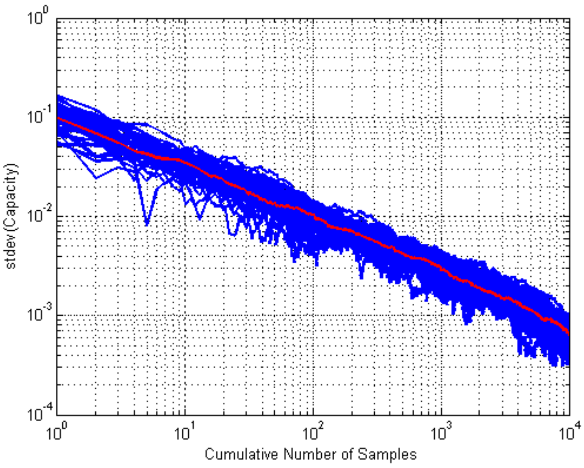

Fig. 8 Each of 50 blue curves shows standard deviation among 9 realizations of MIMO channel capacity using 15 dB SNR level following the port weights in Fig. 7. The red curve is "true" standard deviation computed from 100 realizations

the following formulae respectively:

$$DG_n = P_{div} / P_{branch}$$

$$C_n = log_2\left(\left|I_{M_r\times M_r} + \frac{SNR}{M_t} H_{M_r\times M_t} H^*_{M_r\times M_t}\right|\right)$$

$$\widetilde{C_n} = \left(\sum_{i=1}^{n} C_i\right) / n$$

where $DG_n$ is the diversity gain, $C_n$ is the instantaneous channel capacity at $n^{th}$ sample and $\widetilde{C_n}$ is the average channel capacity after accumulating $n$ samples, $P_{div}$ is the power level after applying selection combination, and $P_{branch}$ is the power level which appears to be the strongest among all antenna branches in a multiport antenna system. Both $P_{div}$ and $P_{branch}$ are typically read at 1% CDF level. $I$ is the identity matrix, $H$ is the channel matrix, and $H^*$ is the complex conjugate of $H$.

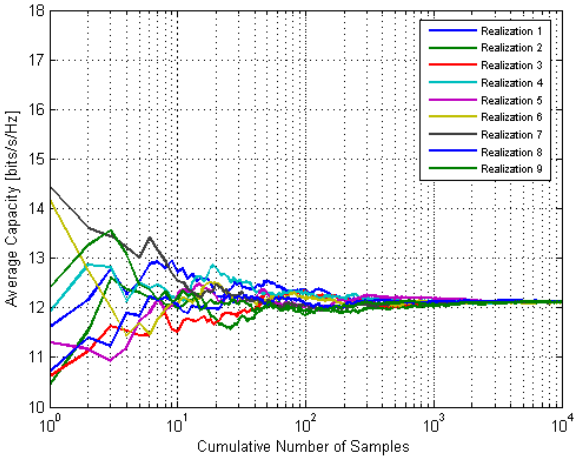

Fig. 9 A 4×4 MIMO channel capacity when two antenna branches have user-defined embedded element efficiency of -2 dB and the other two have -4 dB and user-defined correlation matrix

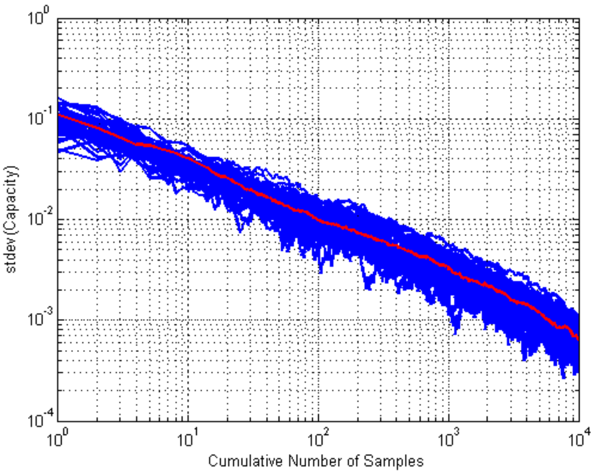

Fig. 10 Each of 50 blue curves shows standard deviation among 9 realizations of MIMO channel capacity using 15 dB SNR level following the port weights in Fig. 9. The red curve is "true" standard deviation computed from 100 realizations

## IV. Case Study of 4×4 MIMO System

In this case study, we will do a side-by-side comparison of the statistical uncertainties and the order of convergence of two 4×4 MIMO systems by choosing different port weights i.e. different embedded element efficiencies and correlations. We will have two cases, namely Case A and Case B for comparative study. Case A represents an ideal MIMO antenna system in which all antenna branches have the same embedded element efficiency of 0 dB and no correlation between channels, i.e. the correlation matrix is the identity matrix. Case B represents a case which is also a user-defined case e.g. two antenna branches having -2 dB embedded element efficiency while the other two antenna braches have -4 dB and the correlation matrix is as follows:

$$\begin{bmatrix} 1.0 & 0.5 & 0.3 & 0.1 \\ 0.5 & 1.0 & 0.2 & 0.4 \\ 0.3 & 0.2 & 1.0 & 0.6 \\ 0.1 & 0.4 & 0.6 & 1.0 \end{bmatrix}$$

All results on the right column i.e. Fig. 1-3 and Fig. 7-8 belong to Case A while all results plotted on the left column i.e. Fig. 4-6 and Fig. 9-10 belong to Case B. It is quite evident from comparative analysis of results that the performance is reduced when correlation exists between different antenna elements with non-ideal values of embedded element efficiencies. Even though the performance is clearly reduced in terms of diversity gain and channel capacity,there is no observable change in the statistical uncertainties and their convergence towards an ergodic value.

Note that all the diversity gains in the above figures have been read from the 1% level of the graphs. They would have converged much faster, already after 100 samples, by using the approaches described in [9]-[10].

## V. CONCLUSIONS

Several realizations of MIMO capacity and diversity gain have shown that: (a) Some hundred samples are needed to be gathered in each realization to find the converged or ergodic value, and (b) There exists an evident standard deviation among different realizations of the results before they converge to their ergodic value. The convergence and statistical uncertainties do not change by changing the port weights (i.e. embedded element efficiencies and correlation).

## ACKNOWLEDGMENT

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