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Safety Evaluation of Concrete Structures with Nonlinear Analysis

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Division of Structural Engineering
Concrete Structures
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THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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Photograph of the new Svinesund Bridge

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ABSTRACT

Concrete is the most widely used construction material in the world. To obtain effective new constructions and to use existing concrete structures in an optimal way, accurate structural models are needed. This requires that good approximations of important model parameters be available and that the nonlinear material response of concrete can be accounted for.

However, uncertain model parameters can significantly influence the structural response modelled which leads to high modelling uncertainty. To estimate uncertain parameters a methodology is proposed and applied to the new Svinesund Bridge to improve the initial finite element model through finite element model updating using on-site measurements.

To account for the nonlinear material response, it is also necessary to have a safety format suited to nonlinear analysis. However, the available safety formats for nonlinear analysis have been questioned and the need to quantify the modelling uncertainty of nonlinear analysis has been highlighted, Carlsson *et al.* (2008).

Therefore, the modelling uncertainty of nonlinear analysis was quantified based on available data. It was found that the uncertainty varies significantly depending on the failure mode obtained and that this uncertainty was often the factor that governed the safety evaluation. Based on this observation, a new safety format is proposed which allows the modelling uncertainty be explicitly accounted for. To facilitate realistic modelling the mean *in situ* material parameters are used in the nonlinear analysis; the reliability is assured by a, so called, resistance safety factor. Apart from the modelling uncertainty, the resistance safety factor depends on the material and geometrical uncertainty. It was found that the material variability can be estimated by using a sensitivity study, which involves two to three additional nonlinear analyses with reduced material strengths.

Applying the safety format to short columns loaded by a normal force and to beam sections loaded in bending, shear, and the combination of bending and shear, led to a reliability level that was in good agreement with the target reliability. Other safety formats for nonlinear analysis, according to EN 1992-2, CEN (2005), and Model Code 2010, fib (2010a), fib (2010b), were found to underestimate the modelling uncertainty of difficult-to-model failure modes, leading to a reliability level below the target reliability.

To study the consequences of assuring the safety on the structural level by an inequality of forces, as proposed in Model Code 2010, four safety formats were applied to a concrete portal frame bridge. It was shown that an inequality of forces on the structural level does not necessarily lead to the intended reliability level, unless the deformation capacity used is reliably available.

Key words: nonlinear analysis; modelling uncertainty, safety format; concrete; reliability; model updating, structural identification, concrete structures

Contents

Preface

The research presented here was carried out from January 2007 until June 2011 at the Chalmers University of Technology, Department of Civil and Environmental Engineering, Division of Structural Engineering. This study was financed by the Swedish Research Council, FORMAS, and the Swedish Transport Administration, Trafikverket.

My initial plan was to study one semester of economics abroad to improve my English and to enjoy the Australian sunshine. However, due to a good personal reason the destination was changed to Gothenburg and the topic was changed to structural engineering.

The stay in Gothenburg was made possible by Silke Olmscheid from the International Office and by Professor Reinhard Maurer from the Technical University of Dortmund who supported me during the struggle with bureaucracy. The interesting lectures given by Professor Björn Engström made me extend my stay by one year to write the Master's thesis at Chalmers under the helpful supervision of Professor Karin Lundgren and Kamyab Zandi Hanjari, Ph.D., together with Armando Soto San Roman. As a doctoral student I enjoyed the warm atmosphere at the Division of Structural Engineering as engendered by my helpful colleagues. I would like to thank my main supervisor and examiner, Professor Kent Gylltoft, for creating a stimulating working environment, giving a sense of stability, and for his helpful guidance. The patience, knowledge and the incredibly helpful attitude of my assistant supervisor Professor Mario Plos is very much appreciated. The great experience of my third supervisor Professor Sven Thelandersson, from Lund University, was very helpful. I have also made friends among my colleagues of whom Rasmus Rempling and my previous office mates, Kamyab Zandi Hanjari and Mathias Bokesjö, deserve special thanks. I would like to thank the members of my reference groups consisting of Poul Linneberg and Thomas Darholm, COWI, and Elisabeth Helsing, Peter Harryson and Ebbe Rosell, Trafikverket, and Professor Raid Karoumi, Royal Institute of Technology. The theoretical knowledge of Thomas Svensson, Ph.D., from the SP Technical Research Institute of Sweden combined with his helpfulness influenced this work significantly. The language editing by Lora Sharp McQueen and Evan Shellshear, Ph.D., and the support from Yvonne Juliusson and Lisbeth Trygg was of great help to me.

I wish to thank outstanding teachers, including Mr. Werner Lödige, Mr. Clemens Quante, Mr. Dieter Caase and Professor Piotr Noakowski who were inspiring, as well as my basketball coaches, Raimund Heggemann and Martin Krüger, who taught me much more than basketball.

Last but not least, I am most grateful to my friends and family who had limited access to me during the past six years. The outstanding encouragement of Simon, Martina and my parents deserves special recognition.

Gothenburg, May 2011

Hendrik Schlune

List of publications

The thesis is based on the work contained in the publications listed below, which are referred to in the text by Roman numerals.

I. Schlune H., Plos M. and Gylltoft K. (2009): Improved bridge evaluation through finite element model updating using static and dynamic measurements. *Engineering Structures*, Vol. 31, No. 7, pp. 1477–1485.

II. Schlune H., Plos M. and Gylltoft K. (2011): Safety Formats for Nonlinear Analysis of Concrete Structures. Provisionally accepted for publication in *Magazine of Concrete Research* April 2011.

III. Schlune H., Plos M. and Gylltoft K. (2011): Testing of Safety Formats for Nonlinear Analysis on Concrete Beams Subjected to Shear Forces and Bending Moments. Accepted for publication in *Engineering Structures*, April 2011.

IV. Schlune H., Plos M. and Gylltoft K. (2011): Safety verification on the structural level using nonlinear finite element analysis: Application to a concrete bridge. Submitted for publication in *Structural and Infrastructure Engineering*, April 2011.

Other publications by the author

Licentiate Thesis

Schlune H. (2009): *Improved Bridge Evaluation : Finite Element Model Updating and Simplified Non-linear Analysis*. Licentiate Thesis, Department of Civil and Environmental Engineering, Division of Structural Engineering, Concrete Structures, Chalmers University of Technology, Gothenburg, 2009.

Scientific Papers

Schlune H., Plos M. and Gylltoft K. (2009): Non-linear Finite Element Analysis for Practical Application. *Nordic Concrete Research*, Vol. 39 No. 5, 1/2009, pp. 75–89.

Lundgren K., Kettil P., Hanjari K. Z., Schlune H. and Roman A. S. S. (2009): Analytical model for the bond-slip behaviour of corroded ribbed reinforcement. *Structure and Infrastructure Engineering*, No. 1, pp. 1–13.

Conference Papers

Lundgren, K., San Roman, A. S., Schlune, H., Zandi Hanjari, K. and Kettil, P. (2007) "Effects on bond of reinforcement corrosion." *International RILEM Workshop on Integral Service Life Modelling of Concrete Structures*, 5-6 November 2007, Guimarães, Portugal, pp. 231–238.

Schlune H., Plos M., Gylltoft K. (2008). "Bridge Evaluation through Finite Element Analysis and on Site Measurements – Application on the New Svinesund Bridge." *Proceedings Nordic Concrete Research*, 8-11 June 2008, Bålsta, Sweden, pp. 70–71.

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Popular Science

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Notation

Roman upper case letters

E	Action effects
E_d	Design action effects
F	Actions
F_{Ed}	Design actions
F_{Rd}	Design structural resistance
F_{Ru}	Ultimate structural resistance
F_{Rum}	Ultimate structural resistance when the mean values of material strengths are used in the nonlinear analysis
F_{Ruk}	Ultimate structural resistance when the characteristic values of material strengths are used in nonlinear analysis
G	Permanent load
K	Constant that takes into account the influence of the redistribution of forces according to Henriques *et al.* (2002)
M_G	Bending moment due to permanent load
M_Q	Bending moment to variable load
N	Number of experiments
P_f	Probability of failure
Q	Variable load
R	Resistance
R_n	Nominal resistance
R_d	Design resistance
V_{f_c}	Coefficient of variation of the concrete compressive strength
V_R	Coefficient of variation of the resistance
V_f	Coefficient of variation to account for the variability of the material strength
V_{f_y}	Coefficient of variation of the yield strength of the reinforcement steel
$V_{f_{c,is}}$	Coefficient of variation of the *in situ* concrete compressive strength
$V_{f_{ct,is}}$	Coefficient of variation of the *in situ* concrete tensile strength
$V_{\beta_{EN}}$	Coefficient of variation of the reliability index when designing according to EN 1992-2
$V_{\beta_{new}}$	Coefficient of variation of the reliability index when designing according to the new safety format
V_θ	Coefficient of variation of the variable to model the modelling uncertainty
$V_{\theta,R}$	Coefficient of variation of the variable to model the modelling uncertainty associated with the critical failure mode
$V_{\theta,E}$	Coefficient of variation to model the modelling uncertainty associated with the loading in the critical section

Roman lower case letters

a_{nom} Nominal value of geometrical parameter
c Step size parameter
d Effective depth of beam
$\tilde{f}_{\mathrm{c}}$ Concrete compressive strength used in the nonlinear analysis according to the global resistance factor method and in the safety format for nonlinear analysis from EN 1992-2
f_{cd} Design value of concrete compressive strength
f_{ck} Characteristic concrete compressive strength (of specially cured cylinders)
f_{cm} Mean concrete compressive strength (of specially cured cylinders)
$f_{\mathrm{cm,is}}$ Mean *in situ* concrete compressive strength
$f_{\mathrm{ctm,is}}$ Mean *in situ* concrete tensile strength
$\tilde{f}_{\mathrm{y}}$ Yield strength of the reinforcement steel used in the nonlinear analysis according to the global resistance factor method and in the safety format for nonlinear analysis from EN 1992-2
f_{yd} Design value of the yield strength of the reinforcement steel
f_{yk} Characteristic value of the yield strength of the reinforcement steel
f_{ym} Mean value of the yield strength of the reinforcement steel
f_{x} Joint probability density function
$g(\dots)$ Limit state function, or function to represent the nonlinear analysis
$r(\dots)$ Resistance function
$\mathrm{s}(\dots)$ Loading or action effect function
x Concrete compressive zone depth

Greek lower case letters

α_{E} Sensitivity factor of action effect side
α_{R} Sensitivity factor of resistance side
β Reliability index
β_{R} Reliability index of the resistance side
$\bar{\beta}_{\mathrm{EN}}$ Mean reliability indexes for the design according to EN 1992-2
$\bar{\beta}_{\mathrm{new}}$ Mean reliability index for the design according to the new safety format
γ_{C} Partial factors for the concrete compressive strength, accounts for variability of material strength, geometrical variability, and resistance model uncertainty
γ_{F} Partial factor for the actions, accounts for variability of the actions, geometrical variability, and action model uncertainties
γ_{f} Partial factor for the actions, accounts for the variability of the actions
γ_{G} Partial factor for permanent actions, accounts for variability of the actions, geometrical variability, and action model uncertainties
γ_{g} Partial factor for permanent actions, accounts for variability of permanent actions
γ_{Q} Partial factor for variable actions, accounts for variability of the actions, geometrical variability, and action model uncertainties
γ_{q} Partial factor for variable actions, accounts for variability of the actions

γ_{R} Partial factor for the resistance

γ_{Rd} Partial factor to account for the resistance model uncertainty and geometrical uncertainty

γ_{RdR} Partial factor to account for the uncertainty of the structural resistance, accounts for geometrical and material variability and the modelling uncertainty

$\gamma_{\mathrm{R,ductile}}$ Partial factor for ductile failure modes according to Six (2001)

$\gamma_{\mathrm{R,brittle}}$ Partial factor for brittle failure modes according to Six (2001)

γ_{S} Partial factors for the yield strength of the reinforcement steel, accounts for variability of the steel strength, geometrical variability, and resistance model uncertainty

γ_{Sd} Partial factors for the action and action effect model

Φ Cumulative distribution function

θ Random variable to model the modelling uncertainty

θ_{R} Random variable to model the resistance modelling uncertainty

γ_{sys} Partial factor to account for system reliability and uncertainty of global structural model according to Six (2001)

θ_{m} Mean ratio of experimental- to predicted strength for the chosen modelling approach

$\theta_{\mathrm{R,m}}$ Mean ratio of experimental- to predicted strength of the critical component

$\theta_{\mathrm{E,m}}$ Mean ratio of experimental- to predicted loading in the critical component of a structure

$\sigma^2_{\mathrm{f_{c,is}}}$ variance of the *in situ* concrete compressive strength

$\sigma^2_{\mathrm{f_{ct,is}}}$ variance of the *in situ* concrete tensile strength

$\sigma^2_{\mathrm{f_y}}$ variance of the yield strength of the reinforcement steel

1 Introduction

1.1 Background

Concrete is the most widely used construction material in the world with an uncountable number of existing structures made of concrete and a production worth at least 35 billion US$ in the year 2010, USGS (2011). For an optimal utilization of existing structures and for efficient new constructions, accurate models for the behaviour of concrete structures are essential.

However, the accuracy of structural models is not always satisfactory. This can be a result of unavoidable modelling assumptions about interactions of structural parts, boundary conditions, and unknown model parameters. Furthermore, it is necessary that the nonlinear material response be accounted for, and this requires safety formats which are suitable to be used in combination with nonlinear analysis.

The thesis presented here consists of two parts which address these two problems.

1.2 Objective, scientific approach and limitations

The general objective underlying the two parts of this thesis is to facilitate more accurate structural evaluations of bridges and concrete structures. The first part deals with uncertainties in structural modelling that can be reduced by combining on-site measurements with finite element (FE) analysis. The aim is to develop methods for improved assessment and maintenance of bridges by means of FE analysis combined with on-site measurements. The aim was approached by:

- Studying available literature and evaluating existing methods for combining FE analysis with on-site measurements for improved bridge evaluation.
- Applying appropriate evaluated methods to the new Svinesund Bridge as a case study, and
- Drawing general conclusions and recommendations from the experience gained through the case study.

The focus of the project was on the modelling error that is introduced by uncertainties in model parameters and boundary conditions.

The aim of the second part is to deepen the understanding of the safety evaluation of concrete structures with nonlinear analysis. This was approached by:

- Quantifying the modelling uncertainty of nonlinear analysis,
- Studying principles and available safety formats for nonlinear analysis of concrete structures,
- Proposing a new safety format which is generally applicable, and
- Testing the new safety format by means of full probabilistic analysis.

The study focused on the resistance side in the ultimate limit state. No spatial variability or system reliability was included in this study. The stochastic models used for the reliability analysis were based on limited information.

1.3 Original features

A methodology for FE model updating to improve bridge evaluation is proposed. Applying the proposed methodology to the new Svinesund Bridge included the updating of a nonlinear FE model using an optimisation algorithm; it was shown that manual model refinements would be important in that compensating for modelling errors by meaningless changes to model parameters could be avoided.

To facilitate the safety evaluation of concrete structures with nonlinear analysis for different kinds of structures and failure modes, the modelling uncertainty for this type of analysis, was quantified. The importance of the modelling uncertainty as the factor that often governs the safety evaluation was highlighted, and it was shown that current safety formats, CEN (2005) and fib (2010b), do not properly account for the modelling uncertainty of difficult-to-model structures and failure modes.

Based on the observation that the modelling uncertainty varies considerably for different types of failure modes, a new safety format proposed here for the nonlinear analysis of concrete structures was successfully tested; this new safety format enables one to explicitly account for the modelling uncertainty.

It was shown that the material uncertainty can be approximated by a sensitivity study. Despite nonlinear response surfaces, it was shown that a linear approximation of the response surfaces based on two to three additional nonlinear analyses offers sufficiently accurate results, provided that an appropriate step size for the sensitivity study is used.

By writing a pre- and postprocessor for Response-2000, Bentz (2000), a tool for full probabilistic analysis of beam sections, subjected to arbitrary combinations of normal forces, shear forces and bending moments, was developed.

1.4 Outline

The thesis comprises four papers and an introductory part that provides a framework for the articles. In **Chapter 2** and ***Paper I*** background information about FE model updating is given, the application of FE model updating in a case study on the new Svinsesund Bridge is described, and general recommendations are drawn from the study. **Chapter 3** is about the safety evaluation of concrete structures with nonlinear analysis. Background information in Section 3.1 is followed by a short description of reliability methods in Section 3.2. The quantification of the modelling uncertainty of nonlinear analysis is provided in Section 3.3; available safety formats for nonlinear analysis are described in Section 3.4 and ***Paper II***. A new safety format is proposed in Section 3.5 and the testing of the safety format is described in Section 3.6 and ***Papers II, III*** and ***IV***. Important aspects of safety evaluations with nonlinear analysis are discussed in Section 3.7. Finally, in **Chapter 4** the main conclusions of this work together with suggestions for future research are given.

2 Finite element model updating

2.1 Background

To model structures, assumptions of unknown properties such as boundary conditions, interaction between structural parts, and material parameters must be made. However, making reasonable assumptions can be difficult and uncertainties in structural modelling have been shown to greatly influence the results from an FE analysis, see Huria *et al.* (1993), Shahrooz *et al.* (1994), Song *et al.* (2002). Hence, even very detailed FE models can be inaccurate: discrepancies between simulated and measured responses of the order of 100% on the global level and 500% for local responses have been found, Bell *et al.* (2011), Enevoldsen *et al.* (2002).

Therefore, current assessment procedures with limited quantitative coupling between on-site inspections and structural modelling often do not provide an accurate structural evaluation. This shortcoming initiated the development of procedures to update structural models based on on-site measurements with the aim to bridge the gap between reality and modelling.

In the following*, FE model updating* is used to denote the complete process of adjusting an FE model to better correspond to measurements. *Structural Identification* has been used in the ASCE state-of-the-art report on applications in civil engineering in Bell *et al.* (2011) to denote the same process.

2.2 Problem description

To make a model agree better with reality, some structural parameters, such as geometric or material ones, can be measured directly and the information gained can be included directly in the structural model. However, other important parameters associated with boundary conditions and the interaction of structural parts can often only be estimated by an inverse analysis, see Figure 2.1. Rather than measuring the uncertain parameters directly, the response of the structure to a given loading is measured. In a second step, the uncertain parameters are tuned in a structural model in order to obtain closer agreement between the measured response and the modelled one. The underlying assumption is that the initial FE model is a good representation of the actual bridge, apart from the model parameters that are tuned.

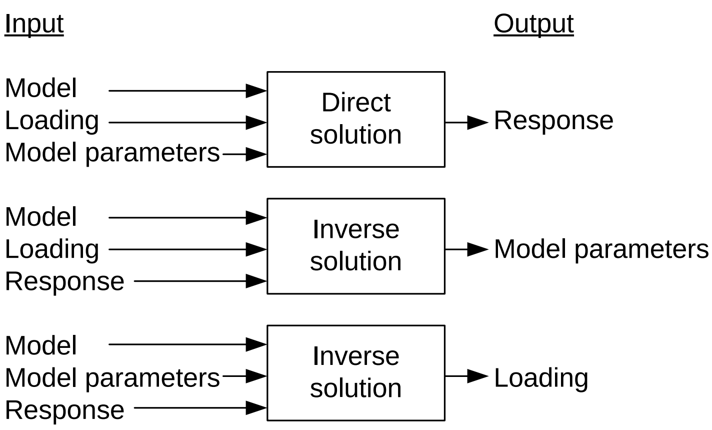

Figure 2.1 Direct and inverse problems, redrawn and adapted from Johansson (2007)

However, as is typical for an inverse problem, estimating structural parameters by inverse analysis has often been shown to be an ill-posed problem. This means that it does not fulfil the requirements of a well-posed problem, namely existence, uniqueness and the stability of a solution, a definition which goes back to Hadamard (1902). In this context "stability" means that small variations of the initial model yield only small variations in the results. Researchers have noticed that many possible combinations of updating parameters in realistic ranges could be found, which have led to more accurate FE models, Zhang *et al.* (2001), Jaishi and Ren (2005); studies have shown that it is difficult to identify model parameters when a rather small amount of artificial noise is added to simulated measurements, Jaishi and Ren (2007), Bakir *et al.* (2008). It has been shown that measurements are often insensitive to structural changes, Brownjohn *et al.* (2001), Huth *et al.* (2005).

This makes it difficult to find updating parameters that are not just an arbitrary combination of model parameters, which conceal the measurement and remaining modelling errors, but are improved estimates of the actual structural parameters.

For bridge applications, two research initiatives tackled the problem of combining on-site measurements with structural modelling for bridge evaluation by using different approaches. The research approaches are here designated *Bridge evaluation by static load testing* and *FE model updating in structural dynamics*.

2.3 Bridge evaluation by static load testing

When bridges are subjected to static loads, strains, forces and deformations are recorded, see Chajes *et al.* (1997), Barker (2001), Huang (2004). Depending on the purpose of the load test, a distinction between a proof load test and a diagnostic load test can be made, Cruz and Casas (2007), A. G. Lichtenstein and Associates (1998).

In proof load tests, the bridge is subjected to very high loads which include the risk of damage and collapse of the structure. The aim is to prove that the bridge has the required capacity. The information gained from the tests can be incorporated in the probabilistic model of the bridge to truncate the theoretical capacity distribution. Applications of proof load tests can be found in Nowak and Tharmabala (1988) and Moses *et al.* (1994).

The other branch of static load testing, i.e. diagnostic load testing, focuses on the structural model instead of the probabilistic model. A lower load level is often chosen and the load tests are used to improve the understanding of the bridge behaviour, as well as to verify and adjust the structural model. Uncertainties regarding material properties, boundary conditions, and interaction between structural parts can be reduced, and shortcomings of the structural model can be eliminated. The changes to the initial FE model are usually introduced manually; they are not limited to parametric changes to the model. Thus, all possible sources of modelling error can be reduced by justified changes.

A major problem of diagnostic load tests is the extrapolation of the measured bridge behaviour to other loads. Bridges can have non-stationary boundary conditions due to temperature and moisture changes, can show sudden releases of movement systems, and show nonlinear geometrical and material behaviour. Findings from the load test may therefore be invalid for other loading configurations. To take these things into account, it is required that the reason for the differences in structural behaviour between the model and the real bridge be found. An overview of the effects in bridges that can lead to a significant different behaviour change from that initially assumed, and a discussion of whether they can be relied upon, can be found in Bakht and Jaeger (1990) and A. G. Lichtenstein and Associates (1998).

2.4 FE model updating in structural dynamics

The term *FE model updating in structural dynamics* is used when an improved agreement between measured and computed modal (dynamic) data is desired. This branch emerged initially in mechanical and aerospace engineering and was later applied to civil engineering structures. When modal data are used to update a model, a distinction between direct and indirect updating methods can be made, Friswell and Mottershead (1995).

- In direct methods, the mass, stiffness and damping matrixes are updated directly. The advantage of direct methods is that they do not require iterations, which eliminates the risk of divergence and excessive computational demands. However, the major drawback of direct methods is that the updated mass and stiffness matrix may lose their physical meaning.
- In contrast to that, iterative methods solve the inverse problem, as the name implies, iteratively. Instead of directly changing the complete FE model, only uncertain model parameters are changed iteratively in order to make the FE model agree better with the measurements. The inverse problem is solved by a repeated solution of the direct problem as part of an iterative optimization procedure. When using optimisation algorithms to solve the inverse problem, the model changes introduced are usually restricted to parameter changes. Hence, only a part of the total modelling error can be reduced.

2.5 Proposed methodology

The proposed and applied methodology for FE model updating aims to combine the advantages of the framework of diagnostic load tests with those of the mathematically more advanced concept of iterative methods for FE model updating in structural dynamics. This led to the methodology for FE model updating which is presented and applied in *Paper I*. It consists of three main steps.

1. The methodology starts from an FE model suited for the design of the bridge. First the lower bound assumptions, which are appropriate for the design of bridges but not for FE model updating, have to be removed. Second, manual model refinements are introduced to the FE model. This facilitates dealing with all kinds of modelling errors. This is followed by a sensitivity study to get an overview of model parameter sensitivities and to find improved estimates of model parameters.

2. Fine tuning of model parameters is done with the help of optimisation algorithms.

3. To make the updated model applicable for modelling untested conditions, it is necessary to evaluate the accuracy of the updated parameters and to find any possible sources of model parameter changes. Before using the model to analyse conditions that are different from the testing conditions, it may be

necessary to eliminate previously introduced model changes which do not hold for the conditions that are to be analysed.

2.6 Application to the new Svinesund Bridge

2.6.1 The new Svinesund Bridge

The new Svinesund Bridge was opened for traffic, in June 2005, as a part of the new European road E6 between Gothenburg and Oslo. The bridge connects Sweden and Norway over the Idefjord. With a total length of 704 m and a main span length of 247 m, it is one of the longest single arch bridges of the world, see Figure 2.2.

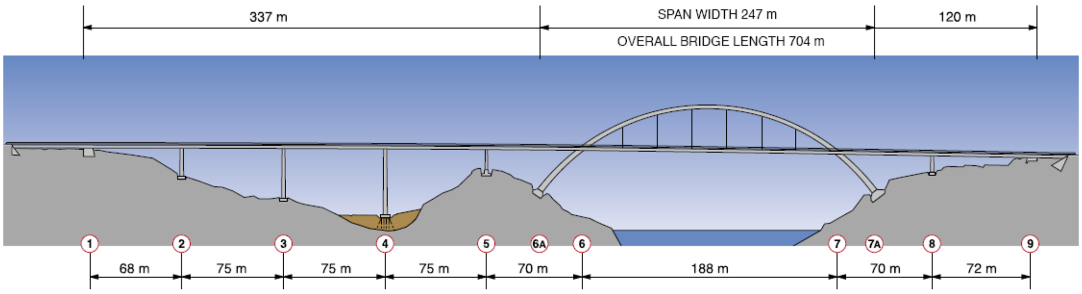

Figure 2.2 Elevation of the new Svinesund Bridge, from Darholm et al. (2007)

The two bridge deck girders carry two lanes of traffic each and are made of steel. In the side spans the bridge deck girders are supported, via cross beams, by concrete columns, while in the main span the cross beams are suspended from the concrete arch. The bridge deck girders are connected to the concrete arch where they pass the arch on either side. Due to the slender columns and the wide spacing of the bridge deck girders, it was necessary to prestress the bridge deck girders onto the columns to avoid uplifting during asymmetric loading.

When the new Svinesund Bridge was constructed, a measurement program was initiated to check and verify the response of the bridge. The programme started during the construction phase, included two days of testing before opening the bridge, and has been running during the first years of service of the bridge, see James and Karoumi (2003), Ülker-Kaustell and Karoumi (2006) and Karoumi and Andersson (2007).

Due to the available measurements, the new Svinesund Bridge was chosen for a case study for FE model updating. The data used to update the FE model included in total 264 measurements of four types. This large amount of data reduced the risk of non-unique solutions of the updating procedure, which can occur when a small number of measurements is used to update a large number of model parameters.

2.6.2 Updating of the finite element model

The FE model that was used for updating is based on the designers' model for the bridge. A grid of beam elements was used to represent the bridge deck girders, with longitudinal beams for the longitudinal walls and transverse beams for the transverse walls, see Figure 2.3. The columns and the arch were also represented by beam elements. Shear deformations were included by using Timoshenko beam theory. Including the temporary supporting structures, the FE model had 11 724 degrees of freedom. A more detailed description of the FE model, including details of the model conversion into the FE software package ABAQUS, can be found in Plos and Movaffaghi (2004).

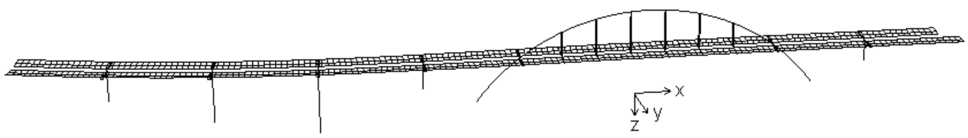

Figure 2.3 FE model of the new Svinesund Bridge

The manual model refinements included:

- Increasing the Young's modulus of the arch to account for the reinforcement in the arch and the further hardening up to the day of load testing,
- Remodelling of the bearing behaviour of the bridge,
- Including the non-structural mass, and
- Increasing the bridge deck stiffness.

For fine tuning of model parameters, the Nelder-Mead simplex algorithm, Nelder and Mead (1965), was used to avoid convergence problems of gradient-based optimisation algorithms. Parameters that were fine tuned were:

- The elastic modulus of the arch,
- The elastic modulus of the bridge deck,
- The static friction threshold of the bearings, and
- The mass of non-structural elements along the bridge deck girder.

2.6.3 Results of updating

An overview of the accuracy of both the initial and updated models is shown by plotting the numerical responses, before and after updating over the experimental counterparts, see Figure 2.4. Good agreement between numerical and experimental responses is obtained when the markers are close to the line of equality. To exemplify the effect of updating, only the responses corresponding to the FE model updated with respect to one objective function, J_3, are shown. This function is defined as the sum of absolute differences between the numerical and experimental responses normalised by the standard deviation of the specific type of measurement.

It can be seen that a significantly improved agreement for the eigenfrequencies, the displacements, and the hanger forces was obtained. For the strains, only small improvements were achieved. Possible reasons for this could be local effects which could not be captured by the FE model due to the simplified representation of the concrete arch using beam elements, the still quite simplified representation of the bearing response, or measurement errors.

The updated model parameters remained within reasonable ranges; the reason could be found for the changes that were manually introduced into the model before the parameter study.

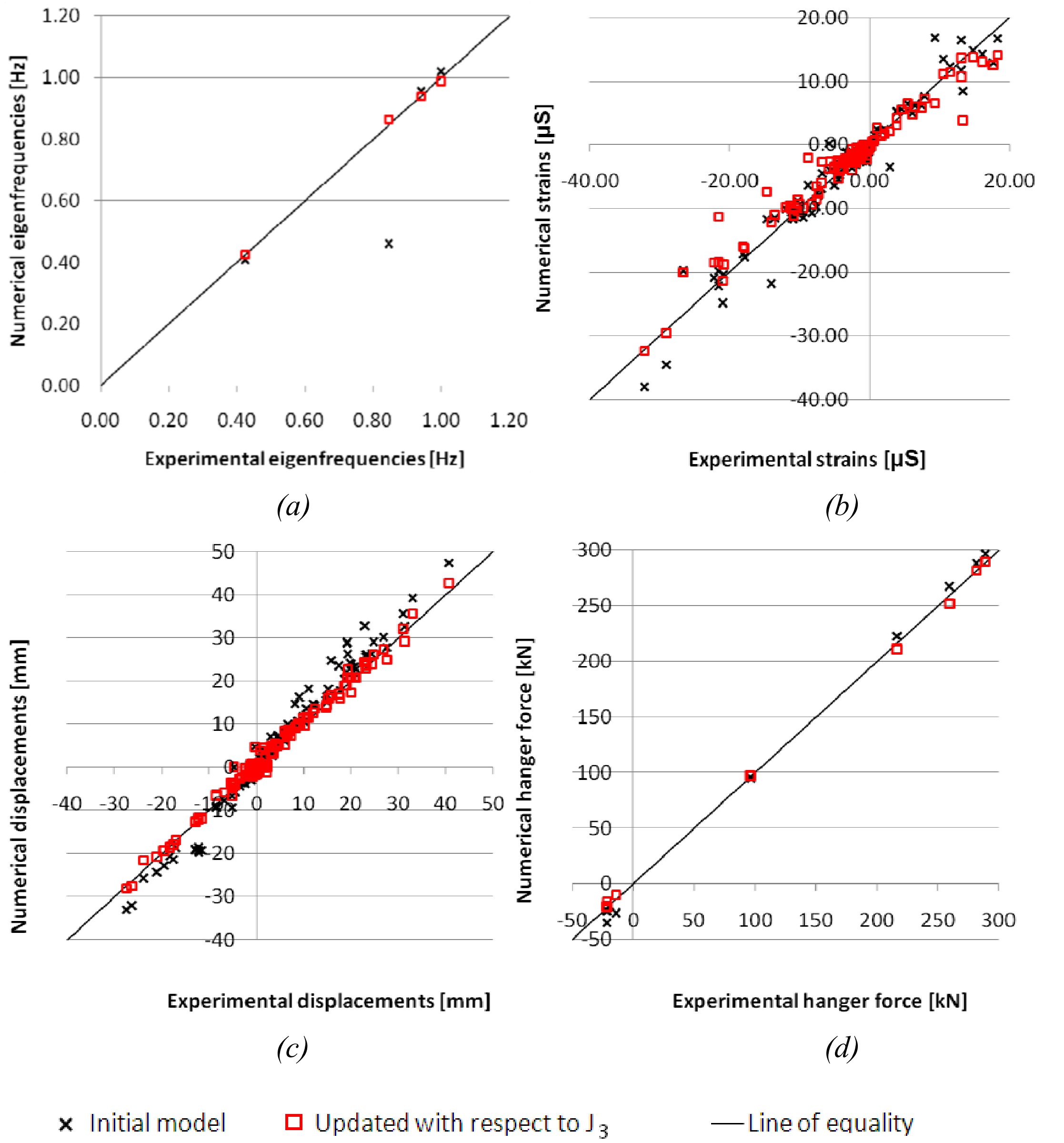

Figure 2.4 A comparison of the accuracy of initial and updated model: (a) Eigenfrequencies, (b) Strains, (c) Displacements, (d) Hanger forces.

2.7 General recommendations

To obtain improved agreement between measured and modelled response, model parameters are often fine tuned. However, in the study of the new Svinesund Bridge, including the nonlinear bearing behaviour, was shown to offer a major improvement of the model accuracy. This showed the importance of the nonlinear response of the structure and modelling assumptions that go beyond uncertain model parameters. Therefore, only changing model parameters by using optimisation algorithms seldom leads to model parameters which are improved estimates of the real structural parameters. Instead the model parameters will be calibrated to conceal inappropriate modelling assumptions.

The combination of static and modal measurements showed that different bearing behaviours during ambient vibrations and under the load test must be assumed. Hence, to update the bearing parameters using modal data and to use these parameters for the evaluation of the bridge under static loading can be impossible.

Despite the large number of measurements, it was not possible to update the rotational stiffness of the arch support. The parameter study showed that the target responses were insensitive to this parameter, which made it impossible to update it parameter by inverse analysis. This shows that a careful choice of measurement program is needed. Prior to determining the programme, a sensitivity study is therefore recommended. By changing uncertain model parameters in the a priori FE model, the sensitivity of measurable responses to model parameters changes can be studied. This can be used to find an appropriate measurement program which allows estimating all uncertain model parameters.

The use of the Nelder-Mead simplex algorithm for fine tuning of the model parameters is recommended to avoid the convergence problems of gradient based optimisation algorithms.

3 Safety Evaluation of Concrete Structures with Nonlinear Analysis

3.1 Background

The verification of a structure or a structural component can be done on three different levels: the structural level, the sectional level or the material level, see Figure 3.1. Depending on the chosen level, the verification is done using an inequality of forces, generalised stresses, or stresses.

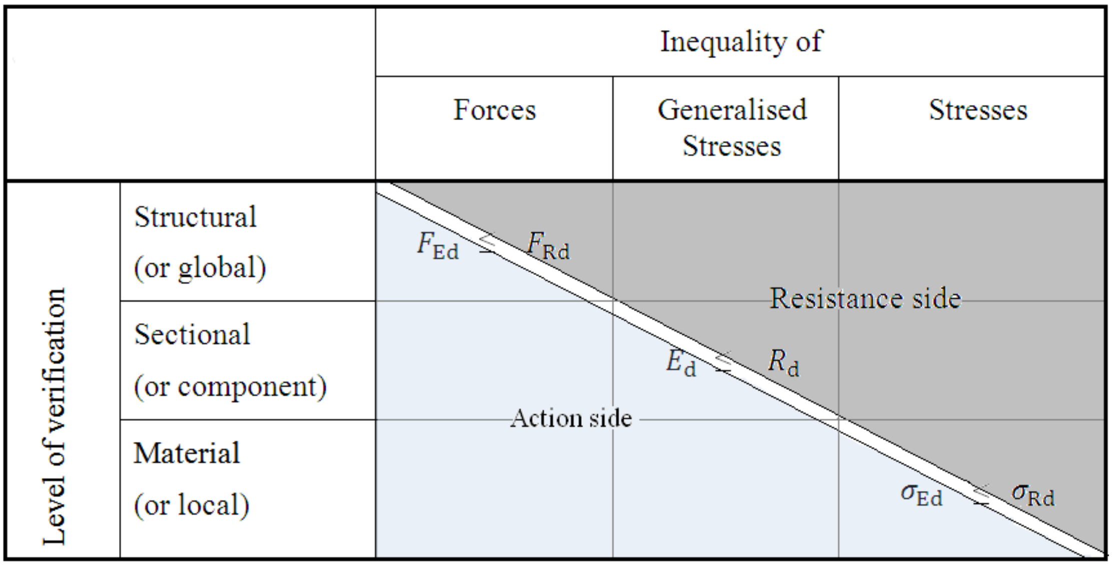

Figure 3.1 Possible levels for verification

Today, the standard design of concrete structures is based on a two-step procedure and the verification is done on the sectional level. In the first step, generalised stresses due to external loading are calculated using a structural model. These generalised stresses are denoted "action effects", E, and the design values are denoted "design action effects", E_{d}. In the second step, the maximum allowable generalised stresses, R_{d}, are calculated. These generalised stresses are denoted "design resistances" and are calculated using sectional models. Each critical section is then verified by showing that the design action effects, E_{d}, are smaller than or equal to the design resistances, R_{d}, see Figure 3.2.

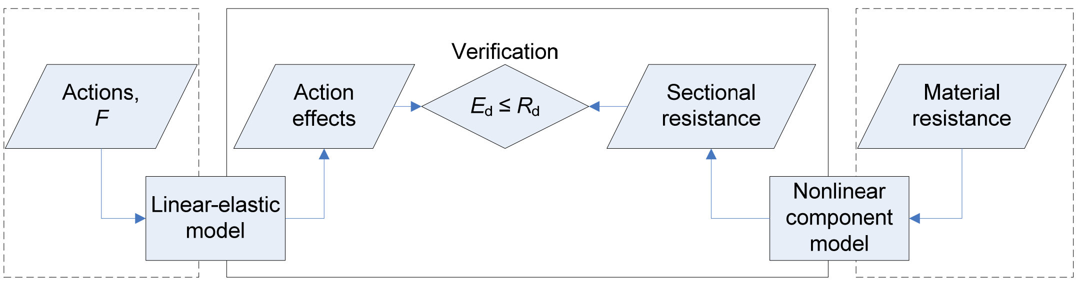

Figure 3.2 Flow chart for the design according to the two-step procedure

This standard two-step procedure is inconsistent because incompatible constitutive relations are assumed in both the structural model and the sectional resistance models. The structural model is usually based on the assumption of a linear-elastic material response, while the sectional resistance models account for the nonlinear material behaviour. Linear-elastic structural models do not allow the structural response to be modelled realistically. Cracking, which occurs even under service loads, yielding of the reinforcement and crushing of concrete for higher loads, cannot be captured. In addition, the release of restraining forces and the deformation increase due to cracking also cannot be modelled. Nor is it possible to utilise the full capacity of structures by redistribution. For advanced structures it can also be difficult to identify critical sections; only highly simplified resistance models are available.

To overcome the drawbacks of the two-step procedure, nonlinear analysis is increasingly used to calculate the failure load directly as part of a one-step procedure. 'Nonlinear analysis' is used here to denote an analysis which accounts for the nonlinear stress-strain relationship of the concrete and reinforcement steel, and allows for redistribution; it can be used to calculate the failure load of a structure directly. The load is usually increased incrementally and the constitutive models employed automatically guarantee equilibrium in all parts of the structures. This means that the nonlinear analysis fulfils the purpose of both the structural model and the sectional models according to the two-step procedure, see Figure 3.3. Additional manual sectional checks are needed only for failure modes that cannot be described by the nonlinear analysis.

The distinction between action effects and sectional resistances is well suited for the two-step procedure as two separate models are used. However, for the one-step procedure, when a single nonlinear analysis is used, it is more common to distinguish between external and internal forces. External forces are acting on the nonlinear model and it checks if internal forces can be found to balance the external forces. Therefore, the verification for the one-step procedure is usually done on the structural level by an inequality of forces.

For the ultimate load, which corresponds to the final load step at which the nonlinear analysis finds equilibrium, there are many expressions, such as "theoretical ultimate load" and "structural load bearing capacity" by König *et al.* (1995), "theoretical carrying capacity of the system" by König *et al.* (1997), the load at which "there is global failure of the structure" in EN 1992-2, and "(global) resistance" by Model Code 2010 fib (2010a) fib (2010b), Cervenka *et al.* (2007), Henriques *et al.* (2002) and Six (2001). In this thesis "ultimate structural resistance" will be used. The following symbols have been used in the past:

- F by König *et al.* (1995),
- R_{sys} by König et al. (1997),
- q_{ud} in EN 1992-2, and
- R by Model Code 2010, fib (2010a) fib (2010b), Cervenka *et al.* (2007), Henriques *et al.* (2002) and Six (2001).

Here, the symbol F_{Ru} is used ("R" was used in *Paper II, III* and *IV*). The design value here is called "design structural resistance" with the symbol, F_{Rd}.

To denote the forces obtained from the Eurocodes which are applied on the nonlinear model, i.e. the design actions, different symbols have been used:

- $\gamma_G \cdot G + \gamma_Q \cdot Q$ by König *et al.* (1995) and in EN 1992-2,
- F_{d} in the Model Code 2010 and by Six (2001), and
- S_{d} by Henriques *et al.* (2002).

In the following, F_{Ed}, is used as the symbol to denote the design actions (in *Paper IV* "F_{d}" was used). The structure can then be verified by showing that the design actions are smaller or equal to the design structural resistance, $F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}}$, see Figure 3.3.

It is equivalent to show that the nonlinear analysis finds equilibrium when the design actions are applied on the nonlinear model. This can simply expressed by $g(F_{\mathrm{Ed}}, \dots) > 0$, where g represents the nonlinear analysis function, and $g(\dots) > 0$ denotes that the nonlinear analysis finds equilibrium (is stable) for the parameters defined within the brackets.

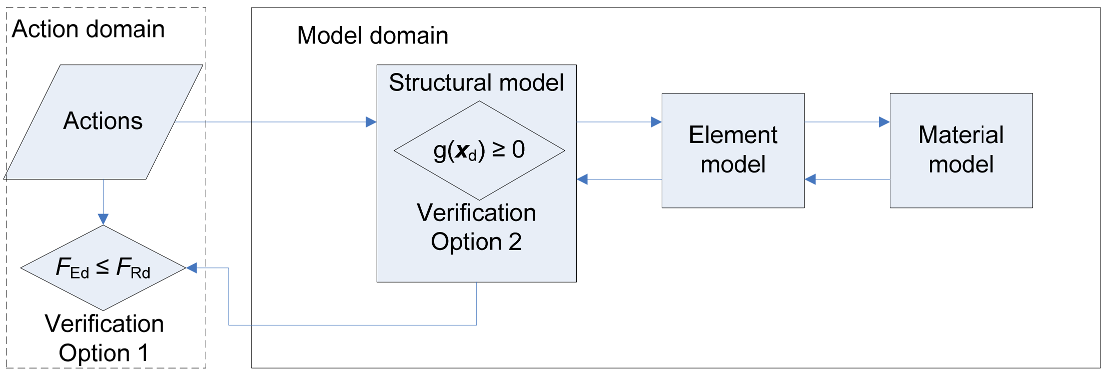

Figure 3.3 Flow chart for the design according to the one-step procedure

To utilise the advantages of the one-step procedure, questions about the safety evaluation need to be answered, Carlsson *et al.* (2008).

3.2 Reliability methods

The general purpose of structural design is to assure that the structure is sufficiently safe and that it will fulfil its intended function. In the context of structural reliability, this is mathematically expressed by using a limit state function, $g(\boldsymbol{x})$, which is dependent on the basic variables, $\boldsymbol{x} = x_1, \dots, x_{\mathrm{n}}$. Inadmissible states are defined by $g(\boldsymbol{x}) \leq 0$ and admissible states are defined by

$$g(\boldsymbol{x}) > 0. \tag{3.1}$$

The failure probability can be expressed by

$$P_{\mathrm{f}} = P[g(\boldsymbol{x}) \leq 0] = \int_{g(\boldsymbol{x}) \leq 0} f_{\mathrm{x}}(\boldsymbol{x}) d\boldsymbol{x} \tag{3.2}$$

where $f_{\mathrm{x}}(\boldsymbol{x})$ is the joint probability density function of $\boldsymbol{x}$.

Even though the performance of the structure itself is of primary interest, the limit state function is often formulated on the component level. To evaluate reliability the different methods available are commonly subdivided into three levels.

3.2.1 Level III methods

Level III methods are fully probabilistic ones which use the failure probability as a reliability measure. Provided that an accurate stochastic model is available, Level III methods allow failure probabilities to be computed accurately. The most common Level III methods are Numerical Integration and Monte Carlo simulations, but there are also more efficient modifications, Waarts (2000).

- Numerical integration is used to approximate the integral from Equation 3.2 numerically. The joint probability function is evaluated for a finite set of integration points to construct an interpolation function. Polynomials which are easy to integrate are usually chosen for the interpolation function. By transforming the random variables from the X-space into the U-space, i.e. the standard normal space, the integral according to Equation 3.2 can be solved by multiple summation as

$$P_{\mathrm{f}} = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \dots \sum_{-\infty}^{\infty} I[g(\boldsymbol{u})] f_{\mathrm{U}}(\boldsymbol{u}) \Delta \boldsymbol{u} \tag{3.3}$$

  where $f_{\mathrm{U}}(\boldsymbol{u})$ is the joint probability function and $I[g(\boldsymbol{x})]$ is the indicator function defined by

$$I[g(\boldsymbol{x})] = \begin{cases} 0 \text{ if } g(\boldsymbol{x}) > 0 \\ 1 \text{ if } g(\boldsymbol{x}) \leq 0 \end{cases}. \tag{3.4}$$

  The efficiency of numerical integration decreases exponentially with an increase of random variables. This makes numerical integration inefficient for problems which involve many random variables.

- Monte Carlo simulations rely on repeated random experiments to approximate the failure probability by the relative number of experiments for which $g(\boldsymbol{x}) < 0$. The failure probability is approximated to

$$P_{\mathrm{f}} \approx \bar{P}_{\mathrm{f}} = \frac{1}{N} \sum_{i=1}^{N} I[g(\overline{\boldsymbol{x}}_{\boldsymbol{i}})] \tag{3.5}$$

  where N is the number of experiments and $\overline{\boldsymbol{x}}_{\boldsymbol{i}}$ is the sample, i, from $\boldsymbol{x}$. Monte Carlo simulations are widely applicable and easy to implement. However, for small failure probabilities, a high number of random experiments is needed to obtain accurate estimates of the failure probability. This makes Monte Carlo simulations computationally expensive in these cases.

  To increase the efficiency of Monte Carlo simulations, a safe area can be excluded from sampling. In *Paper III* previous limit state function evaluations were used to establish bounds to classify a safe set of the sample space. If the bounds allowed placing a new sample in the safe set, no limit state function evaluation was necessary. By this a reduction of computation time was gained.

3.2.2 Level II methods

Level II methods use approximations of the limit state function to calculate the reliability index, β, as a reliability measure, instead of the failure probability. Haldar and Mahadevan (2000) further subdivide Level II methods.

- Mean value first-order second-moment methods which are based on a first-order Taylor series expansion of the limit state function at the mean values of the random variables. Random variables are only represented by the first two moments, i.e. mean and covariance. Examples are the Cornell reliability index and the Rosenbleuth-Esteva reliability index, see Cornell (1969) and Rosenbleuth and Esteva (1972).
- Advanced first-order method for normal variables was used by Haldar and Mahadevan (2000) to denote the Hasofer-Lind method, Hasofer and Lind (1974). To compute the Hasofer-Lind reliability index requires that the limit state function be rewritten in terms of reduced variables, i.e. with random variables of zero mean and unit standard deviation. The reliability index is then defined as the shortest distance from the origin of the coordinate system to the limit state surface. The point on the limit state surface that minimises the distance is called "design point". The design point is usually not known a priori but can be found by an iterative procedure.
- Advanced first-order methods for non-normal variables can be seen as an extension of the Hasofer-Lind method to non-normal random variables. This can be achieved by a transformation of the non-normal random variables into standardized equivalent normal variables, e.g. by the normal trail, Rosenblatt, or Nataf transformation. Detailed information about these algorithms is available, Ditlevsen and Madsen (2007) and Melchers (1999). Due to the use of more than second moment information, advanced first-order methods for non-normal variables have also been classified as Level III methods, Madsen *et al.* (1986).
- Second-Order Reliability Methods (SORM) use a second order approximation of the failure function around the design point. This yields more accurate results for failure functions, which are heavily nonlinear around the design point.

For most structural applications Level II methods can be considered to be sufficiently accurate, CEN (2002a).

3.2.3 Level I methods

Instead of evaluating the failure probability by Level III methods or calculating the reliability index according to Level II methods, Level I can only be used to verify that a sufficient reliability level is obtained. Uncertain parameters are modelled by a single or a few representative values, e.g. upper and lower characteristic values, $x_{i,\mathrm{k}}$, which correspond to predetermined fractile values. Partial factors, γ_i, are used to calculate the design values, $x_{i,\mathrm{d}}$, from the characteristic values to $x_{i,\mathrm{d}} = \gamma_i x_{i,\mathrm{k}}$, or $x_{i,\mathrm{d}} = x_{i,\mathrm{k}}/\gamma_i$. It is then assumed that a sufficient reliability level is obtained if

$$g(\boldsymbol{x}_\mathrm{d}) \geq 0. \qquad (3.6)$$

For favourable parameters, e.g. material strengths, the characteristic value must be decreased to obtain the design value. For unfavourable parameters, e.g. loads, the characteristic values must be increased. Often it is possible to predetermine wheather parameters are favourable or unfavourable a priori. However, generally it is required to check Equation 3.6 for all possible combinations of increased and decreased design values. This requires 2^n checks, where n is the number of random variables.

The partial factor method used in the Eurocodes, CEN (2002a), is a Level I method. To derive the design values according to Eurocodes, fixed sensitivity factors, $\alpha_\mathrm{E} = 0.7$ and $\alpha_\mathrm{R} = 0.8$, for the action/action effect and resistance were assumed, CEN (2002a). The design resistance, R_d, and design action effects, E_d, can then be defined independent with $P(R \leq R_\mathrm{d}) = \Phi(-\alpha_\mathrm{R}\beta)$ and $P(E > E_\mathrm{d}) = \Phi(\alpha_\mathrm{E}\beta)$, where Φ is the cumulative distribution function, and β is the target reliability, i.e. $\beta = 3.8$ for a reference period of 50 years for reliability class RC2 according to CEN (2002a). Equation 3.6 can be expressed by

$$g(E_\mathrm{d}, R_\mathrm{d}) \geq 0. \qquad (3.7)$$

Frequently, when designing according to the two-step procedure, a separation between action effect and resistance calculation can be assumed. In this case Equation 3.7 can be simplified to

$$E_\mathrm{d} \leq R_\mathrm{d}\,. \qquad (3.8)$$

Detailed rules about the calculation of design action effects and design resistances of concrete structures are available, CEN (2002a), CEN (2002b), CEN (2004).

3.2.4 Limitation of reliability methods

For the building industry only extremely small failure probabilities are usually acceptable, which requires that the random variables can be described accurately in the extreme tails of the distributions. This information is not usually available in practical applications. Furthermore, structural failures are often a result of more or less gross human errors, Andersson *et al.* (2010), which are difficult to describe by random variables. Therefore, calculated failure probabilities must be viewed as operational values, for code calibration purposes or the relative comparison of structures, i.e. not as good approximations of actual failure rates, CEN (2002a). Consequently, current codes such as the Eurocodes are based primarily on existing design practices, CEN (2002a); however calibration to superior reliability methods has also been performed.

3.3 Modelling uncertainty of nonlinear analysis

3.3.1 Comparison of two-step and one-step procedures

For the two-step procedure a distinction between a structural analysis to calculate action effects and a component analysis to calculate the sectional resistance can be made. This distinction has been used in the JCSS Model, JCSS (2001), to give separate model uncertainties for the action effects calculations (quantified by a coefficient of variation, $V_{\theta,\mathrm{E}}$, and the mean ratio of experimental- to predicted strength, $\theta_{\mathrm{E,m}}$) and sectional resistance calculations (quantified by $V_{\theta,\mathrm{R}}$, and the ratio, $\theta_{\mathrm{R,m}}$).

The semi-probabilistic approach used in the Eurocodes accounts for the model uncertainty of the resistance model by the partial factor, γ_{Rd}, on the resistance side. The model uncertainty for action effect calculations is covered by the partial factor, γ_{Sd}, on the loading side, see Figure 3.4. According to the Eurocodes the partial factor, γ_{Sd}, accounts also for the uncertainty in action models, but according to Model Code 2010, fib (2010a) and fib (2010b), the uncertainty of action models is covered by, γ_{f}, instead, see CEB (1988).

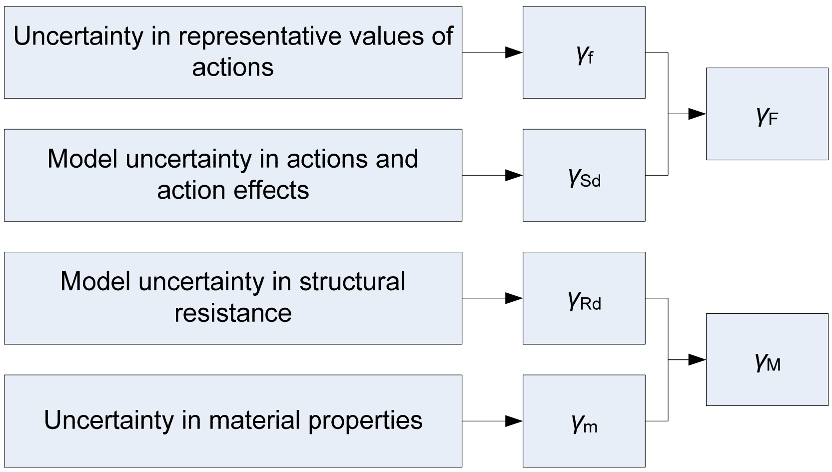

Figure 3.4 Partial factors for two-step procedure, redrawn from CEN (2002a)

For the one-step procedure, the separate calculation of action effects and sectional resistances, using two different models, is substituted by a single nonlinear analysis. The nonlinear model includes the (global) structural analysis, to calculate the loading in the critical parts of the structure, and calculates sectional resistances at the same time. The coupling between the structural calculations and sectional analysis is actually one of the main advantages of the one-step procedure. Therefore, a clear separation of two different model uncertainties is more difficult; the semi-probabilistic approach, which separates action effect calculations and sectional resistance calculations, is not directly applicable to the one-step procedure, see Figure 3.5. Nevertheless, a distinction between the model uncertainty of the failure critical part of the structure and the calculation of the loading in the failure critical part is to some extent possible. To emphasise that the difference between modelled and tested results is a combination of model and user contribution, the term "modelling uncertainty" will be used instead of model uncertainty in the following.

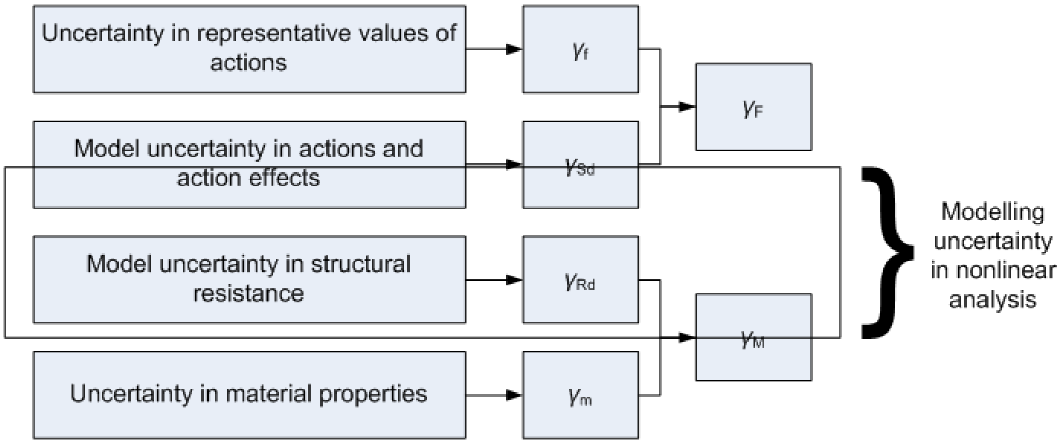

Figure 3.5 Nonlinear analysis and partial factors according to EN 1990, redrawn and modified from CEN (2002a)

3.3.2 Quantification of modelling uncertainty

To derive the modelling uncertainty of nonlinear analysis, the available recommendations for the two-step procedure were reviewed. In addition, available round-robin tests and modelling competitions were analysed (*Paper II*). In these studies, participants where asked to predict the response of specimens before revealing the experimental response. The modelling uncertainty was quantified in terms of the average ratio of experimental- to predicted strength, θ_{m}, and the coefficient of variation, V_{θ}, of the predicted strengths. The specimens were structurally simple, with well defined boundary conditions, but they failed in difficult-to-model failure modes. Consequently, hardly any uncertainty was introduced by the structural idealisation and redistribution of forces. The main difficulty for the participants was to model the response in the critical part of the structures. Therefore, the studies analysed addressed mainly the modelling uncertainty of the critical failure modes; it can be assumed that $\theta_{\mathrm{R,m}} \cong \theta_{\mathrm{m}}$ and $V_{\theta,\mathrm{R}} \cong V_{\theta}$.

Depending on the type of structure and failure mode, coefficients of variation, $V_{\mathrm{R},\theta}$, in the range of $3 - 39\%$, and average ratios of experimental- to predicted strength, $\theta_{\mathrm{R,m}}$, in the range of $0.72 - 1.12$, were found in the round-robin tests and modelling competitions. Comparing these values to the variability of the yield strength of the reinforcement steel, with a coefficient of variation of roughly $V_{f_y} = 5\%$ or to the coefficient of variation of the concrete compressive strength of around $V_{f_c} = 15\%$, makes clear that the modelling uncertainty will often be the parameter that will govern the reliability of structures.

It must be noted that the specimens studied were not representative of real structures due to their structural simplicity; but they exhibited failure modes that are difficult to model. The other aspect that makes the round-robin tests and modelling competitions analysed less representative for the use of nonlinear analysis in practice is the participants. Most of them came from universities and research institutes; a special interest in the failure modes studied can be assumed. Their knowledge and time spent on the task is most likely different from that of engineers working in practice. Therefore, the modelling uncertainties observed can only be seen as a rough approximation of the modelling uncertainty of nonlinear analysis used in practice.

3.4 Available safety formats for nonlinear analysis

Early safety formats for nonlinear analysis of concrete structures were only applicable when moment-curvature relations (M, θ) as constitutive laws were used. However, with the emergence of nonlinear analysis based on stress-strain relations (σ, ε) new approaches were developed. In the following quite recent proposals are summarised. An overview of earlier developments related to CEB can be found in Mancini (2002) and Henriques *et al.* (2002). The safety formats included here focus on the resistance side. The actions or action effects must be treated according to CEN (2002a) and CEN (2002b).

3.4.1 The partial factor method

The partial factor method is primarily used to calculate design sectional resistances of beams and columns that are loaded by bending moments and normal forces when using EN 1992-1-1 and EN 1992-2, CEN (2004), CEN (2005). However, the Model Code 2010, fib (2010a) and fib (2010b), proposes the use design material parameters for nonlinear analysis.

Partial factors, γ_{C} and γ_{S}, are used to calculate design material strengths, f_{cd} and f_{yd}, from the characteristic material strengths, f_{ck} and f_{yk}, according to

$$f_{\mathrm{cd}} = \frac{f_{\mathrm{ck}}}{\gamma_{\mathrm{C}}} \tag{3.9}$$

$$f_{\mathrm{yd}} = \frac{f_{\mathrm{yk}}}{\gamma_{\mathrm{S}}}\,. \tag{3.10}$$

The partial factors, γ_{C} and γ_{S}, can be derived based on the assumption that the relation between the resistance, R, and nominal resistance, R_{n}, can be expressed in multiplicative form. This means that the resistance, R, can be expressed as a product of, a factor to account for the resistance modelling uncertainty, θ_{R}, a geometrical variable, X_{g}, and a material strength variable, X_{f}, and the nominal resistance, R_{n}:

$$R = \theta_{\mathrm{R}} \cdot X_{\mathrm{g}} \cdot X_{\mathrm{f}} \cdot R_{\mathrm{n}}\,. \tag{3.11}$$

This allows the coefficient of variation of the resistance to be approximated:

$$V_{\mathrm{R}} = \sqrt{V_{\theta,\mathrm{R}}^2 + V_{\mathrm{g}}^2 + V_{\mathrm{f}}^2} \tag{3.12}$$

where $V_{\theta,\mathrm{R}}$, V_{m} and V_{f} are the coefficients of variation of the associated parameters. According to the European Concrete Platform (2008), the coefficients of variation according to Table 3.1 can be assumed to be the basis for the partial factors according to CEN (2004).

Table 3.1 Statistical representation which underlies the partial safety factors in EN 1992-1-1, from European Concrete Platform (2008)

Type of uncertainty	Assumed Coefficient of Variation	
	Steel	Concrete
Modelling	$V_{\theta,\mathrm{R}} = 2.5\%$	$V_{\theta,\mathrm{R}} = 5\%$
Geometry	$V_{\mathrm{g}} = 5\%$	$V_{\mathrm{g}} = 5\%$
Material	$V_{\mathrm{f}} = 4\%$	$V_{\mathrm{f}} = 15\%$

The assumption that the resistance is lognormal distributed allows one to calculate the partial factors, γ_{S} and γ_{C}, using equations from the European Concrete Platform (2008) to

$$\gamma_{\mathrm{C}} = 1.15 \cdot \exp(\alpha_{\mathrm{R}}\beta V_{\mathrm{R}} - 1.64 V_{\mathrm{f}}) \qquad (3.13)$$

$$\gamma_{\mathrm{S}} = \exp(\alpha_{R}\beta V_{\mathrm{R}} - 1.64 V_{\mathrm{f}})\ . \qquad (3.14)$$

The factor 1.15 in Equation 3.13 has been introduced to account for the lower concrete strength in real structures than in the specially cured cylinders which are used to determine the characteristic concrete strength, f_{ck}.

Theoretically, the assumption that the resistance can be expressed in multiplicative form does often not hold for reinforced concrete structures in which the resistance is often a sum of the reinforcement steel and concrete contribution. However, by applying the partial factors to the material parameters, instead of to the resistance, the partial factor method is still applicable.

According to Model Code 2010 the structure should then be verified by showing that the design actions, F_{Ed}, are smaller than or equal to the design structural resistance, F_{Rd}. The design structural resistance is defined as the ultimate structural resistance, F_{Ru}, when design material parameters are used in the nonlinear analysis. This leads to the following equation for the structural verification

$$F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}} = F_{\mathrm{Ru}} = g\left(f_{\mathrm{cd}}, f_{\mathrm{yd}}, \boldsymbol{a}_{\mathrm{nom}}\right) \qquad (3.15)$$

where $\boldsymbol{a}_{\mathrm{nom}}$ represents the nominal geometrical parameters. Equation 3.15 can alternatively expressed by

$$g\left(F_{\mathrm{Ed}}, f_{\mathrm{cd}}, f_{\mathrm{yd}}, \boldsymbol{a}_{\mathrm{nom}}\right) > 0 \qquad (3.16)$$

where g represents the nonlinear analysis and, and $g(\ldots) > 0$ denotes that the nonlinear analysis finds equilibrium (is stable) for the parameters defined within the brackets.

3.4.2 The global resistance factor method

Accounting for all resistance uncertainty by a reduction of material parameters results in very low material values. Such low material parameters lead to an overestimation of deformations and underestimation of restraint forces, which means they cannot be used to realistically model the response of concrete structures. Therefore, a modification to the partial factor method was proposed by König *et al.* (1997) and included in the Model Code 2010. König *et al.* (1997) proposed to use more realistic material strengths, $\tilde{f}_{\mathrm{c}}$ and $\tilde{f}_{\mathrm{y}}$ calculated as

$$\tilde{f}_{\mathrm{c}} = 1.1\,\gamma_{\mathrm{S}}/\gamma_{\mathrm{C}}\,f_{\mathrm{ck}} \tag{3.17}$$

$$\tilde{f}_{\mathrm{y}} = 1.1 f_{\mathrm{yk}} \tag{3.18}$$

According to Model Code 2010 the structure should then be verified by:

$$F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}} = \frac{F_{\mathrm{Ru}} = g(\tilde{f}_{\mathrm{c}}, \tilde{f}_{\mathrm{y}}, \boldsymbol{a}_{\mathrm{nom}})}{\gamma_{\mathrm{Rd}}\gamma_{\mathrm{R}}} \tag{3.19}$$

or when using the previously introduced notation:

$$g(\gamma_{\mathrm{Rd}}\gamma_{\mathrm{R}}F_{\mathrm{Ed}}, \tilde{f}_{\mathrm{c}}, \tilde{f}_{\mathrm{y}}, \boldsymbol{a}_{\mathrm{nom}}) > 0 \tag{3.20}$$

where $\gamma_{\mathrm{Rd}} = 1.06$ is the partial factor to account for the resistance model uncertainty, and $\gamma_{\mathrm{R}} = 1.2$ is the partial factor for the resistance. Inserting the recommended values into Equation 3.20 and expressing everything in terms of design material parameters yields

$$g(1.27F_{\mathrm{Ed}}, 1.27f_{\mathrm{cd}}, 1.27f_{\mathrm{yd}}, \boldsymbol{a}_{\mathrm{nom}}) > 0. \tag{3.21}$$

This makes clear that using increased design material parameters requires for actions which must be increased beyond the design values from CEN (2002b) in the nonlinear analysis, i.e. the use of more realistic material parameters comes at the cost of using less realistic actions. In addition, Equations 3.20 and 3.21 make clear that the notation "resistance safety factors" for, γ_{R}, can be misleading as the resistance safety factors are actually used to increase the actions beyond the design values, F_{Ed}.

3.4.3 The safety format according to EN 1992-2

The safety format for nonlinear analysis according to EN 1992-2 CEN (2005) is based on the global resistance factor method, and the same material parameters, $\tilde{f}_{\mathrm{c}}$ and $\tilde{f}_{\mathrm{y}}$, are used in nonlinear analysis. However, objections to verification on the structural level by an inequality of forces have led to a reformulation of the safety format on the sectional level using an inequality of generalised stresses, Mancini (2002) and Bertagnoli *et al.* (2004). Depending on the load level, the generalised stresses from the nonlinear analysis are either denoted action effect, E, or resistance, R, CEN (2005). The EN 1992-2 provides three alternative equations which can be used to assure an intended reliability level:

$$\gamma_{\mathrm{Rd}} E\big(\gamma_{\mathrm{G}} G + \gamma_{\mathrm{Q}} Q\big) \leq R\left(\frac{F_{\mathrm{Ru}}}{\gamma_{\mathrm{R}}}\right) \tag{3.22}$$

$$E\big(\gamma_{\mathrm{G}} G + \gamma_{\mathrm{Q}} Q\big) \leq R\left(\frac{F_{\mathrm{Ru}}}{\gamma_{\mathrm{Rd}}\gamma_{\mathrm{R}}}\right) = R\left(\frac{F_{\mathrm{Ru}}}{\gamma_{\mathrm{O}'}}\right) \tag{3.23}$$

$$\gamma_{\mathrm{Rd}}\gamma_{\mathrm{Sd}} E\big(\gamma_{\mathrm{g}} G + \gamma_{\mathrm{q}} Q\big) \leq R\left(\frac{F_{\mathrm{Ru}}}{\gamma_{\mathrm{R}}}\right). \tag{3.24}$$

where γ_{G} and γ_{Q} are the partial factors for the action effects which include the model uncertainty, γ_{g} and γ_{q} are the partial factors for the action effects without the action effect model uncertainty, $\gamma_{\mathrm{Sd}} = 1.15$ is the partial factor for the model uncertainty of action effects, G and Q represent the permanent and variable actions.

The use of generalised stresses in critical sections to verify an intended reliability level is impractical and can be impossible. Common nonlinear analysis toolboxes do not support the comparison of generalised stresses for different load steps which requires that the verification is done manually. In addition, for more advanced analysis based on nonlinear stress-strain relations, as shown in Figure 3.6, it can be impossible to limit the number of critical sections for which verifications are needed. This leads theoretically to an infinite number of possible sections which must be verified. For most structures in which action effects increase monotonically for increasing loading, Equation 3.23 and Equation 3.19 give the same results.

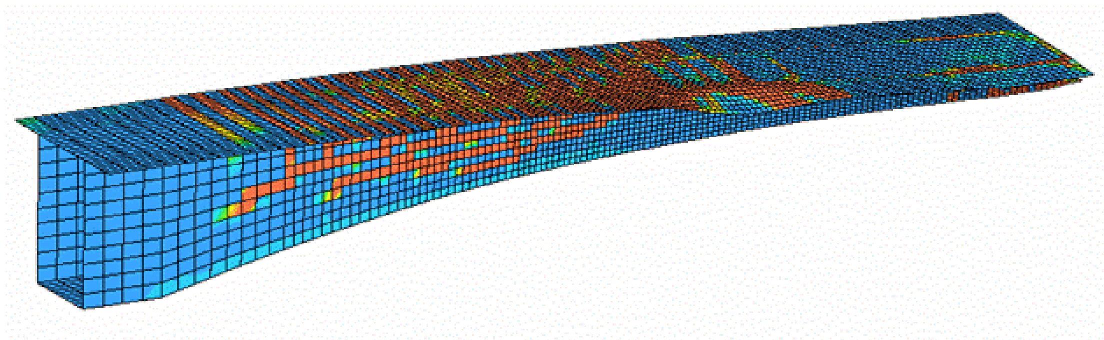

Figure 3.6. Crack pattern for a box girder bridge, from Broo (2008)

3.4.4 The estimated coefficient of variation method

Due to difficulties in verifying the structures on the sectional level when using the safety format according to EN 1992-2, Cervenka *et al.* (2007) revived the idea of verification of the structure by an inequality of forces. To be able to realistically model the structural response, the use of mean material strengths, f_{cm} and f_{ym}, in the nonlinear analysis was proposed. The mean material parameters are used to calculate the ultimate structural resistance, F_{Rum}, at which the structure fails:

$$F_{\mathrm{Rum}} = g\big(f_{\mathrm{cm}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}}\big). \tag{3.25}$$

To quantify the sensitivity of the ultimate structural resistance to material changes, the use of a second nonlinear analysis using characteristic material parameters, f_{ck} and

f_{yk}, has been proposed to calculate the "characteristic" ultimate structural resistance, F_{Ruk}:

$$F_{\mathrm{Ruk}} = g(f_{\mathrm{ck}}, f_{\mathrm{yk}}, \boldsymbol{a}_{\mathrm{nom}}). \tag{3.26}$$

The mean and characteristic ultimate resistances are used to estimate the coefficient of variation of the resistance, V_{f}:

$$V_{\mathrm{f}} = \frac{1}{1.64} \ln\left(\frac{F_{\mathrm{Rum}}}{F_{\mathrm{Ruk}}}\right). \tag{3.27}$$

This is used to calculate a resistance safety factor, γ_{R}:

$$\gamma_{\mathrm{R}} = \exp(\alpha_{\mathrm{R}} \beta V_{\mathrm{f}}) . \tag{3.28}$$

In the initial proposal by Cervenka *et al.* (2007) there was no guidance on how to account for the modelling uncertainty and geometrical uncertainty. However, when the proposal was included in Model Code 2010, fib (2010a) and fib (2010b), the use of the model uncertainty factor, $\gamma_{\mathrm{Rd}} = 1.06$, was recommended to verify the structure:

$$F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}} = \frac{F_{\mathrm{Rum}} = g(f_{\mathrm{cm}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}})}{\gamma_{\mathrm{Rd}} \gamma_{\mathrm{R}}} \tag{3.29}$$

$$g(\gamma_{\mathrm{Rd}} \gamma_{\mathrm{R}} F_{\mathrm{Ed}}, f_{\mathrm{cm}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}}) > 0. \tag{3.30}$$

3.4.5 The safety format according to Six (2001)

Six (2001) employs the probabilistic analysis of slender columns to establish an equation to calculate a resistance safety factor, γ_{R}, for these types of structural elements. The material parameters for the nonlinear analysis are calculated:

$$f_{\mathrm{c}} = 1.1 \alpha_{\mathrm{cc}} f_{\mathrm{ck}} \tag{3.31}$$

$$f_{\mathrm{y}} = 1.1 f_{\mathrm{yk}} \tag{3.32}$$

where α_{cc} is a coefficient taking into account long term effects. The safety factor depends on numerous parameters of the section that causes failure according to two equations and a linear interpolation in between:

$$\gamma_{\mathrm{R,ductile}} = 1.3 \qquad \text{for } \varepsilon_{s1} \geq 0.004 \tag{3.33}$$

$$\gamma_{\mathrm{R,brittle}} = 1.1 \gamma_{\mathrm{C}} \gamma'_{\mathrm{C}} \left(\frac{\rho_{\mathrm{tot}}}{\rho_0}\right)^{-0.085 \frac{\rho_2}{\rho_1}} \qquad \text{for } \varepsilon_{s1} \leq 0 \tag{3.34}$$

where ε_{s1} is the strain of the tensile reinforcement (or the less compressed reinforcement layer), $\gamma'_{\mathrm{C}} = 1/[1.1 - (f_{\mathrm{ck}}/500)] \geq 1$ is an extra safety factor for high strength concrete (f_{ck} in MPa), ρ_{tot} is the total reinforcement ratio, ρ_2/ρ_1 is the ratio of the reinforcement amount of the most compressed layer to the least compressed layer, and $\rho_0 = 1\%$ is a normalisation reinforcement ratio. Besides these safety factors, Six (2001) introduced an additional safety factor, γ_{sys}, to account for the system reliability and the model uncertainty of the structural model. However, no information about how this factor can be quantified is given. The structure should then be verified:

$$F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}} = \frac{F_{\mathrm{Rum}} = g(1.1\alpha_{\mathrm{cc}} f_{\mathrm{ck}}, 1.1 f_{\mathrm{yk}}, \boldsymbol{a}_{\mathrm{nom}})}{\gamma_{\mathrm{R}}\gamma_{\mathrm{sys}}} \tag{3.35}$$

$$g(\gamma_{\mathrm{R}}\gamma_{\mathrm{sys}} F_{\mathrm{Ed}}, 1.1\alpha_{\mathrm{cc}}\, f_{\mathrm{ck}}, 1.1 f_{\mathrm{yk}}, \boldsymbol{a}_{\mathrm{nom}}) > 0. \tag{3.36}$$

3.4.6 The safety format according to Henriques *et al.* (2002)

Henriques *et al.* (2002) used the probabilistic analysis of beam sections and beams with clamped ends subjected to bending moments to derive a safety format. According to their proposal the mean concrete strength, f_{cm}, and the mean steel yield strength, f_{ym}, should be used in the nonlinear analysis. They proposed to define the safety factor, γ_{RdR}, “as a function of a parameter measuring the ductility of the structural response” and mentioned the facture energy as a possible parameter. For frame structures they used the relative position of the neutral axis, x/d, of the section where the failure occurs as a measure to quantify the resistance safety factor, γ_{RdR}, where x is the compressive zone depth and d is the effective depth of the cross section. The resistance safety factor should then be calculated:

$$\gamma_{\mathrm{RdR}} = \begin{cases} \dfrac{f_{\mathrm{ym}}}{f_{\mathrm{yd}}};\ \dfrac{x}{d} \leq 0.35 \\ K\dfrac{f_{\mathrm{cm}}}{f_{\mathrm{cd}}};\ \dfrac{x}{d} > 0.35 \end{cases} \tag{3.37}$$

where K is a constant that takes into account the influence of the redistribution of forces between critical sections and should be calculated according to the concrete compressive class: $K = 0.9$ for C20 and $K = 0.8$ for C40, with linear interpolation in between. This leads to two equations for the structural verification:

$$F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}} = \frac{F_{\mathrm{Rum}} = g(f_{\mathrm{cm}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}})}{\gamma_{\mathrm{RdR}}} \tag{3.38}$$

$$g(\gamma_{\mathrm{RdR}} F_{\mathrm{Ed}}, f_{\mathrm{cm}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}}) > 0. \tag{3.39}$$

3.4.7 Discussion of existing safety formats

The safety formats according to Six (2001) and Henriques *et al.* (2002) have been formulated for special types of structural elements, namely columns subjected to an eccentric normal force and beams subjected to bending. Therefore, they are not applicable to other types of structures.

The safety format according to EN 1992-2, CEN (2005), which relies on an inequality of generalised stresses, i.e. stress integrants over a section, is not suitable for nonlinear analysis based on nonlinear material laws. For this type of analysis it can be difficult to limit the number of critical sections that need to be verified.

The remaining safety formats appear to be more generally applicable. However, they do not account properly for the modelling uncertainty that was found in the analysis of round-robin exercises and modelling competitions, see Section 3.3. Coefficients of variation in the range of $V_{\theta,\mathrm{R}} \cong 10 - 40\%$, and sometimes a significant overestimation of experimental strengths, were found for failure modes that were more difficult to model than under-reinforced beams in bending.

The partial factor method accounts for a coefficient of variation of $V_{\theta,\mathrm{R}} = 2.5 - 5\%$, and does not account for biased results; the same can be assumed for the global resistance factor method which is identical to the partial factor method, apart from the factor 1.27. The estimated coefficient of variation method uses a resistance factor in the range of $\gamma_{\mathrm{Rd}} = 1.0 - 1.1$, with a recommended value of $\gamma_{\mathrm{Rd}} = 1.06$ for nonlinear analysis, to account for the resistance modelling uncertainty. It clear that this value cannot be used to cover the modelling uncertainty and the biased results found in Section 3.3. In addition, the use of the mean and characteristic concrete compressive strengths of specially cured cylinders, f_{cm} and f_{ck}, to determine F_{Rum} and F_{Ruk}, does not account for the lower mean and higher variability of the concrete strengths in real structures when compared with the specially cured cylinders. This can lead to a lower reliability level than intended.

3.5 Proposal of a new safety format

3.5.1 General outline

For the one-step procedure, when nonlinear stress-strain relations are used as constitutive models, a verification of the structure on the sectional level by comparing generalised stresses, i.e. normal forces, shear forces, bending moments and torsional moments for different load steps, is at least impractical, if not impossible due to an infinite number of possible critical sections. Therefore, it is more appropriate to verify the intended reliability level based on Equation 3.6 by showing that the nonlinear analysis must find equilibrium for a given set of unlikely and unfavourable parameters specified by $\boldsymbol{x}_{\mathrm{d}}$. This raises the question of an appropriate set of design parameters, $\boldsymbol{x}_{\mathrm{d}}$.

It is commonly agreed upon that the mean material parameters must be used to model the response of a structure realistically, König *et al.* (1997), Eibl and Schmidt-Hurtienne (1995), Henriques *et al.* (2002). Therefore, the use of mean *in situ* material parameters is proposed. For the reinforcement steel there is no difference between the material parameters observed in the laboratory to determine the mean *in situ* material parameters. However, for concrete the *in situ* compressive strength, $f_{\mathrm{cm,is}}$, has been found to be lower than the strength of the specially cured specimens which are used to determine f_{cm}. According to König *et al.* (1998) the mean *in situ* strength can be calculated:

$$f_{\mathrm{cm,is}} = 0.85 f_{\mathrm{cm}}, \tag{3.40}$$

and similar expressions can be found in JCSS (2001) and Sustainable Bridges (2007).

To account for the resistance uncertainty, which includes the structural modelling, geometrical, and material uncertainty, the use of a single safety factor, γ_{RdR}, is proposed. The actions must be treated according to CEN (2002a) and CEN (2002b).

As in the derivations for the partial factor method, it is assumed that the coefficient of variation of the resistance can be calculated as

$$V_{\mathrm{R}} = \sqrt{V_{\theta}^2 + V_{\mathrm{g}}^2 + V_{\mathrm{f}}^2} \qquad (3.41)$$

where $V_{\theta}, V_{\mathrm{m}}$ and V_{f} are the coefficients of variation to account for the structural modelling, geometrical, and material uncertainty, respectively. The safety factor is calculated in a manner similar to the proposal of Cervenka *et al.* (2007):

$$\gamma_{\mathrm{RdR}} = \exp(\alpha_{\mathrm{R}} \beta V_{\mathrm{R}}) \qquad (3.42)$$

or for a biased modelling approach as

$$\gamma_{\mathrm{RdR}} = \frac{\exp(\alpha_{\mathrm{R}} \beta V_{\mathrm{R}})}{\theta_{\mathrm{m}}}. \qquad (3.43)$$

The structure can then be verified according to one of the two equations:

$$F_{\mathrm{Ed}} \leq F_{\mathrm{Rd}} = \frac{F_{\mathrm{Rum}} = g(f_{\mathrm{cm,is}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}})}{\gamma_{\mathrm{RdR}}} \qquad (3.44)$$

$$g(\gamma_{\mathrm{RdR}} F_{\mathrm{Ed}}, f_{\mathrm{cm}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}}) > 0. \qquad (3.45)$$

The remaining difficulty is to quantify the coefficients of variations, $V_{\theta}, V_{\mathrm{m}}, V_{\mathrm{f}}$, and the mean ratio of experimental- to predicted strength, θ_{m}. This is described in detailed below.

3.5.2 Structural modelling uncertainty

At the time when the global resistance factor method was proposed by König *et al.* (1997), nonlinear analysis was mainly used to analyse simple structures, such as continuous beams and frames that were subjected to bending moments and normal forces. For these types of analysis well established modelling approaches, like the Euler-Bernoulli Hypothesis and one-dimensional material models, are available. This justified assuming the same modelling uncertainty for nonlinear analysis as for the component check of beams and columns for which the same modelling approaches can be used. However, since then, the application of nonlinear analysis has been extended to more complicated structures with more difficult-to-model failure modes. This results in a higher modelling uncertainty which must be accounted for. Although separation between modelling of the critical failure mode and the modelling of the loading in the failure critical part of the structures is less clear for the one-step procedure, this separation is used here.

3.5.2.1 Critical failure mode

In Section 3.3, the modelling uncertainty, which was mainly associated with the critical failure mode, was found to be the parameter that often governs the reliability of structures. Depending on the critical failure mode, the modelling uncertainty was shown to vary considerably. This means that the coefficient of variation, $V_{\theta,\mathrm{R}}$, and the bias factor, $\theta_{\mathrm{R,m}}$, must be chosen according to the critical failure mode. Based on the recommendations reviewed for the modelling uncertainty of the two-step procedure and the studies analysed in *Paper II*, the values in Table 3.2 were recommended in *Paper III*. Due to the lack of available data these values should be seen a rough approximations and do not account for gross human errors.

The wide ranges given make it difficult for the designer to choose an appropriate value, and more research is needed to quantify the modelling uncertainty of nonlinear analysis more accurately. This can be seen as a severe drawback to the proposed safety format. However, without information about the modelling uncertainty, the results of the chosen modelling approach are of very limited value to assure an intended reliability level. Therefore, a lack of information about the modelling uncertainty should be seen rather as a limitation of the modelling approach than of the safety format.

Table 3.2 Coefficients of variation and mean ratios of experimental- to predicted strength factors to account for modelling uncertainty of the critical component

Failure type		Coefficient of variation, $V_{\theta,R}$ [%]	Mean ratio of experimental - to predicted strength, $\theta_{R,m}$
Compression	Normal strength concrete	10 – 20	0.9 – 1.0
	High strength concrete	20 – 30	1.0
Bending	Under-reinforced	5 – 15	1.0 – 1.2
	Under-reinforced, bending reinforcement not aligned in principal moment direction	5 – 15	0.9
	Over-reinforced, normal strength concrete	10 – 15	0.9 – 1.0
	Over-reinforced, high strength concrete	20 – 30	1.0
Shear	Failure due to yielding of the reinforcement	10 – 25	0.9 – 1.0
	Failure due to crushing of concrete, Combination of compression and shear loading, Large members, Bending reinforcement not aligned in principal moment direction	20 – 40	0.7 – 1.0

3.5.2.2 Loading in the failure critical part of a structure

For statically indeterminate structures, the uncertainty of the loading in the failure critical part of the structure must be taken into account. However, the modelling competitions and round-robin tests analysed did not allow quantifying this uncertainty. The uncertainty of loading in the failure critical part of the structure is similar to the uncertainty of action effect calculations for the two-step procedure. This has been quantified with coefficients of variation in JCSS (2001) which recommends values in the range of $5-20\%$, see Table 3.3.

Table 3.3 Recommended probabilistic models for model uncertainties of action effect calculations, from JCSS (2001)

	Coefficient of variation, $V_{\theta,\mathrm{E}}[\%]$	Mean value, $\theta_{\mathrm{E,m}}$
Moments in frames	10	1
Axial forces in frames	5	1
Shear forces in frames	10	1
Moments in plates	20	1
Forces in plates	10	1

3.5.2.3 Two options

There are two options to account for the modelling uncertainty for the one-step procedure. The first option is more in line with the semi-probabilistic approach. The modelling uncertainty of the loading in the failure critical part of the structures is accounted for on the action side by the factor, γ_{Sd}, which is included in the partial factor, γ_{F}, see Figure 3.7. In this case the coefficient of variation to account for the modelling uncertainty can be calculated to $V_{\theta} = V_{\theta,\mathrm{R}}$ and the mean ratio of experimental- to predicted strength to $\theta_{\mathrm{m}} = \theta_{\mathrm{R,m}}$.

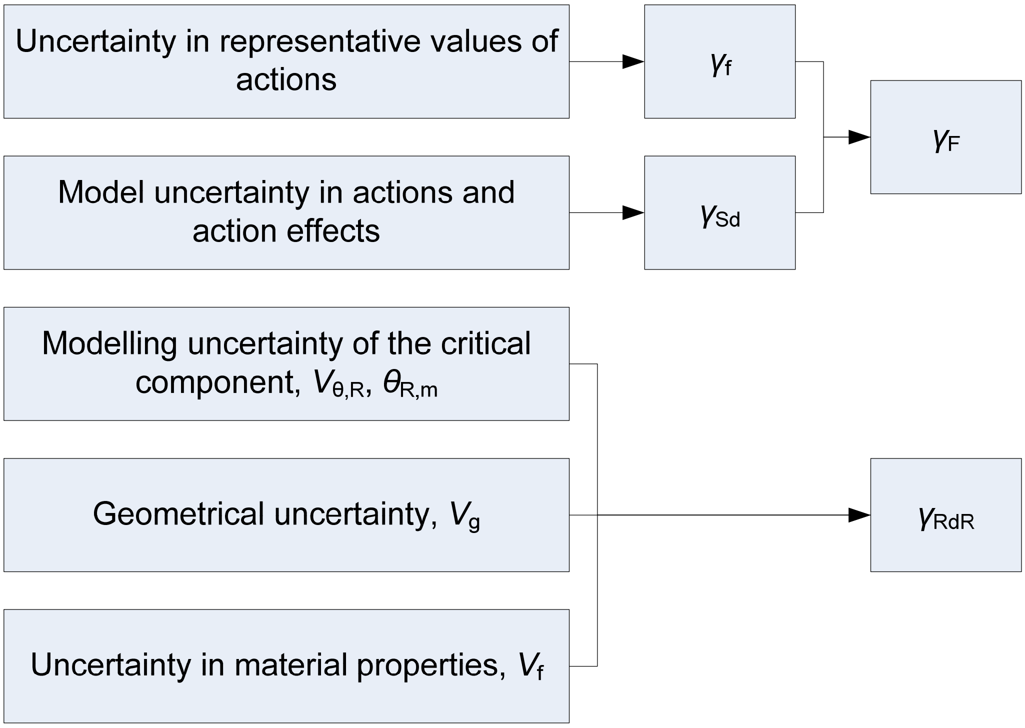

Figure 3.7 Partial factors according to new safety format, Option 1

The second option is to use the partial factor, γ_f, for the design action calculation which does not include the uncertainty of the loading in the failure critical part of the structure, see Figure 3.8. This requires that the coefficient of variation, V_θ, and the mean ratio, θ_m, be calculated:

$$V_\theta = \sqrt{V_{\theta,R}^2 + V_{\theta,E}^2} \tag{3.46}$$

$$\theta_m = \theta_{R,m} \cdot \theta_{E,m} \ . \tag{3.47}$$

The advantage of this approach is that the structural complexity can be taken into account. However, accounting for the loading in the critical component on the resistance side is not in accordance with the semi-probabilistic approach. This can require an adjustment of the assumed sensitivity factors of the load and resistance side, α_E and α_R, of the semi-probabilistic approach.

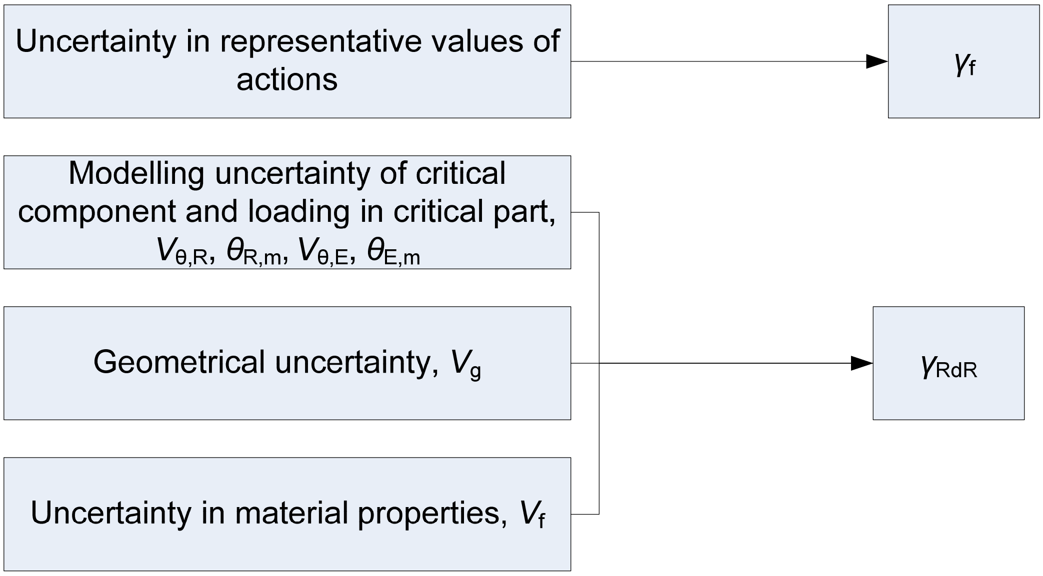

Figure 3.8 Partial factors according to new safety format, Option 2

3.5.3 Geometrical uncertainty

Reinforced concrete structures are often insensitive to geometrical imperfections. Accordingly, the use of a coefficient of variation to account for the geometrical uncertainty, $V_{\mathrm{g}} = 5\%$ is recommended. However, for structures that are sensitive to geometrical imperfections, such as slender columns, it is recommended to include an appropriate imperfection into the nonlinear analysis.

3.5.4 Material uncertainty

In the past, the main problem in formulating a safety format for nonlinear analysis was the influence of the partial factor for concrete, γ_{C}, on the modelled response of the structure, Mancini (2002). To avoid falsifying the modelled response by using the design material parameters, f_{cd} and f_{yd}, a change to more realistic material parameters has been proposed, König *et al.* (1997), Cervenka *et al.* (2007), Six (2001), Henriques *et al.* (2002). For this an additional safety factor must be applied to increase the forces for which the nonlinear analysis must find equilibrium. Due to the different variability of the steel strength and the concrete strength, the additional safety factor must be dependent on the material that causes failure. Six (2001) and Henriques *et al.* (2002) chose formulations which are only suitable for special types of structures to determine the extra safety factor. König *et al.* (1997) proposed the use of roughly the mean steel strength, $f_{\mathrm{ym}} \cong \tilde{f}_{\mathrm{y}} = 1.1 f_{\mathrm{yk}}$, but a reduced characteristic concrete strength, $\tilde{f}_{\mathrm{c}} \cong 0.84 f_{\mathrm{ck}}$, in the nonlinear analysis. This allows one to employ a constant additional safety factor independent of the material that causes failure. More generally applicable than the approaches of Six (2001) and Henriques *et al.* (2002) is the proposal by Cervenka *et al.* (2007) who used a sensitivity study to determine the sensitivity of changes in the material parameters on the ultimate structural resistance. Cervenka *et al.* (2007) used one analysis with the mean material parameters and another one with characteristic material parameters to calculate the coefficient of variation, V_{f}, to account for the material uncertainty.

In this thesis work, instead of decreasing both material strengths at the same time, it was decided to test reducing one material strength at a time. Generally, this required three additional nonlinear analyses to calculate the ultimate structural resistances:

- with decreased concrete compressive strength
  $F_{\mathrm{Ru},\Delta \mathrm{f_c}} = r\left[f_{\mathrm{cm,is}} \exp\left(-\mathrm{c}\, \mathrm{V}_{\mathrm{f_{c,is}}}\right), f_{\mathrm{ctm,is}}, f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}}\right]$,
- with decrease concrete tensile strength to calculate
  $F_{\mathrm{Ru},\Delta \mathrm{f_{ct}}} = r\left[f_{\mathrm{cm,is}}, f_{\mathrm{ctm,is}} \exp\left(-\mathrm{c}\mathrm{V}_{\mathrm{f_{ct,is}}}\right), f_{\mathrm{ym}}, \boldsymbol{a}_{\mathrm{nom}}\right]$ and
- with decreased steel strength,
  $F_{\mathrm{Ru},\Delta \mathrm{f_y}} = r\left[f_{\mathrm{cm,is}}, f_{\mathrm{ctm,is}}, f_{\mathrm{ym}} \exp\left(-\mathrm{c}\mathrm{V}_{\mathrm{f_y}}\right), \boldsymbol{a}_{\mathrm{nom}}\right]$

where $F_{\mathrm{Ru},\Delta \mathrm{f_c}}$, $F_{\mathrm{Ru},\Delta \mathrm{f_{ct}}}$ and $F_{\mathrm{Ru},\Delta \mathrm{f_y}}$ are the ultimate structural resistances when decreased concrete compressive strength, decreased concrete tensile strength and decreased steel strength were used as input parameters; c is the step size parameter; $\mathrm{V}_{\mathrm{f_{c,is}}}$, $\mathrm{V}_{\mathrm{f_{ct,is}}}$ and $\mathrm{V}_{\mathrm{f_y}}$ are the coefficients of variation of the *in situ* concrete compressive strength, *in situ* concrete tensile strength and yield strength of the reinforcement, respectively; and $f_{\mathrm{ctm,is}}$ is the mean *in situ* concrete tensile strength.

In similarity to the Gauss-approximation formula, but with an increased step size, the coefficient of variation to account for the material strength uncertainty can then be calculated as

$$V_{\mathrm{f}} \cong \frac{1}{F_{\mathrm{Rum}}} \sqrt{\left(\frac{F_{\mathrm{Rum}} - F_{\mathrm{Ru},\Delta \mathrm{f_c}}}{\Delta f_{\mathrm{c}}}\right)^2 \sigma_{\mathrm{f_{c,is}}}^2 + \left(\frac{F_{\mathrm{Rum}} - F_{\mathrm{Ru},\Delta \mathrm{f_{ct}}}}{\Delta f_{\mathrm{ct}}}\right)^2 \sigma_{\mathrm{f_{ct,is}}}^2 + \left(\frac{F_{\mathrm{Rum}} - F_{\mathrm{Ru},\Delta \mathrm{f_y}}}{\Delta f_{\mathrm{y}}}\right)^2 \sigma_{\mathrm{f_y}}^2} \tag{3.48}$$

where Δf_{c}, Δf_{ct} and Δf_{y} are the step sizes by which the material strengths were decreased; $\sigma_{\mathrm{f_{c,is}}}^2$, $\sigma_{\mathrm{f_{ct,is}}}^2$ and $\sigma_{\mathrm{f_y}}^2$ are the variance of the *in situ* concrete compressive strength, *in situ* concrete tensile strength and the yield strength of the reinforcement steel, respectively.

In *Paper II* this approach was compared with the proposal by Cervenka *et al.* (2007) on beams in bending for which only the concrete compressive strength and the strength of the reinforcement steel could cause failure. Five step size parameters, c, to decrease the material parameters were tested. It was found that the most accurate results were obtained when Equation 3.48 was used with a step size parameter of $\mathrm{c} = 2.15$.

Paper III examined further a less conservative approach to quantify the material uncertainty and to calculate the coefficient of variation:

$$V_{\mathrm{f}} \cong \frac{1}{F_{\mathrm{Rum}}} \max\left\{\frac{F_{\mathrm{Rum}} - F_{\mathrm{Ru},\Delta \mathrm{f_c}}}{\Delta f_{\mathrm{c}}} \sigma_{\mathrm{f_{c,is}}}, \frac{F_{\mathrm{Rum}} - F_{\mathrm{Ru},\Delta \mathrm{f_{ct}}}}{\Delta f_{\mathrm{ct}}} \sigma_{\mathrm{f_{ct,is}}}, \frac{F_{\mathrm{Rum}} - F_{\mathrm{Ru},\Delta \mathrm{f_y}}}{\Delta f_{\mathrm{y}}} \sigma_{\mathrm{f_y}}\right\}. \tag{3.49}$$

However, only minor differences were found between Equations 3.48 and 3.49.

3.6 Testing of the new safety format

At first a test was conducted to determine whether the safety format leads to the target reliability of the resistance side according to the semi-probabilistic approach of

$\beta_{\mathrm{R}} = 3.04$. For comparison the same test was made for the safety format according to EN 1992-2. Then the loads were also included in the evaluation of the safety format, to see whether the new safety format, in combination with the provisions of EN 1990, CEN (2002a), yields the target reliability of $\beta = 3.8$. In a third step, the new safety format and three safety formats presented in the Model Code 2010 were applied to a concrete frame bridge for which the design structural resistances were compared.

3.6.1 Resistance side

To test whether the safety format leads to the target reliability of the resistance side, according to the semi-probabilistic approach of $\beta_{\mathrm{R}} = 3.04$, it was used to calculate the design structural resistance according to

$$R_{\mathrm{d}} = \frac{R_u = g(f_{\mathrm{cm,is}}, f_{\mathrm{ym}}, a_{\mathrm{nom}})}{\gamma_{\mathrm{RdR}}}. \qquad (3.50)$$

The same was done for the safety format according to EN 1992-2. Then full probabilistic analysis was used to calculate the probability that the resistance is below the design resistance, $P(R < R_{\mathrm{d}})$, and the corresponding reliability index, $\beta_{\mathrm{R}} = -\Phi^{-1}\big(P(R < R_d)\big)$, where Φ is the standard normal distribution function. Testing of the new safety format is presented for:

- Beam sections in bending in *Paper II*,
- Short columns subjected to a normal force in Schlune *et al.* (2011)
- Beam sections subjected to shear forces in Schlune *et al.* (2011), and
- Beam sections subject to a combination of bending moment and shear force in *Paper III*.

The mean reliability indexes for the design according to EN 1992-2, $\bar{\beta}_{\mathrm{EN}}$, and for the new safety format, $\bar{\beta}_{\mathrm{new}}$, and the corresponding coefficients of variation, $V_{\beta_{\mathrm{EN}}}$, and ,$V_{\beta_{\mathrm{new}}}$, are summarised in Table 3.4.

It can be seen that the design according to EN 1992-2 led to a good agreement for beams in bending when a small modelling uncertainty of $V_{\theta,\mathrm{R}} = 5\%$ was assumed. However, when a higher modelling uncertainty was assumed, EN 1992-2 led to reliability indexes that were below the target reliability.

The new safety format led to a much closer agreement with the target reliability regardless of the component analysed. The higher the assumed modelling uncertainty was the closer the agreement was to the target reliability.

It must be noted that the comparison between the safety formats is not entirely fair. To calculate the safety factor, γ_{RdR}, for the new safety format, the same coefficient of variation, $V_{\theta,\mathrm{R}}$, and ratio, $\theta_{\mathrm{R,m}}$, as in the full probabilistic analyses were assumed. The same holds for the mean material strengths and variability of the material strengths. In practical applications of the new safety format only approximations of these values can be used, which results in a higher variability of the reliability indexes obtained.

Table 3.4 Reliability indexes and coefficients of variation of reliability indexes for design according to EN 1992-2 compared with those of the new safety format

Type of component	Number of analysed configurations	Safety format according to EN 1992-2		New safety format	
		$\bar{\beta}_{\mathrm{EN}}$	$V_{\beta_{EN}}$	$\bar{\beta}_{\mathrm{new}}$	$V_{\beta_{\mathrm{new}}}$
Beam sections in bending, $V_{\theta,\mathrm{R}} = 5\%$, $\theta_{\mathrm{R,m}} = 1.0$, see *Paper II*	156	2.95	9.5%	3.08	7.6%
Short columns loaded by a normal force, $V_{\theta,\mathrm{R}} = 10\%$, $\theta_{\mathrm{R,m}} = 1.0$, see Schlune *et al.* (2011)	72	2.51	5.2%.	3.05	4.9%.
Beams in shear, $V_{\theta,\mathrm{R}} = 30\%$, $\theta_{\mathrm{R,m}} = 1.0$, see Schlune *et al.* (2011)	96	1.07	25.0%.	3.04	1.9%.
Beams in shear, $V_{\theta,\mathrm{R}} = 10\%$, $\theta_{\mathrm{R,m}} = 1.0$, see Schlune *et al.* (2011)	96	2.34	11.7%	3.12	6.0%
Beams loaded by shear force and bending moment, $V_{\theta,\mathrm{R}} = 17.4 - 20\%$, $\theta_{\mathrm{R,m}} = 1.07 - 1.13$, see *Paper III*	30	2.08	13.4%	3.16	2.3%
Beams loaded by shear force and bending moment, $V_{\theta,\mathrm{R}} = 12\%$, $\theta_{\mathrm{R,m}} = 1.05$, see *Paper III*	30	2.56	11.5%	3.23	4.0%

3.6.2 Global reliability

To ascertain whether the new safety format, in combination with the provisions of EN 1990 CEN (2002a), leads to the target reliability of $\beta = 3.8$, the loads were also included in the evaluation of the safety format on beam sections in bending.

At first, the design moment resistance, M_{Rd}, was calculated for each beam configuration, according to EN 1992-2 and to the new safety format. The design action effect, M_{Ed}, was calculated with the assumption of an optimal design, $M_{\mathrm{Ed}} = M_{\mathrm{Rd}}$. For the assumed ratios of the characteristic action effect of the permanent loads to the imposed loads of $M_{\mathrm{Q,k}} = 0.4 M_{\mathrm{G,k}}$ and $M_{\mathrm{Q,k}} = 1.0 M_{\mathrm{G,k}}$, Equations. (6.10a) and

(6.10b) from CEN (2002a) were used to calculate the characteristic action effects. In the next step the mean action effects were calculated for a reference period of 50 years according to Grünberg (2002). Finally, Monte-Carlo simulations were used to calculate the failure probability, P_f, using the stochastic model according Table 3.5.

Four concrete compressive strength classes and three effective depths were analysed. Three reinforcement ratios were analysed ($\rho_{tot} = 0.4\%, 1.5\%, 3.0\%$). For each configuration $n = 5 \cdot 10^6$ samples were used to evaluate the limit state function

$$g(\boldsymbol{X}) = \theta_\mathrm{R} M\left(f_\mathrm{y}, f_\mathrm{c}, d_\mathrm{s}\right) - \theta_\mathrm{S}(M_\mathrm{G} + M_\mathrm{Q})$$

where θ_S is a variable to model the uncertainty of the action effect, and M_G and M_Q are the action effects of the permanent and imposed loads, respectively.

Table 3.5 Stochastic model of basic variables

	Symbol	Dimension	Distribution	Mean value, μ_X	Standard deviation, σ_X
Materials					
Concrete, C20	$f_{\mathrm{c,C20}}$	MPa	LN	23.8	$0.214\mu_X$
Concrete, C40	$f_{\mathrm{c,C40}}$	MPa	LN	40.8	$0.126\mu_X$
Concrete, C60	$f_{\mathrm{c,C60}}$	MPa	LN	57.8	$0.097\mu_X$
Concrete, C80	$f_{\mathrm{c,C80}}$	MPa	LN	74.8	$0.084\mu_X$
Reinforcing Steel	f_y	MPa	LN	550	30
Geometrical data					
Effective depth	d_{S1}	mm	N	200	10
Effective depth	d_{S2}	mm	N	275	10
Effective depth	$\mathrm{d_{S3}}$	mm	N	350	10
Action effects					
Permanent load	M_G	kNm	N	$\mathrm{M_{G,k}}$	$0.1\mu_{\mathrm{MG}}$
Imposed load	M_Q	kNm	GU	$\mathrm{M_{Q,k}}$/ $(1+1.87\mathrm{V_{MQ,50}})$	$0.3\mu_{\mathrm{MQ}}$
Model uncertainty					
Resistance	θ_R	–	LN	1.00	0.05
Action effects	θ_S	–	LN	1.00	0.05

Distribution types: N: normal; LN: log-normal; GU: Gumbel.

For the failure probabilities obtained, the reliability indices, β, were calculated. For the design, according to EN 1992-2, a mean reliability index of $\bar{\beta}_\mathrm{EN} = 4.03$ with the coefficient of variation $V_{\beta_\mathrm{EN}} = 4.4\%$ was obtained; for the design according to the new safety format a mean reliability index of $\bar{\beta}_{\mathrm{new}} = 4.14$ with a coefficient of variation $V_{\beta_{\mathrm{EN}}} = 3.9\%$ was obtained. The corresponding histograms are shown in Figure 3.9.

For the stochastic model assumed, the new safety format reached a reliability level that is higher than the target reliability. It is also higher than the reliability of the safety format according to EN 1992-2. The conservative results are caused partly by the conservative assumption of the semi-probabilistic approach. At the same time, the reliability level of the new safety format is more constant than that of the EN 1992-2 safety format.

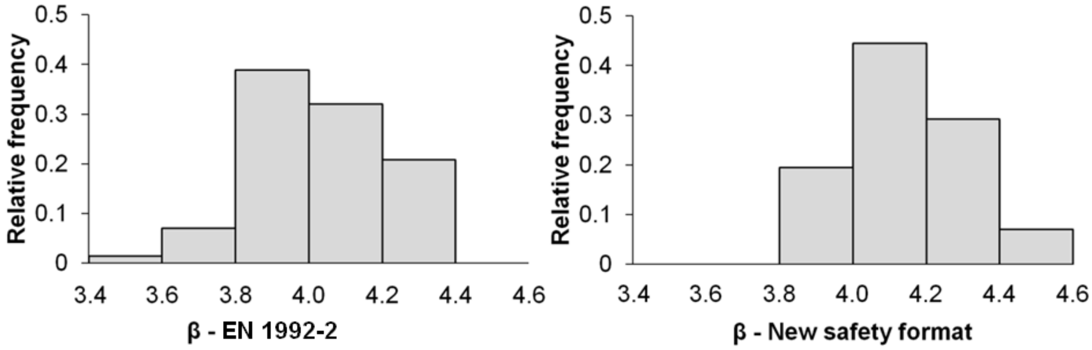

Figure 3.9 Reliability indexes for sample structures

3.6.3 The portal frame bridge

Together with three safety formats proposed by the Model Code 2010, the new safety format was applied to a portal frame bridge, see *Paper IV*. Beam elements were used to model the bridge and the resistance was limited by the strength of the reinforcement steel. For the new safety format, a coefficient of variation, $V_{\theta,\mathrm{R}} = 5\%$, and ratio of, $\theta_{\mathrm{R,m}} = 1.0$, were assumed to account for the modelling uncertainty. This led to good agreement with the design structural resistances according to Model Code 2010 when the model uncertainty factor, $\gamma_{\mathrm{Rd}} = 1.06$ was assumed. When the coefficient of variation, $V_{\theta,\mathrm{R}} = 10\%$, was assumed in the new safety format, the design structural resistance was lower than the design resistances according to Model Code 2010 with the highest recommended model uncertainty factor, $\gamma_{\mathrm{Rd}} = 1.10$, was used. Higher modelling uncertainties could not be covered by the recommended ranges of the model uncertainty factor from Model Code 2010.

3.7 Discussion

3.7.1 Deformation capacity

Almost all of the safety formats described in Section 3.4 and the new proposed safety format verify the structural reliability by an inequality of forces. However, this alone does not result in structures that have the intended reliability level. It is also required that the deformation capacity utilised be reliably available.

This became apparent for the portal frame bridge (*Paper IV*) but can be shown more clearly for the structure given in Figure 3.10. Although, the structure was chosen for illustration purposes, the linear elastic–perfect plastic behaviour of section 1-1 is not so far from that of typical concrete structures, such as under-reinforced beams in bending.

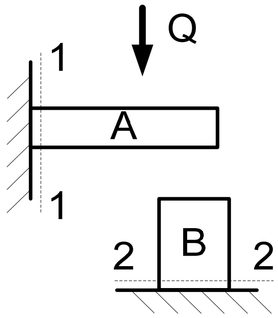

Figure 3.10 Structure to illustrate the importance of deformation capacity

For simplicity it is assumed that the structure is loaded with one load, Q, and the intended reliability level is assured by a single safety factor, γ. Increasing the load causes first a linear increase of the bending moment in section 1-1, before normal forces are introduced in section 2-2, see Figure 3.11. When the gap between beam A and column B is closed, the moment, M_{1-1}, and curvature, θ_{1-1}, in section 1-1 do not increase further, while the normal force in section 2-2 increases rapidly. To verify the structure it is required that the nonlinear analysis find the equilibrium for the load, Q_{k}, which is increased by the factor, γ, that is $Q_{\mathrm{k}} < \gamma Q_{\mathrm{k}}$. This increase does not lead to an increase of the bending moment or curvature in section 1-1, that is $M_{1-1}(Q_{\mathrm{k}}) = M_{1-1}(\gamma Q_{\mathrm{k}})$ and $\theta_{1-1}(Q_{\mathrm{k}}) = \theta_{1-1}(\gamma Q_{\mathrm{k}})$. It is now possible that the ultimate curvature in section 1-1, $\theta_{1-1,\mathrm{u}}$, is nearly reached for the load, Q_{k}, that is $\theta_{1-1,\mathrm{u}} \cong \theta_{1-1}(Q_{\mathrm{k}}) = \theta_{1-1}(\gamma Q_{\mathrm{k}})$. This means that despite the safety factor, γ, section 1-1 could be very close to failure.

One might ague that it does not matter if section 1-1 is sufficiently reliable as long as the structure has sufficient reserves of redundancy. However, to be able to utilise the capacity of column B, it is required that beam A has sufficient deformation capacity so that is does not fail before load, Q, can be redistributed to column B. This is only checked as part of the nonlinear analysis, using realistic material parameters, when the global resistance factor method, the estimated coefficient of variation method, the safety formats according to Six (2001) and Henriques *et al.* (2002) or the new safety format are used. This checking does not reliably guarantee the required deformation capacity, especially as the deformations after peak loading and the deformation capacity are usually quite poorly described by nonlinear models, van Mier and Ulfkjaer (2000), Collins *et al.* (1985) and Jaeger and Marti (2009).

For plastic analysis it is well known that the deformation capacity must be checked. However, very little attention has been paid to this making nonlinear analysis. None of the previously described safety formats accounts for this including the safety format according to EN 1992-2.

To obtain the intended reliability level it is therefore necessary that the deformation capacity used be verified by separate checks. Alternatively, conservative modelling approaches can be used for the deformation capacity, or the resistance uncertainty due to the deformation uncertainty must be appropriately accounted for in the safety format.

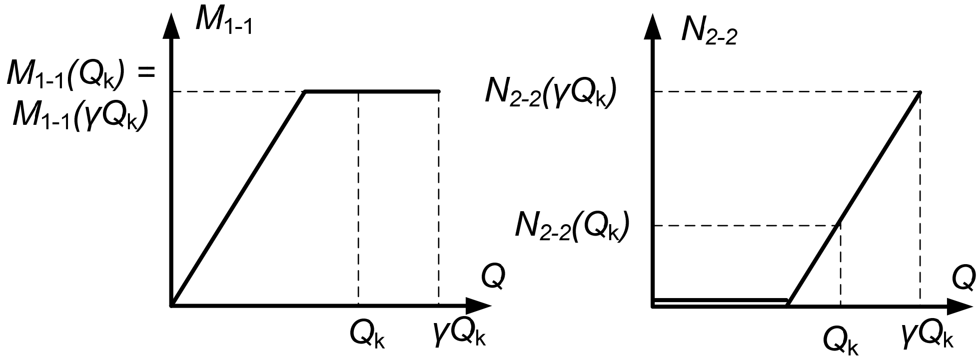

Figure 3.11 Sectional force–load diagram for the structure shown in Figure 3.10.

3.7.2 Human error

When compared with the traditional design approach, nonlinear analysis seems to be more vulnerable to human error. There are many modelling choices and a lot of parameters that need to be defined which results in numerous options to do wrong. As human error were shown to be the main reason for structural failures, Andersson *et al.* (2010), verification procedures and measures for quality assurance for nonlinear analysis need to be developed. In addition, verification procedures for nonlinear analysis software packages are needed.

3.7.3 Verification and the one-step procedure

To verify the structure in the one-step procedure it is not possible to manipulate action effects or component resistances without violating equilibrium which is the key requirement for the nonlinear analysis. The only parameters that can be manipulated are input parameters for the nonlinear analysis, like material parameters, geometrical parameters or actions and the structure must be verified by showing that the nonlinear analysis finds equilibrium for a given set of unfavourable and unlikely input parameters. All previously described safety formats use to some extent the semi-probabilistic approach to calculate the input parameters for the nonlinear analysis. However, to avoid using design material parameters all safety formats, except the partial factor method, use more realistic material parameters which require that the action for which the nonlinear analysis must find equilibrium must also be increased beyond the design values. This violates the principle of the semi-probabilistic approach. Therefore, the question must be raised if it is not time for a more drastic change for the verification procedure with the one-step procedure.

Traditionally, in nonlinear analysis the material properties are keep constant and the actions are increased until failure occurs. However, it was mentioned that for special cases it might be better to keep the actions constant and to decrease the resistance parameters, see Vrouwenvelder (2010). An intermediate solution has been proposed by starting at mean (or representative) values and increasing (or decreasing) them proportional to the standard deviation until failure occurs, Vrouwenvelder (2010).

Another similar solution can be to start again at mean (or representative) values. In the next step a sensitivity analysis is used to determine the importance of model parameters. This can e.g. be done by decreasing each favourable parameter within a reasonable range or until failure is reached. The same must be done for all unfavourable parameter which must be increased. In a final nonlinear analysis the parameters can then be increased or decreased proportional to their sensitivity until failure is reached. This can then be used to calculate an approximate reliability index, β.

4 Conclusions

4.1 Finite element model updating

4.1.1 General conclusions

Uncertainties in the boundary conditions, in interaction between different structural parts, and model parameters can result in inaccurate FE models. These uncertainties can greatly influence the results, and they can seldom be determined directly through tests. Therefore, a methodology for FE model updating to improve bridge evaluation was proposed, which allows estimating these uncertainties implicitly, with on-site measurements coupled to FE models by FE model updating. Applying the methodology to the new Svinesund Bridge improved the understanding of the bridge response and resulted in a significantly more accurate FE model. The updated model parameters are believed to be more accurate estimates of the actual structural parameters.

However, despite the comprehensive measurement program, the number of parameters that could be updated was relatively small. To enable the updating of structural parameters, e.g. for damage detection, further improvements of the updating procedures are needed.

Furthermore, it was found that engineering judgment was paramount for the success of FE model updating. The importance of manual model refinements was highlighted to avoid compensating for modelling errors by meaningless changes to model parameters.

4.1.2 Suggestions for future research

To facilitate estimating more structural parameters by FE model updating, more, as well as more accurate, measurements are needed. In addition, more accurate a priori FE models that allow for capturing nonlinear behaviour and ways to eliminate environmental effects from the measurements need to be further developed. Improvements in the objective function formulation and advances in the field of optimisation would also contribute to the advances in FE model updating. Finally, updating procedures that include an appropriate choice of measurements should be developed for common types of bridge.

4.2 Safety Evaluation of Concrete Structures with Nonlinear Analysis

4.2.1 General conclusions

The modelling uncertainty of nonlinear analysis was quantified, based on available round-robin tests and modelling competitions, and coefficients of variation in the range of $V_{\theta,\mathrm{R}} = 10 - 40\%$ were found to be appropriate for failure modes that are more difficult to model than under-reinforced concrete beams in bending.

Available safety formats for nonlinear analysis, according to EN 1992-2, CEN (2005), and Model Code 2010, fib (2010a) and fib (2010b), do not account for such a high modelling uncertainty: they result in a reliability level which is below the intended.

Unless a small modelling uncertainty can be guaranteed, their use is not recommended.

Based on the observation that the modelling uncertainty is often the parameter that governs the reliability of structures, a new safety format is proposed. The safety format allows one to explicitly account for the structural modelling, geometrical, and material uncertainty. To quantify the material uncertainty, a sensitivity study that requires two to three additional nonlinear analyses with reduced material strengths is recommended. Testing of the safety format on numerous examples showed that it provides to close agreement with the target reliability.

In addition, it was shown that an inequality of forces does not necessarily lead to the intended target reliability. It is further necessary, that the deformation capacity which is utilised during the nonlinear analysis is reliably available.

4.2.2 Suggestions for future research

The data to quantify the modelling uncertainty is scarce and not representative for nonlinear analysis in practical applications. Therefore, it was only possible to quantify the modelling uncertainty in wide ranges which are of limited practical help. To be able to quantify the modelling uncertainty more accurately, further round-robin tests and modelling competitions are needed.

The scarcity of detailed modelling guidelines for nonlinear analysis means that the modelling uncertainty is very high for difficult-to-model failure modes. As a result high safety margins are needed which results in low structural design resistances. Large benefits in design resistances can be obtained by decreasing the modelling uncertainty of nonlinear analysis. This requires for the development of accurate modelling approaches and detailed modelling guidelines.

For complicated structures, such as that shown in Figure 3.6, it is unclear how the deformation used can be reliably assured. Approaches to achieve this are needed.

Today, conservative assumptions for boundary conditions and initial imperfections are often used for the global modelling. However, in similarity to the practice of using design material parameters, this can falsify the modelled response. To allow for realistic modelling, a sensitivity study, such as the proposed approach to quantify the material uncertainty for the new safety format, can be appropriate to account for these kinds of uncertainties. The usefulness of a sensitivity study for other applications also needs to be studied.

Due to the incompatibly of the one-step procedure with the semi-probabilistic approach, a more drastic conceptual change to the verification procedure for nonlinear analysis is needed and should be developed.

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Paper I

Improved bridge evaluation through finite element model updating using static and dynamic measurements. *Engineering Structures*, Vol. 31, No. 7, pp. 1477–1485.

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Improved bridge evaluation through finite element model updating using static and dynamic measurements

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ABSTRACT

The potential of combining finite element (FE) analysis with on-site measurement through finite element model updating is indisputable. However, simplified initial models and too few measurements can lead to updated model parameters which conceal inaccurate modelling assumptions rather than improve estimates of the actual structural parameters. Therefore, the methodology proposed aims primarily to eliminate inaccurate modelling simplification by means of manual model refinements before parameters are estimated by non-linear optimization. In addition, multi-response objective functions are introduced, which allow combing different types of measurements to obtain a solid basis for parameter estimation. The proposed methodology was applied to one of the world's largest single-arch bridges, the new Svinesund Bridge, and disclosed a need to use a non-linear model in order to estimate the structural parameters more accurately. The resultant model could reproduce the measurements with significantly improved accuracy without assigning unrealistic values to model parameters.

1. Introduction

Maintenance, upgrading, repair, and replacement of bridges lead to high costs and considerable disruption of today's traffic. For effective bridge management, accurate and reliable information about the safety and condition of bridges is indispensable. In current practice, however, existing bridges are analysed and evaluated by means of highly simplified structural models. This approach is not entirely satisfactory because inevitable uncertainties in material and structural modelling may have significant effects on the results of the analysis [1–3]. Structural models that are verified, refined, and tuned with respect to actual measurements can reduce these uncertainties and provide a better basis for management decisions.

A significant amount of research has been done to combine measurements with finite element (FE) analysis for the assessment of bridges. Chajes et al. [4] used strain measurements under truck loading to determine the support restraint and the composite section properties of a steel-girder-and-concrete-slab bridge. When these findings were introduced into the initial FE model, a substantially higher load carrying capacity could be shown. Huang [5] described a method of using strain and deflection measurements to improve the accuracy of FE models. Without including unreliable effects, such as unintended support restraint which may not hold for higher loads or during the entire lifetime, he showed a higher capacity for a box-girder bridge than with conventional methods. Enevoldsen et al. [6] changed the stiffnesses of joints manually in an FE model of a truss railway bridge until good agreement with measured strains during a train passage was obtained. The FE model was then used to assess the fatigue life of the bridge for higher axle loads. For an overview of bridge rating based on load tests and FE analysis see [7].

Beside static or quasi-static load tests, modal characteristics have been extensively used to obtain more information about the response of bridges. Daniell and Macdonald [8] introduced manual changes to an FE model of a cable-stayed bridge to obtain a better match with eigenfrequencies from ambient vibration tests. However, FE model updating through non-linear optimization is more common when modal characteristics are used as reference data; see [9–11].

In this study, a methodology is presented which allows combining static and dynamic measurements with an initial FE model to obtain better knowledge of the structural response of existing bridges. To avoid obtaining model parameters which conceal modelling simplifications the importance of manual model refinements is highlighted.

In this article, 'FE model updating' refers to the complete procedure of changing an FE model to better correspond to experimental data. This includes manual model refinements,

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Table 1
Methodology for FE model updating.

Steps		Purpose
1	Manual model refinements	
	1.1 Remove lower bond assumptions	Introduce justified changes
	1.2 Refine model	Disclose the need for model refinements; decrease all types of modelling errors
	1.3 Conduct parameter study	Obtain an overview of model parameter sensitivities; find improved estimates of model parameters
2	Updating through non-linear optimization	
	2.1 Find improved estimates of updating parameters	Obtain more accurate model; obtain more accurate estimates of model parameters
3	Model evaluation	
	3.1 Evaluate model parameter changes	Evaluate the reliability and accuracy of updated parameters; find possible source for model parameter changes
	3.2 Prepare model for further analysis	Eliminate model parameter changes that do not hold for the conditions that need to be analysed

a parameter study, and parameter estimation by non-linear optimization. Furthermore, 'manual model refinement' is used to describe all kinds of changes that are introduced manually to the model. This term better describes comprehensive changes to the model than does 'manual tuning' used by Daniell and Macdonald [8] and Živanović et al. [11] or 'manual calibration' used by Aktan et al. [12].

2. Disagreement between experimental observations and numerical predictions

One of the reasons for FE model updating is the need for an accurate structural model with respect to measurements. If this is the only goal, FE model updating can be quite simple, as only some model parameters of the structural model have to be changed which lead to improved agreement with the measurements. In such a case it is admissible, but not necessary, that the changed model parameters be improved estimates of actual structural parameters as long as they bring about improved agreement with the measurements.

However, when a generally valid FE model that can be used to analyse untested conditions is sought or when FE model updating is used as a global non-destructive testing method to estimate uncertain structural parameters, it is important that the updating procedure corrects wrong assumptions in the FE model, rather than introduces arbitrary changes that make the model agree with measurements. This is considerably more difficult because simplified models, model parameter uncertainties, and inaccurate measurements can easily lead to inaccurate estimates of the structural parameters through FE model updating.

In FE model updating, attention is often drawn to changing model parameters, while less attention is given to other possible sources of disagreement between measurements and FE analysis. The total modelling error accumulates during the whole finite element modelling and analysis process. It starts with the engineer's decisions about which physical phenomena to include into the model. Unavoidable simplifications of the structural representation concerning the level of detailing, the choice of element types and size, boundary conditions, loading, material parameters, and material models can influence the accuracy significantly. Furthermore, numerical errors accumulate due to round-off errors and iterative solutions. Robert-Nicoud et al. [13] illustrate the effect of a small modelling simplification on an estimated structural parameter in a simple example.

In addition, the measurement error and signal processing errors contribute to the total discrepancy between the experimental and numerical results. The challenges that are associated with measuring the response of constructed systems, such as bridges, and the consequences for FE model updating have been described in [14].

The idea underlying structural parameter estimation by FE model updating is to combine measurements with an a priori FE model to gain new information about the structure, typically some of the structural parameters. As the new information is based on the combination of prior information, namely the measurements and the initial FE model, the reliability and accuracy of this new information strongly depends on the quality of the prior information. Therefore, it is essential that the measurement error and the modelling error are small compared with the error which is introduced by the model parameters that are updated. Ill-conditioning due to a low sensitivity of the structural response to changes of model parameters, as observed in [15–17], amplifies this problem.

3. Proposed methodology

Due to the previously described difficulties that are associated with parameter estimation through FE model updating, two main improvements are proposed: manual model refinements before parameter estimation through non-linear optimization; and the use of multiple types of measurements for updating. The steps of the proposed methodology are summarized in Table 1.

3.1. Static and dynamic measurements

Finite element model updating can be deceptively simple for a small amount of experimental data and a large number of uncertain structural parameters [12]. In this instance it is likely that several non-unique solutions exist. To decrease the risk of having an underdetermined or ill-posed problem, a large amount of experimental data is needed. While either static or dynamic measurements are often used for updating, Aktan et al. [12] and Catbas et al. [18] concluded that different types of measurements are needed when a generally valid model is sought. To obtain improved estimates of the actual structural parameters of a bridge, it is important that the uncertain structural parameters determine the choice of measurements, not the reverse. To find a set of measurements that can be used to estimate the main uncertain structural parameters, it can be helpful to simulate the model updating procedure before undertaking expensive on-site measurements.

Furthermore, it is desirable to choose measurement types and loading conditions which are relevant to the final purpose of the updated FE model. This reduces the consequences of inaccurately estimated structural parameters through FE model updating and decreases the uncertainty associated with extrapolating the finding to other loading conditions.

Compared to static measurements, modal characteristics have the advantage of containing information about the global response of bridges. This makes them less sensitive to local structural parameters. The number of identifiable modes limits the information that can be gained about the response of a bridge. The number of static measurements, typically strains, forces, displacements, and inclinations, under known loads is limited only by the number of available sensors. As measured strains and forces are more sensitive to the response in their vicinity, they are better suited to determine local parameters. On the other hand a dense net of sensors is required when information about the global response of a bridge is sought. For these reasons, Aktan et al. [12] and Catbas et al. [18] used the first modal characteristics to update the boundary conditions and then static measurements to update the local model parameters.

3.2. Manual model refinements

To obtain an accurate FE model that can capture all important physical effects and allows for estimation of uncertain structural parameters, manual model refinements can be crucial. The proposed procedure successively reduces the modelling errors of an initial FE model, thereby reducing the risk of concealing the initial modelling simplifications by inaccurately estimated structural parameters. The strength of manual model refinements is that they can be used to decrease all types of modelling errors, while the strength of FE model updating by non-linear optimization lies in tuning uncertain model parameters. Manual model refinements can be done in three steps; however, it may be necessary to apply the second and third steps alternately.

When FE models are used for the design or assessment of bridges, simplifications and assumptions 'on the safe side' are appropriate. However, the FE models used for FE model updating should be as accurate as possible. Therefore, all lower bond assumptions should be replaced by realistic assumptions in a first step. To avoid introducing additional modelling errors, it is recommended to first base all changes on engineering calculations without taking into account the agreement between the measurements and the FE analysis.

In a second step, the FE model can be changed according to agreement between the FE analysis results and the measurements. By comparing the residuals for different load cases, measurement locations and sensor types, invalid or inaccurate assumptions in the FE model can be disclosed. An iterative trial and error procedure based on engineering judgement can be used to introduce realistic changes into the model. As boundary conditions and joint properties often have a high influence on the FE analysis results, their representation in the FE model should be reviewed. Furthermore, one can check whether additional physical phenomena have to be introduced into the FE model to obtain a more realistic description of reality. This also leads, through experience, to an improved understanding of the physical phenomena which have to be included when modelling common bridge types and, hence, to more accurate a priori models.

In the third step, the model and the measurements can be combined to gain more information about uncertain structural parameters. This can be done by changing uncertain model parameters until improved agreement between the numerical and experimental response is obtained. Another way is to conduct a parameter study by formulating an index of discrepancy, called the objective function, and plotting the value of the objective function for parameter variations. An advantage of this procedure is that it gives an idea of the shape of the objective functions, which makes it easier to assess the reliability of the estimated parameters. If the agreement of the model with measurements is satisfactory, a critical assessment of the model should be made before it can be used for further analysis; see Section 3.4. For further improvements of the model and the estimated structural parameters, FE model updating through non-linear optimization can be utilized. An advantage of manually improving estimates of the structural parameters is that a better starting point for the optimization algorithm is obtained, which reduces the risk of getting stuck in local minima.

3.3. FE model updating through non-linear optimization

An increasing number of model parameters are impractical to tune manually. Therefore, optimization algorithms are commonly used to find model parameters that make the FE model agree better with measurements. Finite element model updating by non-linear optimization consists of three main components, namely the choice of updating parameters, the objective (or cost) function, and the optimization algorithm.

Updating parameters are the model parameters that are changed by the optimization algorithm to minimize the objective function. To increase the sensitivity of model parameters, grouping of those which show a high correlation can be done. When estimating the uncertainty of possible updating parameters, one must be aware of the distinction between the certainty of the elastic modulus of a steel beam, for example, and the uncertainty of the elastic modulus of the bridge deck girder, which is used as an updating parameter. In the first example the elastic modulus is a material parameter which is accurately known; see [19]. In the second, the elastic modulus is used as a parameter which describes the stiffness changes of the bridge deck in all directions. In this instance it is used to summarize effects, such as the railing system and the asphalt layer, on the structural performance of the bridge deck girder, which leads to higher uncertainty.

Optimization algorithms are typically required to be stable and fast. The Gauss–Newton algorithm is probably the most often used algorithm for FE model updating; see e.g. [20]. Teughels and De Roeck [21] applied the Gauss–Newton method with a trust region strategy to improve stability. Gentile [22] used Rosenbrock's methods; global optimization algorithms have also been applied [23].

The objective function is an index of discrepancy between the FE analysis results and the measurements. As FE model updating by non-linear optimization using static measurements has seldom been applied, experience in the choice of an appropriate objective function is still limited. When eigenfrequencies are used to update an FE model, the sum of squared differences or the sum of squared relative differences has usually been used:

$$J_1 = \sum_{i=1}^{m} (z_{ni} - z_{ei})^2 = (\mathbf{z}_n - \mathbf{z}_e)^{\mathrm{T}} (\mathbf{z}_n - \mathbf{z}_e) \quad (1)$$

$$J_2 = \sum_{i=1}^{m} \left(\frac{z_{ni} - z_{ei}}{z_{ei}} \right)^2 = (\mathbf{z}_n - \mathbf{z}_e)^{\mathrm{T}} \mathbf{W}_2 (\mathbf{z}_n - \mathbf{z}_e), \quad (2)$$

where $\mathbf{z}_n$ and $\mathbf{z}_e$ are vectors with numerical and experimental eigenfrequencies, respectively, m is the number of eigenfrequencies, and $\mathbf{W}_2$ is a diagonal matrix with the squared reciprocals of the experimental eigenfrequencies as diagonal elements.

When different types of measurements contribute to the objective function, it is important that they are unit-less. Normalization is therefore required to assure that the contribution to the objective function is not dependent on the units chosen. Eq. (2) fulfils this demand and could thus be used as a multi-response objective function when $\mathbf{z}_n$ and $\mathbf{z}_e$ are changed into vectors containing different types of numerical and experimental responses. Problems arise when the normalization factors are close to zero. In this instance the objective function can become dominated by one or a few terms. When eigenfrequencies are used exclusively in the objective function, this problem does not occur. However, when measured displacements, strains, and forces under various types of loadings are used, there is a risk that some measured responses will be very small compared with the difference between the numerical and measured responses.

Bell et al. [24] chose the difference between the measured and the numerical response of the initial model as a normalization parameter. This can cause the objective function to be dominated by a few terms, when some measured and numerical responses happen to agree very well initially.

A way of taking the significance of different kinds of measurements into account and obtaining dimensionless terms is to use the standard deviation, σ_i, of the measurements as a normalization term:

$$J_3 = \sum_{i=1}^{m} \sqrt{\frac{(z_{ni} - z_{ei})^2}{\sigma_i^2}} = \sum_{i=1}^{m} \frac{|z_{ni} - z_{ei}|}{\sigma_i}. \tag{3}$$

A way of keeping the least square form of the objective function is to omit the square root, as in the next equation. This leads to an objective function with lower weights for small residuals and higher weights for residuals that are higher than one standard deviation:

$$J_4 = \sum_{i=1}^{m} \frac{(z_{ni} - z_{ei})^2}{\sigma_i^2} = (\mathbf{z}_n - \mathbf{z}_e)^{\mathrm{T}} \mathbf{W}_4 (\mathbf{z}_n - \mathbf{z}_e), \tag{4}$$

where $\mathbf{W}_4$ is a diagonal matrix with the reciprocals of the variance as diagonal elements; see Fig. 1. Experimental responses that are far from the numerical counterparts may indicate a significant need for improvement of the FE model, but this can also indicate a faulty sensor. To prevent the objective function from being dominated by residuals from faulty sensors, the next equation can be used:

$$J_5 = \sum_{i=1}^{m} erf\left(\frac{|z_{ni} - z_{ei}|}{\sigma_i \sqrt{2}}\right). \tag{5}$$

By taking the error function of the normalized absolute residuals, each term of the objective function is limited to 1.0; see Fig. 1. Another way of composing the objective function is to first calculate an average error for all types of responses separately:

$$J_{6,k} = \frac{\sum_{j=1}^{n_k} |z_{k,nj} - z_{k,ej}|}{\sum_{j=1}^{n_k} |z_{k,ej}|}, \tag{6}$$

where k indicates the response type, n_k is the number of responses for each response type, and $z_{k,nj}$ and $z_{k,ej}$ are analytical and experimental responses of type k. The weighted average error for the different types can then be added to form the objective function:

$$J_6 = \sum_{i=1}^{k} w_k J_{6,k}, \tag{7}$$

where w_k is a weighting factor for the response type k. The shapes of objective functions 3–5 for a single residual are shown in Fig. 1. Objective function J_6 also leads to a linear-shaped objective function, similar to objective function 3, with the slope determined by the weighting factor w_k and the sum $\sum_{j=1}^{n_k} |z_{k,ej}|$.

When there is divergence of the optimization algorithm or if unrealistic values are assigned to the updating parameters, they can be constrained by adding a penalty term to the objective functions. This additional a priori information allows a compromise between good agreement with experimental data and small changes to the updating parameters.

3.4. Model evaluation

Assessing the quality of the updated model comprises two steps. First, the reliability of the changes that were introduced into the model has to be checked and justifications for model parameter changes have to be found. A good agreement between numerical and experimental data as well as realistic model parameters are necessary, but not sufficient, conditions for a physically meaningful updated model. Friswell and Mottershead [25] suggested dividing the measurements in two parts. One part is used for updating while the other is used for verification of the updating procedure. This has the drawback that the available reference data for updating are reduced. In the absence of well developed procedures to assess the estimated model parameters, it is often necessary to rely on engineering judgement. Therefore, it is important that the capability of the model updating procedure is not overestimated.

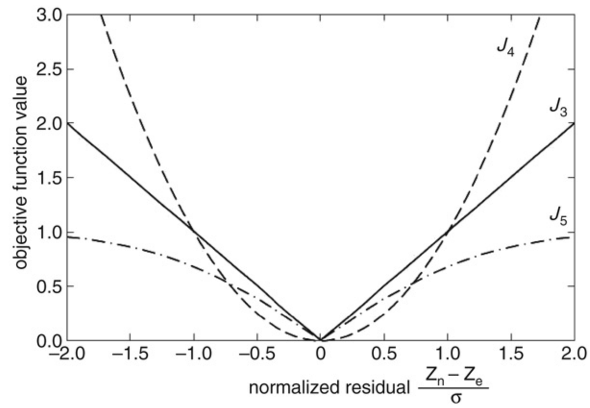

Fig. 1. Shape of objective functions for one normalized residual.

The second step is to be check whether the changes introduced in the model also hold for the conditions which will be analysed with the updated model. This requires that the sources for the model parameter changes can be found. There are guidelines on how to extrapolate findings from diagnostic load tests to higher loads [7].

4. Case study

The methodology proposed here was applied in a practical example on the new Svinesund Bridge. With a total length of 704 m and a main span of 247 m, it is one of the world's longest single-arch concrete bridges; see Fig. 2. The two multi-cell bridge girders consist of steel and carry two lanes of traffic each. The girders are connected by crossbeams which are suspended from the arch in the main span. In the side spans the cross beams are supported by concrete columns. Due to the slenderness of the columns, it was necessary to prestress the crossbeams onto the column heads to avoid uplifting of the bridge deck girders under asymmetric loading. The two bridge girders, passing the arch on either side, were designed to be rigidly connected to the arch in order to provide additional lateral stability to the arch. For a more detailed description of the bridge, see [26].

4.1. Preconditions

A measurement programme was initiated to check that the real bridge behaved as designed. Later, the decision was taken to use the available experimental data also for FE model updating to obtain an accurate model for the management of the bridge and to determine uncertain structural parameters. The locations of the permanently installed sensors are shown in Fig. 3. For a detailed description of the measurement programme, see [27–29].

The full reference data for updating consisted of 10 measured forces in the hanger pair 1, 95 readings of strain gauges in the arch and 155 measured displacements that were collected during static load tests with eight trucks; see Fig. 4. In addition, the first four eigenfrequencies, determined under ambient vibration from June to August, were used. Higher order modes could not be clearly paired due to the lack of mode shape information. All hanger forces under dead weight, which were determined using temporary sensors, were used only when the model was manually refined. As the hanger forces depended mainly on their prestress during the construction process, which was beyond the scope of the study, they were not included in the objective functions.

To illustrate the effect of the changes introduced in the FE model, objective functions J_3–J_6, were used; see Eqs. (3)–(7). The

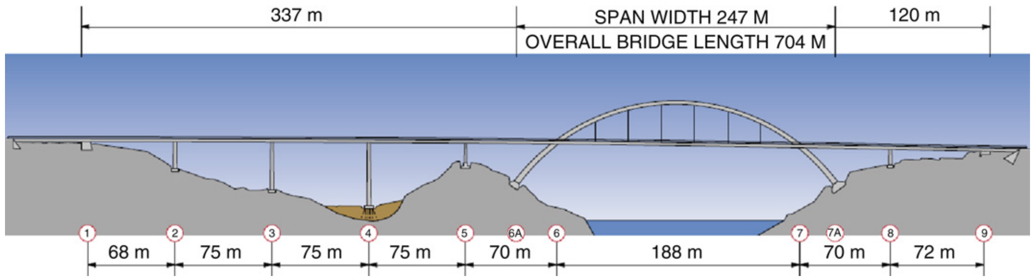

Fig. 2. Elevation of the new Svinesund Bridge [26].

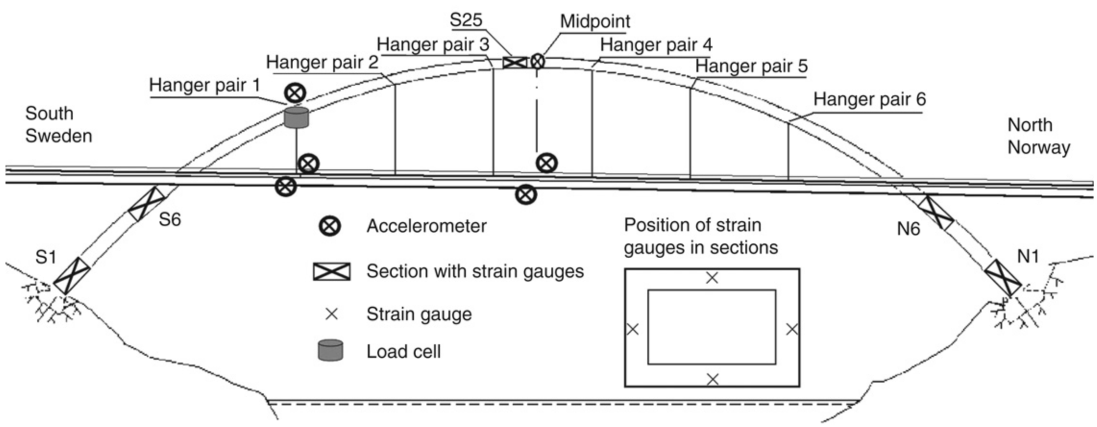

Fig. 3. Sensor locations, modified from Jonsson and Johnson [30].

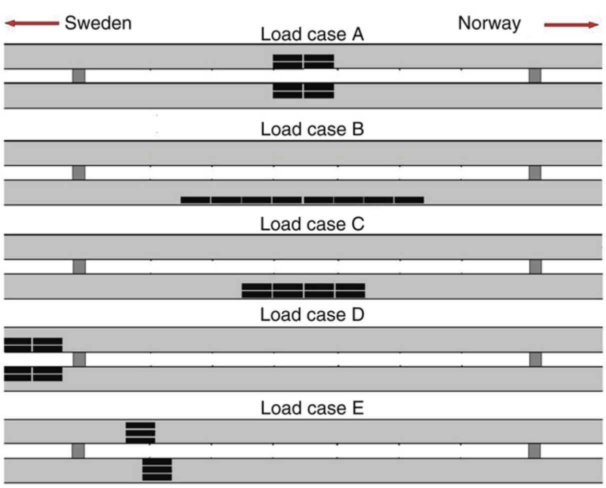

Fig. 4. Truck position for load cases.

standard deviations for the four eigenfrequencies were calculated separately. For each of the other measurement types a common standard deviation was assumed. After removing temperature effects from the strain, displacement, and force measurements, the standard deviations during the tests, $\sigma_{\text{test},k}$, for each measurement type, k, were calculated. This accounted for effects that varied during the load test, such as wind, noise in the data acquisition system and deviations from the intended truck position. Effects that were not covered by $\sigma_{\text{test},k}$, such as deviations from the intended sensor position and sensor accuracy, were estimated and summarized in $\sigma_{\text{sensor},k}$. Assuming Gaussian distributions, both standard deviations were then added to estimate the total standard deviation for the measurement type according to

$$\sigma_{\text{tot},k} = \sqrt{\sigma^2_{\text{test},k} + \sigma^2_{\text{sensor},k}}. \tag{8}$$

Table 2
Accuracy of the initial FE model.

	Average experimental values $\frac{1}{m}\sum_{i=1}^{m}\lvert z_{ei}\rvert$	Average numerical values $\frac{1}{m}\sum_{i=1}^{m}\lvert z_{ni}\rvert$	Average error $\frac{1}{m}\sum_{i=1}^{m}\lvert z_{ei}-z_{ni}\rvert$
Eigenfrequencies (Hz)	0.80	0.71	0.11
Strains (10^{-6})	7.56	7.58	1.54
Displacements (mm)	7.92	9.69	2.18
Hanger forces (kN)	131.89	137.42	5.95

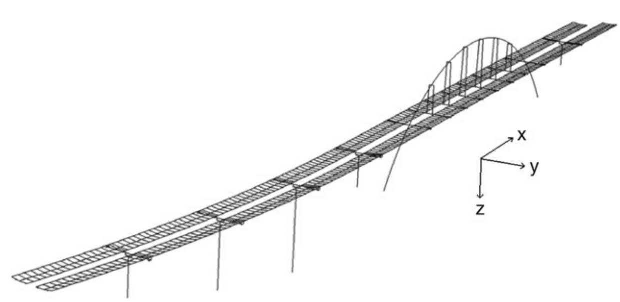

Fig. 5. Finite element model of the bridge.

To calculate the objective function, J_6, weighting coefficients of $w_k = 1$ were chosen.

The initial FE model used for updating was developed by Plos et al. [31]. It was based on the model that was used for the design of the bridge. Timoshenko beam elements were used to model all structural parts. The bridge girders were modelled as a grid of beam elements with longitudinal beams to represent the longitudinal walls and transversal beams to model the transversal stiffening walls; see Fig. 5. The agreement of the initial FE model with the measurements is summarized in Table 2.

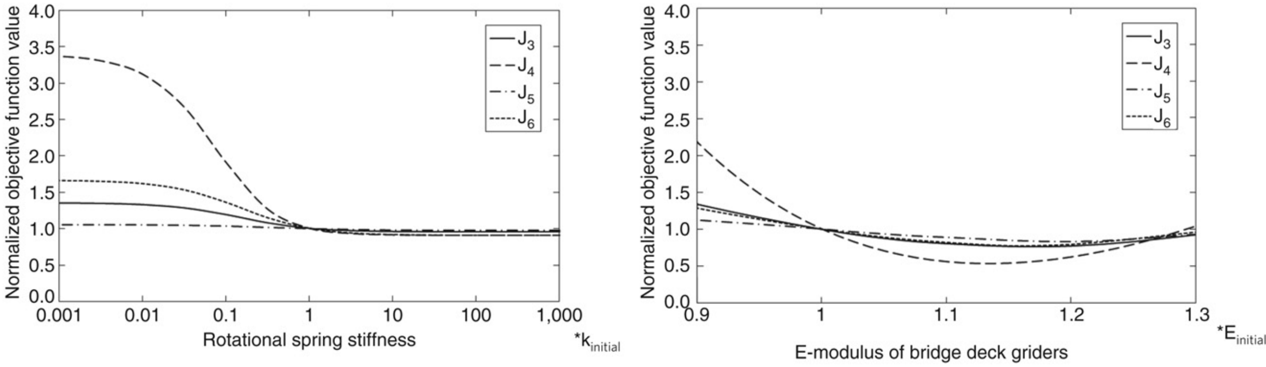

Fig. 6. Development of the normalized objective function values for model parameter variations.

4.2. Manual model refinements

Removal of lower bond assumptions. Mean material parameters had been used in the initial model. However, to take into consideration that the concrete arch was more than one year old when the bridge was tested, the elastic modulus based on a concrete age of 28 days was increased to account for the further hardening [32]. In addition, the contribution of the reinforcement steel to the arch stiffness had not been accounted for. The stiffness increase due to these effects has been calculated for the cross sections for which reinforcement drawings were available. A quadratic curve was than fitted into the available data to obtain an approximation of the stiffness increase for the complete arch; see [30].

Refine model. To increase the agreement with measured eigenfrequencies, Ülker-Kaustell and Karoumi [29] suggested restraining the longitudinal bridge-girder movement over the columns in the FE model. Based on their suggestion the bridge girders were tied to the supporting columns which restrained the bridge-girder rotation and translation. This led to an improved agreement of the eigenfrequencies; however, the contribution of the displacement decreased for three of the four objective functions. At the same time the agreement observed for the strains was less good, while the measured forces remained almost unchanged; see Fig. 7.

When the agreement for each load case tested was evaluated separately, it became clear that sometimes a restrained, and sometimes a free, bridge-girder movement led to better agreement with the tests. This revealed the need for more realistic modelling of the responses of the bearings. Therefore, an elastic-perfect-plastic model was introduced into the model to simulate the response of the bearings under the static load tests while tied bearings were assumed for the eigenfrequency analysis. A linear increase of the horizontal forces in the bearings up to the static friction threshold at a displacement of 1 mm was assumed, followed by constant friction force for increasing displacements. The estimated static friction threshold was based on the specification of the bearing manufacturer. It depends on the temperature, the contact pressure, and the accumulated sliding path before the load test. This change caused all objective functions to decrease significantly.

The hanger forces computed for dead weight were on average 58% lower than the measured ones. This disclosed an underestimated bridge deck girder mass in the FE model. Investigations showed that the mass of non-structural parts such as the asphalt layer and the railings were not taken into account as mass but as additional load cases. Summing up the mass of non-structural parts and introducing this by means of mass points into the FE model led to improved agreement for the eigenfrequencies and the hanger forces under dead weight.

Due to the increased mass, the analytical eigenfrequencies were then lower than the experimental ones. Therefore, whether or not the asphalt layer added stiffness to the bridge deck girders was checked. Using van der Poel's nomograph [33] to estimate the stiffness modulus of the bitumen and then the nomograph of Ugé et al. [34] to estimate the stiffness of the asphalt did not lead to significant stiffness increase of the bridge deck girder. Therefore, changes of the bridge deck stiffness were postponed until the parameter study.

To check whether the non-linear response of the concrete arch, caused by cracking of the concrete, had to be introduced into the model, the stresses in the arch were computed. As the entire arch was found to be under compression for all load cases tested, no changes were made to the initially assumed linear elastic material model.

Parameter study. When the accuracy of the five load cases was compared, it was found that the FE model was less accurate for load cases with loads close to the arch and bridge deck girder connection. Therefore, the initially rigid connection in the FE model was replaced by a spring-connection which allowed rotation of the bridge deck girders around the *y*-axis. Due to the high uncertainty associated with the initial estimate of the rotational spring stiffness, it was varied over a wide range and the values for the four objective functions used were plotted. The best agreement with all measurement types was obtained for high spring stiffnesses, which restored the initial rigid connection; see Fig. 6. Although the sensitivity of the structural response was low for the initial estimate of the spring stiffness, it was still possible to verify the designers' assumption of a rigid connection for this critical detail of the bridge.

Other parameters more closely studied were the elastic modulus of the concrete arch, the elastic modulus of the bridge girders, the static friction threshold for the bearings, the mass of non-structural parts along bridge girders, and the rotational stiffness of the arch supports around the *y*-axis. The parameter study showed the lowest objective function values when the bridge girder stiffness was increased by approximately 15%; see Fig. 6. In addition, the best agreement with the measurements was found when the arch stiffness was increased by an additional 3%. Furthermore, the study indicated that the static friction threshold of the bearing was around 13% higher than initially assumed. The additional mass that was introduced into the model, based on calculations, could be verified through the parameter study. As the rotational stiffness of the arch support was found to have no significant influence on the measurements, it could not be estimated more accurately through the sensitivity study. Therefore, the initial estimate was used. Had the measurement programme been designed according to the recommendation in Section 3, namely that the measurement chosen should allow for the determination of uncertain structural parameters, it would have been possible to gain additional information about the important, but uncertain, soil–structure interaction.

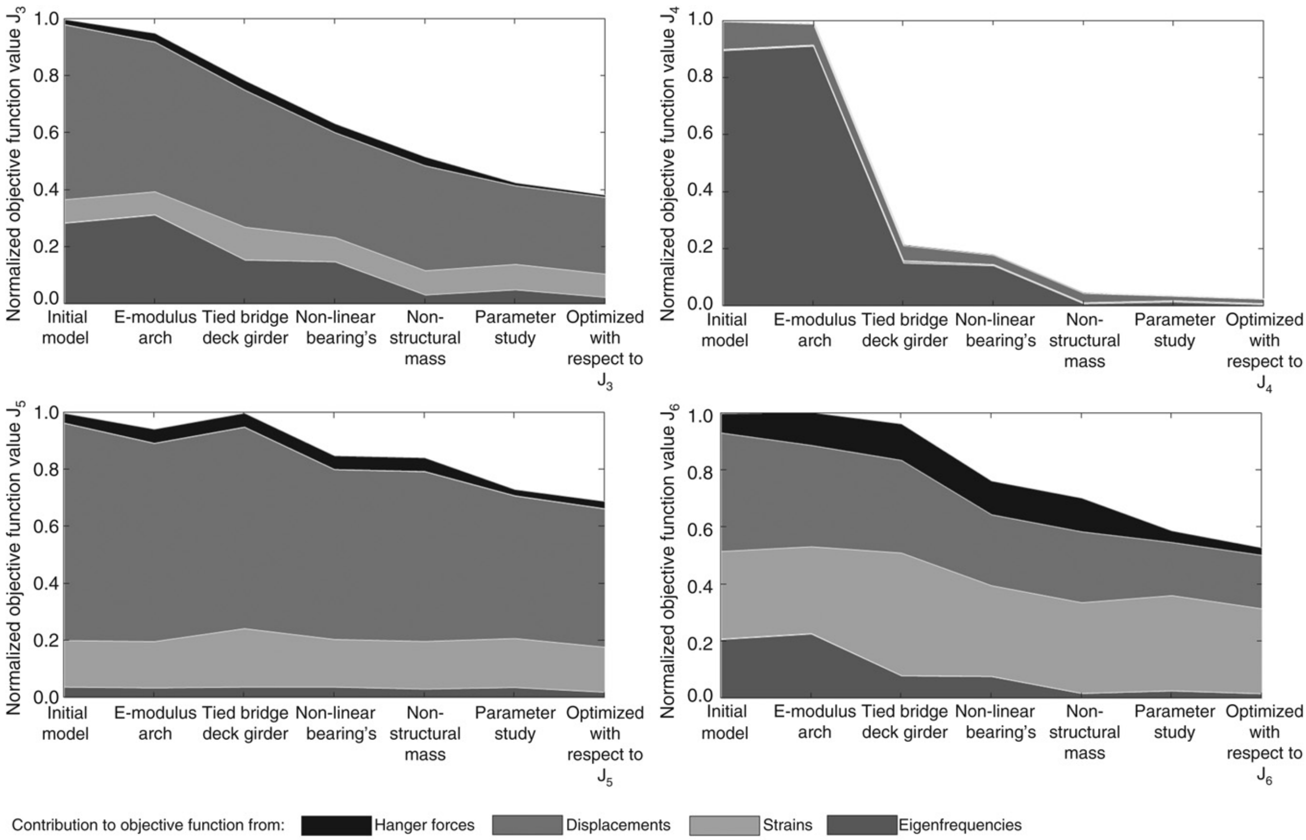

Fig. 7. Objective function development for model evolution steps.

Table 3
Average error development; see also Table 2.

	Initial model	Manual refinements (exclusive parameter study)	Parameter study	Optimized with respect to objective function			
				J_3	J_4	J_5	J_6
Eigenfrequencies (Hz)	0.11	0.01	0.01	0.01	0.01	0.01	0.01
Strains (10^{-6})	1.54	1.60	1.68	1.53	1.55	1.53	1.50
Displacements (mm)	2.18	1.31	0.98	0.96	0.96	0.95	0.99
Hanger forces (kN)	5.95	10.39	3.63	2.68	2.56	4.3	2.37

Table 4
Normalized updating parameters.

	Initial model	Manual refinements (exclusive parameter study)	Parameter study	Optimized with respect to objective function			
				J_3	J_4	J_5	J_6
E-modulus of concrete arch	1.00	1.04–1.17	1.07–1.20	1.03–1.15	1.04–1.17	1.02–1.14	1.02–1.14
E-modulus of bridge deck girders	1.00	1.00	1.15	1.17	1.16	1.32	1.17
Additional mass of non-structural parts	0.00	1.00	1.00	1.06	1.09	1.07	1.03
Static friction threshold of bearings	–	1.00	1.13	0.91	0.86	0.98	0.81

The normalized parameters that led to the lowest objective function values are summarized in Table 4. When the findings from the parameter study were introduced into the model, all of the objective functions decreased; see Fig. 7.

4.3. FE model updating through non-linear optimization

So far only one parameter was varied at the time, which could not account for the interaction between the different types of model parameters. Therefore, parameter estimation through non-linear optimization was done to further increase the accuracy of the FE model and to obtain more accurate estimates of some structural parameters. Due to convergence problems in a prestudy when using gradient based optimization algorithms, the Nelder–Mead simplex algorithm [35] was chosen for optimization. It is robust and does not require the computation of any derivative information. Four structural parameters were chosen for updating, namely the elastic modulus of the concrete arch, the elastic modulus of the bridge girders, the static friction threshold of the bearings, and the mass of non-structural elements along the bridge girder.

The final accuracy of the updated models is summarized in Table 3. The corresponding normalized updating parameters are shown in Table 4. It can be seen that the previously introduced increase of the elastic modulus of the arch stiffness was reversed partially by the optimization algorithm. The elastic modulus of the bridge girders increased further for all objective functions. In addition, the mass of non-structural parts increased by between

3% and 9% and the static friction threshold of the bearings decreased by between 13% and 28%. Even though the same tendency of updating parameter changes could be observed, their final normalized values differed with up to 0.17 depending on the chosen objective function. The optimization algorithm decreased objective functions J_3 and J_6 by approximately 11%, J_4 by 31%, and J_5 by less than 6% compared to the parameter study. These are only small improvements compared to the manual model refinements, despite substantial changes to the updating parameters. The low sensitivity of the objective functions with respect to the updating parameter changes, in combination with the high influence of model simplification, highlights the difficulties which are associated with structural parameter estimation by FE model updating. The final normalized objective function values are presented in Fig. 7.

Although all objective functions clearly show increased model accuracy with the changes introduced, their development is quite different. Objective function J_4 drops to 2.4% of the initial value while objective function J_5 decreases only to 69%. The high drop of J_4 is mainly due to the improved agreement of the second eigenfrequency, which contributes to more than 80% of the objective function value before a restrained bridge girder movement is introduced into the model. On the other hand, the second eigenfrequency adds less than 1% to the value of objective function J_5 for the first two model evolution steps. A good compromise between these two extremes is objective function J_3. A descriptive method of expressing the model discrepancy can be obtained by objective function J_6, which can be interpreted as the average error for all of the measurement types.

4.4. Model evaluation

All of the changes introduced as manual model refinements before the parameter study could be reasoned. Thus, high confidence in the changes introduced was achieved. The parameter changes that were based on the parameter study or the optimization algorithm are more uncertain. All updating parameters remained within realistic ranges without introducing constraints.

The highest uncertainty was associated with the stiffness increase of the bridge girders, which was derived from the parameter study. The reason for this has to be carefully analysed before it can be relied on in further analysis. A possible source for additional stiffness is the railing system.

In addition, the static friction of the bearings was shown to have a significant effect on the measured properties. In particular, the eigenfrequency agreement improved when a restrained bridge deck girder movement was assumed. However, the load test clearly showed that the bearings move under higher horizontal forces. Therefore, it can be questioned whether a restrained bridge girder movement can be assumed for higher excitations such as strong winds or earthquakes. For analysis under static loads, one cannot rely on the static friction in the bearings, as it may not hold for a longer period of time due to thermal changes or excitation caused by passing vehicles.

5. Discussion

In Table 3 and Fig. 7 it can be seen that the agreement of the model with the strain measurements remains almost unchanged for all model evolution steps. A plausible reason is that the strain sensors have been placed in or close to disturbed regions. This could not been accounted for by the beam elements which have been used in the model. Furthermore, other local effects like the elastic modulus variation of the concrete or the reinforcement arrangement around the strain sensors generally limit the accuracy of strain measurements in concrete and hence their value for model updating.

For the other measurements types a significant better agreement was obtained through FE model updating without assigning unrealistic values to the updating parameters. However, a certain inaccuracy of the model remained. To further increase the agreement of the model with the measurements, a less restrictive choice of updating parameters was tested. For example, by using two updating parameter for the arch stiffness, one for the arch bases, one at the crown, and quadratic interpolation in between, further decreasing objective functions could be obtained. However, due to the unrealistic values that were assigned to the concrete cross sections, it is believed that the updating algorithm compensates for the model simplification or measurement errors by unrealistic model parameters, instead of finding improved estimates of the actual structural parameters.

It was also possible to obtain a more accurate model with respect to one type of measurements or to selected parts of the measurements when the others were excluded from the objective function. The challenge however was to obtain a model that could reproduce all 264 measured values with reasonable accuracy.

6. Conclusions

A methodology for FE model updating for improved bridge evaluation has been proposed and applied. Important aspects like the choice of measurements, model simplifications, the accuracy and reliability of updated parameters, and the analysis of untested loading conditions have been included.

While the focus of FE model updating is usually on tuning model parameters, this article highlights the importance of other modelling errors. Manual model refinements prior to FE model updating by non-linear optimization was shown to be decisive for the success of the case study conducted. The manual model refinements decreased the initial modelling error through physically justified changes and allowed the detection of important physical phenomena which had to be introduced into the model. This reduced the risk of compensating for modelling errors by meaningless changes to updating parameters.

Furthermore, it was shown that FE model updating can be applied to non-linear models of real bridges. Besides the slightly increased computational time, no problems were encountered after introducing the non-linear bearing response into the model.

Four multi-response objective functions for FE model updating were proposed and tested. It was shown that improved accuracy for eigenfrequency calculations does not necessarily lead to an improved model for static analysis. The combination of numerous and diverse kinds of measurements should preferably be used when a generally valid and accurate model is sought. As a different bearing behaviour was observed under ambient excitation and during the load tests, it was not possible to update the boundary conditions through modal characteristics and then use the same boundary conditions for static analysis.

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Paper II

Safety Formats for Nonlinear Analysis of Concrete Structures.

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Safety Formats for Nonlinear Analysis of Concrete Structures

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Abstract

For realistic modelling of reinforced concrete structures, nonlinear models are often inevitable, which raises the question of an appropriate safety format for nonlinear analysis. This article gives an overview of available safety formats and discusses their advantages and disadvantages. An analysis of available round robin tests and modelling competitions shows that current safety formats do not properly account for the modelling uncertainty of nonlinear analysis. Based on this observation a new safety format was proposed which allows one to explicitly account for the modelling uncertainty. To avoid any interaction of the modelled response with the safety format, the mean *in situ* material parameters should be used in the nonlinear analysis and a resistance safety factor is used to assure the intended reliability level. The application of the new safety format to beam sections loaded in bending showed that it offers a reliability level that is in good agreement with the target reliability.

Keywords: nonlinear analysis, model uncertainty, concrete, safety format, reliability

Notation

A_c	area of the concrete section
$\boldsymbol{a}_{\mathrm{nom}}$	nominal geometrical parameters
$A_{\mathrm{s,l}}$	area of the longitudinal reinforcement
b	width of a beam
c	step size parameter
d	effective depth of a beam
$\tilde{f}_{\mathrm{c}}$	concrete compressive strength used in the nonlinear analysis according to EN 1992-2
f_{cm}	mean concrete compressive strength of specially cured cylinders
$f_{\mathrm{cm,is}}$	mean *in situ* concrete compressive strength
f_{ck}	characteristic concrete compressive strength
$f_{\mathrm{ck,is}}$	characteristic *in situ* concrete compressive strength
f_{ym}	mean yield strength of reinforcement steel
f_{yk}	characteristic yield strength of reinforcement steel

$\tilde{f}_{\mathrm{y}}$, yield strength of the reinforced steel used in the nonlinear analysis according to EN 1992-2

f_{su} ultimate strength of reinforcement steel

h beam depth

R resistance of a section or a structure

R_{d} design resistance of a section or a structure

R_{m} resistance when the mean in situ material parameters are used in the nonlinear analysis

R_{n} nominal resistance of a section or a structure

V_{f} coefficient of variation of the variable to model the material uncertainty

$V_{\mathrm{f_c}}$ coefficient of variation of the concrete compressive strength of specially cured cylinders

$V_{\mathrm{f_{c,is}}}$ coefficient of variation of the *in situ* concrete compressive strength

$V_{\mathrm{f_y}}$ coefficient of variation of the yield strength of the reinforcement steel

V_{g} coefficient of variation of the variable to model the geometrical uncertainty

V_{R} coefficient of variation of the resistance

V_{θ} coefficient of variation of the variable to model the modelling uncertainty

V_{κ} coefficient of variation of the variable to account for the lower concrete compressive strength in real structures than in specially cured cylinders

x compressive zone depth

X_{g} random variable to model the geometrical uncertainty

X_{f} random variable to model the material uncertainty

α_{cc} coefficient applied on the concrete compressive strength to account for long term effects and the way the load is applied

α_{E} sensitivity factor for the action side according to the semi-probabilistic approach

α_{f} sensitivity factor for the material strength

α_{R} sensitivity factor for the resistance side according to the semi-probabilistic approach

β reliability index

β_{R} reliability index on resistance side

γ_{C} partial factor for concrete

γ_{S} partial factor for reinforcement steel

$\gamma_{\mathrm{O'}}$ resistance safety factor according to EN 1992-2 (CEN, 2004b)

$\varepsilon_{\mathrm{s1}}$ reinforcement steel strain in the tensile reinforcement (or the less compressed reinforcement layer)

$\varepsilon_{\mathrm{su}}$ ultimate strain of reinforcement steel

θ random variable to model the modelling uncertainty

θ_{m} mean ratio of experimental strength over calculated strength

ρ_{tot} reinforcement ratio

ρ_1 reinforcement ratio for tensile reinforcement (or the less compressed reinforcement)

ρ_2 reinforcement ratio for compressive reinforcement

κ variable to account for the lower concrete compressive strength in real structures than in specially cured cylinders

κ_{m} mean value of κ

$\sigma^2_{f_{c,is}}$ variance of the *in situ* concrete compressive strength

$\sigma^2_{f_y}$ variance of the yield strength of the reinforcement steel

Introduction

Today, the classic design and assessment of concrete structures usually involves two steps. In the first step a linear-elastic model is used to calculate the structural response. This analysis is made using mean stiffnesses with the intention to obtain a realistic description of the distribution of internal forces and moments. The second step consists of the design of all critical components of the structure using local nonlinear models. The component design is made using design material parameters, conservative models, or both, in order to ensure the intended reliability level.

The assumed linear-elastic material behaviour of the structural analysis is a gross simplification, especially for concrete structures. This can lead to a modelled structural response and internal force distribution that deviates significantly from reality. To describe the structural response more realistically, nonlinear analysis is becoming more widely used. 'Nonlinear analysis' is in this article used to denote an analysis which accounts for the nonlinear stress-strain relationship of the concrete and reinforcement steel, allows for redistribution and can be used to calculate the failure load directly as the employed material models automatically guarantee equilibrium in critical components. Only for failure modes that can not be described by the nonlinear analysis, additional manual component checks are needed. This reduces the use of incompatible models for the structural analysis and the component checks and the nonlinear analysis fulfils the purpose of both steps according to the classic design approach. This raises the question of an appropriate safety format. The mean material parameters would be needed for a realistic structural analysis, while design material parameters would be needed to ensure the intended reliability level.

In this paper, existing safety formats for nonlinear analysis of concrete structures are presented and their advantages and disadvantages are discussed. Due to the importance of the modelling uncertainty which differs significantly between different types of modelling approaches and failure modes, a new safety format is proposed which allows explicitly accounting for the modelling uncertainty. Rather than using partial factors which are employed to calculate design material strengths a resistance safety factor is employed to calculate the design resistance.

Safety formats for nonlinear analysis of concrete structures

The safety formats discussed here were based on the general principles of structural design according to EN 1990 (CEN, 2002a). According to this Eurocode, a target reliability index in the ultimate limit state (ULS) was chosen, $\beta = 3.8$, for a reference period of 50 years. Furthermore, a fixed separation between action effects and resistance was introduced by assuming fixed sensitivity factors for the action effects, $\alpha_{\mathrm{E}} = -0.7$, and the resistance, $\alpha_{\mathrm{R}} = 0.8$ (Sedlacek and Müller, 2006). For a wide range of applications in structural engineering this semi-probabilistic approach has been shown to yield reliability indexes that are in good agreement with the target reliability index (Grünberg, 2002). Due to this separation between action effects and resistance, the following safety formats apply only to the calculating of the design resistance, with a target reliability of $\beta_{\mathrm{R}} = \alpha_{\mathrm{R}}\beta = 3.04$, while the load must be treated according to EN 1990 (CEN, 2002a) and EN 1991 (CEN, 2002b).

Partial factors according to EN 1992-1-1
The partial factors according to EN 1992-1-1 (CEN, 2004a) were established for the component check, according to the classic design approach. Nevertheless, they can also be used in combination with nonlinear analysis; they are the basis for the safety format for nonlinear analysis according to EN 1992-2 (CEN, 2004b). Therefore, their derivations are briefly summarised. The partial safety factors are based on the assumption that the relation between the resistance, R, and nominal resistance, R_{n}, can be expressed in multiplicative form:

1. $R = \theta \cdot X_{\mathrm{g}} \cdot X_{\mathrm{f}} \cdot R_{\mathrm{n}}$

where $\theta, X_{\mathrm{g}}, X_{\mathrm{f}}$ are random variables to model the modelling, geometrical, and material uncertainty respectively. This allows one to calculate the coefficient of variation of the resistance, V_{R}, according to

2. $V_{\mathrm{R}} = \sqrt{V_{\theta}^2 + V_{\mathrm{g}}^2 + V_{\mathrm{f}}^2}$

where $V_{\theta}, V_{\mathrm{g}}, V_{\mathrm{f}}$ are the coefficients of variation of associated random variables to model the modelling, geometrical, and material uncertainty. The coefficients of variation given in Table 1 can be assumed as the basis for the partial factors given in EN 1992-1-1, (European Concrete Platform, 2008). Inserting the values from Table 1 into Equations 2 and 3 leads directly to the partial safety factor, $\gamma_{\mathrm{S}} = 1.15$, for steel which is used as a divisor to calculate the design steel strength from the characteristic strength:

3. $\gamma_{\mathrm{S}} = \exp(\alpha_{\mathrm{R}}\beta V_{\mathrm{R}} - 1.64 V_{\mathrm{f}})$.

For concrete, an additional factor of 1.15 has been introduced to account for the lower concrete strength in real structures than in the specially cured cylinders that are used to determine the concrete strength class. Inserting again the values from Table 1 in Equations 2 and 4 leads to the partial factor for concrete, $\gamma_{\mathrm{C}} = 1.50$:

4. $\gamma_{\mathrm{c}} = 1.15 \cdot \exp(\alpha_{\mathrm{R}}\beta V_{\mathrm{R}} - 1.64 V_{\mathrm{f}})$.

Theoretically, Equation 1 and the subsequent equations are not valid when the resistance is calculated as the sum of products, e.g. for sections loaded by a normal force when the resistance is the sum of the steel and concrete contribution. However, by applying the partial factors to the material strengths instead of the sectional resistance, the partial factors can still be used for this kind of analysis. Furthermore, it must be noted that the design material parameters are only used to calculate the capacity of sections with respect to bending moments and normal forces. Additional safety factors or conservative assumptions are introduced in the analytical equations to describe other types of failure. Besides these theoretical derivations, the partial factors are also based on experience and full probabilistic analysis (Holický and Marková, 2000). By inserting design material strengths in the nonlinear analysis and using the nominal geometrical parameters, $\boldsymbol{a}_{\text{nom}}$, the design resistance, R_{d}, can be calculated:

5. $R_{\text{d}} = R\{f_{\text{cd}}, f_{\text{yd}}, \boldsymbol{a}_{\text{nom}}\} = R\left\{\frac{f_{\text{ck}}}{\gamma_{\text{C}}}, \frac{f_{\text{yk}}}{\gamma_{\text{S}}}, \boldsymbol{a}_{\text{nom}}\right\}.$

It is a drawback to this approach that in accounting for all types of uncertainties by reducing the material parameters, very low material parameters must be used. In nonlinear analysis of statically indeterminate structures, the use of reduced material parameters can lead to an unrealistic load distribution. Furthermore, for structures in which the structural behaviour is influenced by second order effects, the very low material parameters can result in an over-conservative and uneconomical design.

Table 1. Statistical representation which leads to the partial safety factors in EN 1992-1-1

Type of uncertainty	Assumed Coefficient of Variation	
	Steel	Concrete
Modelling	$V_{\theta} = 2.5\%$	$V_{\theta} = 5\%$
Geometry	$V_{\text{g}} = 5\%$	$V_{\text{g}} = 5\%$
Material	$V_{\text{f}} = 4\%$	$V_{\text{f}} = 15\%$

Table 1. Statistical representation which leads to the partial safety factors in EN 1992-1-1

Safety format for nonlinear analysis according to EN 1992-2

A modification of the previous safety format, better suited for nonlinear analysis, was proposed (König *et al.*, 1997); and after small modifications it has been included in EN 1992-2 (CEN, 2004b). According to this proposal the yield strength, $\tilde{f}_{\text{y}}$, and the concrete compressive strength, $\tilde{f}_{\text{c}}$, which are used as input data in the nonlinear analysis, should be calculated according to

6. $\tilde{f}_{\text{y}} = 1.1 f_{\text{yk}} = 1.27 f_{\text{yd}}$
7. $\tilde{f}_{\text{c}} = 1.1 \frac{\gamma_{\text{S}}}{\gamma_{\text{C}}} \alpha_{\text{cc}} f_{\text{ck}} = 1.27 f_{\text{cd}}$

where α_{cc} is a coefficient taking into account long term effects and the way the load is applied. To obtain the same reliability level as for the safety format used for the cross sectional design, it is then necessary to divide the resistance, $R\{\tilde{f}_{\text{y}}, \tilde{f}_{\text{c}}, \boldsymbol{a}_{\text{nom}}\}$, obtained from the nonlinear analysis, by a resistance safety factor, $\gamma_{\text{O}'} = 1.27$:

8. $R_{\text{d}} = \frac{R\{\tilde{f}_{\text{y}}, \tilde{f}_{\text{c}}, \boldsymbol{a}_{\text{nom}}\}}{\gamma_{\text{O}'}}.$

The safety format is, apart from the factor 1.27, identical to the safety format for the cross sectional design according to EN 1992-1-1. Therefore, it can be assumed that the same modelling uncertainty of $V_\theta = 2.5 - 5\%$ has been accounted for.

Compared to the safety format for the cross sectional design according to EN 1992-1-1, the safety format of EN 1992-2 is a major improvement for the nonlinear analysis of concrete structures. By introducing an additional resistance safety factor, $\gamma_{O'}$, it is not necessary to account for all uncertainty by a reduction of the material strength. This allows the use of more realistic material parameters in the nonlinear analysis, which results in a more realistically modelled response.

A drawback to both of the safety formats presented in EN 1992 is that the modelling uncertainty was chosen with analysis of beams and columns in mind. For the analysis of more complicated structures, with failure modes more difficult to model, a coefficient of variation of $V_\theta = 2.5 - 5\%$ to account for the modelling uncertainty is not valid, see Table 2-4. Hence, for these kinds of structures, both safety formats will lead to unsafe results.

Alternative approaches

Besides the two safety formats mentioned in EN 1992, there have been some alternative approaches to ensure safety in combination with nonlinear analysis of concrete structures. According to one proposal the mean concrete strength, f_{cm}, and the mean steel yield strength, f_{ym}, should be used in the nonlinear analysis, (Henriques *et al.*, 2002). The reliability is ensured by a resistance safety factor, γ_{R}. For beam and column analysis γ_{R} is based on the relative position of the neutral axis x/d, where x is the compressive zone depth and d is the effective depth of the cross section. This allows one to calculate the design resistance according to

9. $$R_{\mathrm{d}} = \frac{R\{f_{\mathrm{ym}}, f_{\mathrm{cm}}, \boldsymbol{a}_{\mathrm{nom}}\}}{\gamma_{\mathrm{R}}(x,d)}.$$

Another safety format was proposed based on probabilistic analysis of slender columns, (Six, 2001). Here, the material strengths used in the nonlinear analysis are the characteristic strengths increased by a factor 1.1. The resistance obtained is then to be divided by a resistance safety factor, γ_{R}. This depends on the reinforcement steel strain in the tensile reinforcement (or the less compressed reinforcement layer), $\varepsilon_{\mathrm{s1}}$, the reinforcement ratio, $\rho_{\mathrm{tot}} = A_{\mathrm{s,l}}/A_{\mathrm{c}}$, and the ratio between the reinforcement amount of the most compressed layer and the least compressed layer, ρ_2/ρ_1, where $A_{\mathrm{s,l}}$ is the area of the longitudinal reinforcement, A_c is the area of the concrete section. This allows calculation of the design resistance as:

10. $$R_{\mathrm{d}} = \frac{R\{1.1 f_{\mathrm{yk}}, 1.1\alpha_{\mathrm{cc}} f_{\mathrm{ck}}, \boldsymbol{a}_{\mathrm{nom}}\}}{\gamma_{\mathrm{R}}(\varepsilon_1, \rho_{\mathrm{tot}}, \rho_2/\rho_1)}.$$

An advantage of both approaches is that realistic material parameters are used in the analysis; consequently, the modelled structural response should be in good agreement with the real structural response. A drawback is that they can only be applied to analysis of beams and columns due to the formulation of the resistance safety factors, γ_{R}.

A general safety format for nonlinear analysis

All of the previously described safety formats are only applicable to nonlinear analysis of beams and columns. This is due either to the assumed modelling uncertainty, which is not appropriate for failure modes that are more difficult to model, or due to the criteria for calculating the resistance safety factor, which are not applicable to other types of analysis. To allow nonlinear analysis to be used for all types of reinforced concrete structures a new safety format is proposed in the following:

To yield a realistic image of reality, the mean concrete *in situ* material strength, $f_{\mathrm{cm,is}}$, the mean yield strength of steel, $f_{\mathrm{ym,is}} = f_{\mathrm{ym}}$, and nominal values, $\boldsymbol{a}_{\mathrm{nom}}$, for the geometrical parameters should be used in the analysis to calculate the resistance:

11. $R_{\mathrm{m}} = R\{f_{\mathrm{ym}}, f_{\mathrm{cm,is}}, \boldsymbol{a}_{\mathrm{nom}}\}$.

The design resistance, R_{d}, is obtained by division of the resistance, R_{m}, by a resistance safety factor, γ_{R}, which depends on the coefficient of variation of the resistance, V_{R}:

12. $R_{\mathrm{d}} = \frac{R\{f_{\mathrm{ym}}, f_{\mathrm{cm,is}}, \boldsymbol{a}_{\mathrm{nom}}\}}{\gamma_{\mathrm{R}}(V_{\mathrm{R}})}$.

Justified by the central limit theorem for products, and in line with EN 1992-1-1 (CEN, 2004a), it is assumed that the resistance is approximately log-normal distributed, which allows approximating the resistance safety factor as:

13. $\gamma_{\mathrm{R}} = \exp(\alpha_{\mathrm{R}}\beta V_{\mathrm{R}})$

Following the principles of the safety format from EN 1992-1-1, the coefficient of variation of the resistance, V_{R}, should be calculated according to Equation 2 to account for the modelling, geometrical, and material uncertainty using, $V_{\theta}, V_{\mathrm{g}}$ and V_{f}. A biased modelling approach can be accounted for by computing the resistance safety factor as

14. $\gamma_{\mathrm{R}} = \frac{\exp(\alpha_{\mathrm{R}}\beta V_{\mathrm{R}})}{\theta_{\mathrm{m}}}$

where θ_{m} is the mean ratio of experimental strength over calculated strength for the chosen modelling approach. The safety format is summarised in Figure 1 and details to quantify the modelling, geometrical, and material uncertainty are given in the following sections.

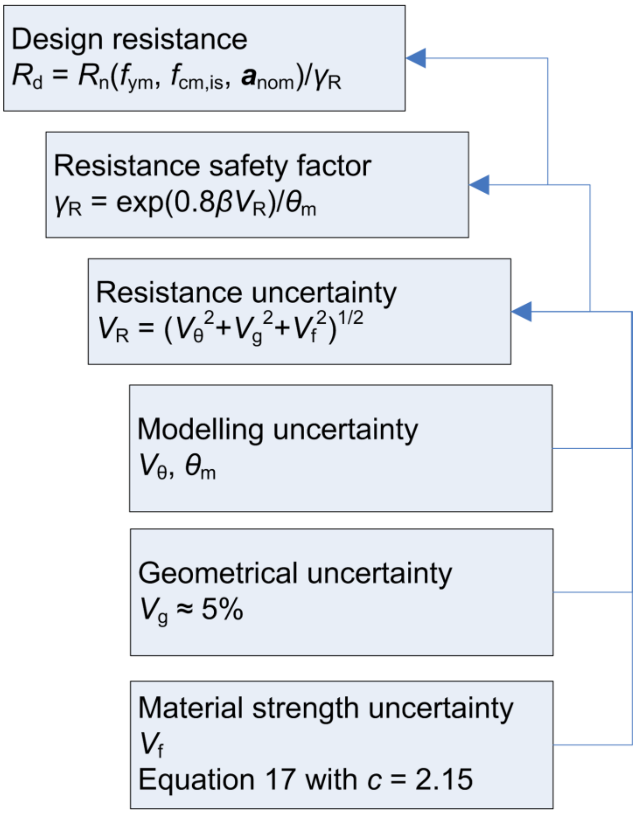

Figure 1. Flow chart for new safety format

Modelling uncertainty
While the variability of the resistance due to material uncertainty and geometrical uncertainty is explicitly accounted for, all other variability must be accounted for by the modelling uncertainty.

Uni-axial bending

For bending failure, the failure load obtained by nonlinear analysis usually agrees very well with the failure load that is obtained with hand calculations according to codes. Both calculation methods are usually based on the Euler-Bernoulli Hypothesis with a material response under uni-axial loading. Hence, for both procedures a similar modelling uncertainties can be assumed. For the classic design approach, the JCSS Model Code (JCSS, 2001) recommends mean ratio of $\theta_m = 1.2$ and a coefficient of variation of $V_\theta = 15\%$ for the component analysis. In Sustainable Bridges (Casas *et al.*, 2007) the coefficient of variation $V_\theta = 6\%$ is assumed.

To quantify the modelling uncertainty of nonlinear analysis, a round robin test of over-reinforced beams under 4-point bending was conducted, (van Mier and Ulfkjaer, 2000). Apart from the concrete, steel and geometrical properties, stress-strain diagrams for the concrete test with high-and low-friction-loading plates were specified in the invitation to the round robin test. Four beam types were tested: one small beam with normal strength concrete (NSC small), one large beam with normal strength concrete (NSC large), one beam made of high strength concrete (HSC) and one beam of fibre reinforced high strength concrete (FRHSC). Each beam type was tested three times. One contribution (Kang *et al.*, 1998) was excluded from this study as a range of solutions was provided. The mean ratio, θ_{m}, and the coefficients of variation of the modelled strengths, V_θ, are summarised in Table 2.

Table 2. Modelling uncertainty of over-reinforced beams

Beam type	θ_{m}	V_θ
NSC small	0.93	10%
NSC large	0.94	11%
HSC	1.02	25%
FRHSC	0.98	26%

It can be seen that for the normal strength concrete beams non conservative results were obtained, whereas the coefficients of variation of the calculated failure loads were around than 10 %. For the high strength concretes much larger coefficients of variation were obtained but hardly any bias.

Shear failure

To quantify the modelling uncertainty for shear type failures, a modelling competition on shear panels from 1985 is available (Collins *et al.*, 1985). Four different shear panels have been tested which were deliberately chosen as difficult to model. Some contributions were based on hand calculations, but no correlation between the complexity of the models and the accuracy of the resulting prediction has been found (Collins *et al.*, 1985). Therefore, all contributions were included leading to Table 3.

Panel A was heavily reinforced and loaded in pure shear. It failed due to sliding of the concrete without yielding of the reinforcement prior to failure. Panel B was nominally identical to Panel A but was loaded in combined bi-axial compression and shear. It failed brittle due to the sliding of the concrete. For Panel C a non-isotropic reinforcement layout has been chosen so that the reinforcement yielded in one direction before the panel failed gradually involving crushing and sliding of the concrete. Panel D was non-isotropic reinforced and was first loaded in shear and later in combined shear and compression. It failed due to the sliding of the concrete.

Table 3. Modelling uncertainty for shear panels

Panel name	θ_{m}	V_θ
Panel A	0.87	16%
Panel B	1.05	35%
Panel C	0.97	21%
Panel D	0.73	39%

For panels which were subjected to pure shear loading (Panel A and Panel C) a smaller variability of the predicted failure loads were obtained than for panels which were subjected to a combination of compression and shear loading. The predicted failure loads for Panel A and Panel D were significantly higher than the experimental ones.

Bending and shear failure in slabs

A modelling competition on slabs included eight slabs of differing thicknesses and reinforcement layout that were tested to failure (Jaeger and Marti, 2009b, Jaeger and Marti, 2009a). Some of these tests resulted in bending failure with yielding of the reinforcement, while others, typically when the slab was not provided with shear reinforcement, resulted in shear failure. One of the eight modelling contributions was based on hand calculations and was therefore excluded here from further analysis of the modelling competition which led to Table 4.

Table 4. Modelling uncertainty for bending and shear failure

Slab name	Slab Thickness [mm]	Direction of bending reinforcement	Shear reinforcement	θ_{m}	V_{θ}
A-1	200	45	No	0.79	27%
A-2	200	45	Yes	0.89	9%
B-1	200	0	No	1.00	24%
B-2	200	0	Yes	1.04	5%
C-1	500	45	No	0.72	34%
C-2	500	45	Yes	0.89	11%
D-1	500	0	No	0.97	29%
D-2	500	0	Yes	1.12	3%

For the slabs without shear reinforcement, coefficients of variation in the range of $V_{\theta} = 24 - 34\%$ were obtained. The use of shear reinforcement made the slabs easier to model and resulted in coefficients of variation on the range of $V_{\theta} = 3 - 11\%$. By trend, the slabs with a thickness of 200 mm could be better modelled than the slabs with a thickness of 500 mm. For slabs with the bending reinforcement not aligned in the principle moment direction, the models overestimated the experimental failure load; most drastically for slab C-1 by 39 % on average!

Other failure modes

The most relevant failure modes of concrete structures are uni-axial bending and shear type failure. For related failure modes such as biaxial bending or torsional failure, higher modelling uncertainties must be expected. For concrete structures primarily under compression, the modelling uncertainty of over-reinforced concrete beams can be appropriate. To quantify the modelling uncertainty of anchor bolts, shells, and concrete containment vessels special studies are available (Elfgren *et al.*, 2001, Krauthammer and Swartz, 1999, Hessheimer *et al.*, 2001).

Summary

Based on the available data, coefficients of variation in the range of $V_{\theta} = 5 - 30\%$ seem appropriate for beams in bending. Values in the lower part can be chosen for failures due to yielding of the reinforcement steel. Intermediate values should be chosen for over-reinforced concrete beams of normal concrete, while values in the higher part should be chosen for over-reinforced high strength beams. Not aligning the bending reinforcement in the direction of the principal moments can lead to an overestimation of the strength which must be accounted for by $\theta_{\mathrm{m}} \approx 0.9$.

For shear type failure the modelling uncertainty can be expected to lie in the range of $V_{\theta} = 10 - 40\%$. Values at the lower end of the range can be used for structures which fail due to yielding of the reinforcement steel. For small structures with simple loading that fail due to crushing of the concrete, values in the middle of the range seem appropriate. For a combination of compressive loading and shear loading and for larger structures higher coefficients of variation must be chosen. Furthermore, shear analysis can lead to a significant overestimation of the experimental failure load which can require a value of $\theta_{\mathrm{m}} \approx 0.7 - 1.0$.

Despite the studies analysed here, the data to quantify the modelling uncertainty of nonlinear analysis is scarce and might neither be representative nor conclusive. The modelling uncertainty will depend on the chosen modelling approach and software and can therefore only be quantifying in quite wide ranges which are of limited practical help. This is especially problematic as the modelling uncertainty will often be the factor that will govern the calculation of the resistance safety factor, γ_{R}. Until more studies are available which allow quantifying the modelling uncertainty more accurately, the choice of the modelling uncertainty will often depend on engineering judgement and can be subjective. Nevertheless, it is the authors' opinion that a deliberate and thoughtful choice of the modelling uncertainty is better than assuming a coefficient of variation of $V_{\theta} = 2.5 - 5\%$ for all kinds of nonlinear models, as is currently done in EN 1992-2.

Geometrical uncertainty

The geometrical uncertainty of reinforced concrete structures is usually quite small and has therefore minor influence on the resistance (JCSS, 2001). In these instances a coefficient of variation of $V_{\mathrm{g}} = 5\%$ can be assumed, analogous to the derivations of the partial factors according to EN 1992-1-1. For special types of structures that are sensitive to geometrical imperfections, e.g. slender columns, this assumption is not valid. In these cases the geometrical imperfections can be explicitly included in the nonlinear model to study their effect on the structural response.

Material uncertainty

For structures with a resistance, $R = R(X_1, X_2, \dots, X_n)$, that depends linearly on the material parameters, $X_1, X_2, \dots, X_n$, the variance of the resistance due to the material uncertainty can be calculated as

$$15.\ \sigma_R^2 = \sum_{i=1}^{n} \left(\frac{\partial R}{\partial X_i}\right)^2 \sigma_i^2 + \sum_{i \neq j}^{n} \frac{\partial R}{\partial X_i} \frac{\partial R}{\partial X_j} \mathrm{Cov}\left[X_i, X_j\right]$$

where $\partial R / \partial X_i$ is the partial derivative with respect to material parameters X_i, σ_i^2 is the variance of material parameter X_i, and $\mathrm{Cov}\left[X_i, X_j\right]$ is the covariance matrix. As the partial derivatives are not usually explicitly available for nonlinear analysis, they have to be replaced by numerically evaluated different quotients. This requires repeated, computationally demanding nonlinear analysis. For example, describing the stress-strain diagram of the reinforcement steel by Young's modulus, the yield strength, ultimate strength and ultimate strain and, for the concrete, by the compressive strength, tensile strength, fracture energy, and the crack bandwidth, would require as many as nine nonlinear analyses. In most situations, it is usually accurate enough to consider only the concrete compressive strength, f_c, and the yield strength of the reinforcement steel, f_y, as random variables. These two variables can be used as reference parameters to which all other material parameters are deterministically related. The concrete compressive strength and the yield strength of the reinforcement are uncorrelated, which allows one to omit the right part of Equation 15 to obtain

$$16.\ \sigma_R^2 \approx \left(\frac{\Delta R}{\Delta f_y}\right)^2 \sigma_{f_y}^2 + \left(\frac{\Delta R}{\Delta f_c}\right)^2 \sigma_{f_{c,is}}^2.$$

For nonlinear response surfaces Equation 16 can lead to inaccurate results, but the inaccuracy can be reduced by an appropriate choice of the linearization interval. For the safety evaluation it is necessary that the linear approximation of the response surface should be in good agreement with the real response surface between the mean point and the design point. The exact design point is not known *a priori* and its determination is computationally expensive. Nevertheless, an appropriate linearization interval can be chosen, which will usually provide sufficient accuracy.

In accordance with the semi-probabilistic approach, it can be assumed that the sensitivity factor of the resistance side is usually in the neighbourhood of $\alpha_R = 0.8$. According to the previous section, the material uncertainty is approximately equal to the modelling uncertainty, e.g. for under-reinforced beams in bending, $V_f \approx 5\%$, $V_m \approx 5\%$. By disregarding the geometrical uncertainty, the sensitivity factor for the material uncertainty, α_f, can therefore be predicted as approximately $\alpha_f \approx \alpha_R / \sqrt{2}$. Hence, it is suggested here that a step size parameter be used, $c = \alpha_R \beta / \sqrt{2} = 2.15$, which allows one to calculate, for the log-normally distributed material strengths, the step sizes as $\Delta f_{X_i} \approx f_{X_i}\left(1 - \exp\left(-c V_{X_i}\right)\right)$. The coefficient of variation of the resistance due to the material uncertainty, V_f, can then be approximated as

$$17.\ V_{\mathrm{f}} \approx \frac{1}{R_{\mathrm{m}}} \sqrt{\left(\frac{R_{\mathrm{m}} - R\{f_{\mathrm{ym}} - \Delta f_{\mathrm{y}}, f_{\mathrm{cm,is}}\}}{\Delta f_{\mathrm{y}}}\right)^2 \sigma_{\mathrm{f_y}}^2 + \left(\frac{R_{\mathrm{m}} - R\{f_{\mathrm{ym}}, f_{\mathrm{cm,is}} - \Delta f_{\mathrm{c}}\}}{\Delta f_{\mathrm{c}}}\right)^2 \sigma_{\mathrm{f_{c,is}}}^2}\ .$$

Structures in which the concrete tensile strength, rather than the compressive strength, limits the resistance the compressive strength and the associated variance used in Equation 17 must be replaced by the tensile strength and its associated variance. Furthermore, for very rare structures for which a decrease of the material parameters results in an increased resistance, the linearization interval can be adjusted, i.e. rather than using a linearization between the mean values and decreased material strength, the response surface should be approximated between the mean values and increased material strengths. To favour structures that fail with a ductile failure mode, due to yielding of the reinforcement steel, rather than a brittle mode due, to crushing of concrete, it is further possbile to introduce different weight factors into Equation 17.

An alternative, computationally less expensive, approach to estimating V_{f} has been suggested (Cervenka *et al.*, 2007). Instead of using three nonlinear finite element analyses, two analyses are required: one with the mean strengths f_{ym} and, f_{cm} and one with the characteristic strengths using f_{yk} and, f_{ck}. To allow a fair comparison between both approaches to estimate V_{f} it was here chosen to use the mean *in situ* strengths f_{ym}, and $f_{\mathrm{cm,is}}$ and the characteristic *in situ* strengths f_{yk} and $f_{\mathrm{ck,is}} = f_{\mathrm{cm,is}} \exp\left(-1.64 V_{\mathrm{f_{c,is}}}\right)$ to yield:

$$18.\ V_{\mathrm{f}} \approx \frac{1}{c} \log\left(\frac{R\{f_{\mathrm{ym}}, f_{\mathrm{cm,is}}, \boldsymbol{a}_{\mathrm{nom}}\}}{R\{f_{\mathrm{yk}}, f_{\mathrm{ck,is}}, \boldsymbol{a}_{\mathrm{nom}}\}}\right).$$

To see how accurately the coefficients of variation can be estimated using the previously described equations, they were tested on a, deliberately chosen, nonlinear response surface which refers to the maximum moment resistance of the beam section shown in Figure 2. The maximum moment resistance, R, was calculated by the layered approach using 15 layers, i.e. discretising the beam over the depth, calculating the corresponding strains and stresses for each discretisation, and integrating the stresses over the depth. For the reinforcement steel the idealised stress-strain diagram, including strain hardening with an ultimate strength of $f_{\mathrm{su}} = 1.08 f_{\mathrm{y}}$ and a corresponding strain of $\varepsilon_{\mathrm{su}} = 5\%$ was used according to EN 1992-1-1, (CEN, 2004a). For concrete in compression the stress-strain relation for the design of cross sections was used, (CEN, 2004a).

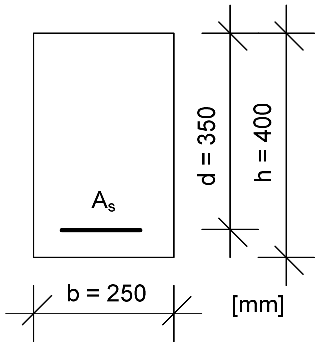

Figure 2. Geometry of beam section

The yield strength of the steel was assumed to be log-normally distributed with a mean value of $f_{\mathrm{ym}} = 550$ MPa and a coefficient of variation, $V_{f_y} = 5.5\%$. According to the stochastic model described in Sustainable Bridges, (Sustainable Bridges, 2007) the mean *in situ* concrete compressive strength was assumed to be log-normally distributed with a mean value of

19. $f_{\mathrm{cm,is}} = \kappa_{\mathrm{m}} f_{\mathrm{cm}}$

where, κ is a log-normally distributed variable with mean value of $\kappa_{\mathrm{m}} = 0.85$ and the coefficient of variation, $V_\kappa = 6\%$, and f_{cm}, is the mean compressive cylinder strength. The coefficient of variation of the *in situ* concrete strength, $V_{\mathrm{f_{c,is}}}$, was calculated according to

20. $V_{\mathrm{f_{c,is}}} = \sqrt{V_{\mathrm{f_c}}^2 + V_\kappa^2}$

where $V_{\mathrm{f_c}}$ is the coefficient of variation of cylinder compressive tests, calculated from f_{cm} and f_{ck} in EN 1992-1-1, to $V_{\mathrm{f_c}} = \log(f_{\mathrm{cm}}/f_{\mathrm{ck}})/1.64$. For the assumed concrete compressive class C20 this resulted in a mean *in situ* compressive strength of $f_{\mathrm{cm,is,C20}} = 23.8\ MPa$ and a coefficient of variation of $V_{\mathrm{f_{c,is,C20}}} = 21.4\%$. Figure 3 (a-c) shows the maximum moment resistance, R, plotted over variations of the steel yield strength, f_{y}, and the concrete compressive strength, f_{c}, for the reinforcement ratio of $\rho_{\mathrm{tot}} = 0.4\%$, $\rho_{\mathrm{tot}} = 1.5\%$ and $\rho_{\mathrm{tot}} = 3.0\%$. The ovals in Figure 3 refer to standard deviations of the log-normal distributed material strengths.

For the beam with the small reinforcement ratio, the maximum moment resistance is limited by a surface that is mainly inclined in the direction of the yield strength of the reinforcement steel. Failure is initiated by yielding of the reinforcement steel, which leads to a decrease of the concrete compression zone depth and, finally, to failure due to crushing of concrete.

For the beam section with the intermediate reinforcement ratio, $\rho_{\mathrm{tot}} = 1.5\%$, the moment capacity is approximately limited by two planes. The left, less steep plane is associated with a failure mechanism that is initiated by yielding of the reinforcement steel, while the right, steeper plane is associated with beams that fail directly by concrete crushing without yielding of the reinforcement steel. When analysing the beam with mean strengths, the beam failure is initiated by yielding of the reinforcement steel. However, when looking at low failure probabilities that are relevant for the structural design, the beam will fail directly in compression of the concrete without yielding of the reinforcement. Hence, a single analysis using the mean material parameters can lead to misleading results concerning the material parameter that causes failure.

For the beam with the high reinforcement ratio, $\rho_{\mathrm{tot}} = 3.0\%$, the concrete compressive strength limits the moment capacity for almost all combinations of material strengths, and failure occurs without yielding of the reinforcement steel. Consequently, the yield strength has almost no influence on the maximum moment resistance.

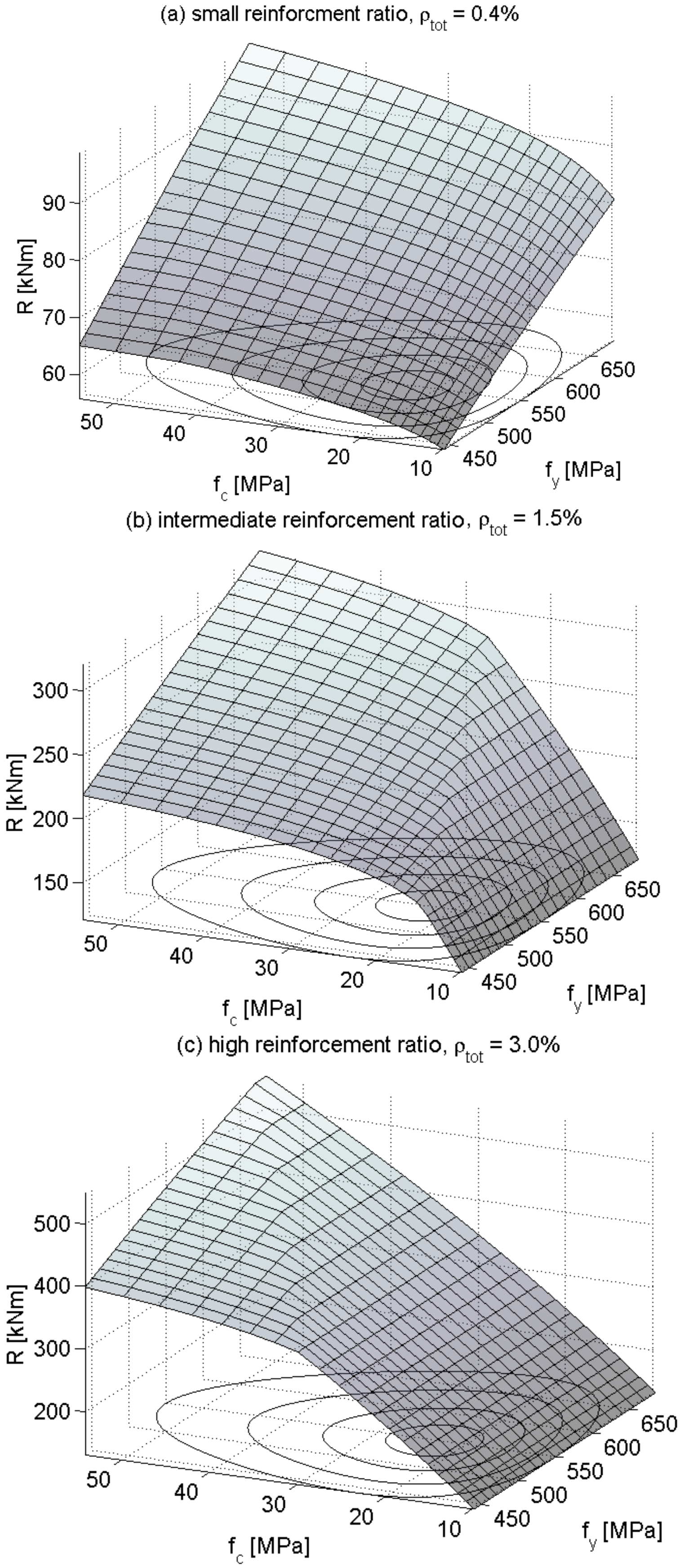

Figure 3. Moment resistance, R, over concrete compressive strength, f_c, and yield strength of reinforcement steel, f_y, for the section according to Figure 2

For the beam section shown in Figure 2, Equations 17 and 18 were applied to calculate the coefficients of variation, V_{f}, for seven reinforcement ratios, ρ_{tot}, and five step size parameters, c, see Table 5. The coefficients of variation increase for an increasing reinforcement ratio. For small reinforcement ratios the coefficients of variation are similar to the yield strength variation, $V_{\mathrm{f_y}} = 5.5\%$; for high reinforcement ratios, it approaches the coefficient of variation of the concrete compressive strength, $V_{\mathrm{f_{c,is,C20}}} = 21.4\%$.

The step size parameter has minor influence for the beams with either the low reinforcement ratios or the high reinforcement ratios. However, for the intermediate reinforcement ratios, $\rho_{\mathrm{tot}} = 1.00 - 1.50\%$, which cause a change in failure mode, the estimated coefficients of variation differs by a factor of up to 2.3 between the smallest step size and the largest. This highlights the importance of an appropriate step size in the case of a nonlinear response surface.

Table 5. Coefficients of variation, V_{f}, [%] according to Equations 17 and 18

	Step size parameter	reinforcement ratio, $\rho_{\mathrm{tot}} = A_{\mathrm{s}}/A_{\mathrm{c}}$						
	c	0.40%	0.80%	1.00%	1.25%	1.50%	1.75%	3.00%
Equation 17	0.50	5.3	5.7	6.1	6.8	7.6	17.6	18.2
	1.64	5.5	6.3	6.9	8.7	14.2	18.2	18.8
	2.15	5.7	6.6	7.3	11.2	15.4	18.5	19.0
	3.08	5.9	7.3	9.8	13.9	16.8	18.9	19.3
	3.80	6.1	8.5	12.1	15.4	17.6	19.3	19.6
Equation 18	0.50	7.8	8.8	9.3	10.4	11.4	20.6	21.8
	1.64	7.3	8.3	9.0	9.9	13.4	18.3	19.5
	2.15	7.2	8.4	9.0	10.0	14.3	18.2	19.3
	3.08	7.3	8.5	9.3	11.8	15.4	18.2	19.3
	3.80	7.3	8.7	9.5	13.0	15.9	18.2	19.3

To evaluate the consequences of the varying coefficients of variation, V_{f}, on the safety analysis, this problem was first isolated for study, i.e. only the compressive strength and the yield strength of the reinforcement steel were assumed to be random variables, while all other parameters were assumed to be deterministically known.

The coefficients of variation, V_{f}, in Table 5 were used to calculate the design resistance, $R_{\mathrm{d}} = R_{\mathrm{m}}/\gamma_{\mathrm{f}}$ with $\gamma_{\mathrm{f}} = \exp(\beta_{\mathrm{f}} V_{\mathrm{f}})$, and an assumed target reliability index of $\beta_{\mathrm{f}} = 2.15$. In the next step Monte-Carlo simulations were used to calculate the probability, $P(R < R_{\mathrm{d}})$, and the corresponding reliability index, $\beta_{\mathrm{f}} = -\Phi^{-1}\big(P(R < R_{\mathrm{d}})\big)$, where Φ is the standard normal distribution function. The concrete compressive strength and the yield strength of the reinforcement steel were sampled from log-normal distributions. For each reinforcement ratio, a sample size of $\mathrm{n} = 10^6$ was used. The reliability indexes obtained for the coefficients of variation according to Equations 17 and 18 and different step sizes are shown in Figure 4.

For a step size parameter of $c = 0.50$, very unsafe results can occur with a minimum reliability index, $\beta_{\mathrm{f}} = 1.3$. For larger step sizes, over conservative results are obtained. The best agreement with the target reliability index is obtained by applying Equation 17 with a step size parameter of $c = 1.64 - 2.15$. Decreasing both material strengths and the associated concrete stiffness at the same time leads to over conservative results for small reinforcement ratios when using Equation 18.

When structures may fail in different failure modes, the resistance of the structure is limited by the failure mode that has the least resistance. For decreasing material parameters, a change in failure mode is only possible if the new failure mode is more sensitive to material changes than the initial one. This means that, for decreasing material strengths, the slope of the response surface often decreases monotonically, see Figure 3. Therefore, for structures that show a change in failure mode, choosing a larger step size usually leads to more conservative results. Based on these results and considerations Equation 17 with a step size coefficient of $c = 2.15$ is recommended to estimate V_{f}. It was shown that this approach can also be used for beams loaded in shear, columns loaded by a normal force and beam sections subjected to a combination of shear force and bending moment (Sykora and Holický, 2011, Schlune *et al.*, 2011b, Schlune *et al.*, 2011a).

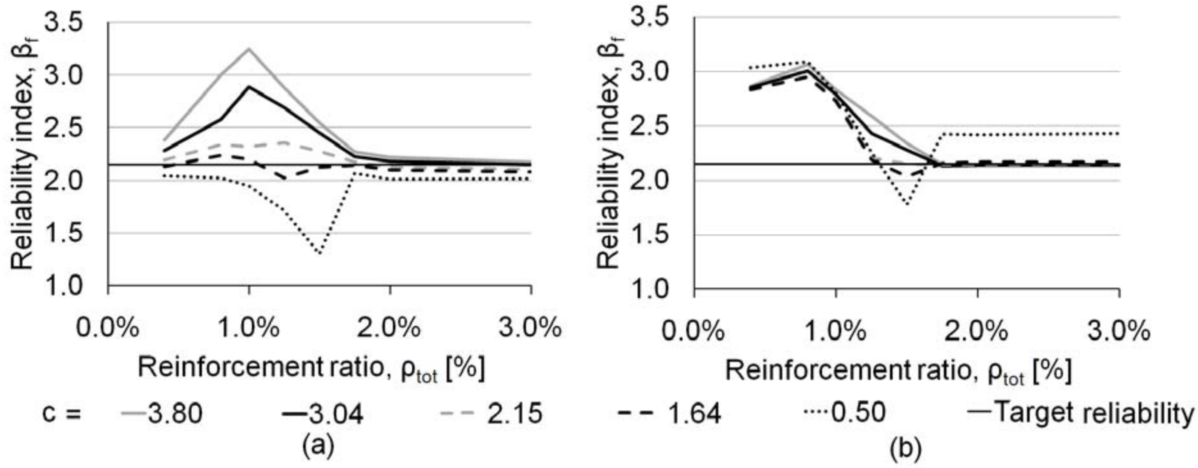

Figure 4. Reliability indexes over reinforcement ratios: (a) V_{f} according to Equation 17, (b) V_{f} according to Equation 18

Testing of the proposed safety format

To see if the proposed safety format leads to the intended target reliability of the resistance according to the semi-probabilistic approach of $\beta_{\mathrm{R}} = 3.04$ it was applied to reinforced concrete beams loaded in bending. For comparison the same was done for the safety format according to EN 1992-2 (CEN, 2004b).

Beams of three depths (h = 250, 325, 400 mm), four concrete compressive strength classes (C20, C40, C60 and C80) and 13 reinforcement ratios, varied from 0.2 - 4%, were analysed. Although not appropriate for high strength over-reinforced concrete beams (van Mier and Ulfkjaer, 2000) the modelling uncertainty in the new safety format and the full probabilistic analysis set at $V_\theta = 5\%$. This choice was made to allow a better comparison between the safety formats and to avoid a bias in the comparison by different underlying assumptions. The geometrical uncertainty was assumed to be $V_g = 5\%$ and the coefficient of variation of the material strength was estimated using Equation 17 with $c = 2.15$.

After calculating the design resistance according to the two safety formats, Monte-Carlo simulations were used to estimate the probability that the calculated resistances are below the design resistances, $P(R < R_d)$, using the limit state function

21. $g(\boldsymbol{X}) = \theta \cdot R(f_y, f_c, d) - R_d$

where θ is a variable to model the modelling uncertainty. The stochastic model and the definition of the variables are given in Table 6. A sample size of n = 10^6 was used for each beam configuration.

Table 6. Stochastic model of basic variables

		Symbol	Dimension	Distribution	Mean value, x_m	Standard deviation, σ_X
Materials	Concrete, C20	$f_{c,C20}$	MPa	LN	23.8	$0.214\mu_X$
	Concrete, C40	$f_{c,C40}$	MPa	LN	40.8	$0.126\mu_X$
	Concrete, C60	$f_{c,C60}$	MPa	LN	57.8	$0.097\mu_X$
	Concrete, C80	$f_{c,C80}$	MPa	LN	74.8	$0.084\mu_X$
	Reinforcing Steel	f_y	MPa	LN	550	30
Geometrical data	Effective depth	d_1	mm	N	200	10
	Effective depth	d_2	mm	N	275	10
	Effective depth	d_3	mm	N	350	10
Modelling uncertainty		θ	–	LN	1.00	0.05

Distribution types: N: normal, LN: log-normal

For the failure probabilities obtained the corresponding reliability indexes were calculated. This yielded a mean reliability index of $\bar{\beta}_{R,EN} = 2.95$ with the coefficient of variation, $V_{\beta_{R,EN}} = 9.5\%$, for the design according to EN 1992-2. For the new safety format, a mean reliability index of $\bar{\beta}_{R,new} = 3.08$ with the coefficient of variation $V_{\beta_{R,new}} = 7.6\%$ was obtained. The histograms in Figure 5 confirm that the new safety format provides a reliability level that is closer to the target reliability and more constant than the current approach in EN 1992-2.

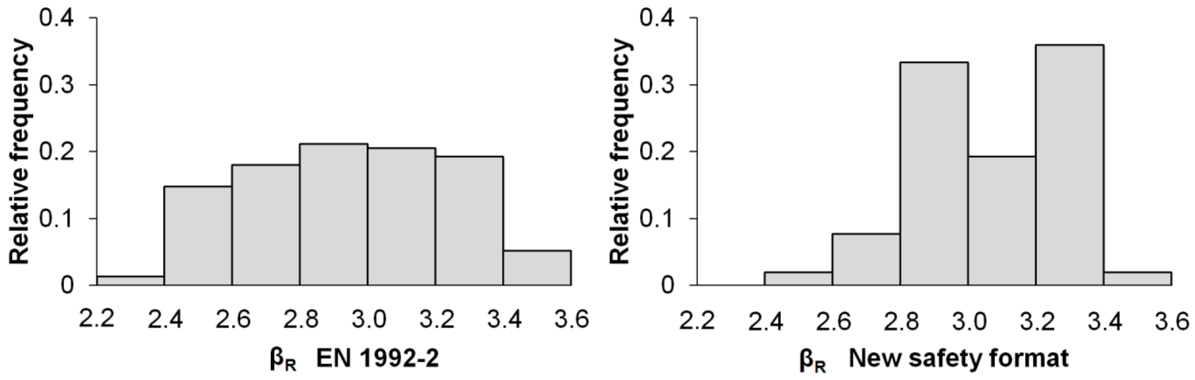

Figure 5. Reliability indices for sample structures

Discussion

According to the classic design approach, boundary conditions and other uncertain structural parameters are often assumed "on the safe side". However, for nonlinear analysis this approach leads to similar problems as using design material parameters, i.e. it cannot lead to a realistically modelled structural response. A possible remedy can be an extension of the approach adopted here to quantify the material uncertainty using a sensitivty study similar to the Gauss-approximation formula. To be better applicable to nonlinear problems, the response surface should not be linearised around the mean value using finite differences, but between the mean values and approximated design values.

Using the classic design approach, i.e. linear elastic analysis (without redistribution) followed by a check or design, of the critical section, to assure the safety on the component level, means that the load carrying capacity of the structure is limited by its most critical component. In contrast, nonlinear analysis reveals the maximum capacity of the entire modelled part. Depending on the extent of the modelled part, this can either correspond to the resistance of a single component or to the resistance of a complex structure with several possible failure modes. In addition, nonlinear analysis automatically redistributes stresses from softening elements to neighboring elements. Failure occures when no further redistribution is possbile. Hence, for nonlinear analysis, system reliability can play an important role, which has not been included in this study. The spatial variabily of the material parameters has not been included either.

Conclusions

The safety format for nonlinear analysis of reinforced concrete structures, according to EN 1992-2 (CEN, 2004b), accounts for a coefficient of variation of the modelling uncertainty of 2.5-5% and does not account for possibly biased results. However, round robin tests and modelling competitions show that coefficients of variation of the modelled resistances lie typically in the range of 5 - 30% and for some structures a systematic overestimation of the strength was found. Hence, in many cases the safety format according to EN 1992-2 (CEN, 2004b) will not lead to the intended reliability level.

Based on the observation that the modelling uncertainty will often be the main factor that will govern the safety evaluation, a new safety format has been proposed which allows one to explicitly account for the modelling uncertainty. To avoid any interaction of the modelled response with the safety format, mean *in situ* material parameters should be used in the nonlinear analysis and a, so called, resistance safety factor is used to assure the intended reliability level. The material uncertainty can be estimated based on a sensitivity study that involves two additional nonlinear analyses, one with decreased steel parameters and the other with decreased concrete parameters.

Testing of the safety format on beam sections loaded in bending showed that it results in reliability indices that are in good agreement with the target reliability. Due to the assumption of a small modelling uncertainty only marginal improvements compared to the safety format according to EN 1992-2 could be shown. However, for nonlinear analysis of more difficult to model failure modes which result in a higher modelling uncertainty mayor improvements can be expected.

Due to the importance of the modelling uncertainty and the lack of data to quantify it, more round robin tests and modelling competitions are needed.

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Paper III

Testing of Safety Formats for Nonlinear Analysis on Concrete Beams Subjected to Shear Forces and Bending Moments.

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Safety formats for nonlinear analysis tested on concrete beams subjected to shear forces and bending moments

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ABSTRACT

Safety formats for nonlinear analysis have mainly been tested on beams and columns subjected to normal forces and bending moments. Therefore, it is unclear, whether available safety formats lead to the intended reliability when they are applied to structures that fail due to shear loading. To test available safety formats for nonlinear analysis, a tool was developed which allows a full probabilistic nonlinear analysis of beam sections subjected to arbitrary combinations of normal and shear forces as well as bending moments. Applying this tool to test the safety format according to EN 1992-2 on beams subjected to a combination of shear forces and bending moments showed that EN 1992-2 led to a reliability level that was lower than the target reliability. The safety format according to Schlune et al. (submitted for publication) [1] led to better agreement with the target reliability.

1. Introduction

In the past, nonlinear analysis of concrete structures has mainly been used to analyse beams, columns or frames subjected to normal forces and bending moments. Available safety formats for nonlinear analysis, [1–5], have therefore been tested mainly on these types of structures, which have quite well understood failure modes due to yielding of the reinforcement steel or concrete compressive failure. However, nowadays nonlinear analysis is increasingly used to analyse structures which fail due to shear or torsional loading. In these cases, the concrete is subjected to a multi-axial stress-state and the structures can fail in multiple failure modes. Factors like friction in cracks and bond properties can become important, which makes it more difficult to describe the structural behaviour numerically. This implies a higher model uncertainty.

Furthermore, the capacity of the structure can be limited by the tensile strength of the concrete which has a large variability. This can lead to a large variability of the structural resistance. Therefore, it is not clear whether available safety formats for nonlinear analysis are appropriate for these types of analyses.

This article describes how the safety formats according to EN 1992-2 [2], and according to Schlune et al. [1] were tested on beam sections subjected to a combination of bending moments and shear forces. Firstly the two safety formats tested are described in Section 2. In Section 3 the development of a tool for full probabilistic nonlinear analysis of concrete sections is explained; this is followed by the application of the tool to test the safety formats. The results and conclusions are given in Sections 5 and 6.

2. Safety formats

The safety formats for nonlinear analysis of concrete structures, according to EN 1992-2 [2], and Schlune et al. [1], use the principles of the semi-probabilistic approach, i.e. fixed weight factors for the action effects and resistance are assumed. According to EN 1990 [6] the weight factor for the resistance can usually be set to $\alpha_R = 0.8$, which provides a target reliability index for the design resistance of $\beta_R = \alpha_R\beta = 3.04$ (Class RC2, ultimate limit state, reference period of 50 years). The load effects for both safety formats must be treated according to EN 1990 [6] and EN 1991 [7].

2.1. Safety format according to EN 1992-2

To apply the safety format for nonlinear analysis according to EN 1992-2, the material strengths that are to be used in the analysis, can be calculated according to

$$\tilde{f}_y = 1.1 f_{yk} \quad (1)$$

$$\tilde{f}_c = 1.1\frac{\gamma_s}{\gamma_c} f_{ck} \quad (2)$$

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where $\tilde{f}_y$ is the yield strength of the reinforcement steel, $\tilde{f}_c$ is the concrete compressive strength, both of which are used in the nonlinear analysis, f_{yk} is the characteristic yield strength, f_{ck} is the characteristic concrete compressive strength, and $\gamma_s/\gamma_c = 1.15/1.5$ is the ratio of the partial factor for steel to that of concrete. The ultimate load, $q_{ud}\{\tilde{f}_y, \tilde{f}_c, \boldsymbol{a}_{nom}\}$, obtained from the nonlinear analysis, is then divided by the safety factor, $\gamma_{O'} = 1.27$ to yield the associated design resistance as:

$$R_d = R\left(\frac{q_{ud}\{\tilde{f}_y, \tilde{f}_c, \boldsymbol{a}_{nom}\}}{\gamma_{O'}}\right) \tag{3}$$

where $\boldsymbol{a}_{nom}$ represents the nominal geometrical parameters. In EN 1992-2 it is stated that: "…non linear analysis may be used provided that the model can appropriately cover all failure modes (e.g. bending, axial force, shear, compression failure affected by reduced effective concrete strength, etc.)". In the context of the safety format for nonlinear analysis, it is further noted that: "It is assumed that the concrete tensile stresses are covered by reinforcement". This means that nonlinear analysis can be used to analyse shear failure modes, but the safety format for nonlinear analysis is not applicable to beams which fail in shear tension failure.

2.2. Safety format according to Schlune et al.

The safety format according to Schlune et al. [1] uses the mean yield strength of the reinforcement steel, f_{ym}, and the mean *in situ* concrete compressive strength, $f_{cm,is}$, as input parameters for the nonlinear analysis. The design resistance is obtained by division of the obtained resistance by a resistance safety factor, γ_R:

$$R_d = \frac{R\{f_{ym},\ f_{cm,is},\ \boldsymbol{a}_{nom}\}}{\gamma_R}. \tag{4}$$

Based on the assumption of a log-normal distributed resistance, the resistance safety factor is calculated as

$$\gamma_R = \frac{\exp(\alpha_R \beta V_R)}{\theta_m} \tag{5}$$

where θ_m is the bias factor, i.e. the mean ratio of experimental to predicted strength for the modelling approach chosen and V_R is the coefficient of variation of the resistance. It can be calculated according to

$$V_R = \sqrt{V_g^2 + V_m^2 + V_f^2} \tag{6}$$

where V_g, V_m, V_f are the coefficients of variation to account for the geometrical, the model, and material uncertainties. The remaining difficulty is to estimate the coefficients of variation V_g, V_m, V_f, and the bias factor θ_m which is described next.

2.2.1. Geometrical uncertainty

To account for the geometrical uncertainty, a coefficient of variation of $V_g = 5\%$ is usually appropriate for structures that are insensitive to geometrical imperfections. However, for slender columns and other structures that are sensitive to geometrical imperfections, it may be necessary to insert an appropriate imperfection into the nonlinear analysis.

2.2.2. Model uncertainty

There is little data available to quantify the model uncertainty of nonlinear analysis. However, based on a review of round robin exercises and modelling competitions [8–14] and on engineering judgement, the values given in Table 1 can be used as a first approximation to quantify the model uncertainty for different failure modes. The round robin exercises and modelling competitions dealt mainly with statically determinate structures, for which the resistance in one critical section is decisive. However, for statically indeterminate structures the deformation capacity to allow redistribution can become important. The deformation capacity is usually more difficult to model than the ultimate strength; this requires higher coefficients of variation than those given in Table 1. Depending on the modelling approach, the model uncertainty could be much higher. Therefore, a deliberate and thoughtful choice of the coefficient of variation is needed. It should be noted that the given values do not account for gross human errors. A more detailed quantification of the model uncertainty is currently being developed [15].

Table 1
Coefficients of variation and bias factors to account for model uncertainty.

Failure type		V_m (%)	θ_m
Compression	Normal strength concrete	10–20	0.9–1.0
	High strength concrete	20–30	1.0
Bending	Under-reinforced	5–15	1.0–1.2
	Under-reinforced, bending reinforcement not aligned in principal moment direction	5–15	0.9
	Over-reinforced, normal strength concrete	10–15	0.9–1.0
	Over-reinforced, high strength concrete	20–30	1.0
Shear	Failure do to yielding of the reinforcement	10–25	0.9–1.0
	Failure due to crushing of concrete, Combination of compression and shear loading, Large members, Bending reinforcement not aligned in principal moment direction	20–40	0.7–1.0

2.2.3. Material uncertainty

In structures for which either the yield strength of the reinforcement steel or the concrete compressive strength can cause failure, one needs to account for the material uncertainty in the coefficient of variation, V_f, and this was estimated using three nonlinear analyses: one with mean material parameters, one with reduced concrete strength and one with reduced steel strength, see [1]. However, in this study beam sections were analysed, for which the tensile capacity of the concrete could also limit the resistance. Therefore, a fourth analysis with the reduced tensile strength of the concrete is needed. The coefficient of variation to account for the material uncertainty was then estimated according to

$$V_f \approx \frac{1}{R_m}\sqrt{\left(\frac{R_m - R_{\triangle f_c}}{\triangle f_c}\right)^2 \sigma_{f_c}^2 + \left(\frac{R_m - R_{\triangle f_{ct}}}{\triangle f_{ct}}\right)^2 \sigma_{f_{ct}}^2 + \left(\frac{R_m - R_{\triangle f_y}}{\triangle f_y}\right)^2 \sigma_{f_y}^2} \tag{7}$$

where R_m, $R_{\triangle f_c}$, $R_{\triangle f_{ct}}$ and $R_{\triangle f_y}$ are the resistances from the nonlinear analysis when using mean material parameters, reduced concrete compressive strength, reduced concrete tensile strength and reduced steel strength; $\triangle f_c$, $\triangle f_{ct}$ and $\triangle f_y$ are the step sizes to decrease the material parameters; $\sigma_{f_c}^2$ is the variance of the *in situ* concrete compressive strength; $\sigma_{f_{ct}}^2$ is the variance of the *in situ* concrete tensile strength; and $\sigma_{f_y}^2$ is the variance of the yield strength of the reinforcement steel. Using the recommended step size, [1], yields the input data (Table 2) for the nonlinear analysis where V_i is the coefficient of variation of the material strength parameter, f_i.

The coefficient of variation according to Eq. (7) accounts for the variation of the concrete compressive strength and the tensile strength as a material uncertainty. Using these two strengths and other related material parameters, like Young's modulus,

Table 2
Input data for nonlinear FEA.

	Concrete compressive strength	Concrete tensile strength	Yield strength of reinforcement steel	Geometrical parameters
R_m	$f_{cm,is}$	$f_{ctm,\ is}$	f_{ym}	$\boldsymbol{a}_{nom}$
$R_{\triangle f_c}$	$f_{cm,is}\ \exp(-2.15\ V_{f_{c,is}})$	$f_{ctm,\ is}$	f_{ym}	$\boldsymbol{a}_{nom}$
$R_{\triangle f_{ct}}$	$f_{cm,is}$	$f_{ctm,is}\ \exp(-2.15\ V_{f_{ct,is}})$	f_{ym}	$\boldsymbol{a}_{nom}$
$R_{\triangle f_y}$	$f_{cm,is}$	$f_{ctm,\ is}$	Log-norm. distr. $f_{ym}\ \exp(-2.15\ V_{f_y})$ Norm. distr. $f_{ym}(1-2.15\ V_{f_y})$	$\boldsymbol{a}_{nom}$

Poisson's ratio and fracture energy, the response under multi-axial stress states is evaluated by the material model. As the variability of these parameters is not accounted for by Eq. (7) this remaining uncertainty is assumed to be part of the (material) model uncertainty.

Based on the assumption that only one of the material strengths causes failure, a less conservative approach to estimate the coefficient of variation to account for the material uncertainty can be calculated as follows:

$$V_f \approx \frac{1}{R_m} \max\left\{\frac{R_m - R_{\triangle f_c}}{\triangle f_c}\sigma_{f_c}, \frac{R_m - R_{\triangle f_{ct}}}{\triangle f_{ct}}\sigma_{f_{ct}}, \frac{R_m - R_{\triangle f_y}}{\triangle f_y}\sigma_{f_y}\right\}. \quad (8)$$

To allow a comparison between Eqs. (7) and (8), both were used in the following.

3. Full probabilistic nonlinear analysis

To test the safety formats on beam sections subjected to a combination of shear forces and bending moments, probabilistic nonlinear analyses were required. In this study the nonlinear sectional analysis program Response-2000 was used for the structural calculations. To allow for probabilistic analysis, Response-2000 was enhanced with pre- and post-processors.

3.1. Nonlinear analysis

The nonlinear sectional analysis tool Response-2000 was developed by Bentz [16]. It allows for the analysis of concrete sections subjected to arbitrary combinations of normal and shear forces and bending moments and is based on the modified compression field theory [17]. Compared to many analytical equations from current codes, Response-2000 leads to better agreement with experiments, [18]. By comparing the computed shear strength of Response-2000 with 534 beams tested, Bentz [16] reported an average ratio of experimental shear strength to predicted shear strength of $\theta_{m,Be} = 1.05$ and a coefficient of variation of $V_{m,Be} = 12\%$. Bohigas [18] compared the shear strength ratio of beams without shear reinforcement for 195 beams and reported a ratio of $\theta_{m,Boh} = 1.13$ and a coefficient of variation of $V_{m,Boh} = 20.0\%$. For 123 beams with shear reinforcement, he reported a ratio of $\theta_{m,Boh} = 1.07$ and $V_{m,Boh} = 17.4\%$.

3.2. Probabilistic analysis

Response-2000 uses a numerical solution procedure with incremental loading for the structural analysis. Hence, the limit state function was only available in implicit form and derivatives could only be approximated using difference quotients. However, Response-2000 does not lead to a perfectly smooth response surface; this is illustrated in Fig. 1 by plotting the obtained resistance of a beam with a longitudinal reinforcement ratio of $\rho_{l,s1} = 4.0\%$ and a shear reinforcement ratio of $\rho_w = 0.8\%$ over variations of the yield strength of the reinforcement steel, f_y, and the concrete compressive strength f_c. All other parameters were kept constant. This roughness is a result of the iterative solver in Response-2000 which only finds an estimate of the maximum resistance. Hence, difference quotients based on small step sizes to approximate derivatives can become misleading, and probabilistic methods which require derivative information will encounter convergence problems. The beam sections to be analysed could fail in differing failure modes, which further limited the number of appropriate methods for the probabilistic analysis. It was therefore decided to use Monte Carlo simulations. To reduce the number of required limit-state evaluations, the dynamic bound approach by Rajabalinejad [19] was enhanced here by using shifted distributions to obtain more effective bounds.

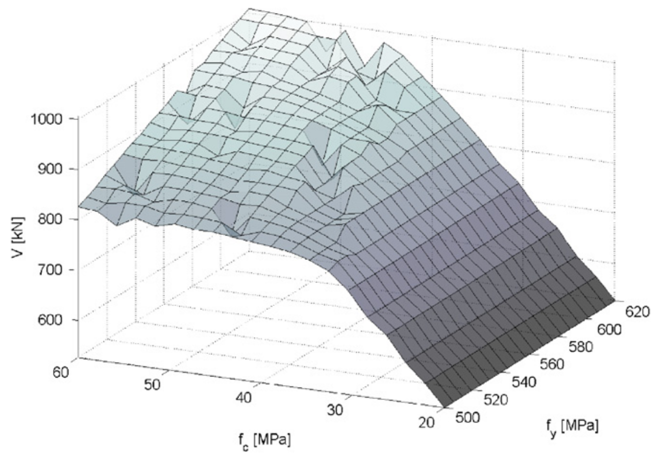

Fig. 1. Shear strength, V, over variations of concrete compressive strength, f_c, and the yield strength of the reinforcement, f_y for a beam with a longitudinal reinforcement ratio of $\rho_{l,s1} = 4.0\%$ and a shear reinforcement ratio of $\rho_w = 0.8\%$.

Rajabalinejad [19] has presented a method to reduce the number of required limit state evaluations in Monte Carlo simulations by using dynamic bounds. The method is applicable to limit state functions, $G(\boldsymbol{X})$, which are monotonic with respect to all their random variables, $\boldsymbol{X} = (\boldsymbol{X}_1, \ldots, X_n)$. The method uses prior limit state evaluations to establish bounds to divide the sample space into a stable set, $S = \{\boldsymbol{X} : G(\boldsymbol{X}) \geq 0\}$, and an unstable set, $U = \{\boldsymbol{X} : G(\boldsymbol{X}) < 0\}$. If previous limit state evaluations allow one to classify the new sample into the stable or unstable set, no limit state evaluation is required. If the sample cannot be classified, the limit state function is evaluated and the sample is used to update the bounds. According to Rajabalinejad [19] this led to a numerical cost reduction factor of 130 in a two-dimensional example and to a reduction factor of nine in four dimensions.

When implementing the approach by Rajabalinejad [19], it was found that the efficiency gain quickly became irrelevant when the number of dimensions increased. As it was intended to analyse problems with 9 dimensions, i.e. 9 random input parameters, it was decided to enhance the approach. Rather than continuously updating the bounds when sampling from the actual probability density functions, $f_{X_i}(x)$, of the random variable X_i,

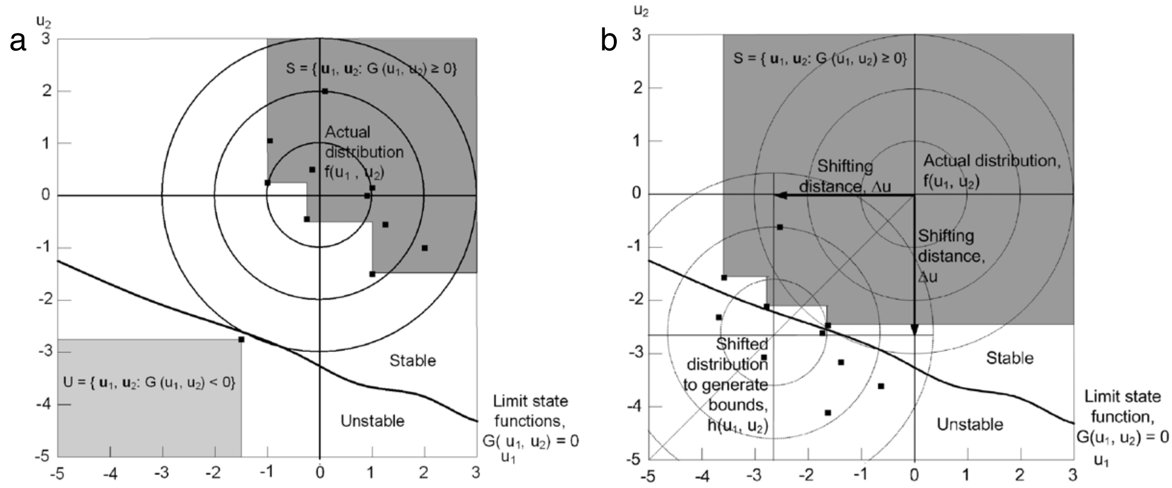

Fig. 2. Bounds according to Rajabalinejad [19] (a) and the approach used here (b).

some importance sampling density functions, $h_{X_i}(x)$, were used to construct the bounds. A sample, $\hat{x}$, of $h_{X_i}(x)$ was generated by first generating a sample, $\hat{u}$, of the standard normal distribution. In the next step the sample was shifted by a constant distance, Δu, closer to the limit state, that is $\hat{u}_s = \hat{u} + \Delta u$ for variables in which the limit state function is monotonically decreasing and $\hat{u}_s = \hat{u} - \Delta u$ for variables in which the limit state function is increasing. Finally, the sample was calculated according to $\hat{x} = F_{X_i}^{-1}(\Phi(\hat{u}_s))$ where F_{X_i} is the distribution function and Φ is the standardised distribution function. Furthermore, for small failure probabilities, which were of interest in this study, hardly any computational benefits are gained by using bounds for the unstable set. Therefore, only the stable set was used here.

In Fig. 2 the Rajabalinejad approach to constructing the bounds [19] and the approach used here are illustrated by a two-dimensional problem in the standard normal space. Eleven and ten samples, respectively, were used to construct the bounds. The grey areas mark the samples for which no limit state evaluations are needed. It can be seen that the approach used here leads to more efficient bounds. The extra computational effort to construct the bounds from the importance sampling density, instead of the actual probability density, is usually quickly compensated.

4. Testing of the safety formats

4.1. Procedure

To test the safety formats, the flow chart shown in Fig. 3 was applied. First, the safety formats were used to calculate design resistances for the structures studied according to both tested safety formats. To apply the safety format according to Schlune et al. [1] a coefficient of variation of $V_g = 5\%$ was assumed to account for the geometrical uncertainty and Eqs. (7) and (8) were used to estimate V_f. The coefficients of variation, $V_{m,Be}$ and $V_{m,Boh}$, and the bias factors, $\theta_{m,Be}$ and $\theta_{m,Boh}$, to account for the model uncertainty according to Section 3.1 were used.

Then 3000–4000 samples from the importance sampling density, $h(\boldsymbol{x})$, were taken to construct the bounds. For the cases analysed here the shifting distances were in the range of $\Delta u = 1.25$–1.55 so that approximately 50% of the samples from $h(\boldsymbol{x})$ were stable.

In a next step 300,000 samples were generated from $f(\boldsymbol{x})$ to estimate the probability $P(R < R_d)$ using the limit state function

$$G(\boldsymbol{X}) = \theta_i R(f_c, f_{ct}, f_y, f_u, \varepsilon_u, b, h, c_{nom}) - R_d \quad (9)$$

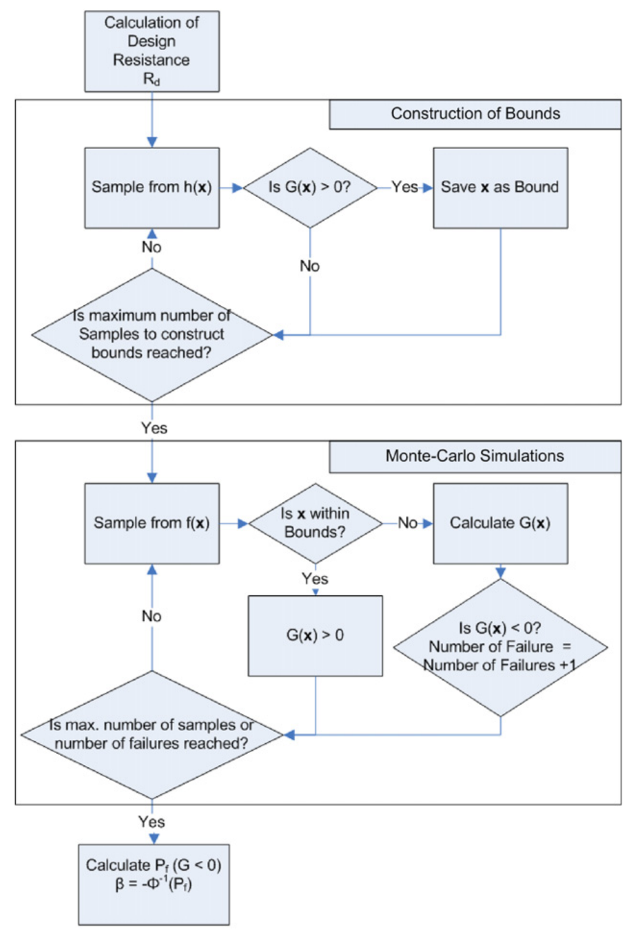

Fig. 3. Flow chart for testing of the safety formats.

where f_u is the ultimate strength of the reinforcement steel, ε_u is the ultimate strain of the reinforcement steel, b is the width of the beam, h is the height of the beam and c_{nom} is the concrete cover at the bottom of the beam. The bounds were used to reduce the number of limit state evaluations. For the 9-dimensional problem, a computation cost reduction by a factor of 5–55 was obtained. To assure that the bounds did not falsify the failure probability, selected structures were analysed using bounds and a function evaluation for each sample. No difference in failure probability was found despite the rough response surfaces. Finally,

Table 3
Stochastic model. Distribution types: N = normal; LN = log-normal.

		Symbol	Distribution	Mean value	Dimension	Cov (%)
Materials						
C25	Compressive strength	f_c		26.9	MPa	16.2
	Tensile strength	f_{ct}		2.7	MPa	32.1
C45	Compressive strength	f_c		36.7	MPa	10.6
	Tensile strength	f_{ct}		3.3	MPa	31.0
Steel	Yield strength	f_y	N	560.0	MPa	5.4
	Ultimate strength	f_u	N	604.8	MPa	6.6
	Rupture strain	ε_u	N	50.0	‰	9.0
Geometrical data						
Width		b	LN	200	mm	2.6
Depth		h	LN	600	mm	1.3
Concrete cover, lower reinforcement		c	LN	50	mm	20.0
Model uncertainty						
Bentz	All beam sections	$\theta_{m,Be}$, $V_{m,Be}$	LN	1.05	–	12.0
Bohigas	Without shear reinforcement	$\theta_{m,Boh}$, $V_{m,Boh}$	LN	1.13	–	20.0
Bohigas	With shear reinforcement	$\theta_{m,Boh}$, $V_{m,Boh}$	LN	1.07	–	17.4

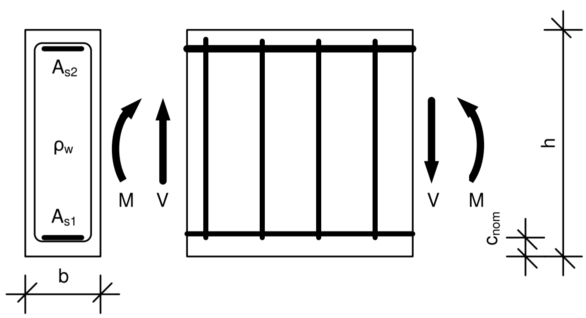

Fig. 4. Beam section.

the corresponding reliability index was calculated, $\beta = -\Phi^{-1}(P)$, where $\Phi()$ is the standard normal distribution function.

4.2. Geometric and Stochastic models

The beam section that was analysed is shown in Fig. 4. The stochastic model of the variables defining the section is given in Table 3. The ratio of the bending moment, M, in kN m, and the shear force, V, in kN, was set to one. This ratio corresponds to the section at the mid-support of a beam, with two equal spans of $l = 5$ m, which is loaded with an equally distributed load. The top reinforcement of $A_{s2} = 440$ mm^2 corresponds to a reinforcement ratio of $\rho_{l,s2} = 0.4\%$, which was assumed to be constant for all structures. The bottom reinforcement amount was varied in three steps, $\rho_{l,s1} = 0.2\%,\ 1.0\%,\ 4.0\%$; two concrete compressive classes C25 and C45 were tested.

For each of the six resulting combinations, five shear reinforcement amounts, $\rho_w = 0\%$–1.2%, were analysed. This made a total of 30 beam sections on which the safety formats were tested. Due to the changes in the reinforcement layout, the beam resistance was limited by: the strength of the shear or bending reinforcement, the concrete compressive strength at the top of the beam or in the shear compressive struts, by the tensile strength of the concrete.

The stress–strain relationship for concrete in compression was modelled by the default equation in Response-2000 according to Collins and Mitchell [20]. Tension stiffening according to Bentz [16] was included which resulted in the stress–strain diagram shown in Fig. 5(a). Compression softening due to transverse tensile strains was modelled according to Vecchio and Collins [17]. The reinforcement steel was assumed to behave linearly until the yield point. Then a constant stress was assumed until strain hardening started, followed by a quadratic curve until the maximum stress was reached, see Fig. 5(b).

The stochastic models of the concrete compressive strength, f_c, and the tensile strength, f_{ct}, were assumed according to the JCSS Probabilistic Model Code, [8]. The strain at peak stress was calculated according to EN 1992-1-1 [21] to $\varepsilon_{c1} = 0.7 f_c^{0.31}$. All other concrete parameters, such as the maximum aggregate size, $max_{as} = 16$ mm, and the tension stiffening factor, $t_{sf} = 1.0$, were set to deterministic values. For reinforcement steel the stochastic models of the yield strength, f_y, the ultimate strength, f_u, and the rupture strain, ε_u, were assumed according to the JCSS Probabilistic

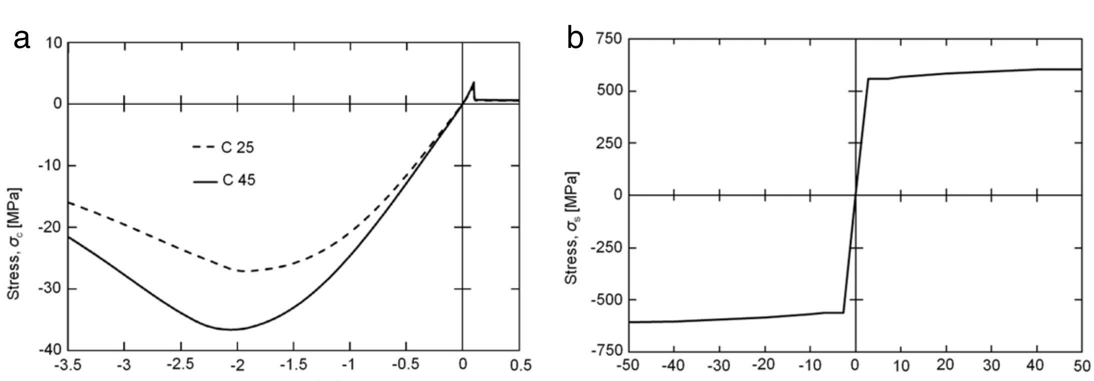

Fig. 5. Stress–strain diagram for concrete and reinforcement steel.

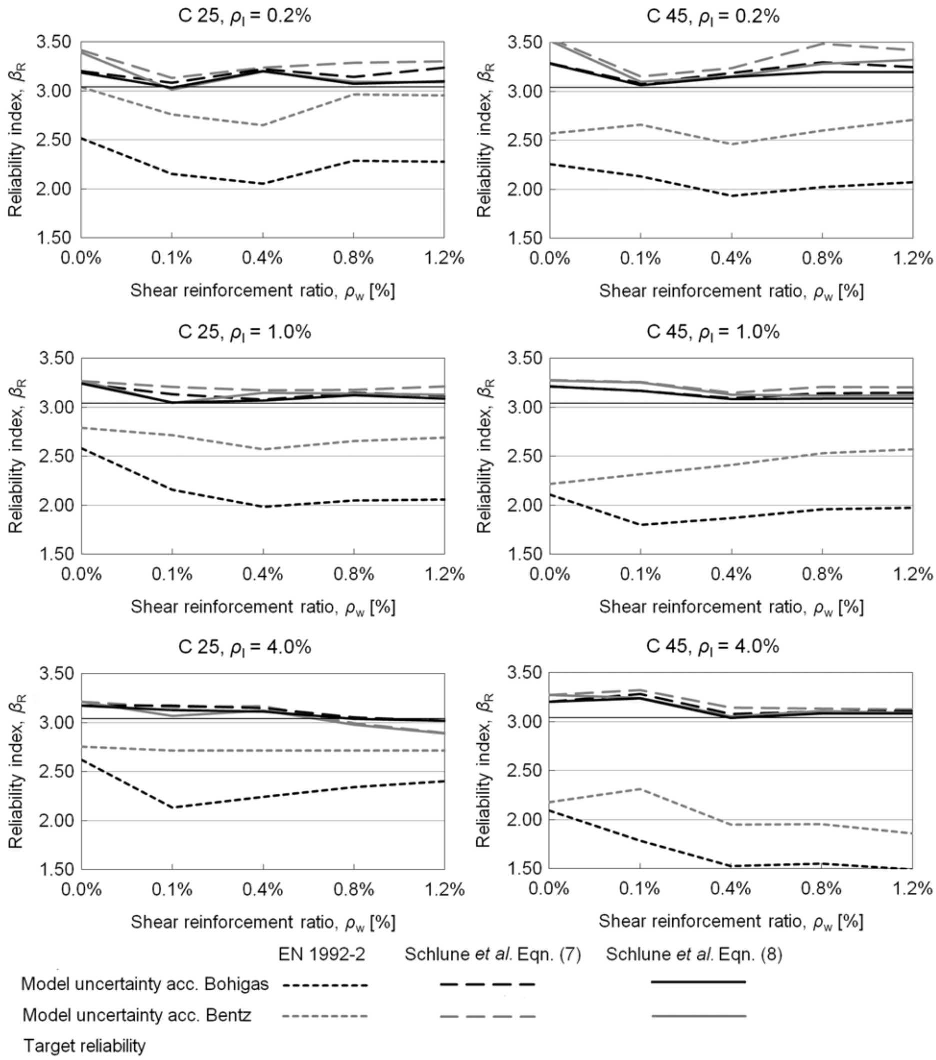

Fig. 6. Reliability indexes for selected beam configurations.

Model Code. The Elastic Modulus was set to the deterministic value of $E_s = 205$ GPa. A mean yield strength, $f_{ym} = 560$ MPa, and a mean ultimate strength of $f_{um} = 1.08 * f_{ym}$ were assumed. The strain at onset of strain hardening was set to the deterministic default value of $\varepsilon_{sh} = 7‰$. For the longitudinal and transversal crack spacing, no values where specified, so that Response-2000 calculated them automatically. The stochastic models for the material and geometrical variables and for the factors to account for the model uncertainty are summarised in Table 3.

5. Results

Fig. 6 shows the reliability indexes obtained for all structures. It can be seen that the safety format according to EN 1992-2 [2] generally led to design resistances below the target reliability. This is apparent especially when the higher model uncertainty according to Bohigas was assumed. The reliability indexes for the beam sections without shear reinforcement, which failed due to tensile failure of the concrete, were not lower than for the other beam sections. This indicates that the too low reliability indexes are caused primarily by the model uncertainty which is not properly accounted for in the safety format according to EN 1992-2.

The safety format according to Schlune et al. [1] showed much better agreement with the target reliability but tended to give overly conservative results, see Table 4. The better agreement is mainly as a result of accounting for a higher model uncertainty. The difference between using Eq. (7) or Eq. (8) to quantify the material uncertainty is negligible.

6. Conclusions

To test safety formats for the nonlinear analysis of concrete structures, a tool for full probabilistic nonlinear analysis of reinforced concrete beams sections was developed. The tool allowed the analysis of beam sections subjected to arbitrary combinations of normal and shear forces and bending moment. This was done by coupling Monte Carlo simulations with Response-2000. To reduce the number of limit-state evaluations required the dynamic bounds approach by Rajabalinejad [19] was enhanced with importance sampling to construct the bounds.

When the tool was used to test the safety format for nonlinear analysis according EN 1992-2, it was found that EN 1992-2 led to design resistances below the intended target reliability. This was despite a nonlinear analysis toolbox that tends to yield conservative results; and compared with other nonlinear analysis approaches, it has a relatively low model uncertainty. Consequently, the safety format for nonlinear analysis according to EN 1992-2 is not appropriate for structures that fail in shear, unless a very small model uncertainty can be guaranteed.

The safety format according to Schlune et al. [1] offers better agreement with the target reliability by accounting more

Table 4
Summary of reliability indexes obtained.

Model uncertainty according to:	Safety format according to:	Mean reliability index, $\overline{\beta}_R$	Coefficient of variation, V_{β_R} (%)	Minimum reliability index, $\beta_{R,\ min}$	Maximum reliability index, $\beta_{R,\ max}$
Bohigas	EN 1992-2	2.09	13.5	1.49	2.62
	Schlune et al., Eq. (7)	3.16	2.3	3.02	3.30
	Schlune et al., Eq. (8)	3.13	2.3	3.02	3.28
Bentz	EN 1992-2	2.56	11.6	1.86	3.04
	Schlune et al., Eq. (7)	3.23	4.0	2.89	3.54
	Schlune et al., Eq. (8)	3.16	4.0	2.89	3.51
Target reliability		3.04			

accurately for the model uncertainty. However, to apply the safety format, a good approximation of the model uncertainty must be available.

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Paper IV

Safety verification on the structural level using nonlinear finite element analysis: Application to a concrete bridge,

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