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Variation Control in Virtual Product Realization
- A Statistical Approach

KRISTINA WÄRMEFJORD

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VARIATION CONTROL IN VIRTUAL PRODUCT REALIZATION
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ABSTRACT

All manufacturing processes are afflicted by variation, which leads to products that may not fulfill assembly, functional or esthetical requirements. The variation in the final product originates from variation in individual parts and assembly processes. In this work, methods and tools aimed to reduce the amount of variation and, more importantly, the effects of the variation in the final product are treated.

Four main areas are treated in this thesis:

- **Statistical process control:** When a deviation or increased variation arises in a process, it is important to discover this as soon as possible. Different methods for statistical process control, based on inspection data, are suggested and compared. When an increased variation or deviation is obtained, it is also important to find the cause. Methods for the diagnosis of assembly fixtures are suggested, compared and evaluated.
- **Inspection:** In the verification phase of product development, a product is normally inspected at a large number of points in order to learn as much as possible about the product and processes. When full production starts, the main purpose of inspection shifts toward statistical process control, which normally requires fewer inspection points. Today, the reduction in demand is usually done manually. This puts great demands on experience and craftsmanship. A method based on cluster analysis is presented and applied to inspection data in order to achieve a systematic reduction. By the clustering of the data, a representative for each cluster or group of points with correlated inspection values can be selected. By inspecting only those representatives, the number of points needed to monitor the product can be reduced. This method leads to larger reductions than manual procedures. The information in a set of inspection points is quantified.
- **Spot welding sequence optimization:** By altering the joining sequence when joining two parts together, using, for example, spot welding, the level of variation in the final assembly is affected. Different strategies to find optimal spot welding sequences (in other words, sequences that minimize the variation in the final assembly) are suggested and compared.
- **Simulation accuracy:** Variation simulation is used to predict variation in the final product or in sub-assemblies. In that manner, the requested number of physical verifications and tests can be reduced, leading to saved resources and increased sustainability. However, if the simulation is to be able to replace physical tests, it is vitally important that the simulation results are accurate and agree with real outcome. Factors affecting this agreement for non-rigid variation simulation are investigated. Four of the factors (spot welding sequence, contact modeling, fixture repeatability and the influence of heat during welding) are further examined and methods to include them in variation simulation are suggested

Keywords: Variation simulation, quality control, inspection point reduction, contact modeling, spot welding sequence, fixture repeatability, welding simulation.

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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CHALMERS UNIVERSITY OF TECHNOLOGY
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APPENDED PAPERS

PAPER A

Wärmefjord, K., 2004, “Multivariate Quality Control and Diagnosis of Sources of Variation in Assembled Products”; Licentiate thesis, Department of Mathematics, Chalmers University of Technology and Göteborg University, ISSN 0347-2809.

PAPER B

Wärmefjord, K., Carlson, J., Söderberg, R., 2010, “An Investigation of the Effect of Sample Size on Geometrical Inspection Point Reduction Using Cluster Analysis”, CIRP Journal of Manufacturing Science and Technology, vol. 3 pp. 227–235, doi: http://dx.doi.org/10.1016/j.cirpj.2010.12.001

PAPER C

Wärmefjord, K., Carlson, J., Söderberg, R., 2009, “A Measure of the Information Loss for Inspection Point Reduction”, Journal of Manufacturing Science and Engineering, 131 (5), doi:10.1115/1.4000105.

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PAPER D

Wärmefjord, K., Lindkvist, L., Söderberg, R., 2008, “Tolerance Simulation of Compliant Sheet Metal Assemblies Using Automatic Node-Based Contact Detection”, Proceedings of ASME International Mechanical Engineering Congress & Exposition, October 31-November 6, 2008, Boston, USA .

PAPER E

Wärmefjord, K., Söderberg, R., Lindkvist, L., 2010, “Strategies for Optimization of Spot Welding Sequence with Respect to Geometrical Variation in Sheet Metal Assemblies”, Proceedings of ASME International Mechanical Engineering Congress & Exposition, November 12-18, 2010, Vancouver, Canada.

Also submitted to ASME Journal of Manufacturing Science and Engineering.

PAPER F

Wärmefjord, K., Söderberg, R., Lindkvist, L., 2010, “Variation Simulation of Spot Welding Sequence for Sheet Metal Assemblies”, Proceedings of NordDesign2010 International Conference on Methods and Tools for Product and Production Development, August 25-27, 2010, Gothenburg, Sweden. ISBN 978-91-633-7064-9.

PAPER G

Pahkamaa, A., Wärmefjord, K., Karlsson, L., Söderberg, R., Goldak, J., “Combining Variation Simulation With Welding Simulation For Prediction of Deformation”, Proceedings of ASME International Mechanical Engineering Congress & Exposition, November 12-18, 2010, Vancouver, Canada.

Also submitted to ASME Journal of Computing and Information Science in Engineering.

PAPER H

Wärmefjord, K., Söderberg, R., Carlson, J., “Including Assembly Fixture Repeatability in Variation Simulation”, Proceedings of ASME International Mechanical Engineering Congress & Exposition, November 12-18, 2010, Vancouver, Canada.

DISTRIBUTION OF WORK

Paper A: This licentiate thesis was written by Wärmefjord under the supervision of Johan Carlson.

Paper B: Wärmefjord, Carlson and Söderberg initiated the idea, and Wärmefjord carried out the investigations, with some guidance from Carlson. Wärmefjord wrote the paper, with Söderberg acting as a reviewer.

Paper C: Wärmefjord, Carlson and Söderberg initiated the idea, and Wärmefjord carried out the investigations, with some guidance from Carlson. Wärmefjord wrote the paper, with Söderberg acting as a reviewer.

Paper D: Wärmefjord, Lindkvist and Söderberg initiated the idea, Lindkvist did the programming, and Wärmefjord carried out the investigations. Wärmefjord wrote the paper, with Söderberg acting as a reviewer.

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ADDITIONAL PUBLICATIONS

Wärmefjord, K., Carlson, J., Söderberg, R., 2007, “Geometrical inspection point reduction for rigid and non-rigid parts using cluster analysis – an industrial verification”, 10th CIRP Conference on Computer Aided Tolerancing, Specification and Verification for Assemblies, March 21-23, 2007, Erlangen, Germany.

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1. INTRODUCTION

This chapter describes the background of the research and introduces the basic concepts within the research area. The goal is to provide an understanding of the research questions.

1.1. BACKGROUND

All manufacturing processes are afflicted by variation. Therefore, no final product will look the same from time to time, and in some cases, functional or esthetical requirement will not be fulfilled. This is of course associated with costs and/or disappointed customers. Completely avoid variation is difficult, if even possible, and extremely expensive. But there are methods and tools aimed at reducing the amount of variation and, more importantly, the effect of the variation. In this thesis, such methods and tools are treated. Most of the examples come from automotive industry, but the methods are of course applicable to other types of products as well.

The variation that affects the final result consists of geometrical variation in the parts included in the assembly and variation in the assembly process. The geometrical variation in the parts shows itself as variation in form and size. The assembly process contributes variation due to variation in the contact between parts and assembly fixtures, variation in welding guns, et cetera.

To reduce variation and its effect, it is important to use all disposable methods and tools, described later in this thesis, in all three phases of the product realization loop, seen in Figure 1. This product realization loop describes, somewhat simplified, the different phases of, for example, the development of a new product in automotive industry.

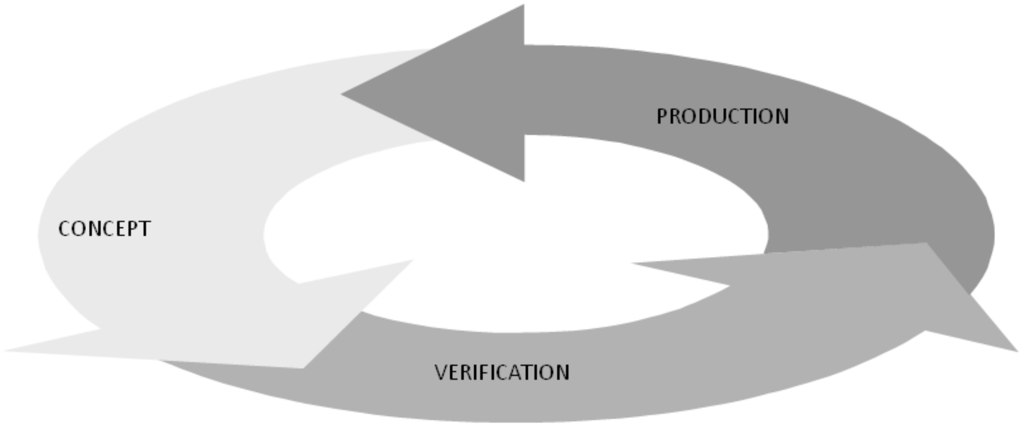


FIGURE 1: THE PRODUCT REALIZATION LOOP

In the concept phase, new design solutions and different concepts are suggested and compared using virtual methods. In the verification phase, a reduced number of suggested concepts are verified using physical prototypes and test series.

In the production phase, the full production starts. Now it is important to monitor the process in order to quickly discover disturbances and increased variation. Another important aspect here is to gather data, and thereby also information about the product and process, in order to be able to transfer this knowledge to future product development processes. Questions related to this topic are how often to measure, what inspection points to use and how to use the inspection data in an effective way.

The research carried out in this thesis is a part of the research conducted by the research group for “Geometry Assurance and Robust Design” at the Wingquist Laboratory at Chalmers University of Technology in Gothenburg, Sweden. This research group deals with methods for minimizing the effect of geometrical variation in assembled products, and there are research activities in all phases of the product realization cycle. The Wingquist Laboratory is an internationally competitive competence center for multi-disciplinary research within the field of efficient product realization.

1.2. GEOMETRY ASSURANCE AND ROBUST DESIGN

The work of aiming to reduce the geometrical variation and its effects are often referred to as geometry assurance. The most important phase from a geometry assurance point of view is perhaps the concept phase. There, it is very important to choose a good concept, which is a concept that is robust (in other words, insensitive to variation). A robust concept suppresses variation in the input parameters while a concept sensitive to variation amplifies the incoming variation instead. The robustness of the concept is to some extent determined by the shape of the included parts and the parts' other characteristics, but most important are the locating schemes. The locating schemes can be optimized with respect to robustness (Lööf et al., 2009), but there are of course also a number of different practical factors that affect the choice of locating schemes.

1.3. LOCATING SCHEMES

A positioning system or locating scheme is a way to fixate parts that will be assembled or inspected in space. A rigid part has six degrees of freedom; three translations and three rotations. For each rigid part, six points are used to position it. Those points are called locators and are realized in the fixture by pins with corresponding holes or slots in the parts or with clamps.

The principles of the so called 3-2-1 locating system are showed in Figure 2. The primary points A1, A2 and A3 define a plane and lock the geometry in space in two rotations (RX and RY) and one translation (TZ). The secondary points, B1 and B2, define a line and lock the geometry in space in one rotation (RZ) and one translation (TY). The tertiary point C1 locks the geometry in space in one translation (TX).

If the part is non-rigid (compliant), additional points can be added to support the part and to avoid a deformation of the part due to gravity or other forces during assembly. Those points are called support points and can be of an arbitrary number.

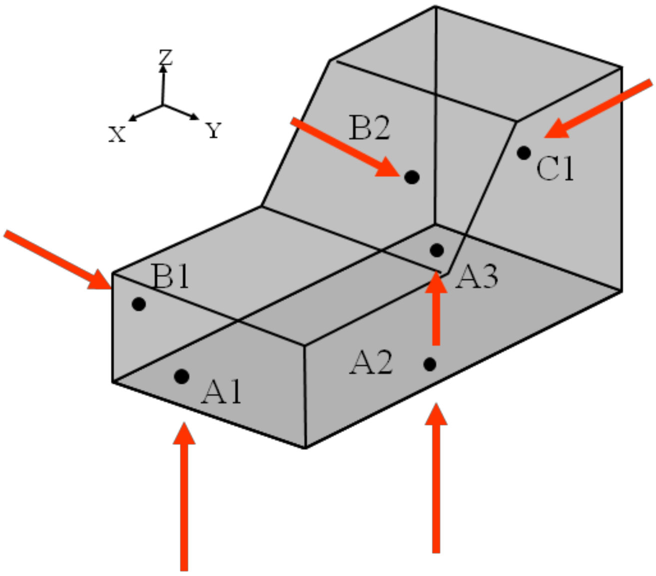


FIGURE 2: THE 3-2-1 LOCATING SCHEME.

The locating schemes are realized by the use of fixtures. In Figure 3, a part, a front fender, is positioned in a fixture during inspection with a coordinate measuring machine (CMM). The locating schemes will be described in detail later on in the thesis.

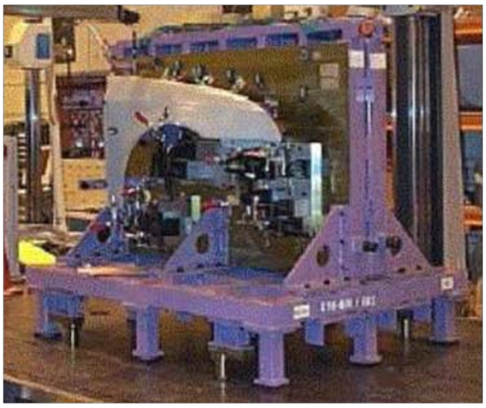


FIGURE 3: POSITIONING OF A FRONT FENDER IN AN INSPECTION FIXTURE.

1.4. VARIATION SIMULATION

The idea of variation simulation is to be able to predict the geometrical outcome of an assembled product or sub-assembly as early on as in the concept phase of the product realization cycle. Variation and offset, i.e. deviation from nominal values, are predicted in a number of important key characteristics or critical dimensions.

The inputs to a variation simulation consist of, among other things, digital models of the parts that are to be assembled, in the form of meshes for non-rigid parts. Further, information about locating schemes for the parts and also their tolerances (i.e., information about what variation can be expected for an individual part) is necessary. If there are any other factors influencing the final result, those should of course also be included. This problem is two-sided: first, those factors should be identified; and second, they should be implemented and included in the simulation model. Good accuracy in variation simulation is necessary if the simulations shall be able to replace physical testing and prototypes. The use of simulations does more than shorten lead times and effect the economics. It is also important from a sustainable point of view. Accurate simulation results lead not only to the increased use of virtual tools, but also to reduced risks of misjudgments and thereby also a reduced scrap rate. A reduced scrap rate will benefit sustainability, with respect to both economical and ecological aspects. Also, the social sustainability is gained by the increased use of virtual tools, since this usually implies improved working conditions.

1.5. RESEARCH QUESTIONS

The research conducted can mainly be grouped under four general research questions:

Research Question 1:
How can statistical methods be used to control variation in production?

When gathering data about the product and production process, it is important to be able to make the most of this data and to quickly discover disturbances in the processes. Further, the methods should be as easy to use as possible and give a good overview of the process. Under this research question, different methods for root cause analysis (in other words, the analysis of inspection data in order to find the reasons of increased variation and offsets) are also treated. This is addressed in Paper A.

Research Question 2:
How can statistical methods be used to reduce the need for inspection?

Inspection is a necessary part of geometry assurance, and the activity should not be eliminated. But it is a costly procedure and with too much data, there is also a risk that it is difficult to get a clear picture of the product/process in question. Therefore, it is important to find methods that can reduce the number of inspection points without losing too much of the information. This is addressed in Papers B and C.

Research Question 3:
How can variation due to joining sequence be reduced?

The geometry assurance process aims to reduce the variation and deviation in the final assembly or product. It can be shown that the joining sequence affects the final outcome. Therefore, it is important not only to include this factor in simulations aiming to predict the final variation but also to develop methods to find sequences that reduce the variation in the final assembly. This is addressed in Papers E and F.

Research Question 4:
How can the accuracy of non-rigid variation simulation be improved?

The accuracy of the variation simulation is vitally important for its ability to replace physical prototypes and test series. An increased accuracy in the variation simulation gives a better basis for the decisions that must be made regarding concepts, tolerances and so on in the early phases of the product realization cycle. Generally, the accuracy is improved by modeling the reality in a better way. This is addressed in Papers D, G and H.

1.6. OUTLINE OF THE THESIS

The work that make up this thesis can be divided into three parts and has been somewhat dispersed in time.

The first part of the work, aiming to answer the first research question, deals with methods for statistical process control and root cause analysis, where the latter one is a method to find causes of geometrical variation in the final product. This work, which constitutes a licentiate thesis, was conducted at the Department of Mathematical Science at the University of Gothenburg and was done in cooperation with Saab Automobile AB.

The second part, which mainly deals with how to reduce the number of inspection points without losing too much information and how to quantify the term information, was done within a MERA-project (Manufacturing Engineering Research Area). This project was a cooperative effort between the Department of Product and Production Development at Chalmers University of Technology and Volvo Car Corporation.

The third part was partly done within the MERA-project mentioned above and partly executed at the same Department of Product and Production Development. This part deals with methods to predict and reduce geometrical variation due to joining strategies and with how to improve the accuracy of the variation simulation.

The outline of the thesis is as follows. In this first section, a brief background and introduction to the research area is given. The research questions and limitations are also described. Section 2 is the frame of reference aiming to give an overview of previous work and knowledge within the research area. The research methodology used in this work is presented in Section 3, and the results are summarized in Section 4. In Section 5, the verification and validation of the results are discussed. Finally, conclusions and thoughts about future work are found in Section 6.

1.7. DELIMITATIONS

Since the parts constituting this thesis have been dispersed in time and cover different areas, there are different limitations for different parts of the thesis. Therefore, the delimitations differ for the different research questions.

- *RQ1: How can statistical methods be used to control variation in production?*

  The work grouped under RQ1 is based on the assumptions that all parts are rigid bodies.

- *RQ2: How can statistical methods be used to reduce the need for inspection?*

  The cluster reduction method is based on the repeated measurements of discrete points, and those points must be evaluated in the same way from time to time.

- *RQ3: How can variation due to joining sequence be reduced?*

  The heat effect from spot welding is neglected. This effect is assumed to only have local influence on the assembly. Further, the material models are linear, all forces are applied to nodes and it is assumed that the changes in the stiffness matrix due to part and positioning variation can be neglected.

- *RQ4: How can the accuracy in non-rigid variation simulation be improved?*

There are a lot of factors affecting the accuracy of a variation simulation. Not all of them are covered here. An attempt to identify the most important ones is done, and methods to include such factors in variation simulation are suggested for three of the factors.

2. FRAME OF REFERENCE

In this chapter, an overview of previous work and knowledge within the research area in question is given.

The outline and content of this chapter follow roughly the product realization loop, shown in Figure 4.

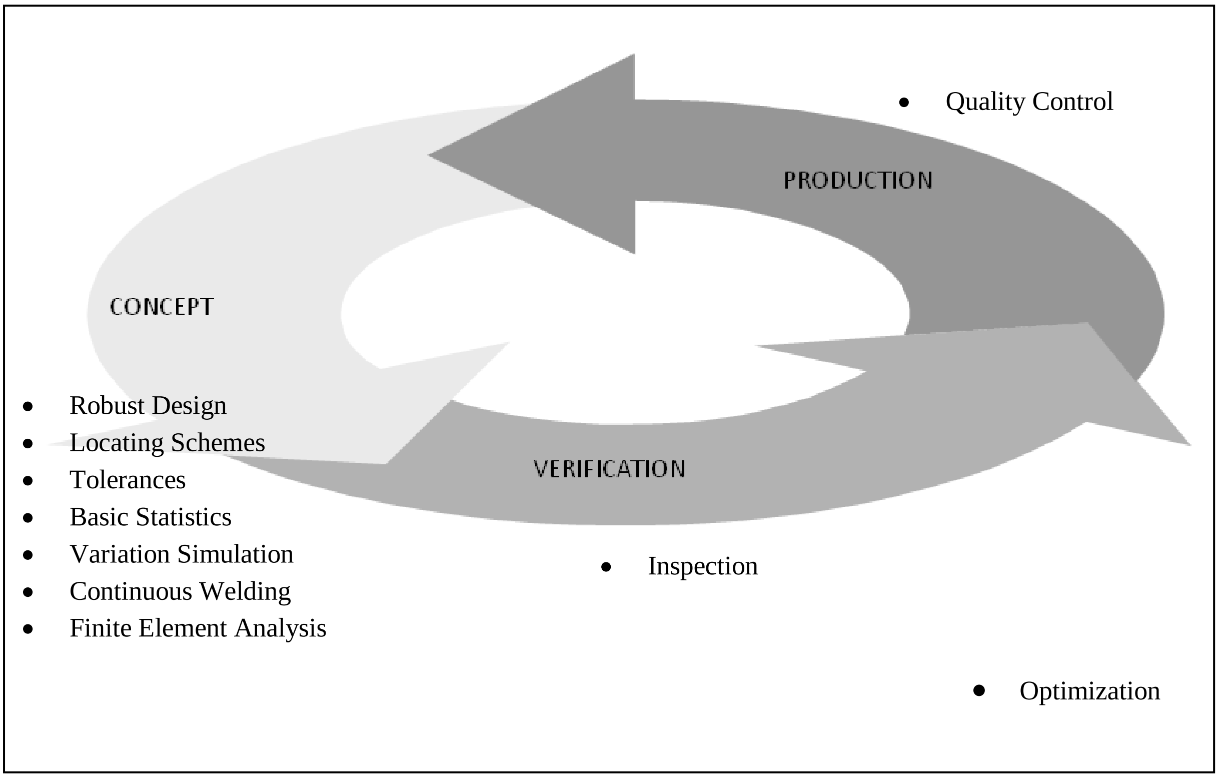


FIGURE 4: THE OUTLINE OF THE FRAME OF REFERENCE IN RELATION TO THE PRODUCT REALIZATION LOOP.

At the concept phase, a number of different concepts are evaluated and compared. Therefore, the frame of reference starts with an overview of the idea of **robust design**. **Locating schemes** turn out to be an important control factor for a robust design and are treated next. Another main input to a variation simulation, namely the **tolerances**, is also considered. When discussing tolerances and variation simulation, a number of statistical terms and concepts are used. Therefore, a brief overview of some **basic statistics** is presented. Given tolerances and locating schemes, the **variation simulation** can be treated. The joining method considered in this work is mainly spot welding. However, since **continuous welding** is also treated in one of the papers, a short introduction to this topic is given. In non-rigid variation simulation, **finite element analysis (FEA)** frequently occurs. That is why on overview of the method is included.

During the verification phase and to some extent in the full production phase, **inspection** is in focus. In the production phase, it is important to check the processes to detect disturbances. This is why **quality control** is discussed.

Finally, there is also a short section about **optimization** in the frame of references, since one of the research question deals with optimization-related issues.

2.1. ROBUST DESIGN

As mentioned in the previous section, a robust design is a design insensitive to variation. The word robust is used in many contexts to describe products and processes insensitive to various kinds of disturbances. In this work, the focus of the robust design is to reduce the sensitivity of a concept to manufacturing variation. The ideas of robust design and quality improvement, however, were originally introduced by Taguchi (1986). The factors affecting a concept are divided into control factors, which are easy to control, and noise factors, which are hard to control. Taguchi's idea is to choose the levels of the control factors in such a way that the expected loss caused by noise factors is minimized. If those principles are applied to geometry assurance, the control factor equals, somewhat simplified, the locating schemes, while the noise factor is the variation in parts (Söderberg, 1998); (Söderberg & Lindkvist, 1999).

All manufacturing processes are afflicted by variation. Therefore, the locating schemes should be chosen in such a way that the design concept is robust. The robustness can be evaluated using a variation simulation tool (Söderberg & Lindkvist, 1999), where different concepts can be compared and iteratively improved. It is also possible to optimize the locating schemes with respect to robustness in critical product dimensions (Cai, 2006); (Lööf et al., 2009) or with respect to the assembly as a whole (Wang, 1999); (Wang & Pelinescu, 2001).

2.2. LOCATING SCHEMES

The idea of location schemes or positioning systems was introduced in the previous chapter, but will be described in more detail here, since it is a central concept in geometry assurance. As previously mentioned, a locating scheme is used to position and support parts during manufacturing operations or inspection. Since a rigid part has six degrees of freedom (three translations (TX, TY, TZ) and three rotations (RX, RY, RZ)), six points are used to lock those translations and rotations. In the physical realization of the locating scheme, i.e. in the fixture, some of those points can coincide, but a minimum of three locators is always necessary.

Jigs are similar to fixtures, but are used to guide cutting tools during boring and drilling operation (Nee et al., 2004).

If there are deviations or variations in the positioning of parts during the joining process, this will lead to errors in the final assembly. Therefore, the robustness of the locating scheme is very important in order to achieve a good geometrical outcome of the final product (Söderberg & Lindkvist, 1999). The variation in positioning is related to the repeatability of the fixture (in other words, with what precision one detail repeatedly can be positioned in a fixture). To examine the repeatability of the fixtures, repeatability studies are made on a regular basis.

Payne and Cariapa (2000) investigate the repeatability of machining fixtures by using a measure to quantify the variability contributed by the fixture in a number of inspection points on a part. Wärmefjord et al. (2010) describe a method for transforming the variations in inspection data into variations in the contacts between workpiece and locators.

Another source of inaccuracy of the final assembly is the non-simultaneous application of clamping forces on the workpiece (Chang et al., 1997).

The design of the fixture affects not only the geometrical outcome of an assembly, but also the production efficiency (Nee et al., 2004). To decrease the manufacturing cycle time, it is important to keep the time for loading and unloading the fixture to a minimum.

Söderberg et al. (2006) describe different kinds of locating schemes. The locating scheme can be orthogonal or non-orthogonal. For an orthogonal system, the locating directions are orthogonal to each other.

- 3-2-1 locating scheme (orthogonal):

  Three points A1, A2 and A3 define the primary locating plane and lock TZ, RX and RY. The second locating plane is defined by the points B1 and B2, locking TX and RZ, while the last point, C, defines the last locating plane and locks TY. All planes are perpendicular to each other. This kind of locating scheme is shown in the left part of Figure 5.

- 3-point locating scheme (orthogonal):

  This is a special case of the 3-2-1 locating scheme using only three different points in the locating scheme, since A1=B1=C and A2=B2. This locating scheme is shown in the right part of Figure 5.

- 3 directions locating scheme (non-orthogonal):

  The points A1, A2 and A3 define the primary locating direction. Those points are not necessarily located in the same plane on the workpiece. The second locating direction is defined by the points B1 and B2. This direction may be non-orthogonal to the A direction. The third locating direction is defined by C1, and the direction C may be orthogonal to the directions A and/or B. This kind of locating scheme is shown in the left part of Figure 6.

- 6 direction locating scheme (non-orthogonal):

  All six points, D1-D6, define the locating directions, perpendicular to the locating surfaces of the parts. This kind of locating scheme is shown in the right part of Figure 6.

For non-rigid parts, the purpose of the locating system is not only to fixate the workpiece in space, but also to support the workpiece to counteract the effect of gravity and other forces. To achieve this, a number of support points exist in addition to the six main locators.

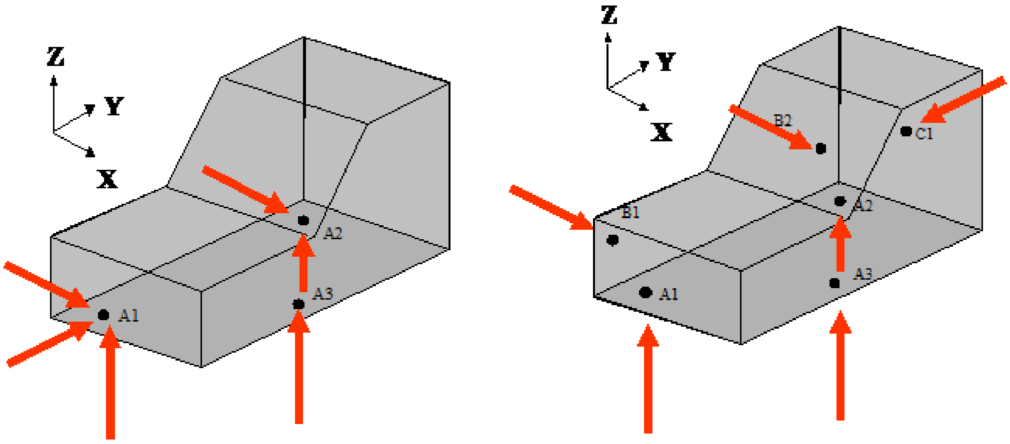


FIGURE 5: LEFT PART SHOWS A 3-2-1 LOCATING SCHEME AND RIGHT PART SHOWS A 3-POINT LOCATING SCHEME (SÖDERBERG ET AL., 2006).

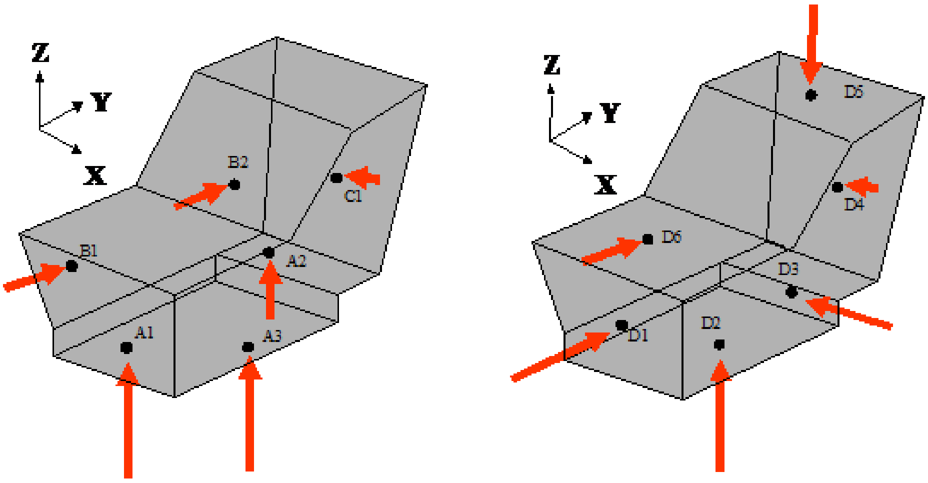


FIGURE 6: LEFT PART SHOWS A 3 DIRECTION LOCATING SCHEME AND RIGHT PART SHOWS A 6 DIRECTION LOCATING SCHEME (SÖDERBERG ET AL., 2006).

2.3. TOLERANCES

According to Zhang & Wang (1994), the tolerance is one of the most important parameters in product and process design and is defined as the maximum deviation from nominal specification within which a part is acceptable for its intended purpose. A tolerance is often defined by specifying an upper specification limit (USL) and a lower specification limit (LSL).

The tolerances are, in addition to the locating schemes, one of the main factors affecting the geometrical outcome of an assembly. In the early stages of the product development phase, the locating scheme can be optimized with respect to geometrical robustness without any additional costs. Tight tolerances are another way to achieve a good geometrical outcome, but this is normally associated with an increased cost and is generally desirable to avoid.

There are tolerances not only for size but also for the form and position of different features.

Tolerancing can be done top-down or bottom-up. The top-down strategy, used at, for example, Volvo Car Corporation (PE Geometry Development AB, 2007), means that a breakdown is made from the specification of the complete product. The most important areas on single part level are given tight tolerances, while less important ones are given wider tolerances. Using this methodology, it is necessary to be able to predict the final outcome of an assembly by calculating the tolerance chains. The problem is illustrated in Figure 7, where the tolerance t of a critical dimension D on an assembly shall be broken down into tolerances t_1, t_2 and t_3 on part level. Different methods for doing this are described in Section 2.5.1.

Top-down tolerancing is also described in (Lööf et al., 2007); (Söderberg, 1993); (Söderberg, 1994); (Söderberg, 1994); (Söderberg, 1995).

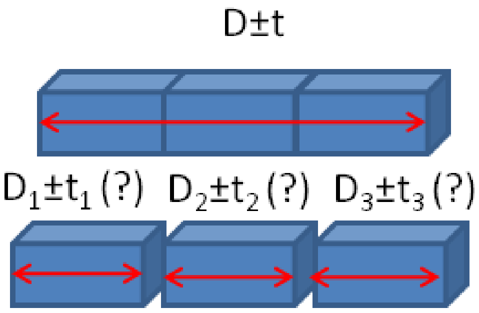


FIGURE 7: TOLERANCE ALLOCATION

Using the alternative, the bottom-up strategy, means that a generic or experience based tolerance is set on different parts. Those tolerances are then leading to final tolerances on the finished product.

In reality, the methods are however quite similar, since a top-down strategy usually implies an iterative bottom-up way of working.

Hong & Chang (2002) give a comprehensive review of different issues related to tolerancing.

2.4. BASIC STATISTICS

When discussing tolerances and variation simulation, a number of statistical ideas and concepts frequently occur. Therefore, a brief overview of some statistical terms used later on in this work will be given here.

Statistics involves the interpretation of data and how to draw correct conclusions about a population, given only a sample of limited size. In order to do this, it is often necessary to create some kind of model of reality. A model that describes the range of possible values for a continuous random variable, X, and the probability that the value of X is within any subset of that range is called a probability density function of X. One of the most common density functions is the normal density function, and a random variable following that model is said to be normally distributed. Inspection data from complex products often follow this distribution. Therefore, only the normal distribution is considered here.

Some basic statistical concepts for a continuous random variable X are given in Table 1. This kind of information can be found in any basic statistical textbook (see, for example, Larsen & Marx (1986)).

Notation	**Formula**	**Remarks**
Expected value	$\mu = E[X] = \int_{-\infty}^{\infty} x f(x) dx$	The mean value of the whole population.
Mean value	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	An estimate of the expected value based on a sample $x_1, x_2, \dots, x_n$.
Variance	$\sigma^2 = E[(X-\mu)^2]$ $= E[X^2] - E[X]^2$	
Standard deviation	$\sigma = \sqrt{\sigma^2}$	
Sample variance	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$	An estimate of the variance based on a sample $x_1, x_2, \dots, x_n$.
Capability index	$C_p = \frac{USL - LSL}{6\sigma}$	Normal distribution is assumed (see also Section 2.9).
Adjusted capability index	$C_{pk} = \min\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\}$	Normal distribution is assumed (see also Section 2.9). This index takes the expected value into consideration.
Normal density function	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	
Distribution function	$P\{a < X < b\} = F(b) - F(a) = \int_a^b f(x) dx$	Those values are usually looked up in statistical tables.

TABLE 1: SOME BASIC CONCEPTS IN STATISTICS.

In Figure 8, the density function *f(x)* of a standardized (i.e., $\mu = 0, \sigma = 1$), normally distributed random variable is plotted together with specification limits equal ±3 standard deviations. This illustrates the situation when $C_p = \frac{USL-LSL}{6\sigma} = 1$, which corresponds to 99.73% of the produced items within the specification limits.

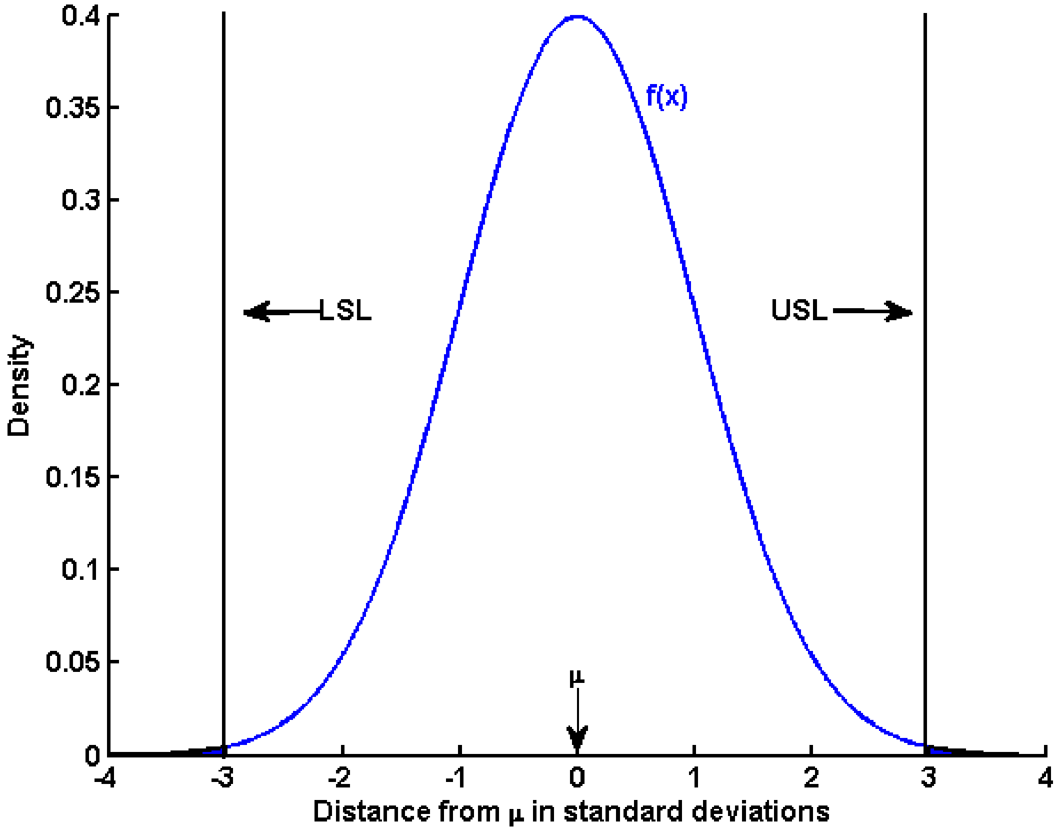


FIGURE 8: A STANDARDIZED NORMAL DISTRIBUTION.

2.5. VARIATION SIMULATION

As indicated in Section 2.3, it is desirable to have a method or tool for predicting the variation in the final product or sub-assembly, given tolerances and other information on part level and information about the assembly process. These kinds of tolerance stack up calculations are usually gathered under terms like variation analysis, variation simulation or tolerance analysis. In this section, some of the different methods will be described and the accuracy of the simulations will be discussed. In the work within this thesis, the software RD&T (RD&T Technology, 2009) has been used and some of the different kinds of analyses supported in this tool will be described, since they illustrate some of the issues within this area.

According to Zhang & Wang (1994), different issues within tolerancing are the following:

- Tolerance allocation/synthesis: How to distribute tolerances to individual parts (top-down).

- Tolerance optimization: How to reduce manufacturing costs, but still produce assemblies within specifications.
- Tolerance analysis/accumulation: How the tolerances on individual parts stack up to affect a final assembly dimension (bottom-up).

Tolerance allocation and tolerance analysis are of course closely related since both are about how tolerances stack up or accumulate.

2.5.1. TOLERANCE ACCUMULATION

There are two main approaches to statistical tolerance analysis: the Monte Carlo simulation-based approach and deterministic methods, often based on Taylor's series expansion. Both approaches aim at finding the resulting tolerance in the final assembly, given tolerances on parameters describing the input.

The Monte Carlo method performs the prediction of the final variation of an assembly by using a random number generator that selects values for each toleranced parameter, based on the type of statistical distribution assigned by the designer. This is repeated a large number of times, thereby generating a distribution for critical dimensions of the final assembly.

The main characteristics of the two respective methods are that the Monte Carlo simulations are said to be exact but time-consuming, while the deterministic methods sometimes lack accuracy, are somewhat complicated to use, but are usually not as computer-expensive as the Monte Carlo method.

Since the key question is how the tolerances on individual parts stack up to affect a critical dimension of the final assembly, the assembly function that describes the relationship between input and output to a variation analysis is of vital interest. Suppose that $\boldsymbol{X}$ is a stochastic variable representing the input to variation simulation, that $Y=f(\mathbf{X})$ is the resulting variable describing the critical dimension and that f is the assembly function. The expected value and variance of Y are sought. The assembly function describes the tolerance accumulation in one, two or three dimensions.

The function f is often approximated, using Taylor expansion, as

$$Y = f(\mu_1, \mu_2, \ldots, \mu_n) + \sum_{i=1}^{n} a_i (X_i - \mu_i), \qquad [1]$$

where the assembly function f relates n inputs X_i to the critical dimension Y. The sensitivity coefficients $a_i = \frac{\partial f}{\partial X_i}(\mu_i)$ describe the sensitivity of the given concept. Each input X_i is a stochastic variable, following a certain distribution and having expected value μ_i and variance σ_i^2. The tolerances of X_i are $\pm t_i$. The accumulated tolerance of the final assembly is denoted T and can be calculated in some different ways. An overview is given in Chase & Parkinson (1991). Some of the most frequently used are listed in Table 2.

In worst case tolerancing, all dimensions are assumed to be in their worst conditions. The worst case technique guarantees assembly and function of finished products regardless of which components are used in the assembly, as long as the components are within their specification limits.

In reality, the probability that all dimensions contributing to the final result of an assembly will be at their worst case condition is very low. Instead, most of the dimensions are distributed close to their mean values. In statistical tolerancing, the possible values for a dimension of a component are described by the distribution function for the dimension in question. Using the root sum square (RSS) method, and different modifications of the RSS method, all component dimensions are assumed to follow a normal distribution.

The worst case method gives results that are too pessimistic, while the RSS gives results that are too optimistic (Nigam & Turner, 1995). Therefore, some modified versions or combinations of the both methods are constructed, as presented in Table 2.

Accumulation Model	Name	Comment
$T=\sum_{i=1}^{n}(a_i t_i)$	Worst case	Dimensions are assumed to be in their extreme tolerance values.
$T=\sqrt{\sum_{i=1}^{n}(a_i t_i)^2}$	RSS	Assuming that the dimensions follow a normal distribution.
$T=c\cdot\sqrt{\sum_{i=1}^{n}(a_i t_i)^2}$	Modified RSS	Using a correction factor *c*, often equal to 1.5, to add an extra safety limit.
$T=\dfrac{\sum_{i=1}^{n}(a_i t_i)+\sqrt{\sum_{i=1}^{n}(a_i t_i)^2}}{2}$	Spotts RSS model	Mean value of worst case and RSS.
$T=\sum_{i=1}^{n}(m_i a_i t_i)+\sqrt{\sum_{i=1}^{n}((1-m_i)a_i t_i)^2}$	Estimated Mean shift model	m_i is the mean shift factor for dimension *i*, where $0 \le m_i \le 1$.

TABLE 2: SUMMARY FROM CHASE & PARKINSON (1991) AND WU ET AL. (1988), CREATED BY LÖÖF (2010).

2.5.2. 3D MODELS FOR APPLIED TOLERANCE ANALYSIS

An assembly model can be constructed to take one, two or three dimensions into consideration. In 1D, only tolerances in the same direction as the evaluated measure contribute to the result. In 3D, rotations also affect the final result. This is illustrated in Figure

9. Here, variation in the *y*-direction is evaluated and illustrated using color coding. The tolerances, illustrated with red arrows, are in the *z*-direction. In the right part of the figure, the result from a 3D model is shown. The rotations of the part lead to variation in the *y*-direction. In the left part of the figure, the result from a 1D model is shown. Here, the tolerances in the *z*-direction do only affect the variation and the *z*-direction. Consequently, there is no variation at all in the *y*-direction. Modern methods for variation simulation take the 3D behavior into consideration, and only such methods will be considered in this work.

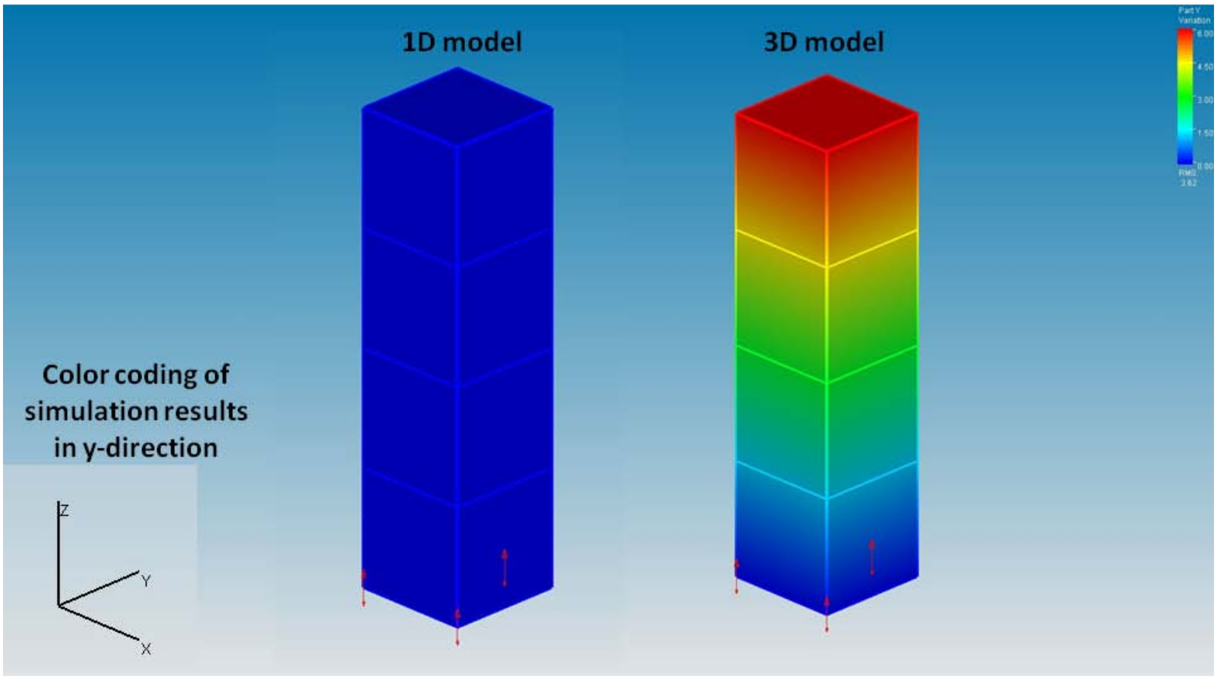


FIGURE 9: VARIATION IN THE Y-DIRECTION IS EVALUATED. THE TOLERANCES, WHICH ARE IN THE Z-DIRECTION, ARE ILLUSTRATED WITH RED ARROWS. IN THE RIGHT PART OF THE FIGURE, A 3D MODEL IS USED WHILE A 1D MODEL IS USED IN THE LEFT PART.

The parts in a variation simulation can be modeled as rigid or non-rigid parts. Thin sheet metal parts, common in the automotive industry, should be treated as non-rigid to achieve good accuracy in the variation simulation, while thicker parts may be treated as rigid. To handle non-rigid parts, variation simulation is usually combined with FEA.

Further, the assembly function *f* can be linear or nonlinear. However, Cai et al. (2006) show that linear models are only valid if the parts do not penetrate and that nonlinear models generally should be used for accurate assembly dimension prediction. The most common approach to handle nonlinear assembly functions is Monte Carlo simulations. The Monte Carlo simulations are usually enclosed in simulation software where a total sensitivity matrix is implicitly defined by a FEA-based simulation model describing all mating conditions, kinematic relations and non-rigid behavior. Most commercial software for tolerance analysis are based on Monte Carlo simulation.

Methods for the tolerance analysis of rigid parts have been investigated by a number of authors. One of the first survey papers was written by Evans (Evans, 1974) and (Evans, 1975) in the middle of the seventies. During the following years, the interest on tolerancing issues increased. An extensive survey on this can be found in Chase & Parkinson (1991). Gao et al. (1998) developed a deterministic model called “direct linearization method” for rigid models, based on linearization of the assembly function.

In the late 90’s, non-rigid tolerance analysis began to develop. Cai et al. (1996) proposed the ‘N-2-1’ locating scheme to handle compliant parts and investigated its impact on geometrical quality. Merkeley et al. (1996) studied part flexibility and demonstrated its significance in tolerance analysis. Hu & Koren (1997) created a model for how dimensional variation propagates in non-rigid assemblies. Hsieh & Oh (1997) developed the software called *EAVS* (Elastic Assembly Variation Simulation) to predict the geometrical variation for multi-station assembly processes.

Hu et al. (2001) considered variation analysis combined with FEA. They took part variation, assembly tooling variation, welding distortion and spring back effects into consideration.

Camelio et al. (2003) developed a methodology to evaluate the dimensional variation propagation in a multi-station compliant assembly system based on linear mechanics and a state space representation. Jaime et al. (2004) presented variation modeling of a compliant assembly based on geometric covariance analysis. Cai (2008) suggested a fixture optimization model, formulated to minimize the assembly dimensional variation under welding gun variation. A deterministic assembly model was used.

Cid et al. (2007) investigated the influence of components’ geometrical variations on functional conditions. Finally, Wang & Ceglarek (2009) presented a beam-based deterministic variation model that can be used in early design phases.

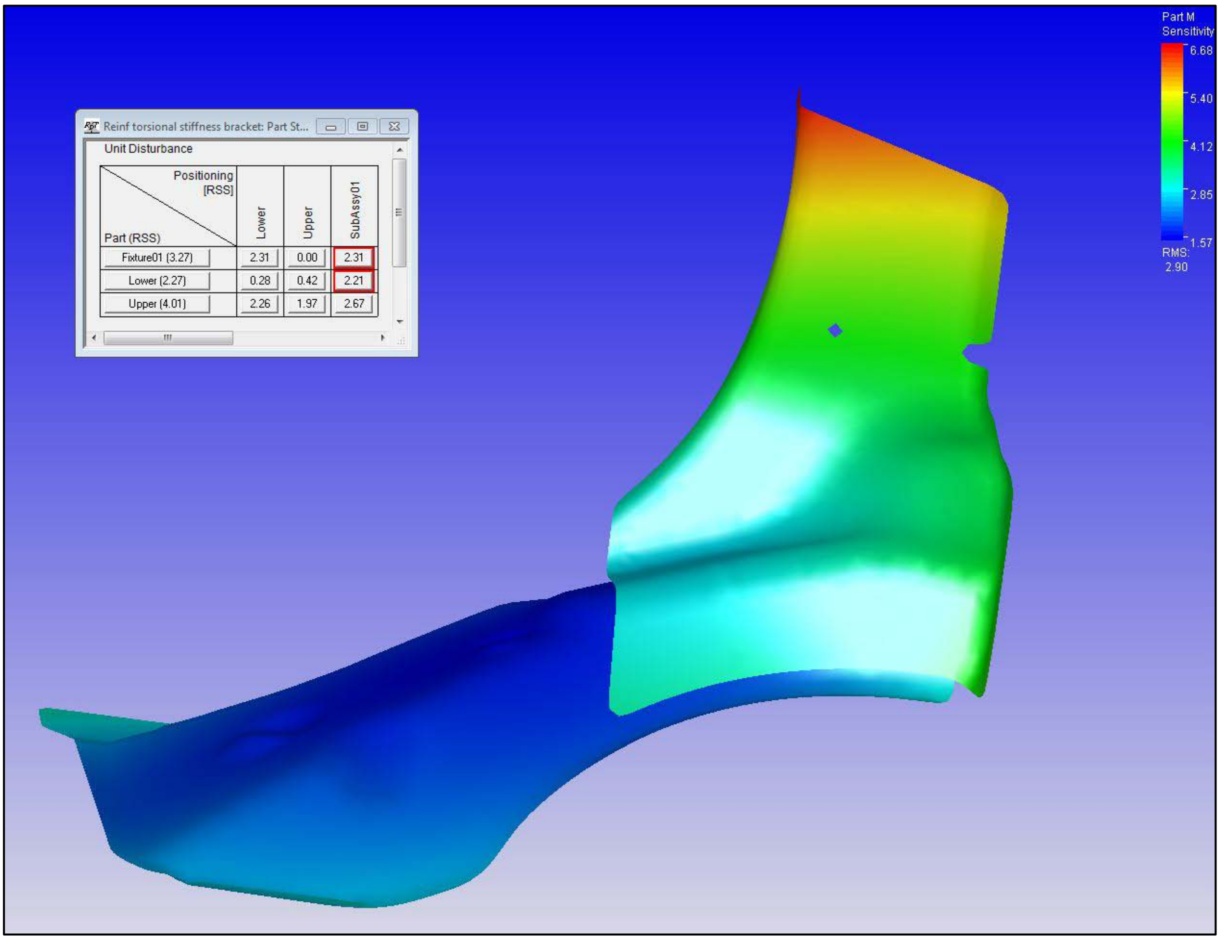
2.5.3. DIFFERENT KINDS OF ANALYSES IN RD&T

There are various commercial software for variation simulation, and most of them are based on Monte Carlo simulations. In this section, the different kinds of possible analysis in one of the software, RD&T (Robust Design and Tolerancing), used in many of the studies in this work, will be described in order to illustrate the possibilities in such software.

RD&T is implemented in the MS Windows environment using Visual C++ and has IGES/VRML/JT and ABAQUS interfaces (RD&T Technology, 2009). It is used for variation simulation at a number of different companies, such as Volvo Car Cooperation, Volvo Trucks, Ford Europe, Jaguar/Land Rover and Renault Trucks. It is based on Monte Carlo simulations and includes a FE solver for non-rigid analyses. To analyze a compliant part or an assembly, meshes for each included part must be imported as files from a FEA system. The stiffness matrices for both parts and assemblies are then calculated in the tool. The unit displacement method is used to create sensitivity matrices, corresponding to specified sets of clamping and welding points. Since FEA meshes are imported and used inside the tool, this solution does not require the FEA system during analysis. The main benefit with this is that once the meshes are imported, the user is free to define and analyze different locating, clamping and welding layouts without communication with a FEA program. However, it is necessary to know the material properties and thickness of the included parts. Of course, it is also possible to run simulations with rigid parts or a mixture of rigid and non-rigid parts.

As described by Söderberg (1998) and Söderberg et al. (2006), there are three main analysis tools in this software:

The **stability analysis**, evaluates the geometrical robustness of a locating scheme (Söderberg & Lindkvist, 1999). The result can be presented as a stability matrix or by using color coding (see Figure 10).

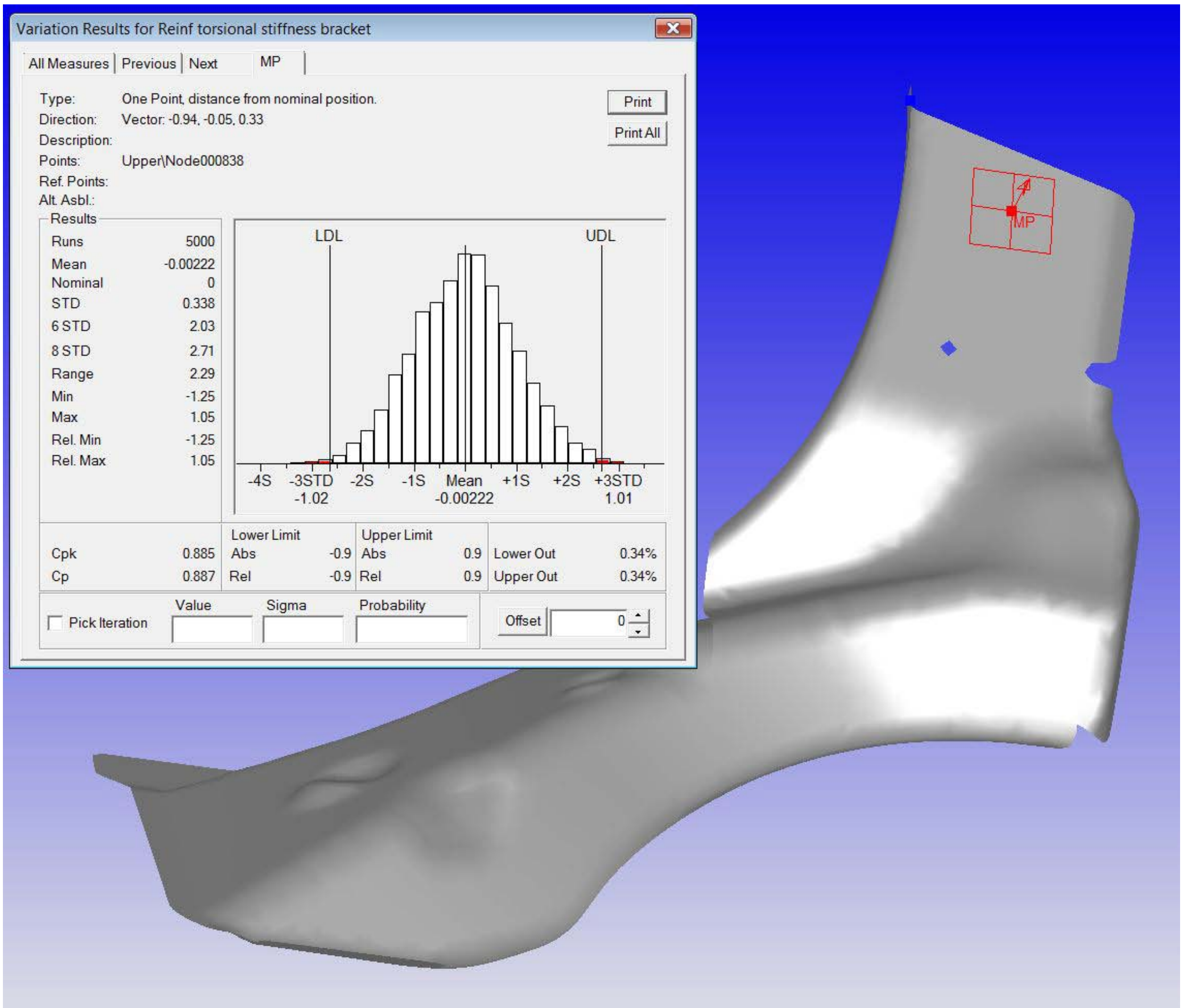


Part (RSS) \ Positioning [RSS]	Lower	Upper	SubAssy01
Fixture01 (3.27)	2.31	0.00	2.31
Lower (2.27)	0.28	0.42	2.21
Upper (4.01)	2.26	1.97	2.67

FIGURE 10: AN EXAMPLE OF A STABILITY ANLYSIS IN RD&T. THE RED COLOR REPRESENTS AREAS WITH A LARGE AMPLIFICATION OF VARIATION, WHILE THE BLUE COLOR REPRESENTS AREAS WITH A LOW AMPLIFICATION LEVEL.

The idea of the stability analysis is to vary each locating point, one at a time, with a small increment Δinput and investigate the resulting quote Δoutput/Δinput in a number of points representing the geometry of the parts. The RSS-value for all points can be shown in color-coding. The blue areas represent stable areas, where the amplification of the variation in the locators are small, while the red areas represent areas with a large amplification. By using this analysis, different locating schemes can be compared and evaluated as early on as in the concept phases of the product realization cycle. It is also possible to optimize the locating schemes with respect to overall robustness (Wang, 1999); (Wang & Pelinescu, 2001) or with respect to some critical dimensions (Lööf et al., 2009).

A **statistical variation simulation** is the very core of the simulation tool. Here, Monte Carlo-based simulations are conducted in order to analyze the tolerance stack up and to predict the variation in the final assembly. In Figure 11, an example of the predicted result in a critical dimension is shown. A lot of information can be extracted from such a simulation, including distribution, mean value, variation and capability. In this case, the capability in the inspection point in question is quite low, and the requirements are not fulfilled.



Variation Results for Reinf torsional stiffness bracket

Type: One Point, distance from nominal position.
Direction: Vector: -0.94, -0.05, 0.33
Description:
Points: Upper\Node000838
Ref. Points:
Alt. Asbl.:

Results

Runs	5000
Mean	-0.00222
Nominal	0
STD	0.338
6 STD	2.03
8 STD	2.71
Range	2.29
Min	-1.25
Max	1.05
Rel. Min	-1.25
Rel. Max	1.05

		Lower Limit		Upper Limit			
Cpk	0.885	Abs	-0.9	Abs	0.9	Lower Out	0.34%
Cp	0.887	Rel	-0.9	Rel	0.9	Upper Out	0.34%

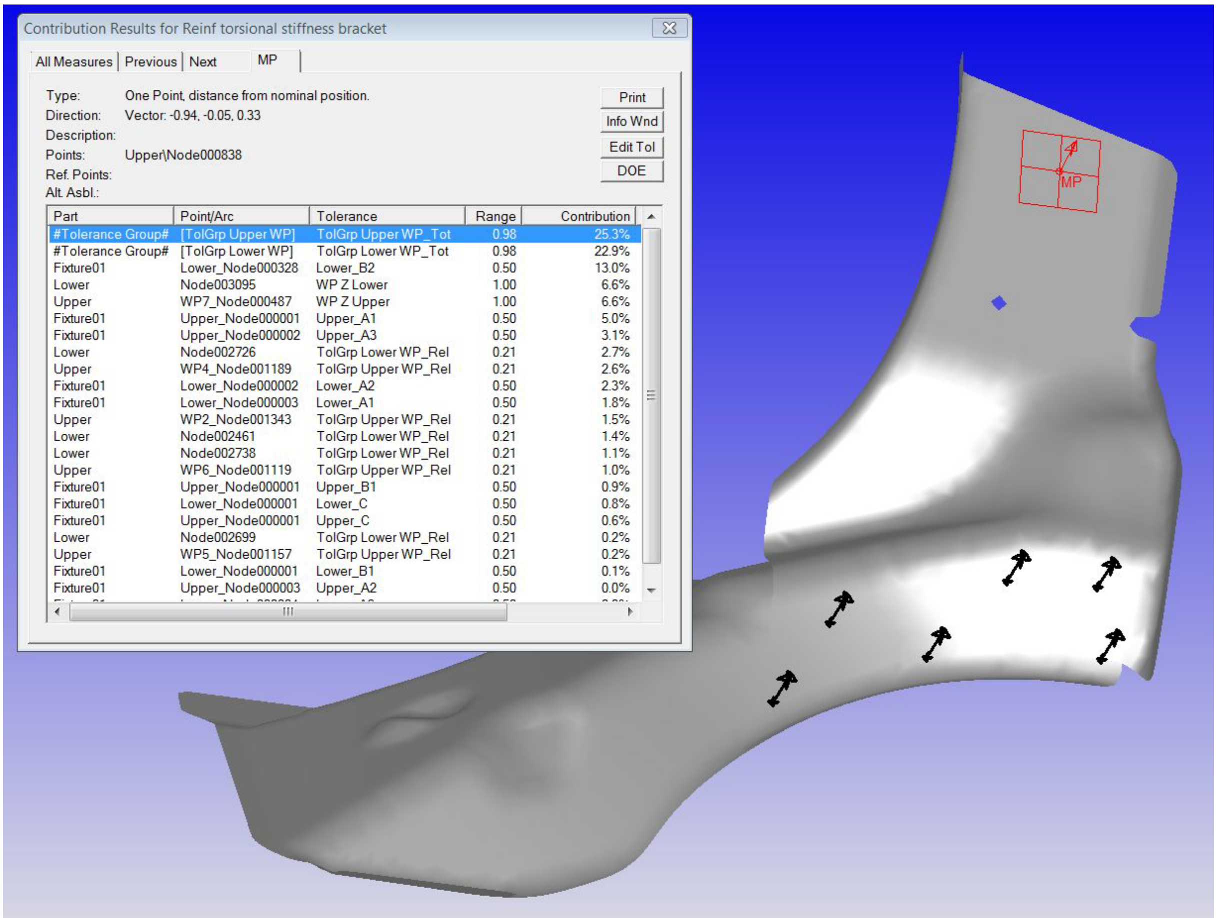
☐ Pick Iteration

Offset: 0

FIGURE 11: THE RESULT OF A VARIATION ANALYSIS FOR A SPECIFIC INSPECTION POINT.

Finally, a **contribution analysis** can be used to calculate the relative importance of each input on the outputs. This means that for a critical dimension, the input parameters (in other words, the tolerances on the different parts of the assembly) can be ranked and their contribution can be determined. This analysis is very useful when a requirement is not fulfilled and some of the tolerances contributing to the result must be reduced. A contribution analysis is run for the same inspection point that was used for illustration of the variation analysis in Figure 11. In Figure 12, the result is shown. Since the variation analysis showed that the requirements on this point will not be fulfilled (i.e., the capability is low) either the concept, for example the locating scheme, should be improved, or the contributing tolerances should be tightened. In the latter case, the contribution analysis shows that among the influencing tolerances, the tolerances in the welding points on the upper part is the largest contributor, with a contribution of 25.3% to the total result. Therefore, it can be a good idea to start with tightening that tolerance.

The contribution analysis is done by varying the influencing parameters, one at a time, at three levels (high, low, mean) and registering the result.


Contribution Results for Reinf torsional stiffness bracket

Type: One Point, distance from nominal position.
Direction: Vector: -0.94, -0.05, 0.33
Description:
Points: Upper\Node000838
Ref. Points:
Alt. Asbl.:

Part	Point/Arc	Tolerance	Range	Contribution
#Tolerance Group#	[TolGrp Upper WP]	TolGrp Upper WP_Tot	0.98	25.3%
#Tolerance Group#	[TolGrp Lower WP]	TolGrp Lower WP_Tot	0.98	22.9%
Fixture01	Lower_Node000328	Lower_B2	0.50	13.0%
Lower	Node003095	WP Z Lower	1.00	6.6%
Upper	WP7_Node000487	WP Z Upper	1.00	6.6%
Fixture01	Upper_Node000001	Upper_A1	0.50	5.0%
Fixture01	Upper_Node000002	Upper_A3	0.50	3.1%
Lower	Node002726	TolGrp Lower WP_Rel	0.21	2.7%
Upper	WP4_Node001189	TolGrp Upper WP_Rel	0.21	2.6%
Fixture01	Lower_Node000002	Lower_A2	0.50	2.3%
Fixture01	Lower_Node000003	Lower_A1	0.50	1.8%
Upper	WP2_Node001343	TolGrp Upper WP_Rel	0.21	1.5%
Lower	Node002461	TolGrp Lower WP_Rel	0.21	1.4%
Lower	Node002738	TolGrp Lower WP_Rel	0.21	1.1%
Upper	WP6_Node001119	TolGrp Upper WP_Rel	0.21	1.0%
Fixture01	Upper_Node000001	Upper_B1	0.50	0.9%
Fixture01	Lower_Node000001	Lower_C	0.50	0.8%
Fixture01	Upper_Node000001	Upper_C	0.50	0.6%
Lower	Node002699	TolGrp Lower WP_Rel	0.21	0.2%
Upper	WP5_Node001157	TolGrp Upper WP_Rel	0.21	0.2%
Fixture01	Lower_Node000001	Lower_B1	0.50	0.1%
Fixture01	Upper_Node000003	Upper_A2	0.50	0.0%

FIGURE 12: A CONTRIBUTION ANALYSIS FOR A SPECIFIC MEASURE.

2.5.4. ACCURACY IN VARIATION SIMULATION

Virtual tools, such as simulation tools, usually aim at replacing physical tests. To do this in a successful way, it is very important that the simulations have good accuracy (in other words, that the simulated outcome and the real outcome are as similar as possible). The gap between the two outcomes should be as small as possible (see Figure 13).

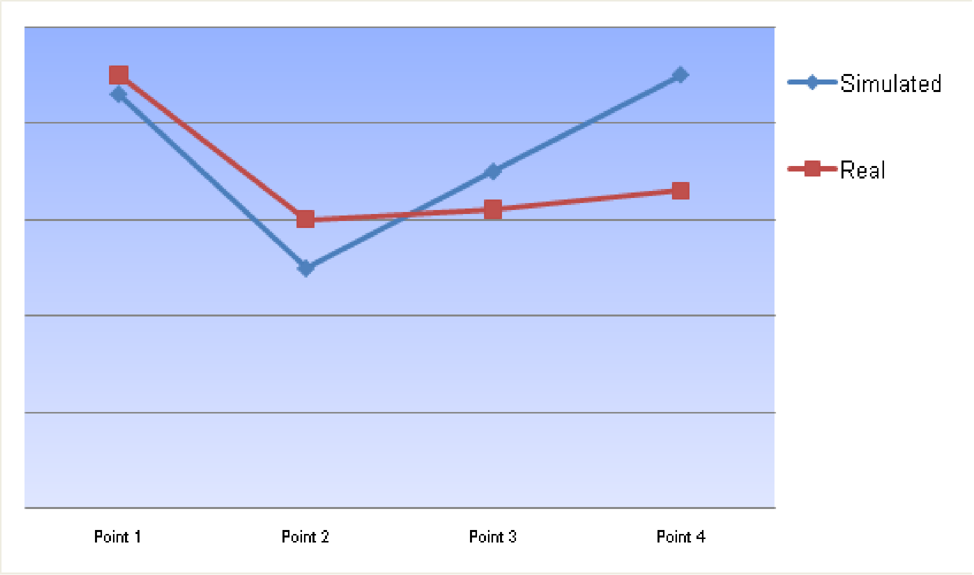


FIGURE 13: SIMULATED AND REAL OUTCOME IN FOUR DIFFERENT INSPECTION POINTS. THE DIFFERENCE BETWEEN THE TWO OUTCOMES SHOULD BE MINIMIZED.

To achieve good accuracy in a Monte Carlo-based variation simulation, it is important to include as many of the factors that affect the outcome of an assembly as possible. Such factors are listed in the chart shown in Figure 14, although the list may not be complete. The most obvious factors, for example representative part geometries and correctly defined locating schemes, are excluded. In the figure, the following factors can be found:

- The assembly sequence is shown to affect the geometrical outcome (Spensieri et al., 2009) and should therefore be included in the simulation.
- The fixture repeatability is treated in Paper G.
- Clamping sequence as well as joining sequence are shown to have effect on the final geometrical outcome by Xie and Hsieh (2002), who use a genetic algorithm to optimize spot welding/clamping sequence and cycle time. Welding sequence for spot welding is treated in Papers E and F. More about welding is found in the next section.
- Forces might arise due to variations in parts and fixture, but also due to other kinds of process deviations, such as variation in welding guns, and those naturally affect the result.
- The influence of heat for continuous welding has substantial influence on the geometrical outcome of the assembly; this is investigated in Paper H.
- Other welding parameters, such as welding speed and direction, might also affect the result.
- If parts or tools are non-nominal, the parts might penetrate during the simulation of an assembly. To avoid this non-realistic phenomenon, contact modeling is used. The inclusion of contact modeling has great influence of the geometric outcome and implies an obvious improvement of the accuracy of the variation simulation (see Paper D).

- In order to be able to predict springback accurately, it is important to have information about the material model, inherent stresses from previous steps and parameters such as material thickness, Poisson ratio and modulus of elasticity.
- The mesh quality is also important.
- When it comes to statistical parameters, it is of course important to have realistic distributions or inspection data of good quality and a sufficient number of samples.

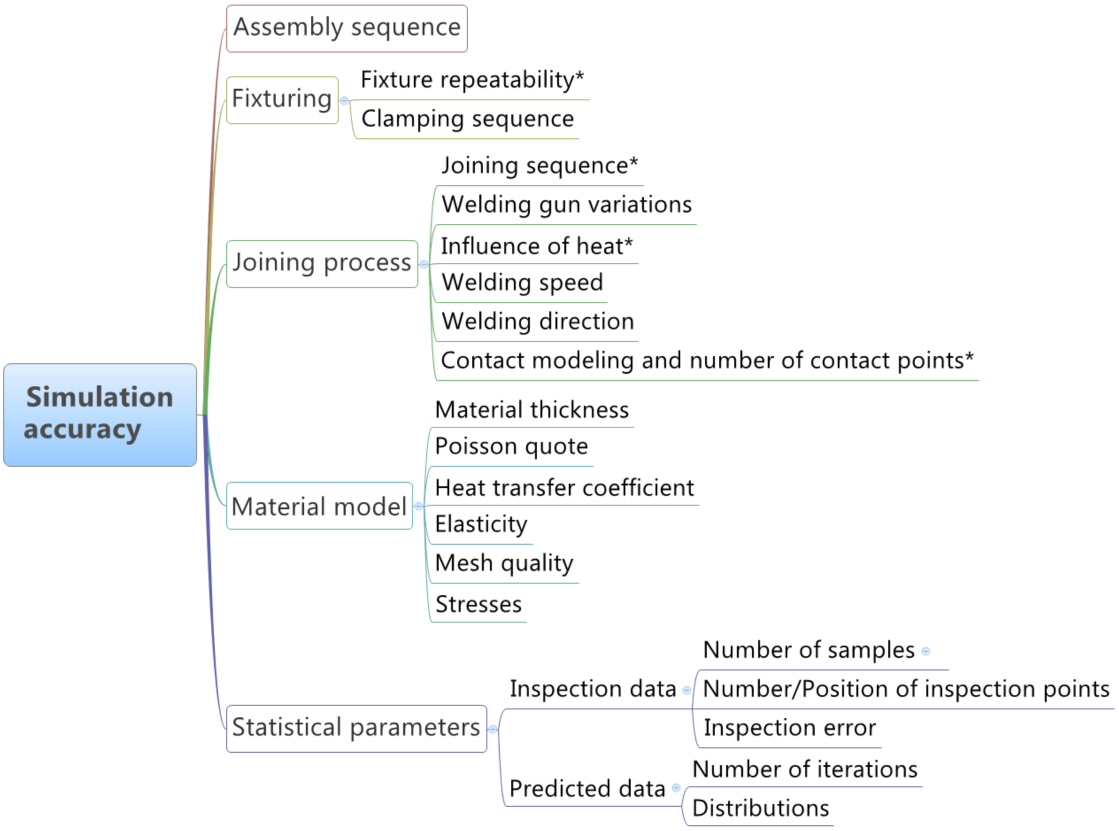


FIGURE 14: FACTORS THAT SHOULD BE INCLUDED IN A VARIATION SIMULATION IN ORDER TO ACHIEVE GOOD ACCURACY. FACTORS MAKED WITH * ARE TREATED IN PAPERS INCLUDED IN THIS THESIS.

2.5.5. THE EFFECT OF WELDING ON TOLERANCE ANALYSIS

One of the factors that should be included in a variation simulation, as outlined in Figure 14, is the joining sequence. The joining process can consist of riveting, gluing, welding, et cetera. In this work, the focus is on welding, and to be more precise, on spot welding. The spot welding sequence (i.e., in what order the weld spots will be executed when two parts are joined together using spot welding) affects the final geometrical outcome of an assembly. The geometrical variation in critical dimensions of a sub-assembly will, in most cases, differ if the welding sequence is altered. Therefore, it is important to find methods, not only to predict the outcome given a certain welding sequence, but also to find an optimal sequence with respect to geometrical variation.

Preferably, this optimal spot welding sequence should be specified in the early phases of the product development process, before the tools and processes are completely determined. By using simulation tools as described in previous sections, different design concepts, including different welding sequences, can be evaluated and compared to each other.

The task of finding a good welding sequence is a fast growing problem. If there are *N* different spot welds, those can be applied in $N!=N*(N-1)*\ldots*1$ different sequences. This means that the option of testing all different possibilities soon becomes unreasonable. Therefore, there is demand for a method for finding a good sequence without testing all possible alternatives.

The effect of the spot welding sequence is investigated by Liu & Hu (1995), but they do not include the phenomenon in variation simulation. They present two principles for minimizing the dimensional variance; namely to weld from weak to strong and to weld simultaneously if possible. Lee et al. (2009) examine how welding sequences for continuous welding can be included in variation simulations by using a pre-generated database. Wärmefjord et al. (2010) (2010) include the welding sequence in variation simulation and verify the result on an industrial case study. They also investigate different strategies for finding an optimal welding sequence with respect to geometrical variation.

Hu et al. (2001) investigate the effect of the welding sequence on a dash panel assembly. They propose a numerical simulation method for compliant assemblies, including the possibility to simulate different welding sequences, and verify their results using experimental data.

Shiu et al. (2000) investigate the relationship between stress build-up due to different spot welding sequences and the resulting dimensional variation. General guidelines for welding sequences are also established.

Genetic algorithms suitable for these kinds of problems are suggested by Bean (1994) and Huang et al. (1997). Liao (2005) describes how to find the optimal number and position of the spot welds using a genetic algorithm for minimizing an objective function, which is the weighted sum of the deviations and/or variation in the inspection points. Xie and Hsieh (2002) also use a genetic algorithm to find spot welding sequences that minimize deformation in user-defined points. They also take cycle time into consideration. The algorithm is implemented in the software EAVS (Elastic Assembly Variation Simulation). However, this work considers only deformation, not variation. Segeborn et al. (2010) apply a genetic algorithm on an industrial case and evaluate its result.

Xiong et al. (2002) built a mathematical model for variation prediction taking locator errors, part errors and welding errors into account. However, the welding errors must be specified as an input to the model and dependency between part error and the resulting welding error is not taken into account.

Not only is the sequence of interest when studying the spot welding process. The number of spot welds and their positions also affect the final geometrical outcome. However, those parameters are usually set by strength requirements and are used as input to the work in this thesis.

Usually, the spot welds are executed in several robot stations in the assembly line. The idea of the spot welds set in the very first joining station is that they will lock the geometry. These welding points are called geometry points. After the geometry points are welded, the assembly can be released from its fixtures and the remaining spot welds can be welded in one of the following stations. However, different welding orders for the geometry points give rise to different forces and, therefore, different displacements in the final assembly as well. For the re-spot points, the welding sequence should be chosen mainly with respect to cycle time.

The spot welding gun has two electrodes, which are applied from either side of the sheet metal parts. When the parts are in contact, an electric current is applied and the result is a small spot, heated to the melting point, in which the parts are joined. The amplitude and duration of the current are chosen to match the specific case. Another important question is how much force is needed to bring the parts to be welded in contact. This depends on the original shape and position of the parts, which may be affected by geometrical variation, and of course the stiffness of the parts. For nominal parts, the parts should be in contact after positioning and clamping. However, this is not always the case for non-nominal parts. If the parts and fixtures are nominal, the welding sequence will not affect the geometrical outcome.

The spot welding gun is usually a balanced or a position gun. Both types have two weld pins that are applied simultaneously from each side of the metal sheets in order to connect the parts. With a balanced gun, equal forces are applied to the welding pins. Therefore, the sheet metal parts will meet in a position of equilibrium. When a position gun is used, the welding pins will meet each other in a fixed position, no matter what the position or stiffness of the sheet metal parts are. Therefore, even if the parts are deflected, they will still be forced to move to that fixed position when the welding gun is applied. The different kinds of welding guns give disparate results, and the type of welding gun must, therefore, be specified. In this work, balanced guns are used.

During the welding process, heat is generated of course. This may lead to the deformation of parts. For spot welding, though, this deformation is of minor importance (Cai et al., 2005). The residual stresses in spot welds are investigated by Henrysson et al. (1999).

2.6. THE EFFECTS OF CONTINUOUS WELDING ON GEOMETRICAL OUTCOME

The effect of continuous welding and how deviations due to welding can be simulated are large research areas, and no claims are made to covering those subjects in this thesis. However, since the effects of welding on geometrical outcome are included in Paper H through the cooperation with another research group, a brief introduction to the area will be given here. This overview is based on Lundbäck (2003), Weman (2007) and The Welding Institute (1968).

There are a large number of different welding methods. The main idea in welding is to melt the workpiece, and in many cases add a filler material, to form a pool of molten material that cools to a strong joint between the workpieces. In resistance welding, which spot welding is an example of, the melting of the workpieces is done by passing a current through the resistance caused by the contact between two or more metal surfaces.

During the welding process, a local area is heated up rapidly. The material expands due to this. However, since it is surrounded by colder and stronger material, the expansion is restricted. This leads to thermal stress, and plastic strains will develop in the weld region. When the weld cools down, the material will shrink and be “too small”, which leads to residual stress. There might also be inherent stress present in the material, caused by operations such as rolling, forming and bending. The heat applied during welding will tend to relieve this stress. The deformations due to welding are driven by thermal expansion (temporary deformation) and residual stress (permanent deformations). From a geometrical point of view, it is desirable to reduce residual stress in the welding process, but that is out of the scope of this work.

Welding simulation aims to predict such things as residual stress, distortions and micro-structural changes after welding. From a geometrical point of view, the distortion is the parameter of main interest.

In Pahkamaa et al. (2010), welding simulation and variation simulation were combined. It turned out that those two kinds of simulations should be combined in order to predict the outcome of a welded assembly accurately. One reason was because the effect of welding is crucial for geometrical deviations and variation. Another was because there is a large difference between applying the welding simulations on nominal or non-nominal parts. The results from welding simulation and variation simulation cannot be superposed to predict the final result either. Lee et al. (2009) use a pre-generated database to include the effect of welding in variation simulation. However, they do not consider the coupling effect between part tolerances and welding distortions.

Just as in spot welding, the welding sequence is an important parameter for the final geometrical result. An approach for the optimization of the welding sequence with respect to displacement was suggested by Voutchkov et al. (2005). They propose a surrogate model to reduce the computational expense and get satisfying results with quite a small number of FEA runs. Genetic algorithms are also used to find optimal sequences for continuous welding (see, for example, Kadivar et al. (2000)). There is, however, no work done to optimize the welding sequence using combined welding simulations and variation simulations.

2.7. THE FINITE ELEMENT ANALYSIS IN VARIATION SIMULATION

Finite element analysis is a numerical technique, used to solve a diversity of problems when finding an analytic solution is too complicated. FEA is also a standard method in non-rigid variation simulation.

The idea of FEA used in non-rigid modeling is to discretize the geometry using a collection of finite elements. Those elements are joined by shared nodes. The finite elements and nodes together form a mesh. In non-rigid analysis, the displacements in the nodes are of fundamental interest. Those displacements depend on forces applied to the geometry, which, for example, can be caused by variation in single parts or fixtures.

It is also necessary to know the stiffness of the material, i.e. how resistant each node is to deflection when exposed to an applied force. This information is included in the so-called stiffness matrix.

Finite element analysis is used to solve equilibrium equations, where the sum of the forces must be in balance. Assembly deviations can be predicted through the use of this method.

The direct Monte Carlo simulation, combined with FEA, is a standard technique for the variation prediction of non-rigid parts. However, since a large number of runs are required to achieve satisfactory accuracy, the method is very time-consuming if a new FEA calculation is executed in each run. Liu & Hu (1997) presented a technique called Method of Influence Coefficients (MIC) to overcome this drawback. The main idea of their method is to find a linear relationship between part deviations and assembly spring-back deviations. A sensitivity matrix, constructed using FEA, describes that linear relationship. This sensitivity matrix is then used in the simulations, and a large number of FEA calculations can be spared. The method was used by Camelio et al. (2003), who applied it to a multi-station system. Dahlström & Lindkvist (2004) investigated how to combine MIC with contact modeling.

2.8. INSPECTION

During the building of physical test series and prototypes, inspection is an important activity in order to learn as much as possible about the product and the processes. When full production starts, the main goal of inspection shifts towards quality control and the detection of deviations and variation. A lot of different aspects of inspection are discussed in Winchell (1996).

Of course, it would be ideal to be able to rely on the premise that only acceptable items are produced and to be able to do that without inspection. However, for most manufacturers, that seems quite unattainable. Instead, inspection is an inevitable part of the manufacturing process. An efficient inspection process can lead to the faster detection of deviations, thereby avoiding costly adjustments and cassations. The advantages of inspection are difficult to put a figure on, according to Kunzmann et al. (2005).

The inspection of a part or assembly can be done by manual inspection, by inline inspection, by the use of checking fixtures or by coordinate measurement machines (CMMs). Each method has its advantages and drawbacks when it comes to accessibility, precision, accuracy and cost. When discussing inspection, the following terms are frequently used, which justifies defining them clearly (Winchell, 1996):

- Accuracy: The closeness of the measurement to the true value of the characteristics (see Figure 15).
- Precision: The degree of agreement of measurements made under the same conditions (see Figure 15). The precision can be judged using:

- Repeatability studies: Repeated measurements made under the same conditions (i.e., precision under the same conditions).
- Reproducibility studies: Repeated measurements using the same test procedure but different operators (i.e., precision under different conditions).

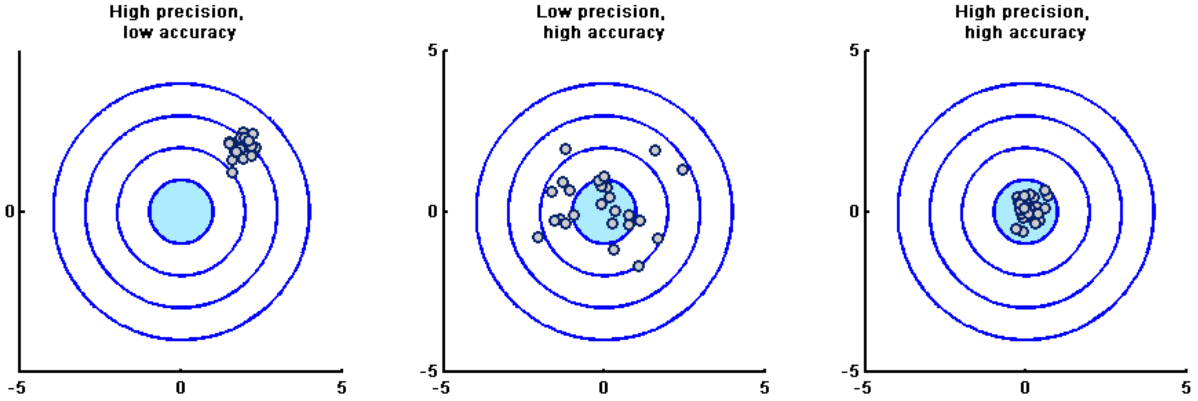


FIGURE 15: ILLUSTRATION OF ACCURACY AND PRECISION.

In a repeatability or reproducibility study, the inspection error is an important aspect. The inspection error should of course be minimized to the utmost possible extent, but this is also a question related to costs. Therefore, it is valuable to compare the size of the measurement error to the magnitude of the standard deviation of the features to be measured. Wetherill & Brown (1991) point out that measurement error is not very serious if the standard deviation of the measurement procedure σ_e is smaller than $\sigma_0/2$, where σ_0 is the standard deviation of the feature to be measured. This may seem to be a large measurement error, but the idea is that the actual observation will then have a standard deviation of

$$\sqrt{\sigma_0^2 + \sigma_e^2} \;=\; \sigma_0\sqrt{1+\frac{\sigma_e^2}{\sigma_0^2}}\,. \qquad [2]$$

Without any measurement error, the standard deviation would be σ_0, so the factor $\sqrt{1+\frac{\sigma_e^2}{\sigma_0^2}}$ is the inflation of the true standard deviation due to measurement error. If $\sigma_e = \sigma_0/2$, then the inflation factor equals $\sqrt{1+\frac{1}{4}} \approx 1.12$ and an error equal or less than 12% is probably acceptable in most cases.

An interesting question related to inspection is where to put the inspection points. Key characteristics are of course interesting to measure, to make sure that the requirements on those are fulfilled. Locators from previous assembly steps might also be interesting to measure, to facilitate potential fault-localization procedures. Inspection points aimed to control the assembly process can, somewhat simplified, favorably be positioned at sensitive areas according to a sensitivity analysis (described in Section 2.5.3). This supports the diagnosis of fixture-related faults. Allocations of sensors for the purpose of diagnosis in a single station are treated by Khan & Ceglarek (2000). For multi-stations assembly process, see Ding et al. (2003).

For a general inspection strategy, a survey is given by Mandroli et al. (2006). Here, a minimization of the total manufacturing costs associated with quality appraisal and failure is considered.

2.9. QUALITY CONTROL AND ANALYSIS OF INSPECTION DATA

Given inspection data of good quality, it is also important to analyze data in a proper way and to learn from this. A concept of frequent occurrence in this area is capability. The capability indices, defined in Table 1, are measures of how many products outside the specification limits can be expected. The capability indices should in most cases be larger than 1.33 and sometimes recommendations of 1.67 are used (Montgomery, 2005).

Capability indices are a good generic measure of the process's capability to produce items within the specification limits. Unfortunately, capability index are also often misused in industry. In order to be able to use the capability indices in a proper way, some general assumptions must be fulfilled:

- The quality characteristic has a normal distribution.
- The process is in statistical control.
- The standard deviation and the expected value must be properly estimated, and this estimate must be based on a sufficient number of inspected items.

The inspection has several purposes, and the way inspection data should be analyzed obviously depends on those purposes. If the purpose is to use inspection data to monitor the process in order to detect disturbances, control charts can be used. The idea of a control chart is to plot a statistic, for example the mean value of the range, for an inspection point. Control limits are plotted as well, and observations outside the control limits imply that the process is out of control. The control limits are estimated from inspection data and correspond to the natural variation that can be expected in the process, on the assumption that no special causes of variation or deviation are present. In Montgomery (2005), different control charts are described. Some of the most widespread charts are also described in the appendix of Section 2, Paper A, in this thesis.

Problems with the traditional charts and ideas about how to solve these problems are presented in Stoumbos et al. (2000).

If several related points should be monitored at the same time, a multivariate control chart can be used. By using a multivariate chart, the number of control charts can be reduced, leading to a better survey of the processes. Furthermore, the relationships between the points are taken into consideration. An overview of multivariate control charts is given in Montgomery (2005) (2005).

Since a multivariate control chart is used to monitor all the inspection points at the same time, it is sometimes complicated to identify the point or points that cause an alarm. Butte and Tang (2010) deal with this problem for T^2, multivariate exponentially weighted moving average and multivariate cumulative sum control charts.

The root cause of an increased variation or deviation can be bad raw materials, worn out machines or fixture faults. According to an investigation performed by Ceglarek and Shi (1995), the major part of all root causes are due to fixture faults.

2.10. OPTIMIZATION

Formal optimization methods are not used in this work and they will not be considered in detail here. However, a brief overview of some different types of methods, mainly based on the work by Rao (2009), will be given to explain why these kinds of methods are not utilized for the problem of optimizing the spot welding sequence.

2.10.1.CONTINUOUS OPTIMIZATION PROBLEMS

A general optimization problem can be formulated like that in Rao (2009):

$$\min_{x \in S} f(x), \tag{3}$$

$$g(x) \leq 0,$$

$$l(x) = 0, \tag{4}$$

where x, in the multi-dimensional case, is the design vector, *f(x)* is the objective function and *g(x)* and *l(x)* are inequality and equality constraints. The optimization problems can be classified in different ways. One important classification is based on the nature of the objective function and the constraints, where the most important classes are linear and non-linear optimization problems.

If there are no constraints and *f(x)* is a function of one variable, defined in the interval $a \leq x \leq b$, the general solution x^* fulfills the conditions:

1. $f'(x^*) = 0$,
2. Let $f'(x^*) = f''(x^*) = \cdots = f^{(n-1)}(x^*) = 0, f^{(n)(x^*)} \neq 0$. If $f^{(n)(x^*)} > 0$ and n is even, $f(x^*)$ is a minimum value of f(x). If $f^{(n)(x^*)} < 0$ and n is odd, $f(x^*)$ is a maximum value of f(x).

Those conditions can also be generalized to the multivariate case, and the derivative is then replaced by the gradient.

Generally, if there is only an equality constraint, Lagrange multipliers can be utilized. This is done by studying the Lagrange function

$$L(x, \lambda) = f(x) + \lambda l(x). \tag{5}$$

Through this method, the constrained problem can be treated as an unconstrained problem (i.e., the Lagrange function is minimized).

In the case of a linear optimization problem, the simplex method is one of the most frequently used methods. The simplex method constitutes of two phases. In the first, the set of feasible values is investigated, and a starting point is generated. In the second, the solution is iteratively improved by a systematic search through the feasible values using matrix manipulation techniques.

For non-linear programming, some kind of interpolation method is a widespread alternative to solve the problem. Among these, the Newton method or Quasi-Newton method are natural choices. Newton's method is an iterative procedure, where a new value is a function of the previous value and of the first and second order derivatives in that point.

2.10.2. DISCRETE OPTIMIZATION PROBLEMS

The methods concerned so far involve continuous variables. Sometimes optimization problems can be discrete. This is frequently referred to as combinatorial optimization, and the variables may even be limited to integer values. In this case, the optimization problem is called integer programming. The discrete optimization methods can also be classified in linear and non-linear optimization problems. In the field of integer non-linear programming, very little work has been done (Rao, 2009), although different penalty function methods and other methods exist. In linear programming problems, the branch-and-bound method is one of the most frequently used. The branch-and-bound method can also be used for solving some types of non-linear problems. The main idea of the method, which originally was developed by Land & Doig (1960) and further developed by Dakin (1965), is to first solve a continuous problem by relaxing the integer restrictions on the variables. With this as a starting point, the feasible solution space is divided (branching) and sub-problems are formulated. By repeating this, the solution space is limited and the solution can be found.

The welding sequence optimization problem (i.e., the problem of minimizing the variation in the final assembly by varying the spot welding sequence) is a combinatorial optimization problem. Another well-known combinatorial optimization problem is the travelling salesman problem (TSP). This problem is about how to find the cheapest way of visiting a number of cities and then returning to the starting point. The problem is illustrated for 15 German cities in Figure 16. The illustrated route is the cheapest one among 43,589,145,600 alternatives. The input to the method is the distance or cost between all different cities. The TSP can be solved by various branch and bound techniques, and there are also a lot of other approaches to the problem. The spot welding sequence problem is similar to TSP. However, a main difference is that the distance or cost between the different cities is known in the TSP, while the contribution to the total variation associated with the execution of a welding point depends on the whole sequence of spot welds. Therefore, the method used for solving the TSP is not applicable to the spot welding problem. Since there is no explicit function for the final variation due to different spot welding sequences, most traditional approaches to optimization are inapplicable.

FIGURE 16: AN ILLUSTRATION OF THE TRAVELLING SALESMAN PROBLEM FOR 15 GERMAN CITIES (WIKIPEDIA, 2011)

There are, however, modern methods of optimization that require only the function values and no derivatives, such as Genetic Algorithms (GA), Simulated Annealing and neural nets.

A GA imitates biological evolution. Over generations, populations evolve according to the principles of natural selection. In each generation, a number of design alternatives are available to reproduce or mate with each other, with bias towards the more fit design alternatives, to form the next generation. This is imitated by the GA, which leads to an iteratively improved objective function value. Although GAs generally find acceptable solutions, they are not guaranteed to find the global optimum. Besides exhaustive calculations, there is no way to determine how close to the actual optimum a GA solution has reached.

Simulated Annealing is based on the simulation of thermal annealing of critically heated solids. When a metal melts, the atoms in the melted metal move freely with respect to each other. When the temperature is reduced, their movements get restricted again. A slow cooling process is desirable in order to avoid defects inside the material. This process of cooling at a slow rate is known as annealing. This process is imitated in the optimization method simulated annealing. The method is based on Boltzmann´s probability distribution, describing the distribution of the energy of a system in thermal equilibrium (which corresponds to the objective function). The energy is dependent on the temperature, and, by controlling the temperature, the convergence of the simulated annealing is controlled.

A neural network is an optimization strategy imitating the human nervous system with a large number of simple processors (neurons) connected to each other. Each neuron takes weighted inputs from other neurons and computes an output, in the form of a sigmoid function, that is propagated to the output nodes. The neural network maps an input vector from one space to another. This mapping is not specified but is learned. The training is done by adjusting the weights on the inputs to minimize the mean-squared error between the actual output and the target output for a given input pattern. Through a suitable formulation of the target output, this method can be used for optimization.

The class of optimization methods that only require a function value should be possible candidates for the solution to the spot welding sequence problem, and GA has also been shown to be successful in this field. However, one disadvantage is that quite a large number of welding sequences must be evaluated in order to train or learn the methods about the problem. In so doing, the methods can learn the relationship between input and output. This is, for a large number of spot welds, a time-consuming task. Therefore, faster alternatives to those methods are sought (see Paper E).

3. RESEARCH APPROACH

In this chapter, the research methodology used in this work is presented.

3.1. BACKGROUND

Within the area of research considered in this thesis, there is no common research model used among researchers, although the Design Research Methodology (DRM) presented by Blessing et al. (1998) frequently occurs. This methodology is also used in this thesis. Other influences exist as well. However, the DRM is identified as being well-suited for computer-aided design tool research (Bracewell et al., 2001).

Blaxter et al. (2006) write that “Research is a systematic investigation to find answers to a problem”. They emphasize the importance of a research methodology, but also indicate the difficulties of choosing a standard method for a certain type of problem. This is also pointed out by Arbnor & Bjerke (2009). They state that there are no such things as a best methodology for a certain kind of problem types; rather, it varies from case to case depending on the previous experiences of the researcher, the techniques used and how the research evolves.

3.2. RESEARCH METHODS IN THIS THESIS

In the Wingquist Laboratory, within whose procedural framework this research has been conducted, the research process illustrated in Figure 17 is used. This means that to start a research project, an industrial need and a research challenge must both exist. Together they give rise to a research idea. The research is conducted using a suitable research method, and the result of the research is used to form a demonstrator, which forms the bases of demonstrations and industrial evaluations. The Wingquist research process has an emphasized implementation strategy. As such, if the evaluations are satisfactory, the demonstrator can be further developed into a product used in industry. This working methodology secures knowledge transfer and has been developed over time. The research in this thesis has mainly been based on these thoughts. Therefore, the research questions are chosen in such a way that complete answers to them meet both an industrial need and a scientific challenge. The research is conducted mainly based on DRM, which is described below. Further, in most cases the research has also formed the basis for some kind of demonstrator, and in some cases even for a product for industrial use. More about industrial implementations of the research can be found in Section 4.3.

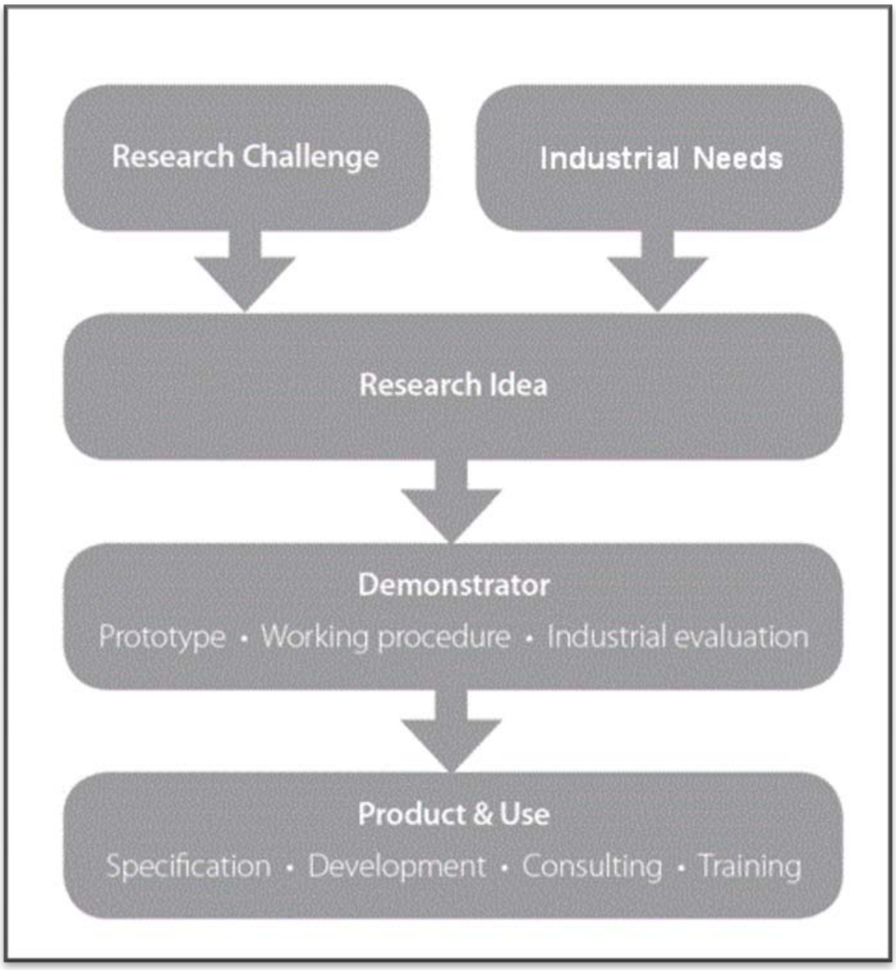


FIGURE 17: THE WINGQUIST RESEARCH PROCESS (WINGQUIST LABORATORY, 2010).

The DRM method, based on the work of Blessing et. al (1998), can in a rough outline be described as a four stage process (see Figure 18). In the first phase, the Criteria are defined. This means measurable success criteria for the research (for example, reduced simulation time or improved quality). In the next phase, Description I, the existing tool/procedure shall be analyzed in order to discover its relationships to the Criteria and thereby also identify where and how the suggested research can lead to improvements. The third stage is the Prescription phase, where insights gained in the Description phase are used as input for the new and improved tool or procedure. Finally, in Description II, the new tool is tested, and its impact on the Criteria is evaluated.

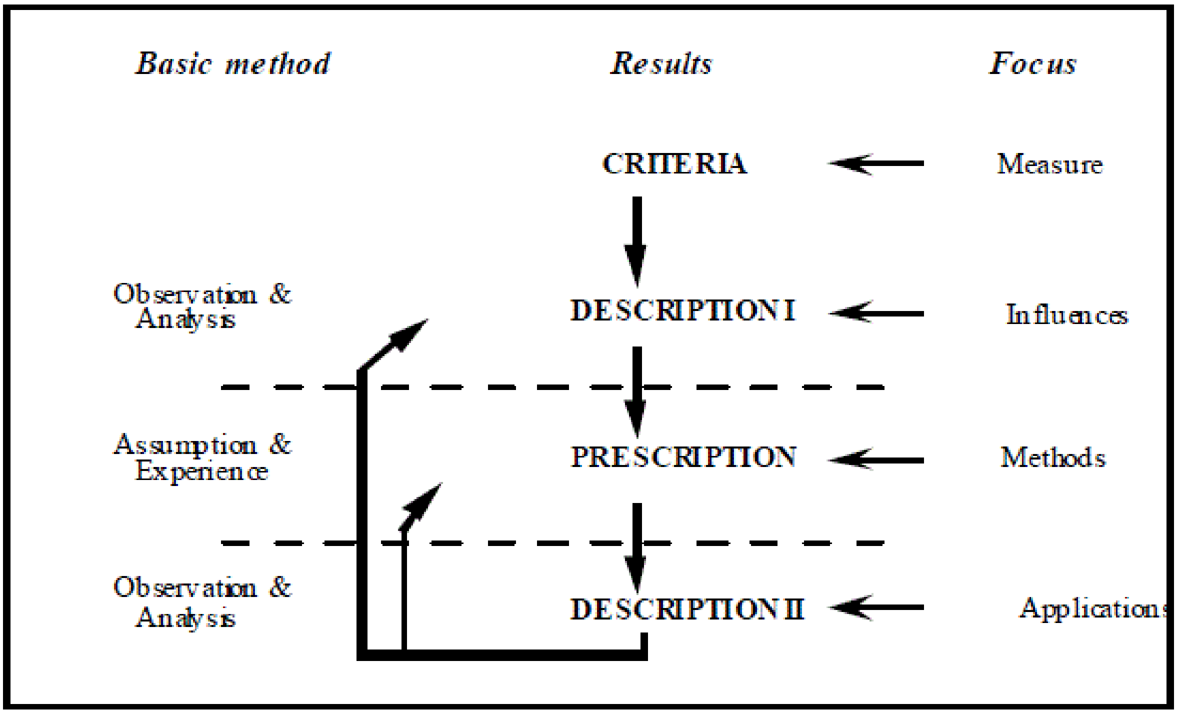


FIGURE 18: THE DESIGN RESEARCH METHODOLOGY FRAMEWORK (BLESSING ET AL., 1998)

The Wingquist Laboratory research process seems very suitable to combine with DRM. The research idea, based on both research challenges and industrial needs, can and should be linked to some criteria for evaluating the possible success of the research. If there are no such criteria, the condition regarding industrial needs is probably not fulfilled either. The DRM helps however to express the need for stating such criteria. The demonstrator, part of the Wingquist Laboratory research process, can be seen as a result of the Prescription phase. By iteratively improving the demonstrator, the process will finally result in a final product, whose effects can be evaluated in Description II phase in the DRM.

3.3. APPLIED RESEARCH APPROACH

The research in this project is mainly based on the DRM and the Wingquist Laboratory research process. In Figure 19, the different phases in the DRM are matched up with the research questions and papers.

For all the research questions, or parts of work in this thesis, the goals have been quite clear. This is perhaps due to the fact that all research questions are linked to industrial challenges, due to the Wingquist Laboratory research process. Since the goals are clear, the formulation of the criteria is facilitated. Clearly stated criteria also make the Description II phase, the evaluation, much easier. In a rough outline, the content of the first description phase can be divided into two main parts; an investigation of the present working procedure at the industrial partner and a literature study to review previous work within the area. This work also provides input and ideas for the prescription phase.

	RQ1: **How can statistical methods be used to control variation in production?**	**RQ2:** **How can statistical methods be used to reduce the need for inspection?**	**RQ3:** **How can variation due to joining sequence be reduced?**	**RQ4:** **How can the accuracy in non-rigid variation simulation be improved?**
Paper	A	B, C	E, F	D, G, H
Criteria	A system for quality control suitable for the kind of processes occurring at Saab, where the number of false alarms is reduced and the probability of detecting significant disturbances is improved.	Paper B: The number of inspection points that can be taken away without losing relevant information. Paper C: A quantification of the information left after a cluster analysis.	Decreased geometrical variation of an assembly.	Improved correlation and reduced mean deviation between inspection data and simulated outcome.
Description I	Control charts for both mean value and variation and for both grouped data and inline measurements are needed. What will a system for quality control of those processes look like?	The number of inspection points needs to be reduced before full production starts. How can this be done in a systematic way?	The spot welding sequence affects the geometrical outcome. How can the sequence be simulated and optimized?	There is a difference between the simulated outcome and reality. How can this difference be decreased?
Prescription	A system for quality control.	B: A method for reducing the number of inspection points. C: An information measure.	Method for simulation of welding sequence and optimization methods for reducing variation due to spot welding sequence.	Inclusion of additional factors affecting the simulation result in the simulation model.
Description II	How are the criteria fulfilled?	How are the criteria fulfilled?	How are the criteria fulfilled?	How are the criteria fulfilled?

FIGURE 19: DRM APPLIED TO THE RESEARCH CONSTITUTING THIS THESIS.

4. RESULTS

In this chapter, a summary of the appended papers is given, and the results are discussed.

4.1. SUMMARY OF APPENDED PAPERS

Short summaries of the appended papers are provided. The papers are presented in the order they were written in this research. However, this order does not necessarily coincide with the order in which they were published.

4.1.1. PAPER A: "MULTIVARIATE QUALITY CONTROL AND DIAGNOSIS OF SOURCES OF VARIATION IN ASSEMBLED PRODUCTS"

This work, which constitutes a licentiate thesis, consists of three major parts. The first part is an introduction, describing the causes and effects of geometrical variation and how this can be handled. The assembly and inspection processes used at Saab Automobile AB are also described. With this as a starting point, the second part of the licentiate thesis deals with a system for acceptance quality control for the kind of processes existing at Saab Automobile AB. Both grouped and ungrouped inspection data are considered. In a typical process in the automotive industry, there are trends and cycles causing variation in the mean value. Some of this variation is very difficult to eliminate at a reasonable cost. As long as the produced items are within their specifications (in other words, the capability of the process is good), this variation might be accepted. If not, the process must be improved. This leads to a system with different types of control charts for grouped and ungrouped data and different kinds of charts depending on the capability of the process. By using the suggested system, trends in the processes are allowed, as long as the produced items are within specifications and traditional control charts are used as a tool to improve processes with low capability.

The third part of this work considers multivariate quality control and diagnosis. All methods are compared and evaluated using two larger case studies. By using multivariate control charts, several inspection points can be monitored using the same chart. A traditional T^2-chart, a T^2-chart based on principal components, regression adjustment, self organizing maps and special methods aimed to control errors due to variation or deviations in the assembly fixtures are all considered. When an error is detected using the control charts, it is of course of vital importance to find the causes of this error in order to be able to adjust it. For this purpose, different methods for root cause analysis are proposed and compared.

Main scientific contribution: Methods specialized for fixture related control and diagnosis were compared and their performances were evaluated. The work also contributed to an increased knowledge of the characteristics of the assembly process in automotive industry.

Main industrial contribution: Increased understanding of the usefulness of statistical methods for quality control and diagnosis was gained. A system for quality control, adapted for Saab Automobile, was proposed. The methods were applied to three industrial case studies.

4.1.2. PAPER B: “AN INVESTIGATION OF THE EFFECT OF SAMPLE SIZE ON GEOMETRICAL INSPECTION POINT REDUCTION USING CLUSTER ANALYSIS”

In order to monitor the process during series production, it is generally not necessary to use as many inspection points as when gathering knowledge about the products and processes in the early phases. Today, this reduction of the number of inspection points is usually done manually and requires great experience and craftsmanship. In this paper, a method for reducing the number of inspection points in a more scientific and methodical way is tested, and the matter of sample size is addressed. The suggested method is based on the cluster analysis of inspection data and finds correlation between inspection points. For each group, or cluster, of correlated inspection points, one point is chosen as representative for the cluster. In that way, the number of inspection points can be reduced. The cluster-based method is complemented by a sensitivity-based method, where inspection points well-suited to monitor the locators are found. Those points are identified by minimizing the covariance matrix of the least square estimator of the movements in the locators. By applying the cluster analysis, combined with the method for finding suitable points for monitoring the locators on three industrial case studies, reductions of 92 %, 72 % and, on an already “manually” reduced case, 42 % can be made.

Main scientific contribution: A statistical method to reduce the number of inspection points was further developed and guidelines for the required sample size were acquired.

Main industrial contribution: A tool for efficient inspection point reduction, leading to saved costs and resources was implemented in commercial software. The method has successfully been applied to several industrial case studies, of which three were included in this paper.

4.1.3. PAPER C: “A MEASURE OF THE INFORMATION LOSS FOR INSPECTION POINT REDUCTION”

In Paper B, methods for reducing the number of inspection points without losing too much information were outlined. In Paper C, the concept of information in this context is considered. The information loss due to the removal of inspection points can be quantified using an efficiency measure based on linear multiple regression. There, the part of the variation in the discarded variables that can be explained by the remaining variables is calculated. This measure can be illustrated graphically and that helps to decide how many clusters should be formed (i.e., how many inspection points can be discarded). It turns out that the stop criterion for clustering used in Paper B (as well as in the work performed by other authors) gives a reduction that is a little too comprehensive, at least if the goal is to retain most of the information.

The suggested information measure is applied to three case studies, and the suggested method appears to give a clear indication of how the information increases with the number of kept points. At some point, the information curve starts to level off, and the corresponding reduction quota can be a suitable choice for the number of remaining points.

Main scientific contribution: A new method to quantify the information contained in a set of inspection points was developed.

Main industrial contribution: The accuracy of the method described in Paper B was further improved. The method has successfully been applied to several industrial case studies, of which three were included in this paper.

4.1.4. PAPER D: "TOLERANCE SIMULATION OF COMPLIANT SHEET METAL ASSEMBLIES USING AUTOMATIC NODE-BASED CONTACT DETECTION"

Contact modeling is a method for preventing the digital parts in a simulation model from penetrating each other during assembly. This increases the accuracy and the degree to which the predictions agree with reality. A simplified, and thereby also timesaving, method for automatic contact detection, well-suited for tolerance simulations, is suggested.

The suggested automatic contact detection is a timesaving procedure, dealing only with contact pairs that consisting of one node from the slave part and one node from the master part. For each contact pair, a plane is defined by the master node and its normal direction. The slave node is not allowed to pass through this plane, and a search algorithm is used to find force equilibrium. To avoid a too large number of contact points close to each other, which leads to time-consuming calculations, an attenuation algorithm for contact pairs is introduced. The method is tested on an industrial case study, and the correlation between simulated outcome and inspection data is greatly improved by including the contact modeling.

Main scientific contribution: A new method for contact modeling, suited for tolerance simulations, was developed.

Main industrial contribution: A method for contact modeling in variation simulation, leading to improved product quality and saved costs, was implemented in commercial software. The method was applied to numerous industrial cases. In this paper, one industrial case study was included.

4.1.5. PAPER E: "STRATEGIES FOR OPTIMIZATION OF SPOT WELDING SEQUENCE WITH RESPECT TO GEOMETRICAL VARIATION IN SHEET METAL ASSEMBLIES"

The spot welding sequence is one of many factors affecting the amount of geometrical variation in a sheet metal assembly. In this paper, a method for including the spot welding sequence in variation simulation is described, which leads to improved simulation accuracy. The methodology is validated on a case study.

Further, the problem of the optimization of the spot welding sequence with respect to geometrical variation is considered. Since this is a fast growing problem - the number of possible sequences for N welding points is N! - testing all possible sequences is not doable. In this work, some different strategies for finding an optimal sequence are tested on several industrial case studies. This should be seen as an alternative to more formal optimization methods, such as genetic algorithms (which have been shown to be successful on this kind of problem). Though successful, genetic algorithms are quite time-consuming for computationally expensive cases with many spot welds. The strategies tested here are based on general guidelines (for example, always weld inside/out, or from left to right), on minimizing variation in each welding step respectively and on calculations of the movements in unwelded points in each step. The strategies based on general guidelines were not successful, nor was the one based on the minimization of the variation in each step. However, the strategy based on movements in the unwelded points seems promising. It resulted in the best or one of the better sequences for all of the eight tested industrial case studies.

Main scientific contribution: A method for including the spot welding sequence in variation simulation was developed. New knowledge about strategies for finding optimal spot welding sequences was gained.

Main industrial contribution: A method for including spot welding sequences in variation simulation, leading to saved costs and increased quality, was implemented in commercial software. The suggested method for including welding sequence in variation simulation was applied to an industrial case study. The investigated methods for finding optimal welding sequences were applied to eight industrial case studies.

4.1.6. PAPER F: "VARIATION SIMULATION OF SPOT WELDING SEQUENCE FOR SHEET METAL ASSEMBLIES"

Just as in Paper E, the spot welding sequence and its effects on geometrical outcome are in focus in this paper. Here, the correlation between variation and deviation for different spot welding sequences is investigated. It is of course interesting to see if a sequence that is good with respect to variation is also good with respect to deviation from nominal (i.e., offset). It turns out that there is quite a strong positive correlation between the both quantities. The study is based on investigations of eight case studies. There is also a mathematical justification for why this relationship is sensible.

Further, an investigation of the number of geometry points (in other words, the spot welds needed to lock the geometry of an assembly) shows that the level of variation and offset level off as early as after two or three executed spot welds. However, it should be noted that there might be other reasons for having a larger number of geometry points, for example, the ability to withstand forces from gravity and misaligned respot guns and so on. In addition, this examination was based on good sequences with respect to offset and variation. Whether similar results can be achieved for sequences resulting in poorer geometrical quality is not yet investigated.

Main scientific contribution: New knowledge about the correlation between mean value and variation of the final assembly for different spot welding sequences was gained.

Main industrial contribution: Applicable knowledge about the correlation between mean value and variation of the final assembly for different spot welding sequences was gained. The investigations were applied to eight industrial case studies.

4.1.7. PAPER G: "COMBINING VARIATION SIMULATION WITH WELDING SIMULATION FOR PREDICTION OF DEFORMATION"

In Papers E and F, the effect of the spot welding process on the geometrical outcome of an assembly was studied. In this paper, continuous welding is considered. The motivation here is to investigate the effect of welding on geometrical variation, how welding simulations based on nominal parts differ from welding simulations based on non-nominal parts and if the effect of variation and the effect of welding in some way could be superposed.

A number of non-nominal parts are generated and used as input to welding simulation software. The results are gathered and compared to the welding of nominal parts. The results before and after welding are also compared. It turns out that the influence of welding on the geometrical outcome is large and that the effect of welding must be included in the variation simulation. This was perhaps not a very big surprise. More interesting is that the difference between welding simulations applied to nominal parts and those applied to non-nominal parts shows evident differences in some areas of the parts. For example, in one node, the added deviation from nominal is in the size of tenths of a millimeter. The welding simulation on nominal parts shows almost no deviation at all after welding, while the welding simulation of the disturbed parts results in a deviation of about four millimeters.

The main conclusion of this work is that variation simulation and welding simulations should be combined. The effect of welding cannot be neglected in variation simulation and welding simulation should also be applied to non-nominal parts. The effects of welding and part deviations cannot be superposed.

Main scientific contribution: Better insight into the need for combined variation simulation and welding simulation was gained. A new method for prediction of the geometrical outcome of a welded assembly was developed.

Main industrial contribution: A method to combine variation simulation and welding simulation was developed. This method led to better accuracy in the predictions, and thereby to decreased costs and improved quality.

4.1.8. PAPER H: "INCLUDING ASSEMBLY FIXTURE REPEATABILITY IN VARIATION SIMULATION"

Sometimes repeatability studies of assembly fixtures are conducted. In them, one single part is positioned a repeated number of times in the fixture, and the part is measured at a number of inspection points. This inspection data usually shows some variation, due to a lack of repeatability in the fixture. As part of the incessant aspiration for better accuracy in the variation simulation, the tolerance due to this lack of repeatability should of course be included in the variation simulation. Traditionally, this is done, by adding this tolerance in the points actually inspected during this study. In this paper, a method for transforming the tolerance from the inspection points to the very origin of the variation. The source of the variation is namely the locators, or to be more precise, the contact between the locator and the part. By applying the tolerances in those points, instead of in the inspection points, they affect not only the inspected points, but also the part as a whole.

Main scientific contribution: A new method for including fixture repeatability in variation simulation in an efficient way was developed.

Main industrial contribution: A method for including fixture repeatability in variation simulation, leading to improved accuracy in variation simulation, and thereby to reduced costs and improved quality, was developed. The method was applied to an industrial case study.

4.2. THE RESULTS IN THE CONTEXT OF A VIRTUAL GEOMETRY ASSURANCE PROCESS

This section aims to position the results described in the appended papers in the virtual geometry assurance loop. The loop, illustrated in Figure 20, has three main areas: the concept phase, the verification phase and the full production phase.

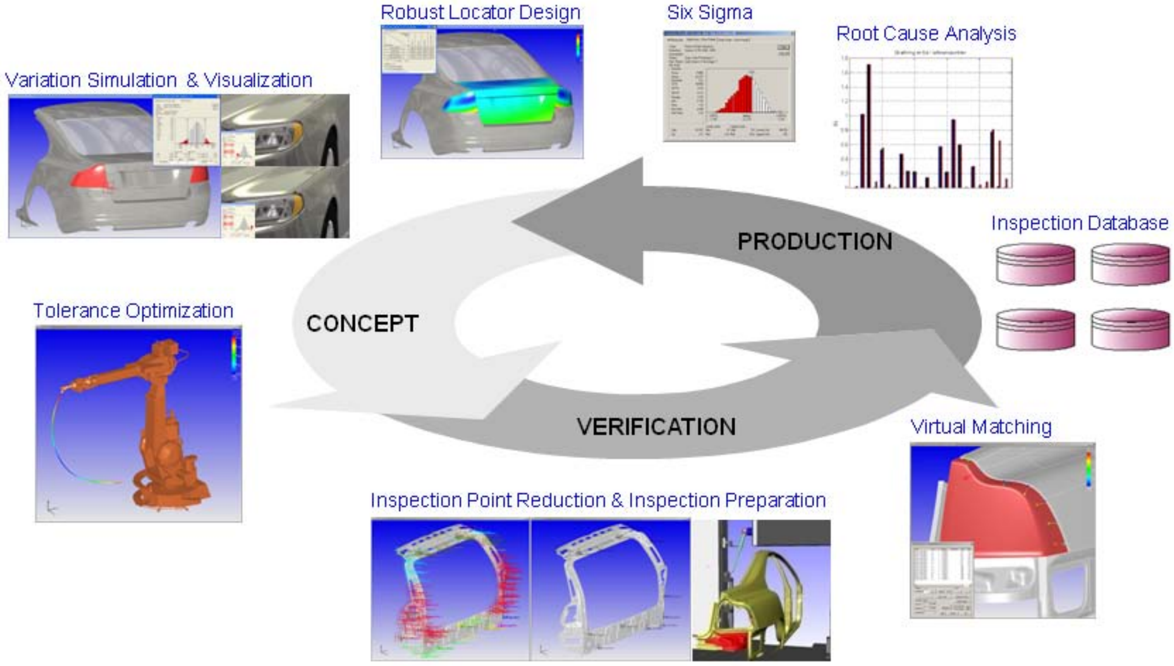


FIGURE 20: THE VIRTUAL GEOMETRY ASSURANCE LOOP.

4.2.1. RESULTS FOR THE CONCEPT PHASE

Papers D and H deal with methods that improve the accuracy of variation simulation. In the concept phase, different concepts are compared and evaluated using variation simulation, aiming to predict the geometrical outcome of different concepts and to compare those results to functional, aesthetical and assembly requirements. When the accuracy of the simulations is improved, the value of the variation simulation is further increased.

Papers E and F also describe methods for improved accuracy in the variation simulation, namely how to include the spot welding sequence in the variation simulation. But those results not only improve the accuracy of the simulations, they also give the user the possibility to test different welding sequences and investigate how those affect the capability of the final sub-assemblies or products.

Paper G, which considers how variation simulation can be combined with continuous welding, also improves the accuracy of the simulations. Thus, it is a result belonging in the concept phase.

4.2.2. RESULTS FOR THE VERIFICATION PHASE

In the verification phase, the concepts are verified and prepared for production. To do this, it is necessary to gather as much information and inspection data about the products as possible. Therefore, it is common practice to inspect the parts at a large number of points. During full production, it is only desirable to inspect the products at a fewer number of inspection points, with a view to monitoring the process and detecting additional variation or offsets.

Papers B and C deal with methods to reduce the number of inspection points without losing too much information.

4.2.3. RESULTS FOR THE PRODUCTION PHASE

As mentioned in the previous section, it is important to monitor the processes during full production in order to detect additional variation or offsets. Paper A suggests a system for acceptance quality control for typical processes in automotive industry. This paper also deals with methods for finding the root causes of such additional variation or offsets.

4.3. INDUSTRIAL IMPLEMENTATION OF RESEARCH RESULTS

Industrial implementation of research results is not only a satisfaction for the researcher; it is also a criterion for evaluating the success of the research result (Eckert et al., 2004). Much of the research in this thesis has been done in close cooperation with industry. In addition, most of the results have been applied to industrial case studies and are included in demonstrators, shown to audiences from various industries to collect feedback on the suggested methods.

- Paper A: This research was done in close cooperation with Saab Automobile AB, and most of the work was done on site at Saab. At the time for this work, all methods were tested on industrial cases, and the company made some changes in its working methodology due to the results from the project.
- Paper B: Implemented in software used at various industrial companies.
- Paper C: Tested at a large number of industrial cases and included in demonstrators.
- Paper D: Implemented in software used at various industrial companies.
- Papers E and F: The method for including spot welding sequence is included in software used at various industrial companies; the optimization strategy is tested on industrial cases and shown at demonstrations.
- Paper G: Not yet implemented or tested on industrial case studies.
- Paper H: Tested on industrial case studies but not yet implemented.

5. DISCUSSION

In this chapter, the answering of the research questions and the relevance of the used research methodology are discussed. The contribution this work makes to new knowledge is also considered.

5.1. ANSWERING THE RESEARCH QUESTIONS

The research questions will be answered one question at a time.

- *RQ1: How can statistical methods be used to control variation in production?*

This question is treated in Paper A, where statistical methods to

1. monitor processes in order to detect deviations or variation leading to products not fulfilling their requirements, and
2. find the root cause of variation in processes

were suggested.

- *RQ2: How can statistical methods be used to reduce the need for inspection?*

This research question was treated in Papers B and C, where a method for reducing the number of inspection points needed to monitor a product was presented.

- *RQ3: How can variation due to joining sequence be reduced?*

A method for including the spot welding sequence in variation simulation was proposed in Papers E and F. This is a prerequisite for finding methods to reduce the variation due to joining sequences using spot welding. In Paper E, several different strategies for reducing the variation due to spot welding sequence were investigated. One of the methods seems promising and resulted in one of the better or even the best sequence for all of the eight tested case studies. In Paper F, the correlation between the best sequence with respect to variation and the best sequence with respect to mean deviation from nominal was investigated.

Joining sequences can of course refer to other joining methods than spot welding, but the open formulation of this question makes it difficult to completely answer. It is, however, likely that the work done on optimizing spot welding sequences can be transferred to other joining methods as well.

- *RQ4: How can the accuracy in non-rigid variation simulation be improved?*

The accuracy of non-rigid variation simulations is highly dependent on the realism of the model. In order to increase the realism of the model, it is necessary to include as many of the factors that affect the geometrical outcome of the assembly in the real world as possible. An outline of such factors is given in Section 2.5.4. Four of those factors are treated in Paper E, D, G and H, and methods for including them in variation simulation are presented.

As a result, the accuracy of the simulations is indeed improved. Nonetheless, openings for future research exist in this case aiming at an even better accuracy of the variation simulation.

5.2. VERIFICATION AND VALIDATION

In this section, the verification and validation of the results from this research will be discussed.

Boehm (1979) states that validation is the process of determining how well a model accurately represents the real world from the perspective of the intended uses of the model. Meanwhile, verification involves how well the model corresponds to its specifications. A way of making the concepts clear is by defining the terms via the following questions (Boehm, 1979):

- Validation: Are we building the right product?
- Verification: Are we building the product right?

In DRM (the research methodology used in this thesis and presented in Section 3), the evaluation of the results is an important activity. In the beginning, some success criteria are formulated. These criteria should be considered as a specification of the research to be conducted. In the last phase, the Description II, how well the criteria are fulfilled (i.e., how well the specifications are fulfilled) is evaluated. This methodology should facilitate the verification, following Boehm’s definition of verification.

According to Buur (1990), results can be verified by *logical verification* or *verification by acceptance*. Logical verification means that there may not be any contradictions between different parts of the suggested theory. It also means that the theory needs to be complete. Further, the result shall be evaluated in relation to other well-established theories and methods. Verification by acceptance means that the suggested methods are accepted by experienced users within the area of application.

In Section 4.3, the industrial implementations of the results in this thesis are outlined. All results are discussed with industrial partners and competent engineers in the area of appliance. Further, in several cases, the methods are also implemented in commercial software accepted and used in industry. Finally, the appended papers have been reviewed and accepted by scientific experts.

Oberkampf et. al (2004) deal with the validation of computational models. They state that the validity of the computational simulation result can be tested by a comparison with experimental data. Instead of just considering a graph, different kinds of metrics are suggested. This topic is also treated in Hills & Trucano (2002) and Easterling (2001), for example.

In the work performed by Oberkampf & Barone (2006), the statistical uncertainty, present due to the limited sample size of the material for comparison, is included in the validation metric. A confidence interval is derived for the deviation between the real world data and simulated data. This deviation or error can also be related to the size of the input variables.

However, the work of Oberkampf & Barone (2006) is focused on statistical interferences about mean values. In most of the work presented here, the focus is on a comparison between variance (or standard deviation) in a number of inspection points and the simulated variance. However, an equivalent confidence interval can be constructed for the difference in variation between simulated and real outcome, using the fact that a $100(1-\alpha)\%$ confidence interval for the population variance σ^2 is given by

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}, \qquad [6]$$

where the sample variance s^2 is estimated from a sample of size n. If the deviation between simulated variance σ^2_{sim} and the population variance σ^2_{pop} is denoted $\partial = \sigma^2_{pop} - \sigma^2_{sim}$, a $100(1-\alpha)\%$ confidence interval for ∂ is given by

$$\frac{(n-1)s^2_{pop}}{\chi^2_{\frac{\alpha}{2},n-1}} - \sigma^2_{sim} \leq \ \partial \leq \frac{(n-1)s^2_{pop}}{\chi^2_{1-\frac{\alpha}{2},n-1}} - \sigma^2_{sim}\ . \qquad [7]$$

This confidence interval is applied to the A-pillar case study used to validate the results in Papers D and E. Only points on the part “extension” are considered. Here, results with and without the inclusion of contact modeling and the spot welding sequence are compared, and the results are shown in Figure 21. For a few of the inspection points, the confidence interval contains zero, implying that there is no significant difference between simulated and real outcome in those points. In other inspection points, the zero is outside the interval. This indicates that there is indeed a significant difference between simulated and real outcome. It should be noted, however, that it is not claimed in the appended papers that the difference between simulated and real outcome will be zero; rather, the difference between the unities will decrease when using the methods suggested in the papers. That can also be seen in the figure.

However, it is important to be aware that both the validation metrics based on confidence intervals for mean value presented by Oberkampf & Barone (2006) and the confidence interval for variation are based on the assumption of normal distribution. Nonetheless, considering the mean values, this is not a very severe assumption. The central limit theorem implies that when n is large, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is approximately normally distributed, regardless of the distribution of x_i .

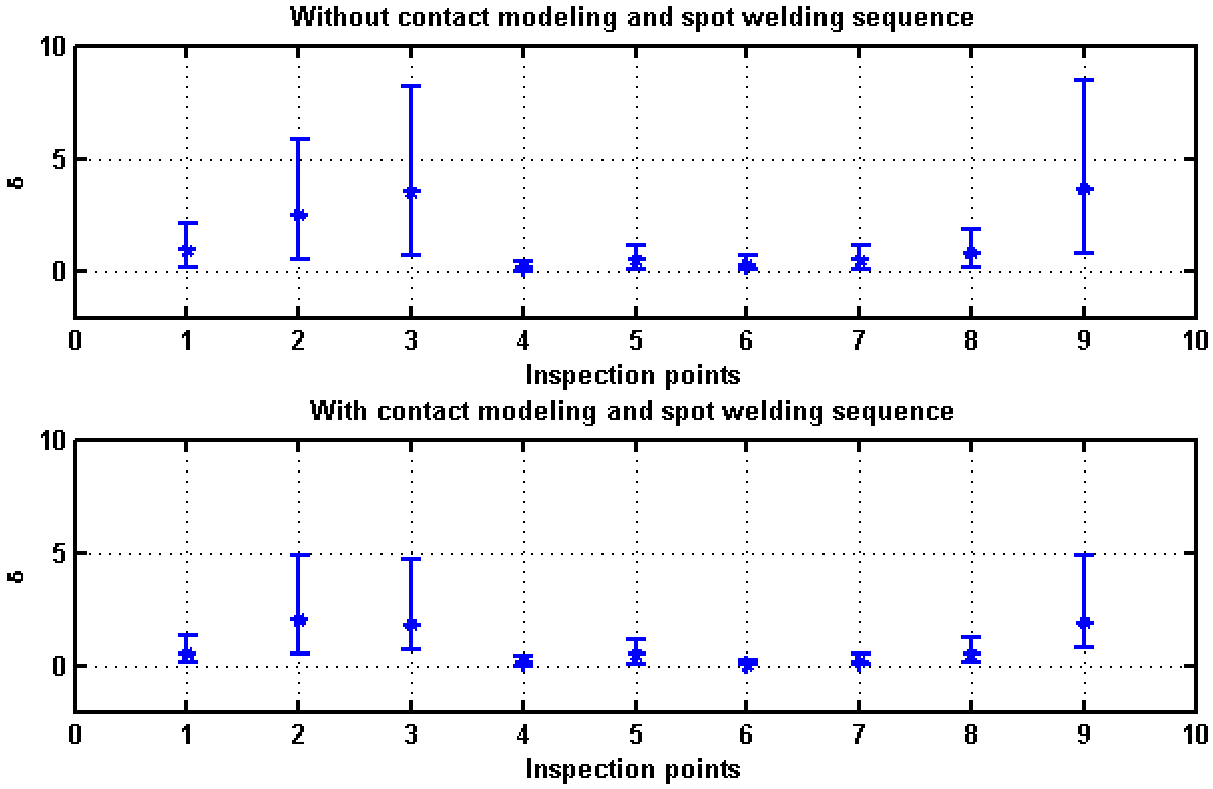


FIGURE 21: THE DIFFERENCE BETWEEN VARIATION IN THE SAMPLE AND THE SIMULATED VARIATION IS ILLUSTRATED WITH "*", AND THE CORRESPONDING ERROR BARS SHOW THE CONFIDENCE LIMITS FOR THIS DIFFERENCE.

5.3. Discussing the Research Methodology

Design Research Methodology has been criticized due to the facts that the success criteria is such a central topic, formulated in the beginning of the study, and that it is problematic to use numerical metrics to assess the success of a new method or computer tool (Eckert et al., 2004). Instead, Eckert et al. (2004) state that the most useful criterion for success might be the perception of value in new methods in industry.

Much of the work within this research has been conducted in close cooperation with industry. In addition, a number of seminars, demonstrators and the actual implementations of the methods in software used in industry should also guarantee the industrial awareness of the suggested methods. Most likely, it will actually be easier to gain a hearing for research ideas in industry, as Eckert et al. (2004) encourage, if the research is connected to a measurable criterion, showing how the research actually can improve a situation or solve a problem.

The Wingquist Laboratory research process also supports the use of demonstrators and industrial cooperation and feedback.

Arbnor & Bjerke (2009) point out that there is no such thing as a best methodology for a certain type of problem. Instead, the best methodology varies from case to case depending on the previous experiences of the researcher, the techniques used and how the research evolves.

However, the chosen methodologies, DRM combined with the Wingquist Laboratory research process, seem to be adequate choices for this kind of research.

5.4. SCIENTIFIC CONTRIBUTION

So far, there has perhaps been more emphasis placed on the industrial benefits of the methods suggested in this thesis compared to the scientific importance. The scientific importance is of course also vital in this kind of work. The contribution in that respect of this research is enhanced knowledge about how to handle geometrical variation in the product realization process. The results have been presented at scientific conferences, at seminars and in scientific journals. All appended papers are reviewed by scientific referees. It is of course important to spread knowledge about scientific results in industry, and that has been done here, through a close cooperation with industrial partners. But increasing the awareness in academia of problems occurring in industry and what industrial working procedures look like is also important, and this work has contributed to improve that awareness.

To be more precise, scientific contributions have been made regarding:

- New knowledge about the characteristics of assembly processes in automotive industry and methods for control and diagnosis.
- A method to reduce the number of inspection points and to quantify the information contained in a set of inspection points.
- Increased understanding of parameters affecting the accuracy in variation simulation and methods to improve this accuracy.
- Increased knowledge about the effect of the spot welding sequence on assemblies and a deeper understanding of methods suitable to optimize the spot welding sequence.
- New knowledge about the effect of combining variation simulation and welding simulation.

5.5. INDUSTRIAL CONTRIBUTION

The industrial benefits of this work have been indicated in Section 4.3, but will be summarized here:

- Increased knowledge about statistical methods for controlling variation in production, potentially leading to increased product quality.
- An implemented tool for efficient inspection point reduction, leading to saved costs and resources.
- An implemented method for contact modeling in variation simulation, leading to improved product quality and saved costs.
- An implemented method for including the spot welding sequence in variation simulation, leading to improved product quality and saved costs.

6. CONCLUSIONS AND FUTURE WORK

In this final chapter, the results are summarized and future work is outlined.

6.1. CONCLUSIONS

This work has been divided into four parts, corresponding to the research questions. The conclusions will be presented one part at a time.

- *RQ1: How can statistical methods be used to control variation in production?*

  - A system for statistical process control for the kind of processes that can be found in automotive industry is presented. By using the suggested methods, the processes with a low capability can be improved, and the processes with satisfying capability can be monitored in order to maintain their performance.
  - Different methods for multivariate process control are applied to case studies, and their performances are compared. The benefit of multivariate control charts is that by using such a method, a large number of univariate control charts can be replaced by one single chart, leading to a better overview of the processes. Further, since the correlation between different points/characteristics can be taken into account, the sensitivity to process changes is improved. The probability of false alarms is also reduced.
  - When a variation or deviation is detected by the control charts, it is important to find the root cause of this problem. Here, the focus is on fixture-related root causes, and methods for identifying which locator caused the problem are compared. The methods are compared and applied to the same case studies as the multivariate methods for process control.

- *RQ2: How can statistical methods be used to reduce the need for inspection?*

  - A structured way of reducing the number of inspection points when going from the verification phase to full production is suggested. The method, demonstrated on three industrial case studies, leads to larger reduction than the ones done by “manual” methods, based on experience and craftsmanship.
  - The term “information” in a set of inspection points is quantified using methods based on linear regression.

- *RQ3: How can variation due to joining sequences be reduced?*

  - A method to include the spot welding sequence in variation simulation is proposed and tested on industrial case studies.
  - Different methods for finding a spot welding sequence leading to low levels of variation in the final sub-assembly are investigated.

  - The number of spot welds needed to ensure that the geometry of an assembly is fixed has been investigated.

- *RQ4: How can the accuracy in non-rigid variation simulation be improved?*

  - A method for handling contact modeling in variation simulation is presented. The method is verified on an industrial case study, and the accuracy is significantly improved.
  - A method for deriving the variation in inspection points, obtained during a repeatability study, going back to the very origin of the variation (namely, the contact between the workpiece and the locators), is presented. By using this method, the obtained variation influences all points in the model, not only the inspected ones.
  - Variation simulation is combined with welding simulation. It is shown that those two simulation methods should be combined in order to improve the accuracy of the predictions of the geometrical outcome of an assembly.

To sum up the results for all four research questions, the proposed methods and procedures have the potential of leading to a more efficient and cost effective product realization procedure. The focus in this work is on automotive industry, but the methods should be possible to apply to all kind of manufacturing industry.

6.2. FUTURE WORK

Finally, much of the work in this thesis has involved how to reduce variation without increased cost. That kind of research has of course not come to an end with this thesis; it will continue in the future.

The accuracy in the non-rigid variation simulation can be further increased by including more and more of the phenomena that affect the result.

Examples of future work are:

- To further investigate, and possibly improve, the suggested method for spot welding sequence optimization.
- To find more efficient methods for including the effect of different joining methods, such as continuous welding, in variation simulation.
- To find optimal welding sequences for continuous welding.
- To apply variation simulation on other types of materials than sheet metal part (for example, molded parts in aluminum or plastic). Within this area, it might also be possible to include variation in material parameters and so on in the variation simulation.
- To investigate the impact of assembly order for non-rigid parts and find optimal assembly sequences.
- To combine the optimization of welding sequences and assembly sequences for non-rigid multi-station assemblies.

- To include the effect of the weight of the parts and fixtures in variation stability analysis and variation simulation.

7. REFERENCES

Arbnor, I. & Bjerke, B., 2009. *Methodology for creating business knowledge*. 3rd ed. London: SAGE Publications Ltd.

Bean, J.C., 1994. Genetic algorithms and random keys for sequencing and optimization. *ORSA Journal of computing*, 6(2), pp.154-60.

Blaxter, L., Hughes, C. & Tight, M., 2006. *How to research*. 3rd ed. Buckingham, England: Open University Press.

Blessing, L.T.M., Chakrabarti, A. & Wallace, K.M., 1998. An Overview of Descriptive Studies in Relation to a General Design Research Methodology. In Ed.: Frankenberger, E., Badke-Schaub, P. & Birkhofer, H. *Designers - The Key to Successful Product Development*. Springer. pp.42-56.

Boehm, B.W., 1979. Guidelines for Verifying and Validating Software Requirements and Design Specifications. In Samet, P.A., ed. *Proceedings of the EURO IFIP 79*., 1979. North Holland Publishing Company.

Bracewell, R.H. et al., 2001. A Methodology for Computational Design Tool Reseach. In *Proceedings of International Conference on Engineering Design, ICED'01*. Glasgow, 2001.

Butte, K. & Tang, L., 2010. Multivariate charting techniques: a review and a line-column approach. *Quality and Reliability Engineering International*, 26(5).

Buur, J., 1990. *A theoretical approach to mechatronics design*. Ph. D Thesis, Technical University of Denmark, Lyngby.

Cai, W., 2006. Robust pin layout design for sheet-panel locating. *Int J Adv Manuf Technology*, 28, pp.486-94.

Cai, W., 2008. Fixture optimization for sheet panel assembly considering welding gun variations. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 222(2).

Cai, W. et al., 2006. Digital panel assembly methodologies and applications for compliant sheet components. *ASME Journal of Manufacturing Science and Engineering*, 128(1).

Cai, W., Hu, S.J. & Yuan, J.X., 1996. Sheet metal fixturing: principles, algorithms and simulations. *ASME Journal of Manufacturing Science and Engineering*, 118(3), p.318–324.

Cai, W., Wang, P.C. & Yang, W., 2005. Assembly Dimensional Prediction for Self-Piercing Riveted Aluminum Panels. *International Journal of Machine tools and Manufacuture*, 45(6), pp.695-704.

Camelio, J.A., Hu, S.J. & Ceglarek, D., 2003. Modeling Variation Propagation of Multi-Station Assembly Systems with Compliant Parts". *Journal of Mechanical Design*, 125, pp.673-81.

Ceglarek, D. & Shi, J., 1995. Dimensional Variation Reduction for Automotive Body Assembly. *Manufacturing Review*, 8, p.139–154.

Chang, T.-C., Wysk, R. & Wang, H., 1997. *Computer-Aided Manufacturing*. Upper Saddle River, NJ, USA: Prentice Hall.

Chase, K. & Parkinson, A., 1991. A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies. *Research in Engineering Design*, 3, pp.23-37.

Cid, G., Thiebaut, F., Bourdet, P. & Falgarone, H., 2007. Geometrical Study of Assembly Behaviour, Taking into Accounts Rigid Components' Deviations, Actual Geometric Variations and Deformations. In Davidson, J.K. *Models for Computer Aided Tolerancing in Design and Manufacturing*. Springer Netherlands. pp.301-10.

Dahlström, S. & Lindkvist, L., 2004. Variation simulation of Sheet Metal Assemblies Using the Method of Influence Coefficients with Contact Modeling. In *Proceedings of ASME 2004 International Mechanical Engineering Congress and Exposition*. Anaheim, California, 2004.

Dakin, R.J., 1965. A tree-search algorithm for mixed integer programming problems. *The Computer Journal*, 8, pp.250-55.

Ding, Y., Kim, P., Ceglarek, D. & Jin, J., 2003. Optimal sensor distribution for variation diagnosis in multistation assembly processes. *IEEE Transactions on Robotics and Automation*, 19(4), pp.543- 556.

Easterling, R.G., 2001. *Measuring the Predictive Capability of Computational Models: Principles and Methods, Issues and Illustrations*. Sandia National Laboratories SAND2001-0243.

Eckert, C., Clarkson, P.J. & Stacey, M., 2004. The Lure of the Measurable in Design Research. In *International Design Conference - Design 2004*. 18-21 May, Dubrovnik, Croatia, 2004.

Evans, D.H., 1974. Statistical tolerancing: the state of the art, Part I: background. *Journal of Quality Technology*, 6, pp.188-95.

Evans, D.H., 1975. Statistical tolerancing: the state of the art, Part II: methods for estimating moments. *Journal of Quality Technology*, 7, pp.1-12.

Gao, J., Chase, K.W. & Magleby, S.P., 1998. Generalized 3-D tolerance analysis of mechanical assemblies. *IIE Transactions*, (30), pp.367-77.

Henrysson, H.F., Abdulwahab, F., Josefson, B.L. & Fermér, M., 1999. Residual stresses in resistance spot welds: finite element simulations, X-ray measurements and influence on fatigue behaviour. *Welding in the World*, (43), pp.55-63.

Hills, R.G. & Trucano, T.G., 2002. *Statistical Validation of Engineering and Scientific Models: A Maximum Likelihood Based Metric*. Sandia National Laboratories.

Hong, Y. & Chang, T.C., 2002. A comprehensive review of tolerancing research. *International Journal of Production Research*, 40(11), pp.2425-59.

Hsieh, C. & Oh, K.P., 1997. A framework for modeling variation in vehicle assembly processes. *Int J Vehicle Des*, 18(5).

Huang, M.-W., Hsieh, C.C. & Arora, J.S., 1997. A genetic algorithm for sequencing type problems in engineering design. *International journal for numerical methods in engineering*, 40(17), pp.3105-15.

Hu, J. & Koren, Y., 1997. Stream-of-Variation Theory for Automotive Body Assembly. *CIRP Annals - Manufacturing Technology*, 46(1), pp.1-6.

Hu, M., Lin, Z., Lai, X. & Ni, J., 2001. Simulation and analysis of assembly processes considering compliant, non-ideal parts and tooling variations. *International Journal of Machine Tools and Manufacture*, 41(15), pp.2233-43.

Jaime, A., Hu, S.J. & Samuel, P., 2004. Compliant Assembly Variation Analysis Using Component Geometric Covariance. *Journal of Manufacturing System*, 3(126).

Kadivar, M.H., Jafarpur, K. & Baradaran, G.H., 2000. Optimizing welding sequence with genetic algorithm. *Computational Mechanics*, 26, pp.514-19.

Khan, A. & Ceglarek, D., 2000. Sensor Optimization for Fault Diagnosis in Multi-Fixture Assembly Systems With Distributed Sensing. *J. Manuf. Sci. Eng.* , 122.

Kunzmann, H. et al., 2005. Productive Metrology-Adding Value to Manufacture. *Annals of the CIRP*, 54(2), pp.691-713.

Land, A.H. & Doig, A.G., 1960. An automatic method of solving discrete programming problems. *Econometrica*, 28(3), pp.497-520.

Larsen, J.L. & Marx, M.L., 1986. *An introduction to mathematical statistics and its applications*. New Jersey: Prentice-Hall.

Lee, D. et al., 2009. Tolerance Analysis Considering Weld Distortion by Use of Pregenerated Database. *J. Manuf. Sci. Eng.*, 131.

Liao, Y.G., 2005. Optimal design of weld patterns in sheet metal assembly based on a genetic algorithm. *International Journal of Manufacturing Technology*, 26, pp.512-16.

Liu, S.C. & Hu, S.J., 1995. Spot Weld Sequence in Sheet Metal Assembly: Its Analysis and Synthesis. In *ASME IMECE Proceeding, Manufacturing Science and Engineering-1995, MED-Vol. 2-2.*, 1995.

Liu, S.C. & Hu, S.J., 1997. Variation Simulation for Deformable Sheet Metal Assemblies Using Finite Element Methods. *J. of Manufacturing Science and Engineering*, 119, pp.368-74.

Lundbäck, A., 2003. *Finite element modelling and simulation of welding of aerospace components*. Luleå, Sweden: Licentiate Thesis, Department of Applied Physics and Mechanical Engineering / Computer Aided Design, Luleå University of Technology.

Lööf, J., 2010. *Managing Geometrical Variation through Optimization and Visualization*. Gothenburg, Sweden: Ph D Thesis, Chalmers University of Technology.

Lööf, J., Hermansson, T. & Söderberg, R., 2007. An Efficient Solution to the Discrete Least-Cost Tolerance Allocation Problem with General Loss Functions. In J. Davidson, ed. *Models for Computer Aided Tolerancing in Design and Manufacturing*. Dordrecht, The Netherlands: Springer. pp.115-24.

Lööf, J., Söderberg, R. & Lindkvist, L., 2009. Optimizing Locator Position to Maximise Robustness in Critical Product Dimension. In *Proceedings of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, IDETC/CIE*. San Diego, California, 2009.

Mandroli, S.S., Shrivastava, A.K. & Ding, Y., 2006. A survey of inspection strategy and sensor distribution studies in discrete-part manufacturing processes. *IIE Transactions*, 38(4), p.309–328.

Merkeley, K.G., Chase, K.W. & Perry, E., 1996. An introduction to tolerance analysis of flexible systems. In *MSC World Users' Conference*. Newport Beach, CA, 1996.

Montgomery, D., 2005. *Introduction to statistical quality control*. John Wiley & Sons.

Nee, A., Tao, Z. & Senthil Kumar, A., 2004. *An Advanced Treatise On Fixture Design And Planning (Series on Manufacturing Systems and Technology)*. Singapore: World Scientific Publishing.

Nigam, S. & Turner, J., 1995. Review of statistical approaches to tolerance analysis. *Computer-Aided Design*, 27(1), pp.6-15.

Oberkampf, W.L. & Barone, M.F., 2006. Measures of agreement between computation and experiment: Validation metrics. *Journal of Computational Physics*, 217, pp.5-36.

Oberkampf, W.L., Trucano, T.G. & Hirsch, C., 2004. Verification, validation, and predictive capability in computational engineering and physics. *Applied Mechanical Review*, 57.

Pahkamaa, A. et al., 2010. *Combining Variation Simulation with Welding Simulation for Prediction of Deformation*. Vancouver, British Columbia, Canada: Proceedings of the ASME 2010 International Mechanical Engineering Congress & Exposition.

Payne, J. & Cariapa, V., 2000. A fixture repeatability and reproducibility measure to predict the quality of machined parts. *International Journal of Production Research*, 38(18), pp.4763-81.

PE Geometry Development AB, 2007. *Drawing Interpretation VCS/ASME*.

Rao, S., 2009. *Engineering Optimization: Theory and practice*. 4th ed. Hoboken, New Jersey: John Wiley & Sons.

RD&T Technology, 2009. *RD&T Manual*.

Segeborn, J., Torstensson, J., Carlson, J.S. & Söderberg, R., 2010. Evaluating Genetic Algorithms that Optimize Welding Sequence with Respect to Geometrical Assembly Variation. In *Proceedings of the NordDesign 2010 Conference, August 25 – 27*. Gothenburg, Sweden, 2010.

Shiu, B.W., Shi, J. & Tse, K.H., 2000. The dimensional quality of sheet metal assembly with welding-induced internal stress. *Proc. of the I MECH E Part D Journal of Automobile Engineering*, 214(7), pp.693-704.

Spensieri, D. et al., 2009. A Method to Optimize Geometrical Quality and Motion Feasibility of Assembly Sequences. In *Proceedings of the 11th CIRP International Conference on Computer Aided Tolerancing*. Annecy, France, 2009.

Stoumbos, Z., Reynolds, M., Ryan, T. & Woodall, W., 2000. The State of Statistical Process Control as We Proceed into the 21st Century. *Journal of the American Statistical Association*, 95(451), pp.992-98.

Söderberg, R., 1993. Tolerance Allocation Considering Customer and Manufacturing Objectives. In *Proceedings of the ASME Deign Automation Conference, 2 (ASME DE-VOL 65-2).*, 1993.

Söderberg, R., 1994. Robust Design by Tolerance Allocation Considering Quality and Manufacturer Cost. In *Proceedings of the ASME Design Automation Conference*., 1994.

Söderberg, R., 1994. Tolerance Allocation in a CAD Enviroment Considering Quality and Manufacturer Cost. In *Proceedings of Lean Production: from Concept to Product Conference, IMC-11*. Belfast, Northern Ireland, 1994.

Söderberg, R., 1995. Optimal Tolerance Band and Manufacturing Target for Monotonic Loss functions with Functional Limits. In. *Proceedings of the ASME Design Automation Conference*., 1995.

Söderberg, R., 1998. Robust Design by Support of CAT Tools. In *Proc. of the ASME Design Automation Conference*. Atlanta, USA, 1998. DETC98/DAC-5633. Fiskbensdiagrammet.

Söderberg, R. & Lindkvist, L., 1999. Computer Aided Assembly Robustness Evaluation. *Journal of Engineering Design*, 10(2), pp.165-81.

Söderberg, R., Lindkvist, L. & Carlson, J.S., 2006. Managing physical dependencies through location system design. *Journal of Engineering Design*, 17(4), pp. 325-346.

Söderberg, R., Lindkvist, L. & Carlson, J., 2006. Virtual geometry assurance for effective product realization. In *Proceedings of 1st Nordic Conference on Product Lifecycle Management - NordPLM'06*. Gothenburg, Sweden, 2006.

Taguchi, G., 1986. *Introduction to quality engineering : designing quality into products and processes*. Tokyo: Asian Productivity Organisation.

The welding institute, 1968. *Control of distortion in welding fabrications*. Unwin Brothers Limited Working and London.

Wang, M., 1999. An Optimum Deign Approach to Fixture Synthesis for 3D Workpieces. In *IEEE International Conference on Robotics and Automation*. Detroit, MI, USA, 1999.

Wang, H. & Ceglarek, D., 2009. Variation propagation modeling and analysis at preliminary design phase of multi-station assembly systems. *Assembly Automation*, 29(2).

Wang, M.Y. & Pelinescu, D.M., 2001. Optimizing Fixture Layout in a Point Set Domain. In *IEEE Trans. on Robotics and Automation*., 2001.

Weman, K., 2007. *Svetshandbok Karlebo (In Swedish)*. Stockholm: Liber.

Wetherill, G.B. & Brown, D.W., 1991. *Statistical Process Control*. London: Chapman & Hall.

Wikipedia, 2011. *Wikipedia.com*. [Online] Available at: http://en.wikipedia.org/wiki/Traveling_salesman_problem [Accessed 8 February 2011].

Winchell, W., 1996. *Inspection and measurement in manufacturing: keys to process planning and improvement*. Society of Manufacturing Engineers.

Wingquist Laboratory, 2010. *Wingquist Laboratory webpage*. [Online] Available at: http://www.chalmers.se/ppd/wingquist-en/research/implementation-strategy [Accessed 05 December 2010].

Voutchkov, I., Keane, A.J., Bhaskar, A. & Olsen, T.M., 2005. Weld Sequence Optimization: The use of surrogate models for solving sequential combinatorial problems. *Comput. Methods Appl. Engrg.*, 194, pp.3535-51.

Wu, Z., Elmaraghy, W.H. & Elmaraghy, H.A., 1988. Evaluation of Cost-Tolerance Algorithms for Design Tolerance Analysis and Synthesis. *ASME Manufacturing Review*, 1, pp.168-79.

Wärmefjord, K., Söderberg, R. & Carlson, J., 2010. Including Assembly Fixture Repeatability in Rigid and Non-Rigid Variation Simulation. In *Proceedings of the ASME 2010 International Mechanical Engineering Congress & Exposition*. Vancouver, British Columbia, Canada, 2010.

Wärmefjord, K., Söderberg, R. & Lindkvist, L., 2010. Strategies for Optimization of Spot Welding Sequence With Respect to Geometrical Variation in Sheet Metal Assemblies. In *Proceedings of the ASME 2010 International Mechanical Engineering Congress & Exposition*. Vancouver, British Columbia, Canada, 2010.

Xie, L.S. & Hsieh, C., 2002. Clamping and welding sequence optimization for minimizing cycle time and assembly deformation. *International Journal of Materials & Product Technology*, 17(5/6).

Xiong, C. et al., 2002. Geometric variation prediction in automotive assembling. *Assembly Automation*, 22(3), pp.260-69.

Zhang, C. & Wang, H.-P., 1994. Tolerancing for Design and Manufacturing. In R., D. & Kusiak, A. *Handbook of Design, Manufacturing and Automation*. Hoboken, NJ, USA: John Wiley & Sons.

APPENDIX WITH APPENDED PAPERS

PAPER A

Multivariate Quality Control and Diagnosis of Sources of Variation in Assembled Products

Wärmefjord, K., 2004; Licentiate thesis, Department of Mathematics, Chalmers University of Technology and Göteborg University, ISSN 0347-2809.

THESIS FOR THE DEGREE OF LICENTIATE OF PHILOSOPHY

Multivariate Quality Control and Diagnosis of Sources of Variation in Assembled Products

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Abstract

Variation in key geometrical characteristics in assembled products is a usual problem in automotive and other industries. Geometrical variation and its causes and effects are described and different methods to reduce this variation are considered.

In order to control the variation, a control chart is traditionally used to detect if the process is out of control and therefore should be adjusted. However, some of the variation is very difficult to eliminate at a reasonable cost. Therefore, a quality system that allows for trends in the processes as long as the produced items are within specifications is introduced. This is done by using traditional charts to improve low capability processes, while high capability processes are controlled by acceptance control charts, in order to see that the produced items still are will within specifications.

If a variation is detected it is essential to find the root cause of the problem. Different methods for root cause analysis are applied to industrial data and their performances are compared. Methods for multivariate statistical process control are also considered. The most successful method for root cause analysis is based on a sensitivity matrix. This matrix relates the movements of the inspection points to those of the locators.

Keywords: geometrical variation, quality control, acceptance control charts, multivariate statistical process control, root cause, rigid body, fixture diagnosis

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Contents

Chapter 1

Introduction

This work is a part of a two-year long project between Saab Automobile AB, Chalmers University of Technology and Fraunhofer Chalmers Research Centre. It aims to develop principles, working procedures and tools for finding fixture related root causes of geometrical variation in assembled products. The work is contained in a research project, called "Three Dimensional Tolerance Management" (3DTM), going on at Chalmers. It deals with methods for minimizing geometrical variation in assembled products.

The thesis is divided into three major parts. The first part is this introduction, where general ideas and principles concerning geometrical variation in assembled products are considered. The introduction gives the motivation of the methods described in later chapters, and it also gives a basis for concepts and ideas used in those chapters.

In the remaining two parts of the thesis, topics related to geometrical variation are discussed; namely how to detect variations and deviations using statistical process control and how to identify root causes of the variation. In Chapter 2 a suggestion of how to use process control in order to get a process able to meet specifications is given. This means that an acceptance chart can be used to control a stable process with high capability, while a traditional control chart is used to improve a process with low capability.

The last part of the thesis, Chapter 3, contains a study of methods used for multivariate statistical control and methods for root cause analysis of geometrical variation in assembled products. The methods are applied on case studies and their performances are compared.

The methods described are tested on data from automotive industry. However, most of the methods should be applicable to any kind of rigid assembled product, provided that key geometrical characteristics of the product are measured.

1.1 Goal of the project

The goal of the project is to develop and adopt methods for process control and diagnosis that support a tool based on geometrical inspection data, which may be used in everyday work with the assembly processes. The tool shall

- Be easy to use and enable quick identification of root causes in complex assemblies.
- Translate variations and deviation in geometric data to adjustable process parameters.
- Make it possible to simulate and verify the effects of actions taken in the process.
- Be a support in evaluation of different inspection point layouts.

1.2 The assembly process

In order to discuss geometrical variation, considered in Section 1.5, it is crucial to have a knowledge of the assembly process, which is described in this section.

The position of inspection and positioning points are described using a coordinate system of the car. A point on the car body is completely determined by its coordinates. The coordinate system comprises three mutually perpendicular planes, where:

- The X axis runs in the longitudinal direction of the car, with its origin in front of the car.
- The Y axis runs in the transverse direction of the car, with its origin in the centre line of the car.
- The Z axis describes the height in the car.

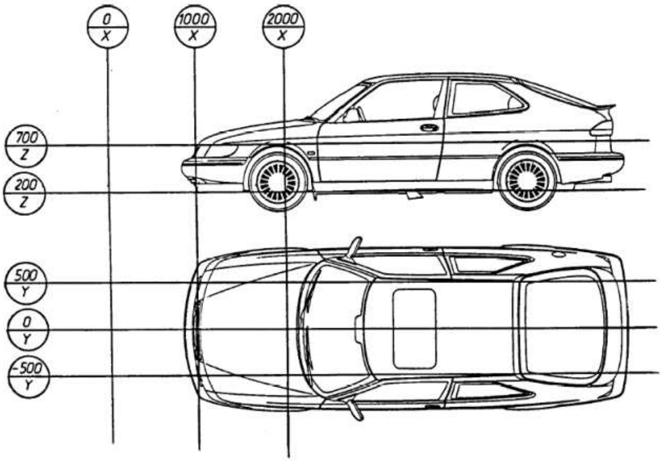


Figure 1.1: *The coordinate system of a car, Saab standard (28)*

The coordinate system is illustrated in Figure 1.1.

To position a part or subassembly during assembly and inspection a positioning frame (P-frame) is used. In automotive industry a 3-2-1 locating scheme is a usual choice to lock the six degrees of freedom of a part. Three master locating points, usually called $A1$, $A2$ and $A3$, are used to form a plane locking one translation and two rotations, two points, $B1$ and $B2$, lock one translation and one rotation and the last point, $C1$, lock the remaining translation. The part is assumed to never loose contact with the locators. This is illustrated in Figure 1.2. The part is positioned in its fixture or joined to another part by bringing its P-frame in contact with a mating P-frame on the target, see Figure 1.3. In addition to the master location points, supplementary points can be required to provide a complete guidance of a part, due to slenderness or spring back factors. Planes, holes and slots are used to represent the locator points in practise.

The selection of master location points is in high extent based on experience, but there are some guidelines in Saab standard (27);

- The manufacturing variations within restricted master location surfaces shall be possible to regard as negligible.

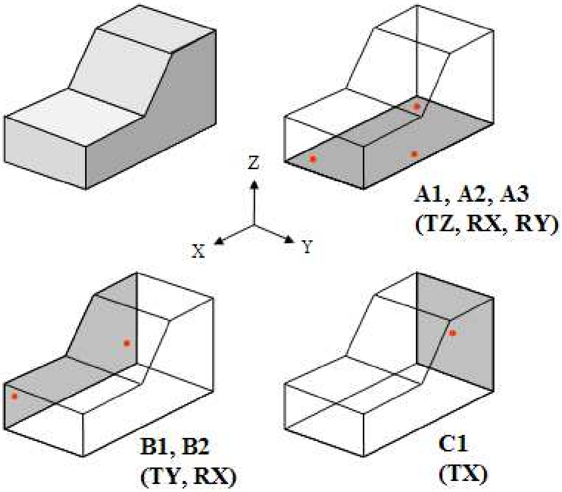


Figure 1.2: *A 3-2-1 locating scheme, Söderberg and Lindkvist (30)*

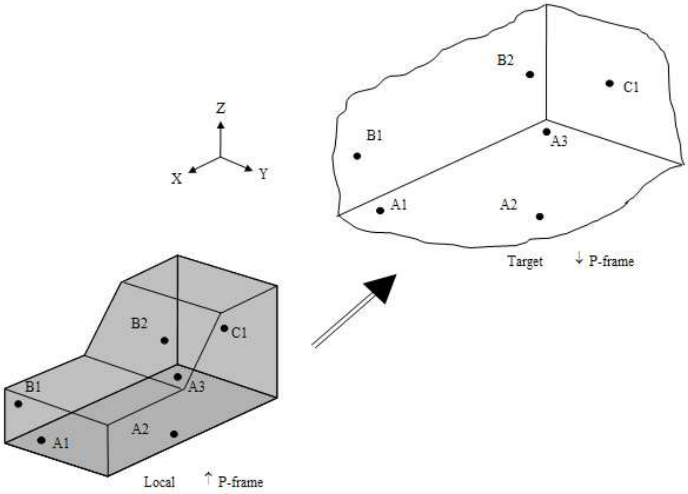


Figure 1.3: *Positioning of a part using 3-2-1 locating scheme, Söderberg and Lindkvist (30)*

- The master location points on mating parts shall, if possible, be positioned directly opposite each other.
- The master location points shall be selected so that the locating features permit accessibility of welding equipment and assembly equipment.
- The range of the master location points on a coordinate shall be as wide as possible.

A complete car is made up of many subassemblies. In every assembly step, it is crucial that the parts are joined with as good precision as possible. This is supported by a robust locating scheme, i.e. a positioning that suppresses variation in the resulting assembly. However, there is always variation between the local P-frame and the target P-frame, and this variation propagates through the assembly. When the assembly is measured, this variation will be detected. At this stage, it may though be a difficult task to identify the root cause of the variation. This is illustrated in the following example. Consider the assembly in Figure 1.4. It consists of two parts, and both parts are positioned using a hole and a slot. The parts are joined and finally measured. The inspection points are represented by arrows in Figure 1.4. The arrows indicate the evaluation direction. Hence, only the deviations in the indicated directions are determined. During the inspection process the assembly is positioned using hole P1 from Part 1 and slot P4 from Part 2. If there is variation in P4 during assembly this perturbation will result in a departure from nominal in the inspection points as shown in the figure. Considering inspection data only, it is not obvious what caused the deviation.

It is important to realize that if there is variation in P4, the only way to achieve a correct assembly is to reduce this variation. If there is a deviation in P4, there are two possible corrections opportunities. The first is to correct the position of P4. The second one is to compensate the deviation in P4 by moving the positions of the locators P1, P2 and P3.

1.3 The inspection process

To detect deviations and variations in parts and subassemblies it is necessary with a continuous control of the processes. The inspection data, used for this purpose, belong to one of two categories; ungrouped or grouped data. The ungrouped data, also called “one at a time”-data, come from inline measurements. An example of inline data can be seen in Figure 1.5. The measurements come from parts produced after each other. Every item

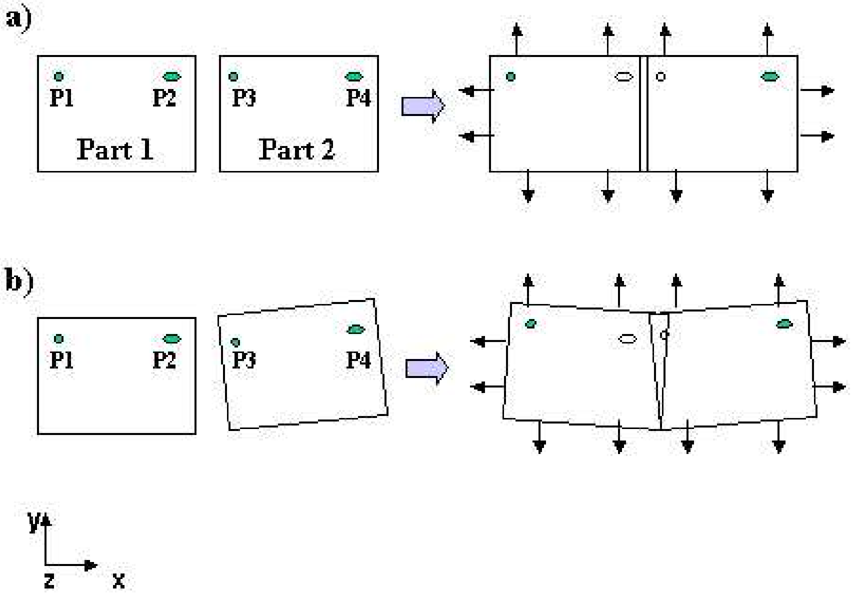


Figure 1.4: *An assembly consisting of two parts. In a) the parts are positioned correctly, in b) there is a perturbation in locator P4.*

produced in a line containing an inline measurement machine is measured as a stage in the production line.

The grouped data consist of samples of n items each. At Saab, a sample size of $n = 3$ is used. These samples are usually taken once or twice a week and the items are measured in coordinate measurement machines (CMMs). A CMM can be seen in Figure 1.6.

In Figure 1.7 an example of CMM data is given. Compared to the inline data in Figure 1.5, those data are sampled during a much longer period of time. There are often trends and long-term variation in a typical process. Much of this long-term variation is not included in the inline data, which are measured during a day or two, but can be seen in the plot of the CMM-data, that are collected during several months. In this example, a sample size of three observations is used. A larger sample size would of course give more accurate information about the process but this must be weighted against an increased cost.

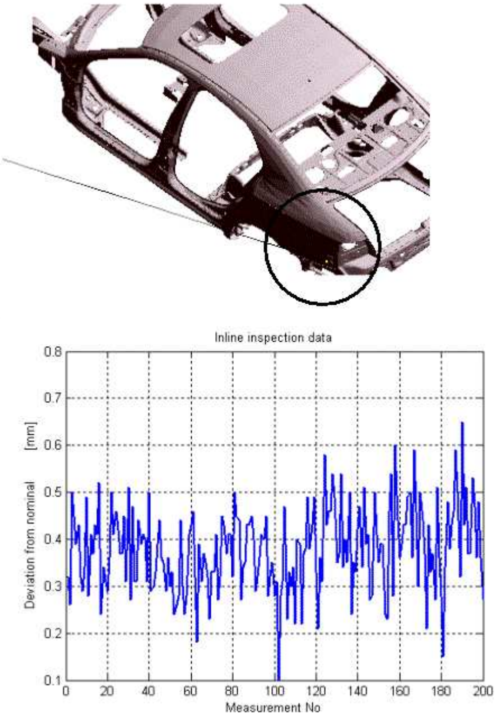


Figure 1.5: *In the top of the figure the inspection point in question is encircled. Below, an example of inline data for this inspection point is plotted. There are 200 items measured.*

The sampling frequency is a question related to the sample size. The frequency should depend on how quick the process may be expected to change, Montgomery (25). If the process may be assumed to vary quickly the sampling should be more frequent then if the process varies slowly. A process that is essential to the final product should be sampled more frequent then a process that only give a minor contribution to the final result. However, just as with the sample size, this issue is a question of balance between costs for inspection and costs for undetected changes in a process.

An inline measurement machine uses laser beams to measure possible

Figure 1.6: *A front fender is measured in a CMM.*

deviation from the nominal coordinates of a point. The measurements have a good precision, i.e. there is a high degree of conformity between independent measurements under the same conditions. The agreement between real value and the value given by the measurement machine, i.e. the accuracy, is lower than for a CMM. Inline measurements are though valuable since they give continuous information about the process. However, while every produced item is measured, it is too time consuming to measure as many points as in the CMM's. There is also some lack of accordance between the CMM and the inline measurements. The CMM is considered as the more reliable measurement device. It is important to be aware of the possible drift in the inline measurement machine and first and foremost use it as a tool for detecting increased short-term variation.

The inspection data is monitored using Statistical Process Control (SPC). SPC is a tool aimed at controlling and, hopefully, improving a process

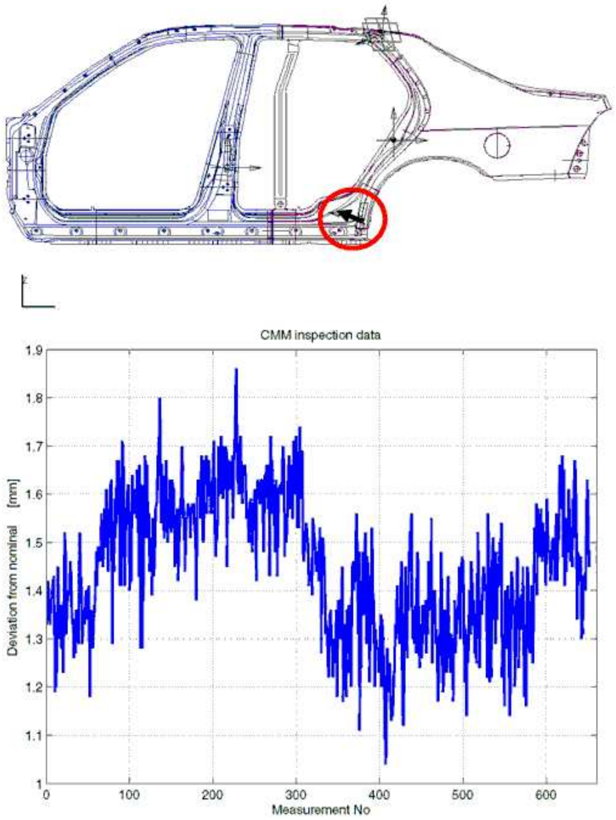


Figure 1.7: *In the top of the figure the inspection point in question is encircled. Below, CMM data for this point. There are 217 samples, where each sample consists of three objects.*

through statistical analysis of inspection data. More about SPC can be read in Chapter 2.

1.4 Finding root causes

Sampling, measuring machines and SPC-methods are tools for detecting deviations and variations in a process. When a variation is detected, it is essential to identify the cause of the problem. Sometimes the cause is obvious. Sometimes finding the variation source is a very demanding and time-consuming work, since variation propagates in a complex way during assembly. Today, much of this work is based on experience and a good process-knowledge. However, some problems may still be difficult to solve; an illustration of this was given in Figure 1.4. Further, merely depending on a small number of experienced problem-solvers makes the organization vulnerable. The methods for root cause analysis (RCA) presented in the following chapters are a set of tools for identifying fixture related causes of variations. Very concisely, the first step in RCA is to find a relation between variations in the P-frames of the parts and the resulting variations in the inspection points of the final subassembly. Using this relation, variation in inspection data can be translated to variation in one or more of the locators.

In Figure 1.8 a future RCA working procedure at Saab is outlined. If the SPC chart indicates increased variation and the reason of this phenomenon is unknown, then the user orders a root cause analysis. The sensitivity matrix A, containing product and process knowledge, is a part of the analysis and is calculated from a virtual model of the assembly. The RCA can be based on inline data or CMM data. The inline data is quickly available and is usually the first choice. However, since only a reduced number of points are measured here, that may not give enough information for a RCA. In that case, a RCA based on CMM data is performed. This gives usually a satisfactory result that is the base of an action to reduce the variation in the process. In some cases the method for RCA requires a modification to suit the current case, like excluding inspection points that not reflect the fixture related errors. RCA will be described more thoroughly in Chapter 3.

1.5 Geometrical variation

Geometrical variation in assembled products is a general problem in automotive industry. In this section, the causes and effects of geometrical variation, as well as different possibilities to reduce and handle the effects in different stages of the process development cycle, will be discussed.

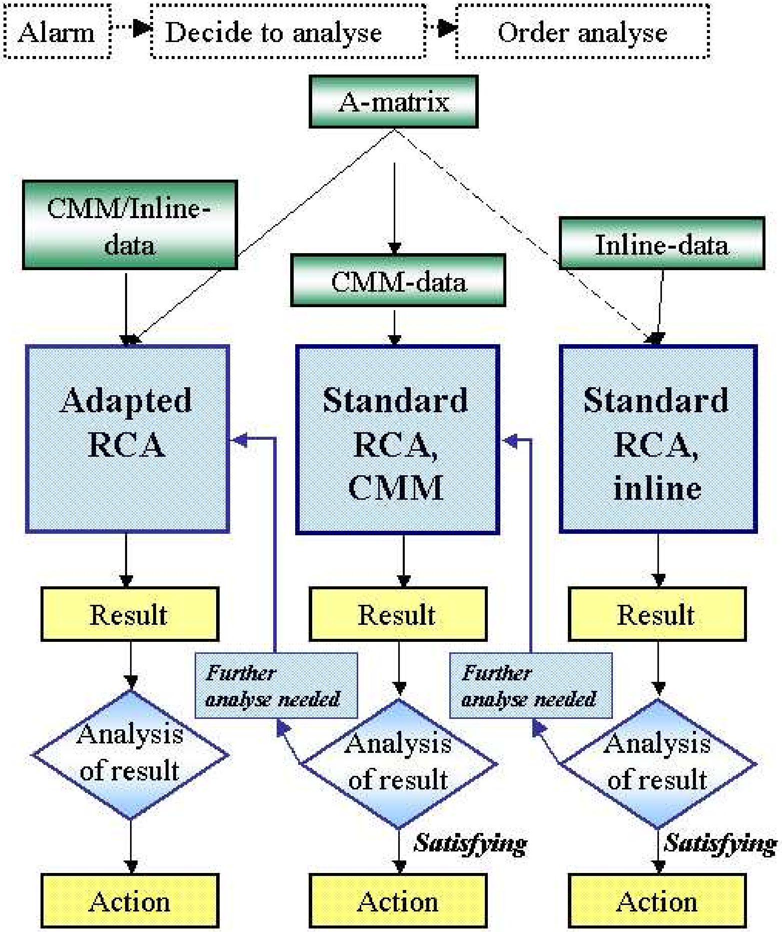


Figure 1.8: *An outline of a possible working procedure at Saab for RCA.*

1.5.1 Causes and effects of geometrical variation

Geometrical variation in parts and assembly process results in variation in size, shape and position of subassemblies or final products. This may lead to difficulties in assembling parts or products not fulfilling functional and esthetical requirements. In Figure 1.9 examples of areas that can be affected by geometrical variation are shown. To the left, variation in camber- and caster angles can affect the driving characteristics of the car. To the right, variation during assembly can give rise to non-nominal flush between for example lamp and applica. Problems caused by geometrical variation are often discovered quite late in the product development cycle, maybe during pre-production or even when the product and the process are prepared for full-scale production. A correction of the problem at this phase is often very costly and time-consuming.

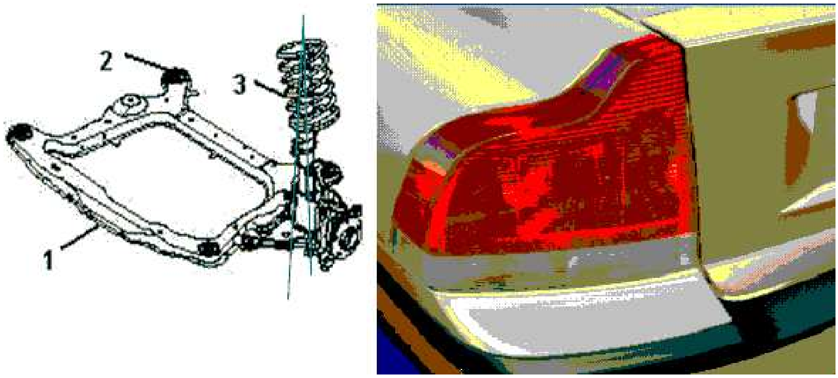


Figure 1.9: *Geometrical variation can cause problems with cumber/caster-angles and poor fit.*

There are usually a number of different sources of geometrical variation in key characteristics of the assembled product; variation in parts and assembly process is thought two major contributors, see Figure 1.10.

Geometrical variation is controlled by locating schemes and by tolerances. The locating schemes describe how parts are positioned during assembly and was described in the Section 1.2. The tolerances are allocated with respect to assembly sensitivity, process variation and cost.

1.5.2 How to minimize the effects of geometrical variation

The principles of this section are based on the results within the 3DTM project.

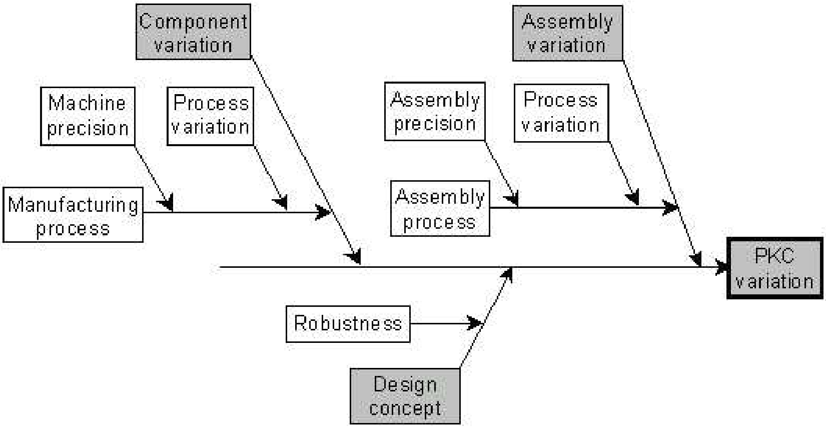


Figure 1.10: *Major sources of variation in a Product Key Characteristic (PKC) of an assembled product, Carlson et al. (7)*

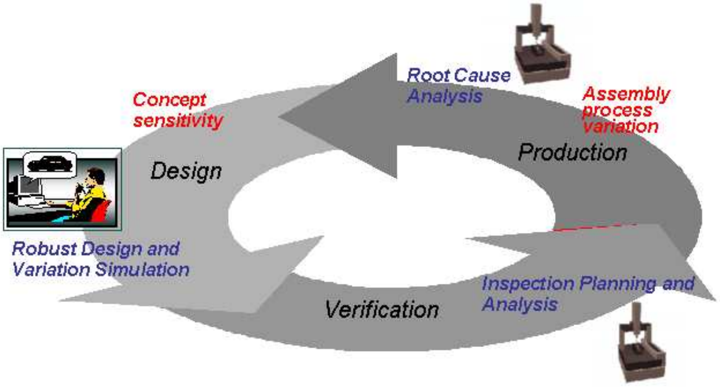


Figure 1.11: *Geometrical variation and tolerance management in a development cycle of products, Carlson et al.(7).*

The development cycle can be divided into three main parts; the design phase, the verification phase and finally the full production phase, see Figure 1.11.

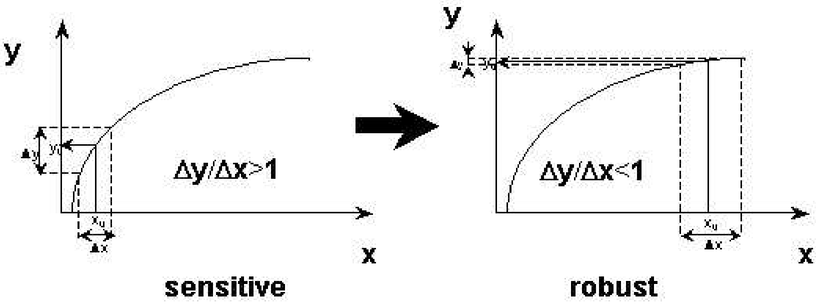


Figure 1.12: *A robust design is characterized by the fact that its important output characteristics are insensitive to disturbance, i.e. variation in input parameters, Söderberg and Lindkvist (31)*

During the first phase, the design phase, it is important to find a robust design concept. A robust design suppresses incoming variation, i.e. the design makes the variation in the output less than the variation in the incoming parts, see Figure 1.12 for an illustration. In order to find a satisfying design concept, it is usually necessary to test different concepts and evaluate their robustness and characteristics. One way of doing this is using prototypes and full-scale models. However, this is expensive and time-consuming. Further, it is not possible to try as many concepts as may be desirable. Using a virtual model is a much more effective way of testing different concepts. However, there are high demands on the software. It must of course be user friendly and offer suitable analysis tools. Further, it is interesting to examine the difference in perception of virtual and physical models. Wickman and Söderberg (37) showed that usually the physical model is experienced as better than an equivalent virtual model, analysing physical requirements.

Within the 3DTM research project, a software called *Robust Design and Tolerancing* (RD&T) is developed. This software offers different types of analysis of an assembly, like

- Stability analysis: Evaluates geometrical robustness and degree of coupling.
- Variation analysis: Statistical analysis of variation in critical dimensions.
- Contribution analysis: Ranking of variation contributors.

The analyses show how chosen key characteristics are affected by different tolerances and perturbations in the locating scheme. These facilitate the design of a robust concept and the allocation of tolerances with respect to assembly sensitivity, process variation and cost. Using the contribution analysis it is also possible to get a ranking list of the tolerances contribution to the variation in a chosen point. This ranking list is helpful if the variation in a specific point must be reduced. These analyses are illustrated on the assembly shown in Figure 1.13. It is a rear wheelhouse that is assembled in two stages. In the first station three reinforcements (labelled 1,2 and 4) are put together with the wheelhouse panel (labelled 3). In the next step, this subassembly is moved to another station. In this station the subassembly is positioned using the same locators that were used to hold the panel in the first station. The support for the parcel shelf (labelled 1 in the right part of the figure) is put together with the subassembly, and finally the complete assembly is measured in an inspection station. During inspection the assembly is again positioned using the locators of the

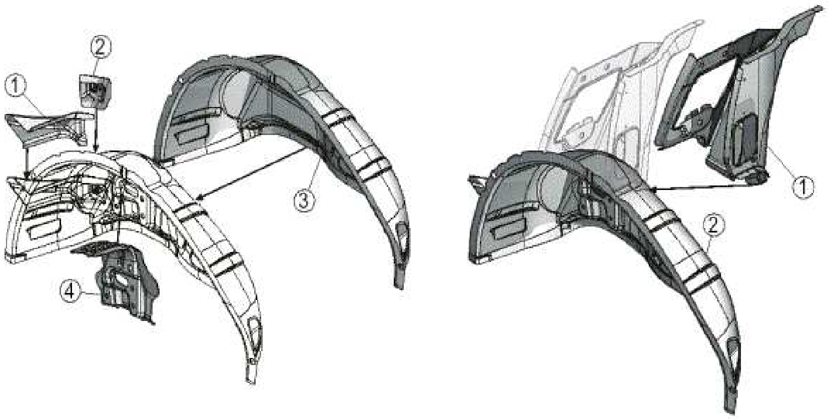


Figure 1.13: *The rear wheelhouse consists of five parts. The parts are assembled in two stages.*

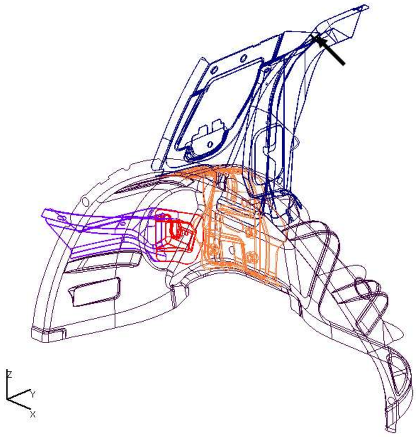
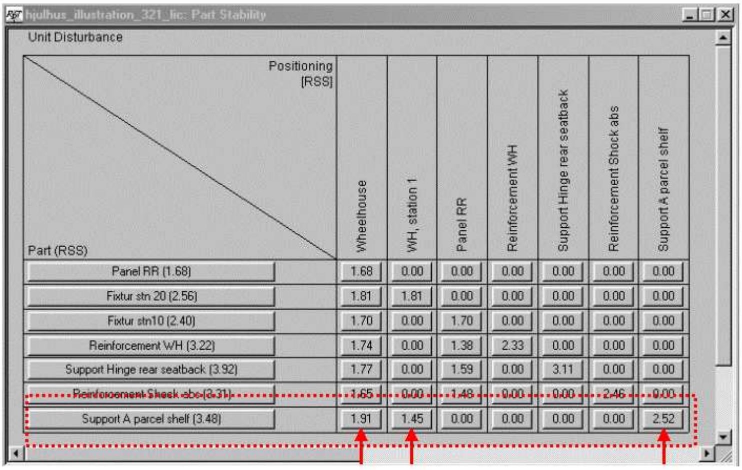


Figure 1.14: *The position of the inspection point labelled "Mea14" is illustrated by an arrow.*

wheelhouse panel. This assembly is analysed using the different kinds of analysis available in RD&T. For the variation and contribution analyses an inspection point called "Mea14", located on the upper part of the parcel shelf support, is utilized. The exact position of "Mea14" is illustrated in Figure 1.14.

The stability analysis for the wheelhouse is shown in Figure 1.15. The stability matrixes reflect the robustness and the degree of geometrical coupling in the assembly. The matrix elements relate the input columns, the P-frames, to the output parameters, the parts. A high value of a matrix element indicates that the input P-frame has a high influence on the part position. The value shown in the matrix is the root sum square (RSS) value of the six individual points of the P-Frame. For stability analysis, the only information needed is the nominal position of locators for parts and fixtures. Therefore, this analysis is a usable tool in the early design phase.

On the last row of the stability matrix shown in Figure 1.15, the degree of robustness for the parcel shelf support is shown. The measured position of this part is depending on the positioning of the wheelhouse in the inspection station, on the positioning of the subassembly from station one in



Unit Disturbance

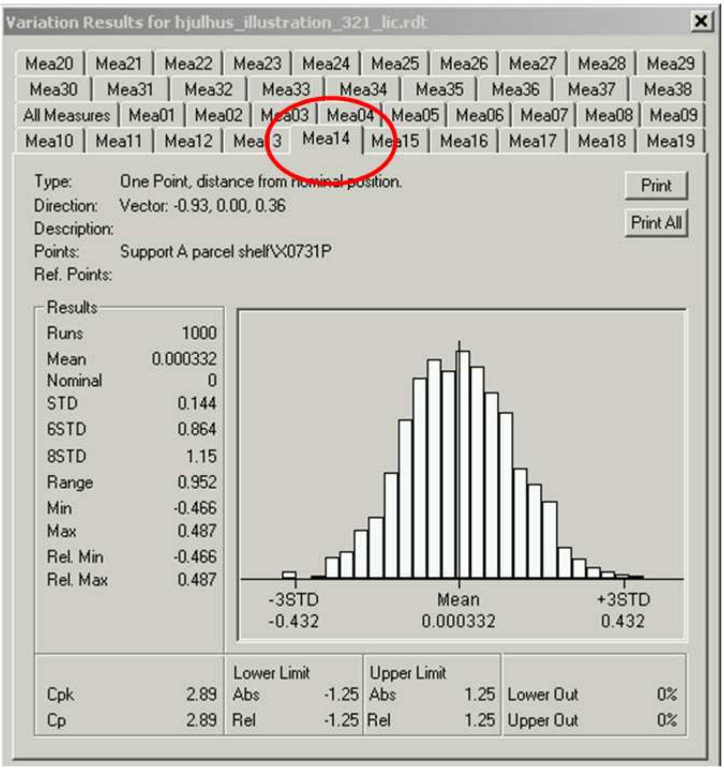
Part (RSS) \ Positioning [RSS]	Wheelhouse	WH, station 1	Panel RR	Reinforcement WH	Support Hinge rear seatback	Reinforcement Shock abs	Support A parcel shelf
Panel RR (1.68)	1.68	0.00	0.00	0.00	0.00	0.00	0.00
Fixtur stn 20 (2.56)	1.81	1.81	0.00	0.00	0.00	0.00	0.00
Fixtur stn10 (2.40)	1.70	0.00	1.70	0.00	0.00	0.00	0.00
Reinforcement WH (3.22)	1.74	0.00	1.38	2.33	0.00	0.00	0.00
Support Hinge rear seatback (3.92)	1.77	0.00	1.59	0.00	3.11	0.00	0.00
Reinforcement Shock abs (3.31)	1.65	0.00	1.48	0.00	0.00	2.46	0.00
Support A parcel shelf (3.48)	1.91	1.45	0.00	0.00	0.00	0.00	2.52

Figure 1.15: *Stability analysis, rear wheelhouse.*

stage two when the parcel shelf support is assembled, and of course, on the positioning frame for the part itself.

A variation analysis of the wheelhouse assembly is shown in Figure 1.16. The variation analysis uses Monte Carlo simulation technique to analyse variation in specified points. Tolerances for contacts between parts and fixtures are chosen by the user. Figure 1.16 shows the simulation results for the inspection point "Mea14", which position was illustrated in Figure 1.14. The specification limits for this inspection point are set to 0 ± 1.25 mm. The variation analysis shows the mean value, standard deviation, capability index et cetera for the simulations. These results give an indication on how well tolerance demands can be satisfied. In this case there are high capability indices; $C_p = C_{pk} = 2.89$, and these tolerances will most likely not cause problems in production.

The result of the contribution analysis of the wheelhouse is shown in Figure 1.17. The same inspection point as in the variation analysis is considered. The contribution analysis presents a ranked list of all points and tolerances contributing to measure variation. This analysis may be used in the work of optimising the selection of tolerances, and for trouble shouting during production. In this case, the locator A2 on the panel is the major



Type: One Point, distance from nominal position.
Direction: Vector: -0.93, 0.00, 0.36
Description:
Points: Support A parcel shelf\X0731P
Ref. Points:

Results

Runs	1000
Mean	0.000332
Nominal	0
STD	0.144
6STD	0.864
8STD	1.15
Range	0.952
Min	-0.466
Max	0.487
Rel. Min	-0.466
Rel. Max	0.487

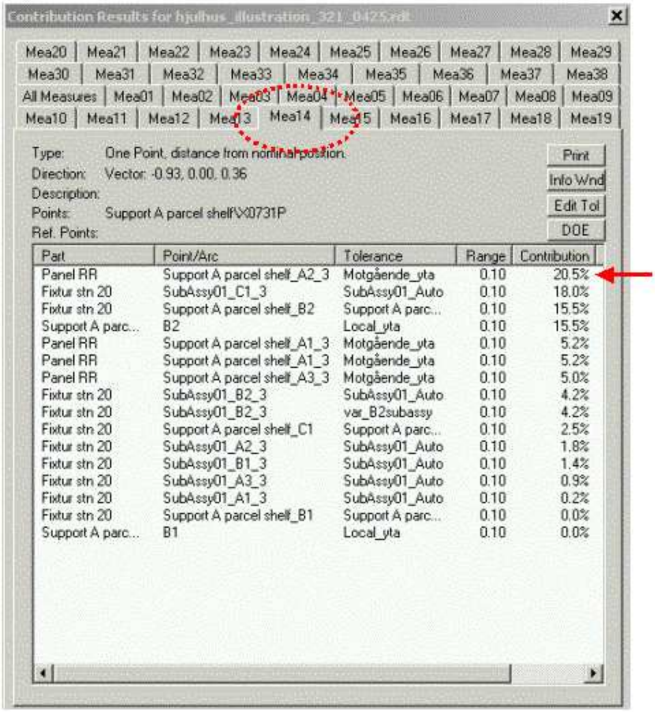
		Lower Limit		Upper Limit			
Cpk	2.89	Abs	-1.25	Abs	1.25	Lower Out	0%
Cp	2.89	Rel	-1.25	Rel	1.25	Upper Out	0%

Figure 1.16: *Variation analysis, rear wheelhouse.*

contributor, since this locator give rise to 20.5% of the variation in this inspection point.

RD&T and the different analyses available in the program are further described by Lindkvist and Söderberg (24) and Söderberg and Lindkvist (30).

When a satisfactory design concept is chosen, the verification phase starts, see Figure 1.11 on page 13. During this stage, the design concept will be confirmed through tests and different pre-production series. It is important to keep this phase as short as possible. Today new car models are launched frequently and a requirement for doing this is short verification phases. Important activities at this stage are inspection planning and



Type: One Point, distance from nominal position.
Direction: Vector: -0.93, 0.00, 0.36
Description:
Points: Support A parcel shelf\X0731P
Ref. Points:

Part	Point/Arc	Tolerance	Range	Contribution
Panel RR	Support A parcel shelf_A2_3	Motgående_yta	0.10	20.5%
Fixtur stn 20	SubAssy01_C1_3	SubAssy01_Auto	0.10	18.0%
Fixtur stn 20	Support A parcel shelf_B2	Support A parc...	0.10	15.5%
Support A parc...	B2	Local_yta	0.10	15.5%
Panel RR	Support A parcel shelf_A1_3	Motgående_yta	0.10	5.2%
Panel RR	Support A parcel shelf_A1_3	Motgående_yta	0.10	5.2%
Panel RR	Support A parcel shelf_A3_3	Motgående_yta	0.10	5.0%
Fixtur stn 20	SubAssy01_B2_3	SubAssy01_Auto	0.10	4.2%
Fixtur stn 20	SubAssy01_B2_3	var_B2subassy	0.10	4.2%
Fixtur stn 20	Support A parcel shelf_C1	Support A parc...	0.10	2.5%
Fixtur stn 20	SubAssy01_A2_3	SubAssy01_Auto	0.10	1.8%
Fixtur stn 20	SubAssy01_B1_3	SubAssy01_Auto	0.10	1.4%
Fixtur stn 20	SubAssy01_A3_3	SubAssy01_Auto	0.10	0.9%
Fixtur stn 20	SubAssy01_A1_3	SubAssy01_Auto	0.10	0.2%
Fixtur stn 20	Support A parcel shelf_B1	Support A parc...	0.10	0.0%
Support A parc...	B1	Local_yta	0.10	0.0%

Figure 1.17: *Contribution analysis, rear wheelhouse.*

measurement machine programming.

In the last phase in Figure 1.11 the production starts and during this phase it is important to monitor and control the process in order to detect offsets and variations, which may result in non-conforming products and big costs. A suggestion of a system for detecting such problems is given in Chapter 2. Of course, it is also necessary to identify the root cause of a detected problem. Methods for finding root causes of geometrical variation will be considered in Chapter 3.

Chapter 2

A System for Acceptance Quality Control

2.1 Introduction

Companies with complex production systems and products have often problems with process variation affecting the products key characteristics. Traditionally, a control chart is used to detect if the process is in statistical control and therefore should be left alone, or if there are reasons for process adjustments. However, some of this variation is very difficult to eliminate at a reasonable cost. In this chapter we propose a quality system that allows for trends in the processes as long as the produced items are within specifications. Industrial data are used to illustrate the different charting methods.

2.1.1 Outline

The outline of the chapter is as follows. A background to the topics discussed is given in Section 2.1.2. In Section 2.2 problems related to a system for quality control are concidered. Methods proposed to solve those problems for different types of data are described in Section 2.3. This is followed by a discussion about when to use an acceptance control chart instead of a traditional chart in Section 2.4. Finally, the conclusions can be found in Section 2.5. In the Appendix an overview of some frequently used control charts and capability indices are given.

2.1.2 Background

During the car manufacturing process, many parts are joined together. The geometries of the resulting subassemblies are controlled by measuring deviations from nominal values in a set of inspection points. Often, many inspection points on each subassembly are utilized to give a good understanding of the assembly process. The inspection data is grouped or ungrouped.

The demands on the quality system are that the charts will be easy to interpret and the same kind of chart will be used for all inspection points belonging to the same category (grouped or ungrouped data). Further, the estimates required will be calculated in the same way for all data belonging to the same category. This is necessary since there is a great number of inspection points to which the charts will be applied and it is far too time consuming to find special solutions for every point. There are also many different users of the charts, and not all users are aware of the characteristics of different estimates and charting methods.

In a typical process, there are trends and cycles that result in variation in the mean value. Some of this variation is very difficult to eliminate at a reasonable cost. This variation may correspond to seasonal variations in temperature, different workers, different batches of raw material and also some unknown factors. For each inspection point there is an upper specification limit (USL) and a lower specification limit (LSL). The tolerance limits are product rejection/adjustment limits, so it is vital that the produced items are within the specified limits. As long as that is fulfilled, the group means may be allowed to vary over time. Of course, it is always good to keep the process in control and to improve the process. This is illustrated by Taguchi's loss function, see Figure 2.1. Taguchi (33) claims that every deviation from the target represents a loss, and the size of the loss is increasing with the size of the deviation. However, if the resources are limited, the first priority is to produce items within the specification limits. That means that under these circumstances, an acceptance chart may be preferable over a usual control chart.

So, the traditional control charts might not always be the best choice when the resources are limited. This issue, with the belonging questions about what acceptance charts to use, is partially discussed by Woodall (38). One of the methods discussed is pre-control. Pre-control is based on the tolerance limits and means that the range of the tolerance limits is divided into four parts of equal length. The middle two parts constitute the green zone, the outer two parts are the yellow zone and the area outside the tol-

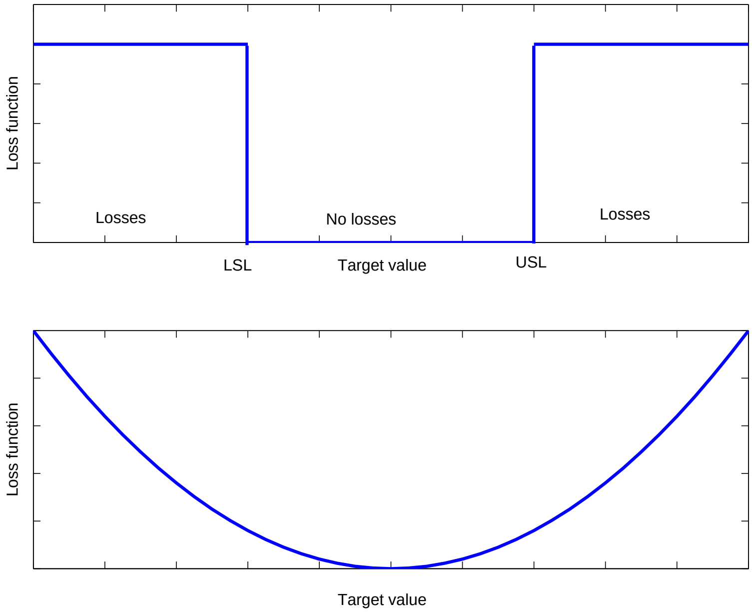


Figure 2.1: *Above: A usual way of thinking. All items within specification are considered equal. Below: Taguchi's loss function.*

erance limits is called the red zone. The process is allowed to operate as long as the inspection data do not fall into the red zone or into the yellow zone too often. There is a range of sampling and decision rules to ensure this. Woodall (38) among others, points out that the pre-control believers promote the idea with a lot of exaggeration. However, a pre-control chart gives no information about the statistical control of a process.

There are other methods beside pre-control that are based on the tolerance limits and can be used for controlling the mean value of a process. These so called acceptance control charts may be used when the process has a high capability. Consider a normally distributed variable with expected value μ and variance σ^2. The idea is to allow the mean value, $\bar{x}$, to vary over an interval $(\mu_{lower}, \mu_{upper})$, such as the fraction non-conforming produced items is at most δ, see Montgomery (25). Further, it is desirable to have a probability α of a type I error, i.e. a false alarm. This is achieved by using an upper control limit

$$UCL = USL - (Z_\delta - \frac{Z_\alpha}{\sqrt{n}})\sigma,$$

and a lower control limit

$$LCL = LSL + (Z_\delta - \frac{Z_\alpha}{\sqrt{n}})\sigma.$$

A quantity $Z_p\sigma$ is a value such that $p = 1 - \Phi(Z_p)$, where $\Phi(x)$ is the value of the standard normal cumulative distribution at the point x. By using these limits the chart gives an alarm when the mean value is so close to a tolerance limit that the expected fraction non-conforming exceeds δ. The positions of the UCL and USL are illustrated in Figure 2.2.

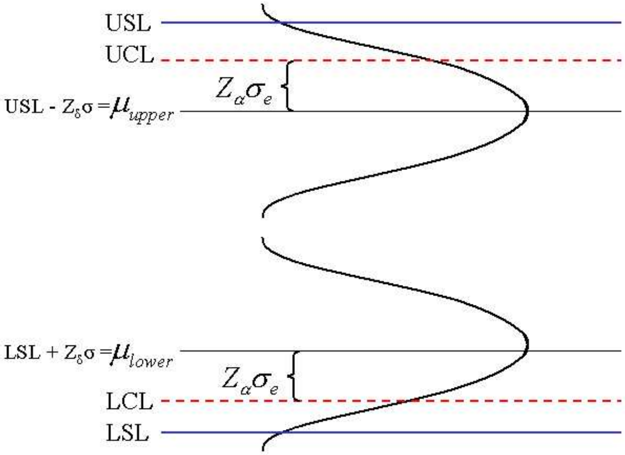


Figure 2.2: *Positions of control and specification limits in an acceptance chart for controlling mean value of a process, related to μ_{upper} and μ_{lower}, the largest respective smallest permissible value of μ. The notation $\sigma_e = \sigma/\sqrt{n}$ is used.*

Chang and Gan (9) use the same method as Montgomery, but express the fraction non-conforming as a capability value. They test the method on data from an integrated circuit assembly and calculate the power function for a one-sided acceptance control chart.

Considering ungrouped data, an EWMA-chart is a better alternative for controlling the mean value than the $\bar{x}$-chart, since the EWMA-chart is creating a group-structure in data, making the chart more sensitive to small deviations. A brief overview of the EWMA-chart is given in the Appendix. An acceptance control chart based on the usual EWMA-chart is considered by Holmes and Mergan (15). The principles are the same as for the acceptance chart for $\bar{x}$. Using the same notations as before, the process mean value, $\bar{x}$, is allowed to vary between μ_{lower} and μ_{upper}, such as the fraction non-conforming produced items is at most δ. If the probability of type I error is α, the control limits are given by

$$UCL = USL - Z_\delta \sigma + Z_\alpha \sigma_m,$$

and

$$LCL = LSL + Z_\delta \sigma - Z_\alpha \sigma_m.$$

The quantity σ_m is the standard deviation of m, the weighted exponentially moving average, and can be expressed as

$$\sigma_m = \sqrt{\frac{p}{2-p}}\sigma,$$

where σ is the standard deviation of the originally variables and p is the amount of weight put to the current value in the EWMA-chart. Holmes and Mergan (15) illustrate the method using simulated data.

2.2 Problem

This section describes problems related to quality control and gives motives for a new system for acceptance control, based on as well traditional control charts as the acceptance control charts introduced in the previous section.

The manufacturing process is often quite complex with many sources of variations. Since many of the causes of variation depend on long-term, but recurring, external conditions, operators and other unknown factors the data contains trends, see Figure 2.3 for an example. Those trends must be taken into consideration when the quality control system is designed.

The terms within-group variation and between-group variation are used to describe different kinds of variation for grouped data. The within-group variation is the variation in each sample, while the between-group variation can be seen as a factor determining the locations of the group means. In Figure 2.4 the group means are plotted together with the individual observations. The sizes of the group ranges, illustrated for the first three samples

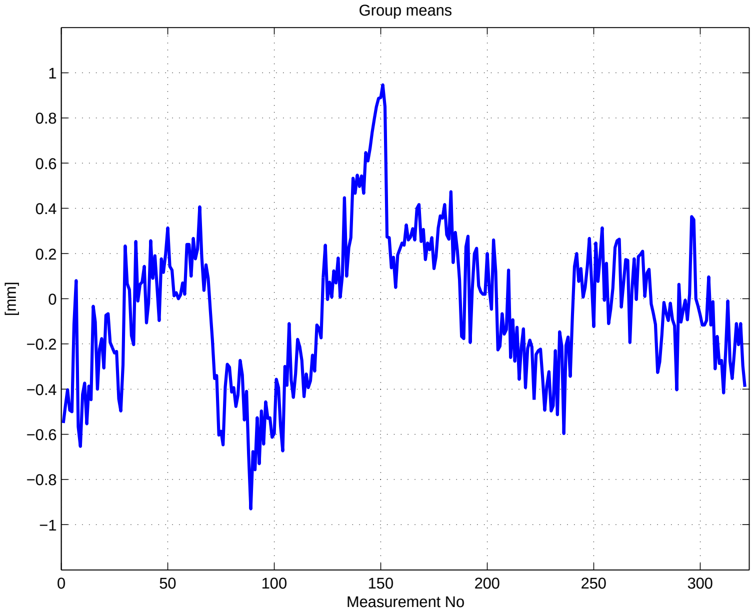


Figure 2.3: *The sample group means for a typical process. Deviation from nominal value are measured*

in the figure, are connected to the within-group variation. The group means differ though more than what can be expected due to within-group variation. Those are namely also affected by the between-group variation caused by external conditions, operators et cetera. If the data are ungrouped, the corresponding terms short-term variation and long-term variation are used.

Another problem is that the inspection data in the automotive industry are often of two different categories; ungrouped or grouped data. The ungrouped data originate from inline measurements. Every produced item is measured in the inline measurement machine as a step in the production. The grouped data consists of samples of n items each measured offline in coordinate measurement machines (CMM's).

The reason for using both inline- and offline-inspection is that the two approaches complement each other. The inline-measuring machine is fast enough to measure every produced item and it gives therefore a very good picture of the process. It is designed for detecting variations in the process quickly. The precision, i.e. the repeatability, in the inline measurement is

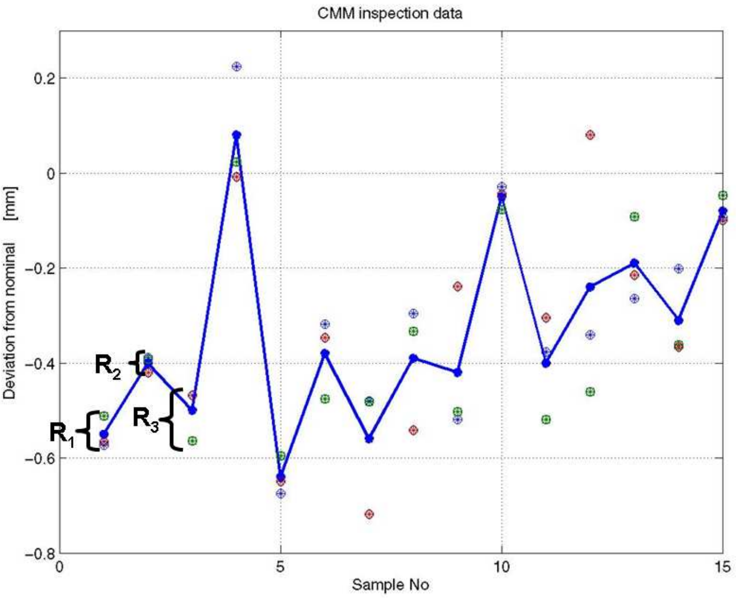


Figure 2.4: *The groupmeans are plotted together with the individual observations.*

good. There may though occur long-term variation in the measurement machine and the measured value may not be in agreement with the true value, i.e. the accuracy is not so good. Therefore, the inline measurement machine is best suited for controlling the short-term variation. The CMM on the other hand, has a very good accuracy. The precision of the CMM is also good. Since the CMM is used offline, there is also time to measure more inspection points on each item then during the inline-inspection.

At Saab Automobile AB, the sample size is $n = 3$, and the samples are usually taken once or twice a week. Each inspection point has an upper and a lower tolerance limit, and today these tolerances are the basis for the process control. The alarm limits for the mean value are set to 70% of the tolerance limits. The same method is used for the range; the range must not exceed 70% of the tolerance width. If these requirements are not met, there is an alarm. These alarms initiate fault localization and possibly a correction of the process.

There are many arguments against process control based on tolerance limits. Since the tolerances do not always reflect the characteristics of the process, problems may occur. In all kind of process control, there are two types of mistakes. The first one, the Type I error, is to take action when the process has not changed. The second one, called Type II error, is to not take action despite a change has occurred. The tolerance limits are set due to functional and design requirements and of course, also due to process performance. Despite this, the tolerance limits are not related to the mean and variance of the process in such a way that the probability of the different types of errors can be controlled. Nevertheless, the tolerance limits are a very important factor in a quality control system.

Consider a control chart based on the tolerance limits. If the capability of the process is low, the probability of type I error will be high. If an already centered process is adjusted due to these false alarms, the process will perform even worse and even more items outside tolerance will be produced. For a low capability process, a traditional control chart is the best chart in order to analyse and improve the process.

On the other hand, when the capability of the process controlled by using tolerance limits is high, the probability of type I is small and the probability of type II error is high. A type II error is though preferable to the type I error and might not be a major concern, as long as the process is capable and produces conforming products. Actually, it might be desirable to avoid alarms as long as the produced items are well within the specifications, in order to cut down the costs. One way of handle this is to use acceptance control charts, mentioned in the introduction. Such a chart allows trends and variation, provided the produced items are within specifications.

To summarize these thoughts, it would be desirable to have a system for quality control that consists of two different types of chart. Traditional control charts should be used to improve and control low capability processes. This kind of chart helps bringing the process in statistical control and can also be used as a tool for reducing trends and variations in the process. For a high capability process on the other hand, eliminations of these trends are not always economically justifiable. In those cases, an acceptance control chart is suitable, since it allows variation provided that the produced items are well within specifications.

2.3 Proposed methods

As mentioned, there are two different kinds of data, grouped and ungrouped measurements. In the grouped data the group structure is utilized for increasing the sensitivity of the chart. For ungrouped data some kind of artificial group structure is usually created. Therefore, different charting methods are used for these two categories of data. Further, charts for controlling the mean value can be of two types; a traditional $\bar{x}$-chart if the process has a low capability or an acceptance control chart if the process capability is high. In this section all those different types of charts are described. Range-charts for controlling the variation in the process are also discussed for the different types of data.

2.3.1 Grouped data

For control of the mean value of the process, there are two different alternatives, depending on the capability of the process. If the capability is low, it is important to improve the process and avoid Type II errors. In this case, a traditional control chart is used. If the capability is high, some trends may be allowed, provided the produced items are within the specification limits. In this case, some kind of an acceptance chart may be used.

Traditional control charts

This alternative is suitable for a process with low capability and is a tool for improving the process.

The grouped data available for estimating the parameters of the process consist of k samples of size n. In the examples, $n = 3$. Let x_{ij} be the j:th observation in the i:th sample for $i = 1, 2, ..., k$ and $j = 1, 2, ..., n$. For each group the sample group mean,

$$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$$

and the sample group variations

$$s_i^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \bar{x}_i)^2$$

are calculated. The within-group variation σ_w^2, is estimated by the mean of the group variances:

$$s_w^2 = \frac{1}{k}\sum_{i=1}^{k} s_i^2.$$

Further, the total sample mean $\bar{\bar{x}}$ and the sample variation of the group means, s_B^2, is determined in the following way:

$$\bar{\bar{x}} = \frac{1}{k}\sum_{i=1}^{k}\bar{x}_i$$

$$s_B^2 = \frac{1}{k-1}\sum_{k-1}^{k}(\bar{x}_i - \bar{\bar{x}})^2.$$

The estimated variation of the group means, s_B^2, will contain contributions from both within-group variation, σ_w^2, and between-group variation, σ_B^2, since

$$V(\bar{x}_i) = \sigma_B^2 + \sigma_w^2/n.$$

The process is controlled by a $\bar{x}$-chart and a range-chart. The control limits of the $\bar{x}$-chart are usually given by

$$CL = \bar{\bar{x}} \pm 3\frac{s_w}{\sqrt{n}}.$$

If it would be desirable to allow between group variation the following control limits can be used, Wetherill and Brown (36),

$$CL = \bar{\bar{x}} \pm 3s_B.$$

By using these control limits as well within-group variation as between-group variation are permitted. Using this kind of control limits allow for trends, but unlike the acceptance control charts, the specification limits are not taken into consideration. Therefore, this chart does not necessarily alarm, even if the trends cause the products to be out of specifications.

The estimates should be based on data representing a satisfying part of the process. It is necessary that the data is representative for the process and covers a period long enough to reflect the behaviour of the process.

If this procedure leads to control limits close to, or even outside, the tolerance limits the cause of this must be examined. There are two possible reasons; one is that the process is not centred in the tolerance band and the other is that the variation is too big compared to the tolerance width. If the problem is due to offset, the consequences of this offset must be examined. If the offset does not affect the product negatively, it will usually be accepted, and the tolerance limits will be updated. Otherwise, it must be corrected. If the problem is due to variation, actions to reduce this variation should be taken. Of course, this problem may have been caused by a temporary deviation in the process and in such case it is not appropriate to include these data in the parameter estimation of the control chart limits.

It is also necessary to control the within-group variation for a sample. To do that a range-chart can be used. In the range-chart the group ranges are plotted. The upper control limit is given by

$$UCL = D_1 s_w$$

and the lower control limit, if such one is used, is given by

$$LCL = D_2 s_w.$$

The values of the constants D_1 and D_2 depend on n, the number of observations in each group, and can be found in for example Wetherill and Brown (36). When $n = 3$, $D_1 = 0.06$ and $D_2 = 5.06$. There is no relation between specification limits and the range of a group. Further, the ranges are not affected by trends or between-group variation.

Acceptance control chart for mean value

Another alternative for controlling the mean value is to use some kind of acceptance control chart, if the process has a high capability. The benefit is that alarms are avoided when the items produced are far enough from the specification limits. The alarm limits for an acceptance chart, described in the introduction, are based on the maximal fraction non-conforming units, δ, that can be tolerated. The fraction non-conforming corresponds to the process capability. In the automotive industry $C_p > 1.33$ is often used as a target. This corresponds, as the following calculations show, to a fraction non-conforming $\delta < 6.61 * 10^{-5}$ if the process is centred, i.e. $C_p = C_{pk}$, since

$$\frac{USL - \mu}{\sigma} = \frac{\mu - LSL}{\sigma} = 3 * 1.33$$

and

$$\begin{aligned} P\{non-conforming\} &= 1 - P\{LSL < X < USL\} = \\ &= 1 - \{\phi(\frac{USL - \mu}{\sigma}) - \phi(\frac{LSL - \mu}{\sigma})\} = \\ &= 1 - \{2\phi(3 * 1.33) - 1\} = 6.61 * 10^{-5}. \end{aligned}$$

This fraction of non-conforming items corresponds in a one-tailed distribution to $Z_\delta = 3.82$.

The control limits for an acceptance chart for the mean value, described in Section 2.1, are given by

$$UCL = USL - (Z_\delta - \frac{Z_\alpha}{\sqrt{n}})\sigma$$

and

$$LCL = LSL + (Z_\delta - \frac{Z_\alpha}{\sqrt{n}})\sigma.$$

The probability of Type I error is determined by α, and a usual choice is $\alpha = 0.0013$. This value corresponds to $Z_\alpha = 3$. The standard deviation σ is estimated by the within-group standard deviation, s_w. The within-group variation is used since the mean value is allowed to vary within an interval $(\mu_{lower}, \mu_{upper})$, such that when $\mu = \mu_{upper}$ or $\mu = \mu_{lower}$, the fraction non-conforming is δ. The position of μ is examined for each group and for the observations in a specific group the variance is given by σ_w^2. When using this chart, it is important that the variation σ_w^2 of the process is in control, this is examined by using a range-chart.

When an acceptance chart is used, there is no alarm if the process is out of control, as long as $\mu_{lower} < \mu < \mu_{upper}$. This is a way to reduce costs, since when μ belongs to this interval the probability of producing a non-conforming item is less then δ. However, if $\mu = \mu_{upper}$ or $\mu = \mu_{lower}$ the chart is designed to detect an increase or decrease of the process mean. Hence, an alarm is always the result of the process being out of control, on the other hand there is not an alarm every time the process is out of control. In other words, the probability of Type II error is big when the process mean is in the interval $(\mu_{lower}, \mu_{lower})$, because then it is not desirable with an alarm, since the fraction nonconforming units is very small. But when the fraction non-conforming units increase, i.e. the capability decreases, the probability of Type II error decreases. This is illustrated using a power function, see Figure 2.5. The power is defined as $1 - \beta$, where β is the probability of Type II error, i.e. the power is the probability of detecting a change in the process.

In Section 2.4 there are examples of as well traditional $\bar{x}$-chart as acceptance control charts.

2.3.2 Ungrouped data

The ungrouped data origin from an inline measurement machine, i.e. all produced items are measured. Just as with the grouped data, it is possible

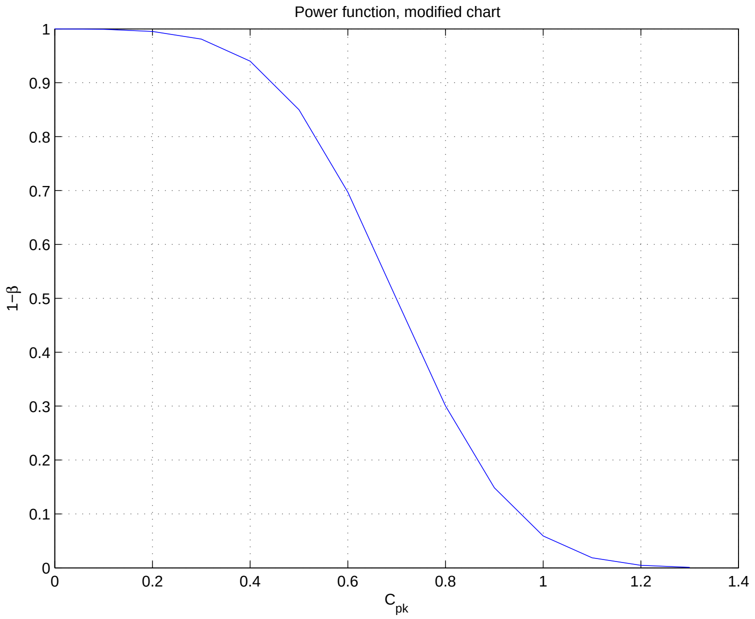


Figure 2.5: *Power function for an acceptance chart. When the capability is high, the probability of an alarm for a change in the process is small.*

to use traditional charts when the capability of the process is low and acceptance charts for controlling the mean value of high capability processes.

Data consist of a number of individual observations x_i. The estimates of necessary parameters are based on n observations.

Traditional control charts

To control the short-term variation in the process a moving-range chart is used. In the chart the range of a small number of consecutive measurements is plotted. The upper control limit is given by

$$UCL = D_3 * s.$$

The constant D_3 depends on the number of observations included in the moving range, and is tabulated by for example Wetherill and Brown (36). In order to control the short term variation, the moving range is usually based on two successive measurements. In that case, $D_3 = 4.65$. The sample standard deviation, s, is calculated from data using a moving range method. This is done by calculating the moving ranges, R_j, for groups of size k, determine the mean range and then divide this number by a constant depending on the value of k, i.e.

$$s = \frac{\sum_j R_j}{\#ranges}/d_k,$$

where d_k is a constant depending on the size of the moving ranges and can be found in tables in for example Wetherill and Brown (36). Often, $k = 2$ is used. By using $k = 3$ slightly more variation than what could be expected in the moving ranges in the chart is permitted. If $k = 2$, then $d_k = 1.128$ and if $k = 3$, then $d_k = 1.683$.

It may also be desirable to use a chart for the mean value. However, the main potency of an inline measurement machine is to control the short-term variation in the process. If control of the mean value is required, then some kind of moving average chart should be used in order to enhance the sensitivity of the chart. An EWMA-chart is often a good choice, since compared to an ordinary moving average chart, more weights are paid to the last observations than to the earlier ones.

In an EWMA-chart

$$m_i = px_i + (1-p)m_{i-1}$$

is plotted. The constant p is the weight given to the most recent observation. A usual choice is $p = 0.4$. The control limits are given by

$$\bar{x} \pm A_1 s.$$

If $p = 0.4$ is used, then $A_1 = 1.545$ for ungrouped data. Values of A_1 for different values of p can be found in for example Wetherill and Brown (36). The sample standard deviation, s, is calculated by the moving range method. For the grouped data, we noticed that it sometimes may be desirable to allow between group variation. For the ungrouped data this corresponds to allowing-long term variation and the estimate of the standard deviation should in that case be based on all data. To calculate s^2 in that case, the following formula is used:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

Acceptance chart using EWMA

Just as with the grouped data, it is possible to use traditional charts when the capability of the process is low and an acceptance chart for controlling the mean value of a high capability process. The benefit is that alarms are avoided when the items produced are far enough from the specification limits. The alarm limits for an acceptance EWMA-chart, described in the introduction, are based on the maximal fraction non-conforming units, δ, that can be tolerated. Just as in the case with the $\bar{x}$-chart the maximal fraction non-conforming products, δ, is set to $6.33*10^{-5}$, which corresponds to capability of 1.33. The probability of false alarm, α, is 0.0013.

The control limits are given by

$$UCL = USL - Z_{\delta}\sigma + Z_{\alpha}\sigma_m,$$

and

$$LCL = LSL + Z_{\delta}\sigma - Z_{\alpha}\sigma_m,$$

where

$$\sigma_m = \sqrt{\frac{p}{2-p}}\sigma,$$

and σ is the standard deviation of the originally variables. The standard deviation σ is estimated by the moving range method described in the previous section. In Figure 2.6 are acceptance and traditional EWMA-charts plotted for an inspection point with high capability. The acceptance chart gives no alarms, since the plotted values are far enough from the specification limits. The traditional chart alarms, indicating that the process is out of control. However, in this case, the process being out of control does not result in an unacceptable fraction non-conforming items.

2.3.3 Multivariate data

To control several related inspection points at the same time it may be convenient to use a multivariate control chart. It is easier to only have one chart, instead of one for each point, and the multivariate chart takes the relationship between different inspection points into consideration. Further, by using a multivariate chart the total probability of a type I error is controlled. The disadvantage of a multivariate chart is that since it is used to control all the inspection points at the same time, it is sometimes difficult to identify the point or points that cause an alarm. Different kinds of multivariate control charts are discussed in Chapter 3.

The p-variate inspection data vector is supposed to follow a p-variate normal distribution $N(\mu, \mathbf{\Sigma})$.

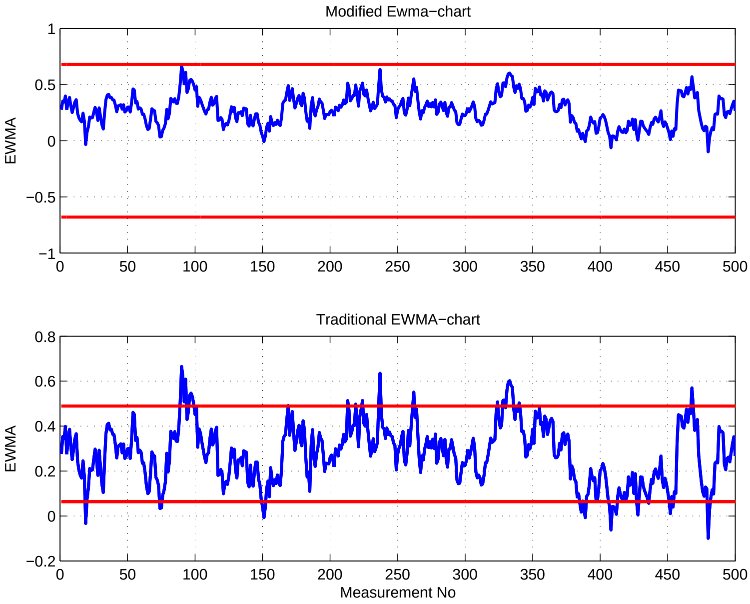


Figure 2.6: *EWMA-charts for a high capability process. The charts are based on measurements of deviation from nominal geometry at one inspection point.*

Multivariate capability indices

There are several different suggestions of multivariate capability indices. Three of these indices are compared by Wang et al. (35). These are a multivariate capability vector by Shahriari et al. (29), a multivariate capability index MC_{pm} by Taam et al. (32) and finally a multivariate capability index MC_p by Chen (10).

Naturally, all of these three indices have their advantages and disadvantages. Here, we have chosen to use the multivariate capability index MC_{pm} by Taam et al. (32). The index is defined as the ratio between the volume of R_1, the modified tolerance region, and the volume of R_2, the scaled 99.73 percent process region,

$$MC_{pm} = \frac{vol.(R_1)}{vol.(R_2)}.$$

The modified tolerance region, R_1, is the largest ellipsoid centred at the process target and completely within the tolerance region. Since data is normally distributed R_2 is also an ellipsoid. The capability index is estimated by

$$\hat{MC}_{pm} = \frac{\hat{C}_p}{\hat{D}},$$

where

$$\hat{C}_p = \frac{vol.\ (tolerance\ region)}{vol.\ (estimated\ 99.73\%\ process\ region)} = \frac{vol.\ (tolerance\ region)}{|\mathbf{S}|^{1/2}(\pi K)^{p/2}(\Gamma(p/2)+1)^{-1}},$$

where K is the 99.73% percent quartile of a χ^2 distribution. The denominator $\hat{D}$ is given by

$$\hat{D} = \left(1 + \frac{n}{n-1}(\bar{\mathbf{X}} - \mu_0)^T \mathbf{S}^{-1}(\bar{\mathbf{X}} - \mu_0)\right)^{1/2},$$

The quantity $1/\hat{D}$ takes values between zero and one and measures the deviation from target. The closer $1/\hat{D}$ is to one, the closer is the processes to their targets. If the mean vector equals the target vector μ_0, $1/\hat{D}$ equals 1 and accordingly $\hat{MC}_{pm} = \hat{C}_p$. The quantity $\hat{C}_p$ is interpreted just like the univariate process capability, i.e. a value $\hat{C}_p = 1$ implies that 99.73% of the produced items are within the specification limits.

Multivariate acceptance control chart

The idea of a multivariate acceptance chart is analogous to the univariate acceptance chart. The starting point is to decide an allowable region for the mean vector of the p points, given an upper limit for the fraction nonconforming produced items. Thereafter, this process area is transferred into an allowable region for the mean value vector, given a type I probability α. The tolerance region is usually formed as a hypercube, at least when the specification settings are independent. This region must in some way be transformed into the same shape as the process region, which is an ellipsoid so the regions can be compared to each other. In the multivariate capability index by Taam et al. (32) the tolerance region is transformed to the largest ellipsoid completely within the tolerance region and centered at the process target. Using this procedure as a starting point, it should probably be possible to construct a multivariate acceptance control chart. This area is though subject to future research.

2.4 When to use an acceptance control chart?

A usual $\bar{x}$-chart gives with a high probability an alarm when the process mean changes. If there is no alarm, the mean value is stable and the process is in statistical control. An acceptance chart gives with high probability an alarm if the process is out of control and the mean value is to close to the tolerance limits. Another way of saying this would be to define a group capability index, $C_{pk}^{i} = \frac{min\{USL-\bar{x}_i,\bar{x}_i-LSL\}}{3\sigma} \quad i = 1...k$, for every group. Then the acceptance chart gives alarm when the group capability index is too low. It is important to note that the group capability index says nothing about the future process performance, since the mean is allowed to vary.

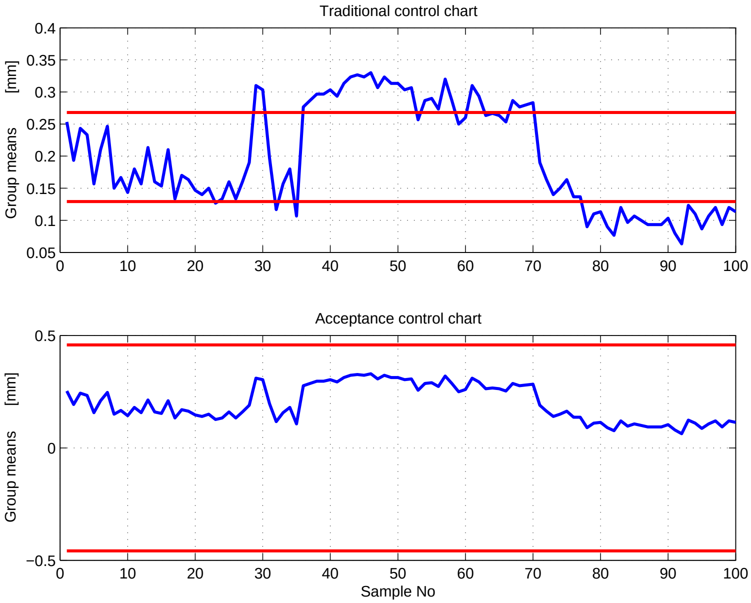


Figure 2.7: *Above: A traditional control chart for the mean value. Below: An acceptance control chart for the same data. The specification limits are ± 0.5 mm.*

In Figure 2.7 an example of control charts for mean value of a high capability process is shown. The upper plot is based on the traditional methods, while the lower one is an acceptance control chart. The acceptance chart gives no alarms since the process is far from the specification limits. The upper chart indicates that the process is out of control. In this case, the process being out of control is not regarded as important, since the process still produces items well within specifications, and the acceptance control chart is the preferable chart here.

In Figure 2.8 the circumstances are reversed. In this case the process has a low capability, and therefore the traditional chart is preferable. In the upper plot, the traditional chart indicates that the process is out of control. The reasons of this should be examined. Since the capability is low, the acceptance control chart in the lower part of the figure is not suitable.

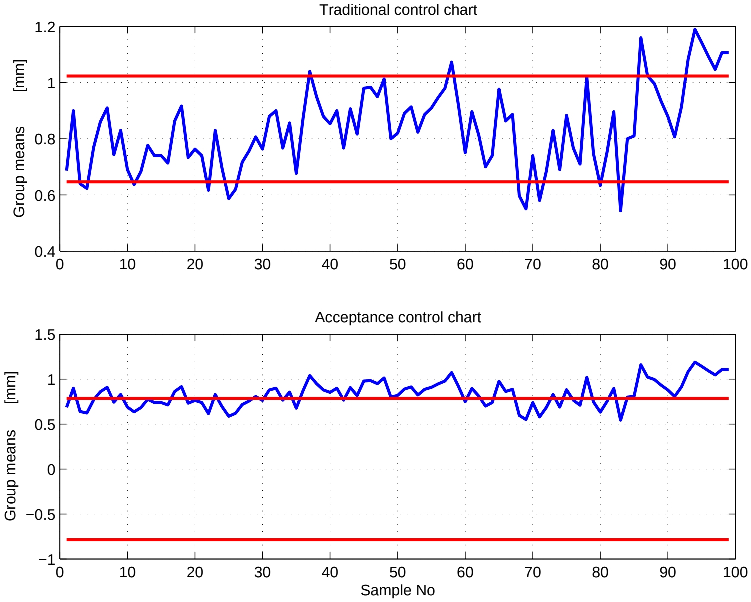


Figure 2.8: *Top: Traditional control chart for an inspection points with* $C_p = 2.76$ *and* $C_{pk} = 0.20$ *The specification limits for this point are* ± *0.9 mm. Below an acceptance control chart for the same data.*

The question is when to use traditional charts and when to use acceptance charts. At some point the control limit for a usual control chart and the control limit for an acceptance control chart coincide. This point can be expressed as a certain value of the capability of the process. The value is determined assuming $min(USL-\mu, \mu-LSL) = USL-\mu$;

$$\begin{aligned} UCL_{acceptance} &= UCL_{\bar{x}} \\ USL-(Z_\delta-\frac{Z_\alpha}{\sqrt{n}})\sigma &= \mu+3\frac{\sigma}{\sqrt{n}} \\ \frac{USL-\mu}{\sigma} &= \frac{3}{\sqrt{n}}+Z_\delta-\frac{Z_\alpha}{\sqrt{n}} \\ C_{pk} &= \frac{1}{\sqrt{n}}+\frac{Z_\delta}{3}-\frac{Z_\alpha}{3\sqrt{n}}. \end{aligned}$$

Using $n=3$, $Z_\delta=3.82$ and $Z_\alpha=3$ this gives $C_{pk}=1.27$. If the value of the adjusted capability index is below this value a usual $\bar{x}$-chart should be used. Otherwise, an acceptance chart is preferable.

It would also be possible to combine these two kinds of charts if the capability index $C_p > 1.27$ and the adjusted capability index $C_{pk} < 1.27$, by using a modified lower control limit and a usual upper control limit if $min(USL-\mu, \mu-LSL) = USL-\mu$ and vice versa.

2.5 Conclusions

In a typical process in automotive industry, there are trends and cycles causing variation in the mean values. Some of this variation is very difficult to eliminate to a reasonable cost. For each inspection point there are an upper specification limit (USL) and a lower specification limit (LSL), and sometimes it may be desirable to allow the variation in mean value as long as those specifications are fulfilled. The specifications can be fulfilled, despite the variation, if the process has a high capability. A low capability process on the other hand, must be improved, to avoid products out of specifications.

An overview of the different charting methods is given in Figure 2.9. Here the relation between what kind of data, process capability and what charts to use for controlling mean and variance of process are illustrated. The different methods were described and the question about when to use an acceptance chart instead of a traditional chart was also discussed.

If resources are unlimited, the ideal is perhaps to use some kind of traditional chart in order to analyse and improve the process. Unfortunately, this is not often the case, and therefore the system for quality control described in this chapter may be a way to improve processes to a level where

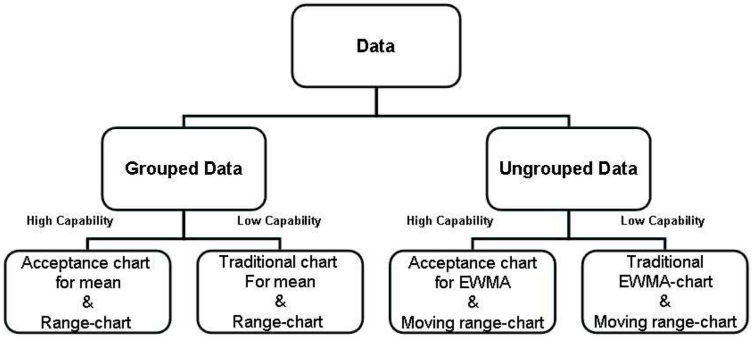


Figure 2.9: *An illustration of the different charting-alternatives.*

the items produced are within specifications, and to control that this level then is remained.

2.6 Appendix - Frequently used control charts and process capability

A control chart consists of a statistic, which is plotted in the diagram for each measurement. If the statistic plots outside the calculated values called control limits, there are assignable causes and the process is considered to be out of control. This means that the process is believed to have changed. The probability that the statistic plots outside the limits, despite the process remains in control, and causes a false alarm is denoted α. A common choice is $\alpha = 1\%$ or $\alpha = 0.1\%$.

To control the mean value of a process an $\bar{x}$-chart can be used, Wetherill and Brown (36). This chart is of Shewhart-type and is probably one of the most frequently used control charts today. This method assumes that the distribution of the plotted data is approximately Normal and uses the fact that most of the dispersion is included within $\pm 3\sigma$ from the mean. The upper control limit (UCL) and the lower control limit (LCL) of the chart are given by

$$UCL = \mu + 3\sigma_e \text{ and } LCL = \mu - 3\sigma_e,$$

where σ_e is the standard deviation of the group mean, i.e. if the standard deviation within a group of size n is σ_w, then $\sigma_e = \sigma_w / \sqrt{n}$. The expected value, μ, can be estimated by the mean $\bar{x}$.

Other methods used to control the mean value of a process are cumulative sum procedures (CUSUM) and exponentially weighted moving average (EWMA) charts. These procedures are especially useful to detect small shifts. In a CUSUM chart the cumulative sum, S_n, of the observations $x_1, x_2, ...$ is plotted, i.e.

$$S_n = S_{n-1} + (x_n - T),$$

where $S_0 = 0$ and T is a target value, often the mean is used as target. In the CUSUM chart a so-called truncated V-mask is generally used. An out of control signal is given when the arms of the mask cross the previous trace of CUSUM values.

The EWMA chart is basically formed by determination of a new moving-average at each sampling point by calculating a weighted average of the new value and the previous moving-average. The moving-average, m_i, is calculated using the formula

$$m_i = p\bar{x}_i + (1 - p)m_{i-1}$$

where p is the amount of weight put on the current value. In an EWMA chart only action limits are used, and they are placed at $\mu \pm A_1\hat{\sigma}_e$, where A_1 is a tabulated value, depending on the group size.

The dispersion of a process can be controlled by using a s- or r-chart. The procedure is similar to the construction of a $\bar{x}$-chart. The standard deviation (or range) of each group is plotted and the control limits in this case are given by σ_w times a constant, see for example Wetherill and Brown (36).

The capability index of a process is reflecting the process's ability to produce items within the specification limits. The capability index, C_p is a comparison between the specification width and the width of the distribution. For a normal distribution 99.7% of the distribution is covered by 6σ and the process capability index is defined by

$$C_p = \frac{USL - LSL}{6\sigma}$$

Usually, $C_p > 1.33$ is recommended. A high capability process may still produce many non-conforming items if the mean is not appropriately centred. Therefore, another capability index sometimes called the adjusted capability index, C_{pk}, is defined,

$$C_{pk} = \frac{min\{USL - \mu, \mu - LSL\}}{3\sigma}$$

The definitions of the indices above are both based on a normal distribution assumption. Further, the process must be in control. Otherwise, the indices cannot be used as a prediction of the process performance. The indices can be calculated no matter what distribution the data have. However, if the indices will be used to predict the process performance it is crucial that the process is in control, otherwise the only information given is what the process performs at the moment when the data is collected.

Chapter 3

Multivariate Quality Control and Diagnosis

3.1 Introduction

In the auto body assembly process, fixtures are used to position parts during assembly and inspection. Geometrical variation in parts and in the assembly process results in variation in size, shape and position of the final product. This may lead to difficulties in assembling parts or products not fulfilling functional and esthetical requirements. Geometrical variation is controlled by locating schemes and tolerances. The locating schemes describe how parts are positioned during assembly. The tolerances are ideally allocated with respect to assembly sensitivity, process variation and cost.

Parts and subassemblies are measured many times during the manufacturing process in order to detect offsets and variations as soon as possible. In order to use data in an optimal way, statistical process control (SPC) may be used. It is a statistical analysis of inspection data aiming at controlling and, hopefully, improving the process. There is also a multivariate equivalence of SPC, the namely the MSPC suited for simultaneous analyse of data from several inspection points.

If an offset or variation is detected, it is of course desirable to find its root cause. For example bad raw materials, worn out machines or fixture faults can cause variation. But a major part of all root causes are due to fixture faults, according to an investigation performed by Ceglarek and Shi (8). In this chapter, methods for MSPC and methods for diagnosing variation in fixtures by using process knowledge and inspection data are considered.

There are many different methods used in multivariate quality control. The purpose of this chapter is to illustrate and compare some of these methods by applying them to given data sets, collected from industrial case studies. The procedures for quality control are then put together with methods for fixture fault diagnosis. Different methods to diagnose the fixture or fixtures causing the error are illustrated using the same data sets.

3.1.1 Outline

This chapter is outlined as follows. In Section 3.2 the two case studies are presented. The methods described in subsequent sections will be applied to the data from these case studies.

In Section 3.3 different kinds of multivariate control charts are considered. Two special methods, aimed for detecting fixture related faults are also illustrated. For each type of chart the theory of the chart is described and thereafter the chart is tested on data from the case studies.

When a process is found to be out of control, it is obviously of main importance to find the root cause of the erroneously state. This topic is discussed in Section 3.4. The methods are demonstrated in the same way as in Section 3.3; the description of each method is followed by a test on data from the case studies.

Among the methods discussed in Section 3.4 one of the techniques are tested further on an additional case study. The assembly in the case study is adjusted in accordance with the results of the RCA. The assembly, the analyses and the results of the adjustment are described in Section 3.5.

Finally, in Section 3.6 the different methods and techniques for quality control and root cause analysis are discussed and compared.

3.2 Data and models

The methods outlined in the following sections will be applied on two case studies. The assemblies and the corresponding measurement data are presented in this section.

3.2.1 Case study 1

The first case is an assembly consisting of two parts called outer side panel and doorframe. The parts can be seen in the top of Figure 3.1. This assembly is only analysed in $x-$ and z-direction, since the assembly is not rigid in the y-direction, which is a necessary condition for some of the diagnosis methods considered in Section 3.4. A fixture is used to position both parts. In Figure 3.1 the locators in xz-direction are marked with black triangles. Both parts are fixed in z-direction using a pin/hole contact (labelled B1/C) and a pin/slot contact (labelled B2). The pin/hole contacts are also used for positioning the parts in x-direction. A pin/slot is a pin placed in a slot, i.e. an oval hole. Therefore the part is only restricted in one direction using this kind of locator. A pin/hole locator restricts the part in two directions.

After the positioning, the parts are welded together. When the assembly is measured, it is fixed in zx-direction using the pin/hole contact on the doorframe (B1) and the slot/pin contact on the side panel (B2), as shown in the middle part of Figure 3.1.

Four inspection points are used, marked by arrows in Figure 3.1, and three of these points are measured in z-direction as well as the x-direction. The fourth inspection point is only measured in x-direction.

The inspection data consist of 217 groups, where each group contains measurements from three consecutive cars. The data contain trends, see Figure 3.2. These trends may partially be caused by fixture faults, but if there are fixture faults causing variation, these faults will cause within group variation as well. Therefore, it is possible to estimate the variation caused by the fixtures by concentrating on the short-term variation only. The trends are therefore eliminated and the estimate of the covariance matrix is based on the within group variation.

The measurements are denoted $\boldsymbol{x}_{ij}$, $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$, where $\boldsymbol{x}_{ij}$ is a vector consisting of inspection data for p inspection points on the jth item in the ith group. Here, $m = 217$ and $n = 3$. The n observations in each group are put together in the group mean,

$$\bar{\boldsymbol{x}}_i = \frac{1}{n} \sum_{j=1}^{n} \boldsymbol{x}_{ij}.$$

The inspection data can be decomposed into an overall mean, $\boldsymbol{\mu}$, a group effect, $\boldsymbol{\tau}_i$, and a error component, ϵ_{ij}, i.e.

$$\boldsymbol{x}_{ij} = \boldsymbol{\mu} + \boldsymbol{\tau}_i + \epsilon_{ij},$$

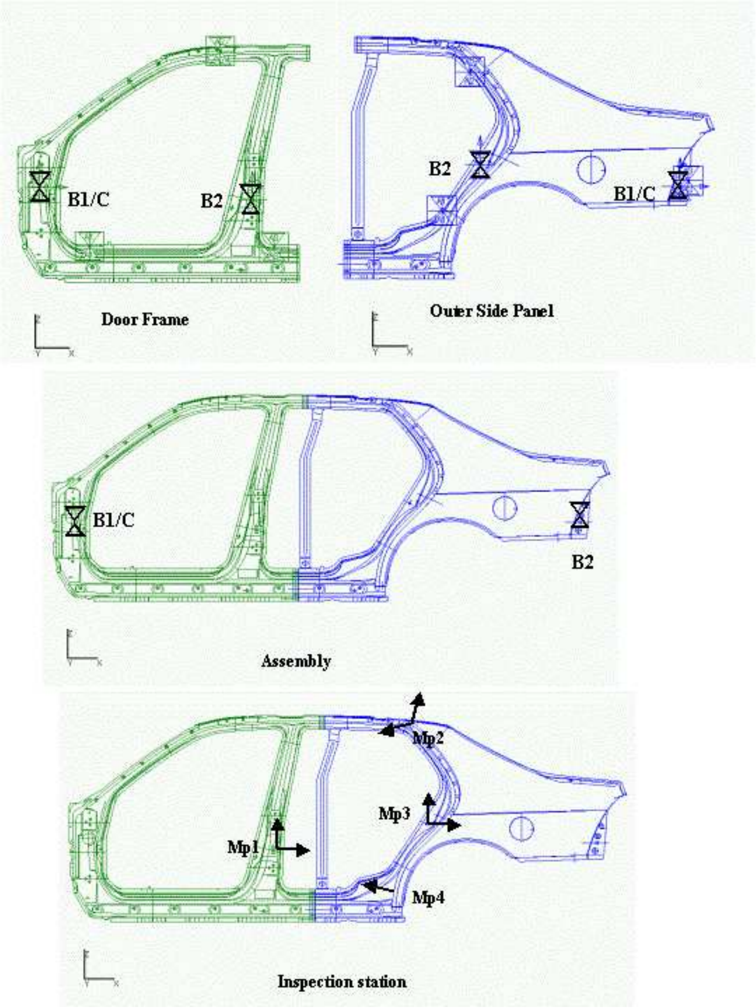


Figure 3.1: *Top: The side panel assembly consists of the door panel and the outer side panel. Middle: The assembly is positioned in xz-direction using the locators labelled B1 and B2. Bottom: The assembly is measured in four inspection points.*

and the covariance matrix of the data can consequently be expressed as the sum of the between group variation, Σ_τ, and the within group variation, Σ_ϵ, i.e.

$$\Sigma = \Sigma_\tau + \Sigma_\epsilon.$$

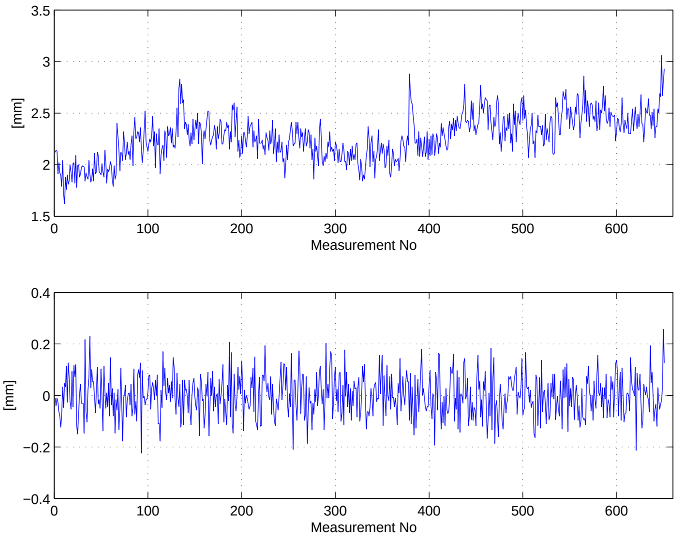


Figure 3.2: *Top: The original data from inspection point 2 in x-direction, side panel assembly. Bottom: The same data but with the trends eliminated.*

As mentioned before, the variation caused by the fixtures can be estimated by the within group variation, Σ_ϵ. To eliminate the trends, the group mean is subtracted from each observation in every group,

$$\boldsymbol{z}_r = \boldsymbol{x}_{ij} - \bar{\boldsymbol{x}}_i, \text{where } r = 1, 2, ..., mn,$$

This procedure gives $m * n$ measurements without trends and the p-variate vector $\bar{\mathbf{z}} = \mathbf{0}$, so the within group variation can be estimated as

$$\hat{\Sigma}_\epsilon = \frac{1}{m(n-1)} \sum_{r=1}^{mn} (\boldsymbol{z}_r)(\boldsymbol{z}_r)^T. \tag{3.1}$$

3.2.2 Case study 2

The second case study deals with an assembly where a rear bumper is joined with a vehicle floor, see Figure 3.3. The bumper is in yz-direction positioned by a fixture, using the locators labelled B1, B2 and C, and in x-direction by the contact (the contact points are labelled A1, A2 and A3) with the floor.

To monitor the assembly process 14 inspection points on the bumper are measured after the two parts are joined. During inspection the locators

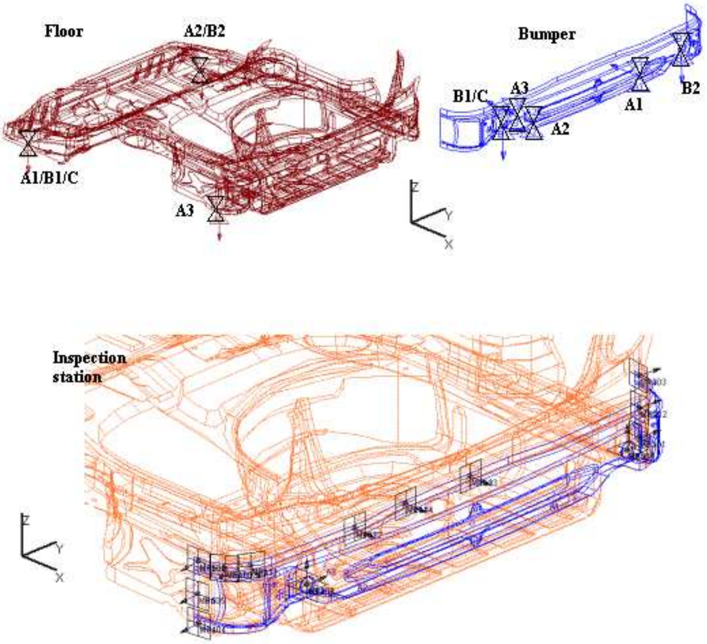


Figure 3.3: *The bumper and floor are assembled. Finally 14 points on the bumper are measured.*

of the floor are used to position the assembly. Hence, inspection points on the floor, if any, will not show any assembly variation even if the floor was incorrectly positioned during assembly. This variation will on the other hand be observed in the inspection point on the bumper. Therefore, the assembly process can only be monitored using inspection points on the bumper. The inspection data are, unlike case study 1, not arranged into subgroups. The data consist of 36 measurements of each inspection point, see Figure 3.4. It can be seen from the figure that there are considerable changes in the process after 16 measurements. There is much more variation in measurement 17 to 36, than in measurement one to 16. It is known that this variation is due to a variation in the contact between the bumper and the locator controlling translations in y-direction. That knowledge make the case study very suitable for testing different methods for RCA, since the results can be compared to this information. The case study is also well suited for testing and evaluation of MSPC-methods.

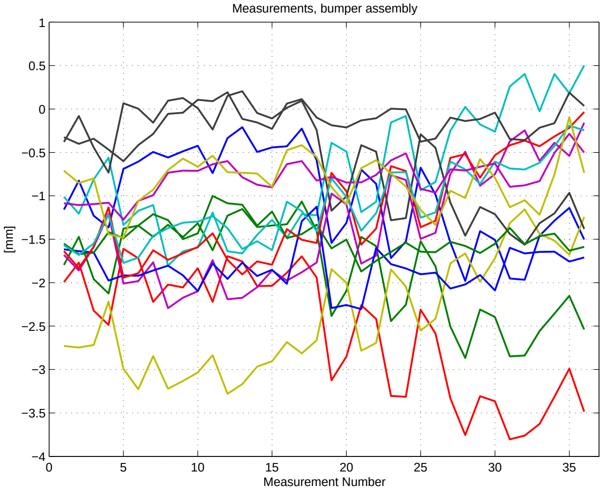


Figure 3.4: *Inspection data for 36 assemblies in 14 inspection points.*

3.3 Multivariate Statistical Process Control

In order to improve and maintain the quality of a product it is important to detect any changes in the process as soon as possible. There can be various causes of extra variation in a process. One of these possible reasons is fixture failure and in this section, an overview of methods to detect this type of variation is given. In Section 3.3.1-3.3.4 general methods to detect variation and offsets in the process are given, in Section 3.3.5 and Section 3.3.6 special methods designed to detect variation caused by fixture faults are considered.

The methods for statistical process control can be divided into univariate and multivariate procedures. The univariate methods are aimed at controlling measurements of one quality variable or inspection point. To control several related points at the same time it is convenient to use a multivariate control chart, and it is this kind of charts that is considered in this chapter.

A control chart consists of a statistic, which is plotted in the diagram for each observation, and corresponding control limits. If the statistic plots outside the control limits, the process is assumed to be out of control, and this implies that the process has changed. The probability of false alarm, α, i.e. the probability that the statistic plots outside the limits despite the process remains in control, is depending on the significance level of the control limits. A usual choice is $\alpha = 1\%$ or $\alpha = 0.1\%$.

Since a multivariate control chart is used to control all the inspection points at the same time, it is sometimes complicated to identify the point or points that cause an alarm. In the regression adjustment method and the self-organizing map method, considered later in this chapter, this problem is partly solved. There are also many other methods that deal with this question, see for example Runger et al. (26), Jackson (19) and Hayter and Tsui (14).

3.3.1 T^2-chart

One of the most frequently used multivariate control charts is the T^2-chart. It is used to control the mean value of p inspection points. It is also sensitive to increased process variation.

The statistic

$$\chi_0^2 = n(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)^T \boldsymbol{\Sigma}_0^{-1} (\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0),$$

where $\boldsymbol{\mu}_0$ is a $p \times 1$ vector of in-control means and $\boldsymbol{\Sigma}_0$ is a $p \times p$ in-control covariance matrix, follows a χ^2-distribution with p degrees of freedom, see e.g. Montgomery (25). When the true population parameters are not known, the following statistic is used to form a Hotelling's T^2 control chart:

$$T^2 = n(\bar{\boldsymbol{x}}_i - \bar{\bar{\boldsymbol{x}}})^T \boldsymbol{S}^{-1} (\bar{\boldsymbol{x}}_i - \bar{\bar{\boldsymbol{x}}}).$$

This statistic was developed by Hotelling (16). Alt (2) showed that T^2 (times a constant) follows an exact F-distribution, and the upper control limit (UCL) is therefore given by

$$UCL = \frac{p(m+1)(n-1)}{n(mn-m-p+1)} F_{\alpha,p,mn-m-p+1},$$

where n is the number of observation in each sample, m is the number of samples taken and $F_{\alpha,p,mn-m-p+1}$ is the inverse of the F distribution function with p and $mn-m-p+1$ degrees of freedom, at the value of α. If the sample mean $\bar{\bar{\boldsymbol{X}}}$ and the sample covariance matrix $\boldsymbol{S}$ are estimated from a relatively large number of samples (at least 20 or 25) it is customary to use $\chi^2_{\alpha,p}$ as an upper control limit on the Hotelling T^2 chart.

Test on data

When using the data from the side panel assembly, with $p = 7$ inspection points, we concentrate on controlling the within group variation, so $\hat{\Sigma}_\epsilon$ from

Equation (3.1) on page 49 is used instead of S. In Figure 3.5

$$T_i^2 = (\boldsymbol{z}_i)^T \hat{\Sigma}_\epsilon^{-1} (\boldsymbol{z}_i)$$

is plotted for $i = 1, ..., 651$. Here, $\chi^2_{0.001,7}$ is used as an upper control limit. There are several observations above the control limit, so the process is said to be out of control. Since α, the probability of type I error is chosen to be 0.001 the expected number of false alarms is $0.001 * 651 = 0.651$ when the process is in control.

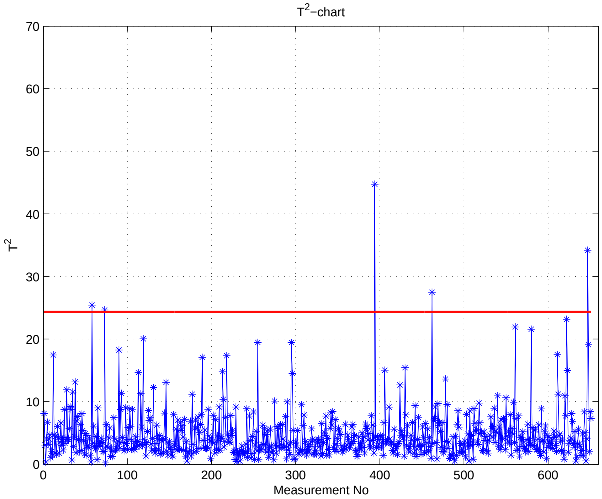


Figure 3.5: *Multivariate T^2-control chart based on within group variation for side panel assembly.*

The method is also applied on the bumper assembly data. Since the process is in an unacceptable stage during measurements 17 to 36, these data are not included in the estimates of the parameters. The multivariate T^2-chart, see Figure 3.6, shows a considerable change in measurement 17. However, the covariance matrix for the 14 inspection points is nearly singular. This fact makes the T^2-values after the 16th measurement very big. Often, a principal component analysis is recommended for this kind of data. That method is considered in the next section.

This example shows that the T^2-chart is an effective tool when it comes to detect changes in a process, especially when the changes affects several inspection points.

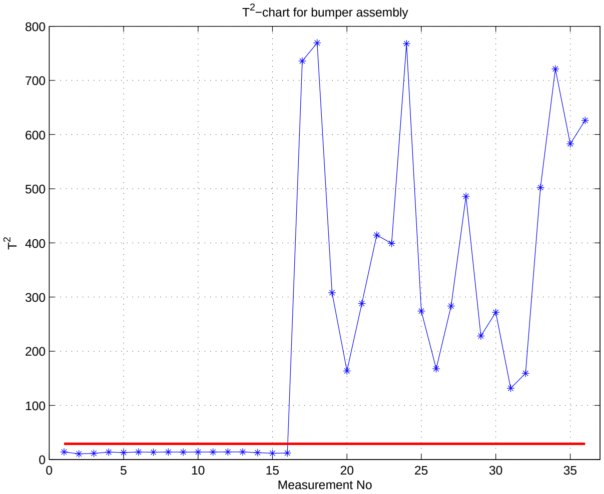


Figure 3.6: *Multivariate T^2-control chart, bumper assembly*

3.3.2 Principal Components and SPE

Principal Component Analysis (PCA) can be used to control a process as well as for diagnosing sources of variation, which is considered in Section 3.4.2.

The idea of PCA is to form a set of new variables, which are linear combinations of the old ones. The new variables, the principal components, are independent of each other. These principal components display different amounts of variance and usually, the variance of some of the components will be so small that they can be considered negligible. Therefore, the variation in the original variables can be described by a smaller number of new variables. The general objectives of PCA are reduction and interpretation of data.

In short, the PCA is performed by computing eigenvalues and eigenvectors of the covariance matrix, $\mathbf{\Sigma}$, of the original variables. An eigenvalue, λ_i, is a root of the characteristic equation

$$|\mathbf{\Sigma} - \lambda_i \boldsymbol{I}| = 0$$

and a the corresponding eigenvector is a non-zero vector $\mathbf{v}_i$, satisfying

$$\mathbf{\Sigma} \boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i.$$

The eigenvalues and their corresponding eigenvectors are sorted in order of size. The principal components, $\boldsymbol{Y}$, are formed in the following way

$$Y_i = \boldsymbol{v}_i^T \boldsymbol{X},\ i = 1, 2, ..., p$$

where $\boldsymbol{\Sigma}$, the covariance matrix of $\boldsymbol{X}$, has eigenvalues

$$\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_p,$$

with corresponding eigenvectors $\boldsymbol{v}_i$) with unit length. Often, a few of the principal components contain the main part of the total variance in the population. The ith principal component, $\boldsymbol{Y}_i$, contains

$$100 * \frac{\lambda_i}{\lambda_1 + \lambda_2 + ... + \lambda_p}$$

percent of the total variation. If the major part of the variance is contained in the first k principal components, then these components may replace the p original variables, without loosing too much of the information. To determine k, it is possible to perform a χ^2-test, see Jackson (18).

A more elaborate discussion of principal component analysis will be found in Johnson and Wichern (21).

To control the process using PCA the statistic

$$T^2_{pca}(i) = (\bar{\boldsymbol{X}}_i - \bar{\bar{\boldsymbol{X}}})^T PP^T \Sigma^{-1} PP^T (\bar{\boldsymbol{X}}_i - \bar{\bar{\boldsymbol{X}}})$$

is used, see Jackson (20). Here, $P = [\boldsymbol{v}_1|\boldsymbol{v}_2|...|\boldsymbol{v}_k]$ is the matrix of the first k eigenvectors. The statistic is χ^2-distributed and the upper control limit is given by $\chi^2_k(\alpha)$. The k principal components used span a subspace containing the variation described of these principal components, and the T^2_{pca}-statistic is used to control the quantity of this variation. If the nature of the variation changes, for example there is increased variation outside the subspace spanned by the principal components, the control diagram does not detect that. This means that there is need for a chart controlling the size of the residual, i.e. the distance between an observation and the subspace, see Figure 3.7 for an illustration of the residual. Since $PP^T\boldsymbol{x}$ is a projection of an observation $\boldsymbol{x}$ on the subspace, the following statistic can be used:

$$SPE_{pca} = (\boldsymbol{x} - PP^T\boldsymbol{x})^T(\boldsymbol{x} - PP^T\boldsymbol{x})$$

According to Jackson (20), the SPE statistic is Q-distributed and the control limit is given by

$$Q_\alpha = \phi_1 (c_\alpha \frac{\sqrt{2\phi_2 h_0^2}}{\phi_1} + \frac{\phi_2 h_0 (h_0 - 1)}{\phi_1^2} + 1)^{\frac{1}{h_0}},$$

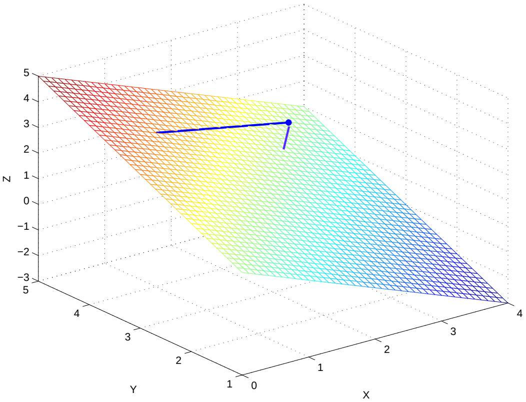


Figure 3.7: *An observation (*) plot outside the principal component subspace. The* SPE_{pca}*-statistic measure the distance between the observation and the principal component subspace.*

where c_α is the inverse of the standard normal cumulative distribution function,

$$\phi_1 = \sum_{i=k+1}^{p} \lambda_i, \qquad \phi_2 = \sum_{i=k+1}^{p} \lambda_i^2, \qquad \phi_3 = \sum_{i=k+1}^{p} \lambda_i^3$$

and

$$h_0 = 1 - \frac{2\phi_1\phi_3}{3\phi_2^2}.$$

Test on data

In Figure 3.8, the T^2_{pca} and SPE_{pca}-statistics for the side panel assembly are plotted. No between group variations are included, i.e. the PCA is based on Σ_ϵ. The matrix P consists of the three first principal component vectors, $P = [\boldsymbol{v}_1|\boldsymbol{v}_2|\boldsymbol{v}_3]$, and these three together contain 86% of the total variation. The UCL is given by $\chi^2_3(0.001)$.

This chart gave some fewer alarms compared to the usual T^2-chart, used in Section 3.3.1. The reason for this is that only 86% of the total variation is included in the principal components. Despite this, there are indications that the process is out of control. The usual T^2-chart alarmed five times, while the T^2_{pca}-chart alarmed three times.

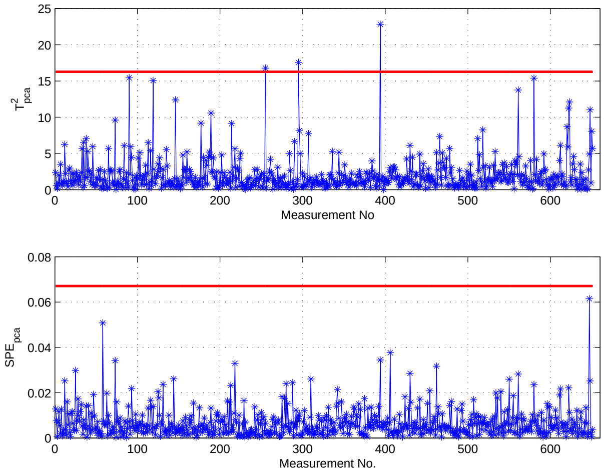


Figure 3.8: *On top a* T^2_{pca}*-chart and below a* SPE_{PCA}*-chart for the side panel assembly.*

The method is also tested on the bumper assembly, and the result can be seen in Figure 3.9. The mean value and the covariance matrix are estimated from the first 16 measurements. The chart gives an alarm in measurement 19. The SPE-chart gives an alarm as well, which indicates that the variation no longer is contained in the subspace spanned by measurement one to 16. In this case, the fixture related fault caused an increased variation after the 16:th measurement. But the fault also implied a different kind of variation compared to the one spanned by the first 16 measured objects, and that causes an alarm in the SPE_{PCA}-chart. When comparing this chart to the usual T^2 chart a major difference is that the increase after observation 16 is much more moderate when PCA is used. That is because the problem with the almost singular covariance matrix is avoided using PCA.

3.3.3 Regression adjustment

Regressing one variable on all the others and then control the regression residuals is an approach for MSPC, considered by Hawkins (12). In regression adjustment separate charts for controlling mean value and variation can be used, which may be advantageous. The method is especially well suited when only a shift in some of the variables is expected. This is usually not the case if the error is fixture related, since a movement in one locator often affects many inspection points, but the method is nevertheless tested on the case studies. An overview of the method is also given by Mont-

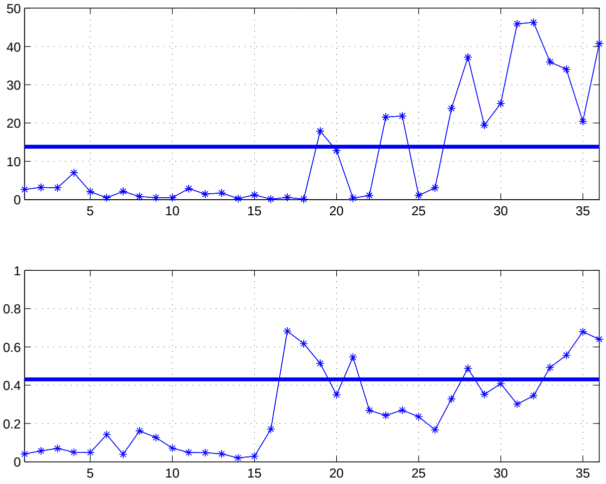


Figure 3.9: *On top a* T^2_{PCA}*-chart and below a* SPE_{PCA}*-chart for the bumper assembly.*

gomery (25).

The residuals, $\mathbf{r}_i = \hat{\mathbf{X}}_i - \mathbf{X}_i$, $i = 1, ..., p$, are calculated for each inspection point i using a usual multiple linear regression for each inspection point, i, $i = 1, ..., p$, i.e.

$$\hat{\mathbf{X}}_i = \boldsymbol{\alpha} + \sum_{i \neq j} \boldsymbol{\beta}_j \mathbf{X}_j.$$

The standardized residual of the regression of one variable on the other variables will follow a $N(0,1)$-distribution when the process is in control. Therefore, the control charts are similar to univariate control chart. But since the regression residuals are plotted the correlation between different variables is taken into account.

Test on data

The regression analysis for the side panel assembly is based on the measurements with removed trends, i.e. $\boldsymbol{z}_i$, $i = 1, ...651$, is considered. The residuals are controlled by an EWMA-chart, see Figure 3.10, and a moving range chart, see Figure 3.11. The EWMA-charts give only a few alarms, while the moving range-charts signals more often. This indicates that the

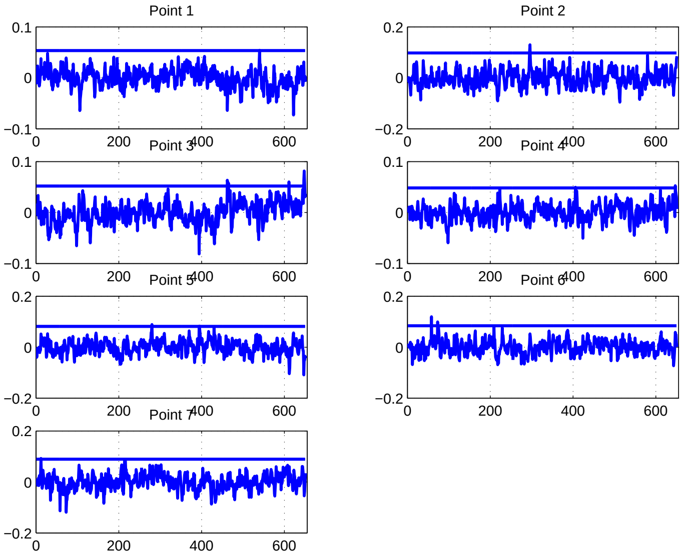


Figure 3.10: *Regression adjustment case study 1, EWMA chart for controlling the regression residuals.*

major problem in the side panel assembly is too much variation, not shifts in the mean value.

The regression adjustment is also applied to nine of the inspection points on the bumper assembly. In this case it is known that the major problem is caused by increased variation after measurement 16. The residuals are therefore controlled with a moving range chart, see Figure 3.12. The numbers of alarms are not increasing distinctly after the 16:th measurement. A reason of that may be that the root cause affects several variables, not only one or two. These kinds of charts perform best when only one variable is likely to be affected by the variation.

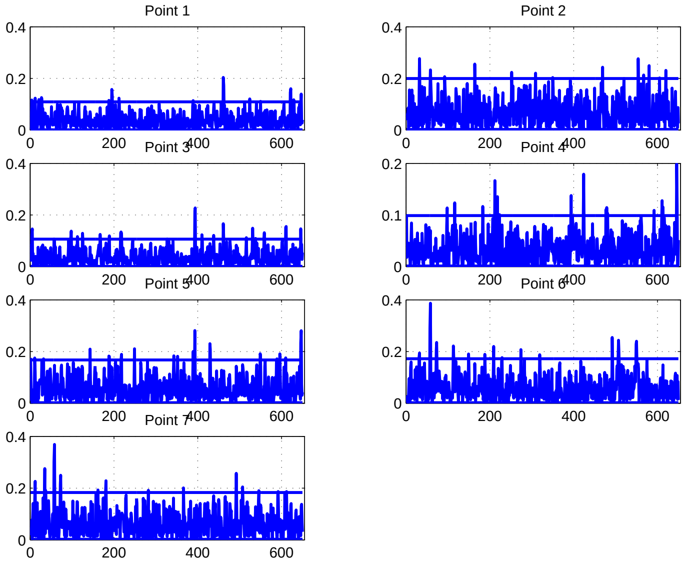


Figure 3.11: *Regression adjustment case study 1, moving range chart for controlling the regression residuals.*

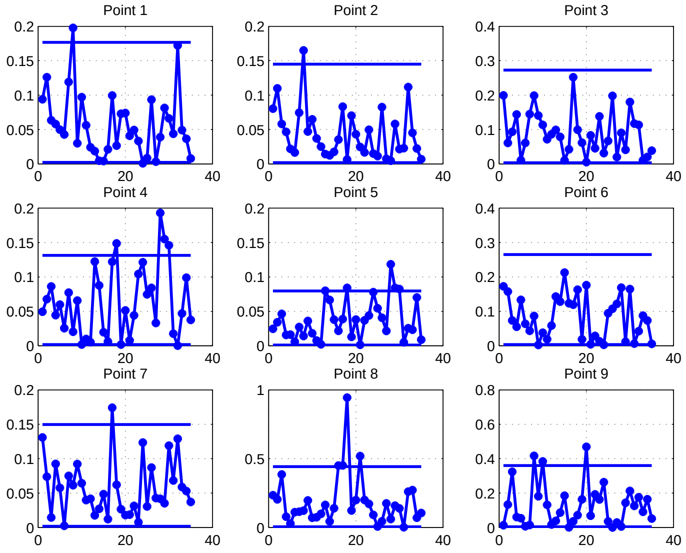


Figure 3.12: *A moving range chart of the regression residuals, bumper assembly.*

3.3.4 Self Organizing Maps

A Self Organizing Map (SOM) is a special type of artificial neural network (ANN) that can be used for multivariate process control and in some cases also for fault identification. A neural network is an adaptive model for non-linear multivariate data. It can learn from the data and generalize the things learned. An ANN consists of a number of neurons in different layers. Each neuron has an individual weight vector and all neurons have connections to other nodes. There are two different kinds of ANN; the supervised ANN and the unsupervised ANN. In supervised learning the system directly compares the network output with a known correct or desired answer, whereas in unsupervised learning the desirable output is not known. ANN is studied by for example Haykin (13).

The SOM was developed by Kohonen (23), and is one of the most popular network models. It is based on unsupervised, competitive learning. It provides a topology preserving mapping from high dimensional input vectors to a low dimensional (usually two dimensional) grid of neurons. Each neuron is represented by a weight vector of the same size as the input vector.

The SOM is trained iteratively, Ahola et al. (1). In each step the Best Matching Unit (BMU) for the input vector is found by comparing the input vector, $\boldsymbol{x}$, with the weight vector, $\boldsymbol{m}$, of every neuron in the net. The neuron closest to the input vector wins, i.e. if the BMU is labelled $\boldsymbol{m}_c$,

$$||\boldsymbol{x} - \boldsymbol{m}_c|| = \min_i\{||\boldsymbol{x} - \boldsymbol{m}_i||\}.$$

Usually, the Euclidian norm is used. The weights of the BMU as well as the weights of the neighbours of the BMU are updated to be more similar to the input vector,

$$\boldsymbol{m}_i = \boldsymbol{m}_i + \alpha(t)h_{ci}(t)(\boldsymbol{x} - \boldsymbol{m}_i),$$

where $\alpha(t)$ is the learning rate and h_{ci} is a neighbourhood function around the winner unit c. Both the learning rate and the neighbourhood function are decreasing function of time. By this procedure the net is formed to estimate the distribution of the input data.

Ultsch and Siemon (34) use a unified distance matrix (u-matrix) to visualize the structure of a SOM. The mean difference between a neuron and its neighbours is calculated. The result of these calculations is presented using a two-dimensional grey-scale picture. A dark area can for example mean that there are small differences between the neurons in the region, while a bright area means that the neurons in that region are not very similar to each other. By this procedure the dark areas can be identified as clusters.

If the SOM is trained on data from the normal operation state as well as on data from different erroneous states the clusters corresponding to these faulty states can be labelled with the fault type or even better, the root cause of the fault.

After the SOM is trained then the net can be used for process control. The inspection data vectors are fed in to the net and the BMU is identified. When the BMU is a node labelled "undesired state" the process is out of control. Plotting the trajectory of the BMU for each measurement vector can be a way of monitoring the process. Since the undesired states are labelled with root causes this procedure may help in fault detection.

If the SOM is trained using measurement vectors describing the normal state of the process only, then the net is forming a mapping of the "normal operation" input space. In order to detect a faulty situation the quantization error can be studied, Ahola et al. (1). The quantization error for unit i, $q_i = \sqrt{\sum_{k=1}^{p}(x_k - m_{ik})^2}$, is the distance between the input vector and the BMU. A large quantization error implies that the process no longer is in the "normal operation" space. This method gives no information of the root cause of the fault.

Test on data

When it comes to applying SOM to data in order to perform process control the results seem to be highly dependent on the number of nodes chosen, what subset of the data that are used for training and so on. Perhaps, SOM is best suited for use by an expert in the area, who can analyse those questions and find the appropriate settings.

Having this in mind, the method is tested only on the bumper assembly. The net is trained using measurement one to 16, i.e. data representing the normal state of the process are used. These data are also used to estimate the mean and standard deviation of the process in order to standardize data. The measurements used for calculating the quantification error are not included during the training phase. Usually a SOM is supposed to be trained on a much larger data set then the one used here, but still, it gives an idea of how the SOM works.

In Figure 3.13 is the quantization error plotted. The first four bars is the quantification error for data before measurement 16, while the remaining ones are the quantification errors for data after the 16:th measurement. The quantification errors obviously increase after measurement 16, indicating that the process is no longer in the normal operation state.

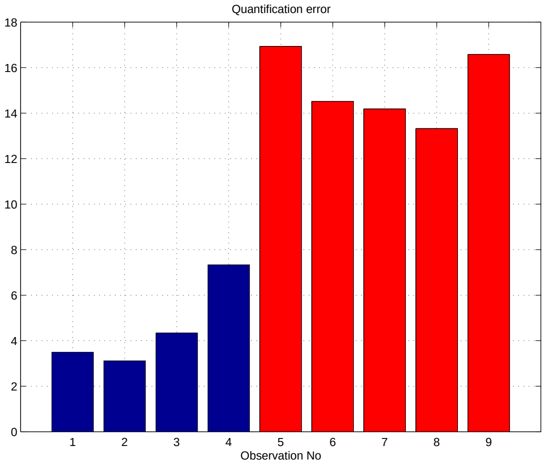


Figure 3.13: *Quantization error for a SOM for bumper assembly. The first four bars correspond to observations before the fixture failure occurred, while the remaining five bars correspond to observations after that fault.*

3.3.5 Fixture failure index

In this section, as well as in the following one, special methods for examining the occurrence of fixture faults are considered.

Carlson et al. (4), introduce a fixture failure index in order to determine if a fixture failure is present. To calculate this index the sensitivity matrix, A, must be known. The sensitivity matrix describes the connection between a displacement in the locators and the resulting displacement in the inspection points. This means that a displacement, $\mathbf{d}$, in the inspection points can be expressed as

$$\mathbf{d} = A\delta,$$

where δ is a small displacement in the locators. The matrix A is calculated analytical or numerical, see Carlson and Söderberg (5).

In order to calculate the fixture failure index the observations are split into two orthogonal subspaces, the failure subspace and the noise subspace.

The failure subspace contains the fixture errors and the residual errors, and the noise subspace contains no fixture errors. A measurement $\mathbf{x}$ can accordingly be written as

$$\mathbf{x} = U_r U_r^T \mathbf{x} + U_{p-r} U_{p-r}^T \mathbf{x},$$

where U_r is an orthonormal basis for the r-dimensional column space of the sensitivity matrix A and U_{p-r} is an orthonormal basis for the $(p-r)$-dimensional null space of A^T.

This decomposition makes it possible to calculate the fixture failure variation index, Ψ, by comparing the amount of variation in the failure subspace with the total variation;

$$\Psi = \frac{\mathrm{Trace}(U_r U_r^T \Sigma_x U_r U_r^T)}{\mathrm{Trace}(\Sigma_x)}.$$

A value of Ψ close to one indicates a fixture fault. When there is no fixture fault variation, the expected value of Ψ is r/p.

Since an estimate of the covariance matrix must be used, an uncertainty in the calculations arise. This uncertainty is taken care of by introducing an approximate confidence interval, derived by Carlson et al (4). The confidence interval for the fixture failure variation index is, when $\alpha = 0.05$, given by

$$\hat{\Psi} \pm 1.96\sqrt{\tau^2} \quad \text{where}$$

$$\hat{\tau}^2 = 2\frac{(1-\hat{\Psi})^2 \mathrm{Trace}(U_r^T S_x U_r)^2 + \hat{\Psi}^2 \mathrm{Trace}(U_{p-r}^T S_x U_{p-r})^2}{(n-1)(\mathrm{Trace}(S_x))^2}.$$

Test on data

In Figure 3.14 the variation index, Ψ, is plotted for the side panel assembly. The index is calculated for each group of observations using a moving estimate of the within variance-covariance matrix over seven groups.

If there is no fixture fault the expected value of the index is r/p, where r is the rank of the sensitivity matrix A and p is the number of inspection points. This value corresponds to the horizontal limit in Figure 3.14. In Figure 3.14 the fixture failure index and the 95% confidence interval are plotted. The index is based on the within group variation, i.e. $S_x = \hat{\Sigma}_\epsilon$. The index indicates that a fixture failure may be present.

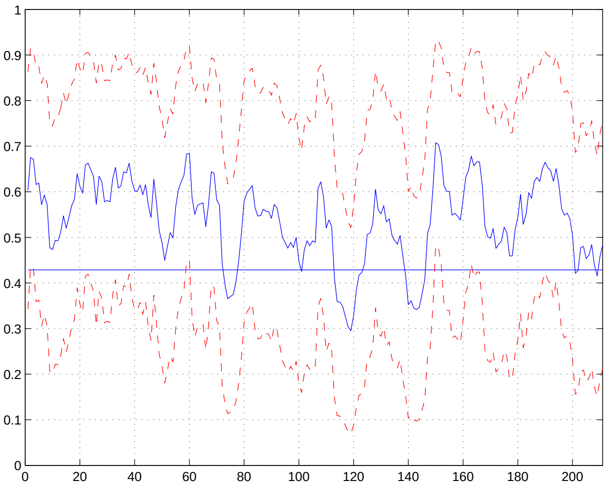


Figure 3.14: *Fixture failure variation index, side panel assembly.*

This method is also applied on the bumper assembly. Since the data is ungrouped the covariance matrix is calculated by a moving estimate. The index is above the limit for all groups, see Figure 3.15. The index is even closer to one after the 16:th measurement. The increased variation after measurement 16 is consequently probably due to fixture faults. This conclusion is in accordance with the information known; there is a noticeable fixture fault in measurement 16 to 36.

3.3.6 Fixture failure subspace chart

To control the variation that originates from the fixtures, the amount of variation in the fixture failure subspace can be studied. The fixture failure subspace is spanned by U_r, the orthonormal basis for the r-dimensional column space of the sensitivity matrix A. The vectors that span A can be collected into the matrix P, and the same method as in Section 3.3.2 can be used by considering the statistics

$$T^2_{fixture} = (\boldsymbol{x} - \bar{\boldsymbol{x}})^T PP^T \Sigma^{-1} PP^T (\boldsymbol{x} - \bar{\boldsymbol{x}}),$$

and

$$SPE_{fixture} = (\boldsymbol{x} - PP^T \boldsymbol{x})^T (\boldsymbol{x} - PP^T \boldsymbol{x}).$$

The control limit for the $T^2_{fixture}$-chart is $\chi^2_r(\alpha)$. For the SPE-chart a control limit is not yet developed.

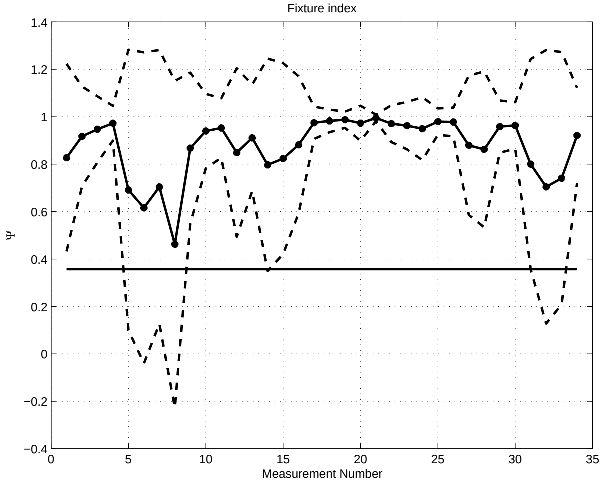


Figure 3.15: *Fixture failure variation index, bumper assembly.*

Test on data

In Figure 3.16 the $T^2_{fixture}$-statistic for the outer panel assembly is plotted. As a control limit $\chi^2_r(0.001)$ is used. The $SPE_{fixture}$-statistic measure the amount of variation that are not contained in the fixture failure subspace.

The chart in Figure 3.16 is based on measurements with the trends eliminated, i.e. z_i, $i = 1, ..., 651$. The chart indicates that there is too much variation in the fixture failure subspace.

In Figure 3.17 the $T^2_{fixture}$ is plotted for the bumper assembly. The chart alarms before measurement 16 (i.e. before the known fixture related error) and indicates, just like the fixture index, that there are some fixture related errors in measurement one to 16 as well. After measurement 16 there is a considerable change in $T^2_{fixture}$ and it is obvious that there is increased variation in the fixture failure subspace.

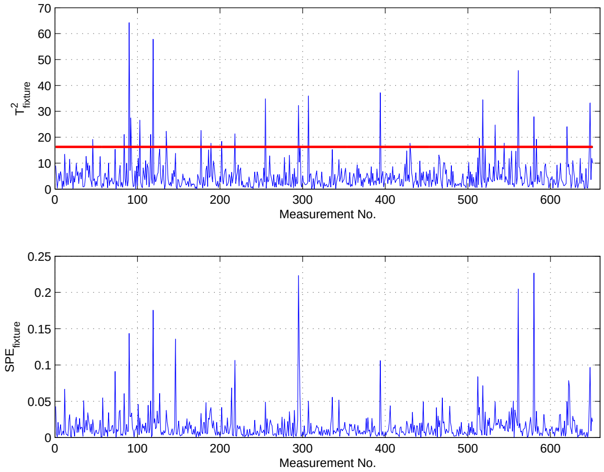


Figure 3.16: *$T^2_{fixture}$- and $SPE_{fixture}$-chart to control the variation in the fixture failure subspace, side panel assembly.*

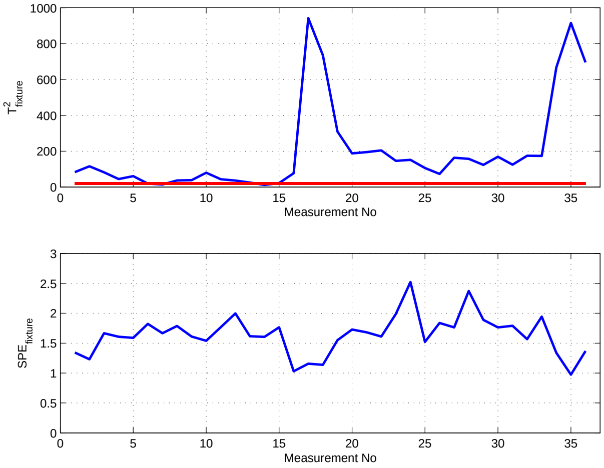


Figure 3.17: *$T^2_{fixture}$- and $SPE_{fixture}$-chart to control the variation in the fixture failure subspace, bumper assembly.*

3.4 Fixture diagnosis

The methods described in Section 3.3 are used to control the process. Usually, the root cause of a problem detected by statistical process control methods is not known. The fault can depend on material, external circumstances, fixture faults and so on. In the previous section some methods for discovering fixture related faults were given. If a process is out of control and the fixture fault index is below the corresponding limit the fixtures can be excluded from the list off possible root causes. If the index is above the limit the root cause is probably fixture related and then it is of course desirable to find out what fixture and what locator that caused the problem. In this section some suggestions of how to identify the cause of fixture related faults are given.

The methods of fixture diagnosis can be divided into two separate groups of approaches; the methods that require knowledge of the assembly process, coordinates of inspection points et cetera and the data based methods that only utilize inspection data for diagnosis.

3.4.1 Root Cause Analysis

One way to find the reason of the unwanted variation in the inspection points would be to estimate the variation in each locator of the fixtures involved. The locator or locators affected by most variation is said to be the root cause of the variation. This approach is considered by Carlson and Söderberg (6), and requires knowledge of the assembly process.

The inspection data are supposed to have covariance matrix Σ_x. When the fixture failure index is large, we will assume that Σ_x can be written as

$$\Sigma_x = A\Lambda_\delta A^T + \sigma^2 I_p, \tag{3.2}$$

where A is the sensitivity matrix and Λ_δ is a diagonal locator covariance matrix. The model

$$\Sigma_x = A\Sigma_\delta A^T + \sigma^2 I_p,$$

can also be used. Here, the locator covariance matrix Σ_δ is a full matrix. However, the condition that the locators will be independent, and consequently that the covariance matrix will be diagonal, is usually no limitation, since variation in one locators seldom affects other locators. Therefore, the model described in Equation (3.2) will be used.

The measurements are supposed to follow a multivariate normal distribution. If it is possible to estimate the elements in Λ_δ, the diagnosis can be accomplished. A maximum likelihood estimate of Λ_δ and σ^2 can be found by maximizing the likelihood function

$$\begin{aligned} l(\boldsymbol{x};\boldsymbol{\mu},\Sigma_x) = &-\frac{n}{2}\log\det(2\pi\Sigma_x) - \frac{n}{2}\text{Trace}(\Sigma_x^{-1}S_x) \\ &- \frac{n}{2}\text{Trace}(\Sigma_x^{-1}(\bar{\boldsymbol{x}}-\boldsymbol{\mu})(\bar{\boldsymbol{x}}-\boldsymbol{\mu})^T). \end{aligned}$$

The maximization may be done numerically by Fishers scoring method; more about this method can be read in Jöreskog (22). Large sample confidence regions for the estimates may be constructed.

Unfortunately, it is not always possible to separate variation from different locators. The reason for this is that two locators can cause the same dimensional deviation in the inspection points. If this is the case, the assembly is said to be incomplete diagnosable. The conditions for complete diagnosability implies the following relation

$$A\Lambda_1 A^T + \sigma_1^2 I_p = A\Lambda_2 A^T + \sigma_2^2 I_p \Leftrightarrow \Lambda_1 = \Lambda_2 \text{ and } \sigma_1 = \sigma_2.$$

This condition can be rewritten as $T = A \otimes A$ have full rank, Carlson and Söderberg (6). Further, the number of inspection points must exceed the number of locators analysed. If a full locator covariance matrix is used, the condition on A for complete diagnosability is strengthened to A having full rank. If the assembly is not completely diagnosable, it is still possible to perform a diagnosis. By solving the linear programming problems

$$\max_\lambda \boldsymbol{\lambda}_k$$

$$L\boldsymbol{\lambda} = L\boldsymbol{\lambda}^*,\ \boldsymbol{\lambda} \geq 0$$

and

$$\min_\lambda \boldsymbol{\lambda}_k$$

$$L\boldsymbol{\lambda} = L\boldsymbol{\lambda}^*,\ \boldsymbol{\lambda} \geq 0$$

for each k, the minimal and maximal possible locator variance can be found. Here, L is an orthonormal basis matrix, V_r, for the r-dimensional column space of T^T and λ^* is a particular solution of the problem.

The estimation of locator variances is also considered by Ding et al. (11). They rewrite Equation (3.2) as

$$vec(\Sigma_x) = T\,vec(\Lambda_\delta) + vec(I)\sigma^2.$$

Using the notation $\mathbf{B} = [T\,vec(I_p)]$ and $\mathbf{d} = vec(\Sigma_x)$ this can be written as

$$\mathbf{B}\lambda^* = \mathbf{d},$$

and the the equation is solved by multiplication with the inverse of $\mathbf{B}$.

Test on data

The outer side panel assembly considered in Section 3.2 is incompletely diagnosable, and the minimal and maximal variation for each locator can be seen in Figure 3.18. The method by Carlson and Söderberg (6) is used for this case. The variation is calculated from measurement 53 to measurement 60, which is a period with a high fixture failure index. The horizontal limit in the figure correspond to $6\sigma = 0.5$ mm, which seems to be reasonable to use as a an upper limit for the allowable variation in a locator. As seen in Figure 3.18, the interval from minimal to maximal variation for locator 1 is the only one which is above the limit. Therefore, this locator is pointed out as the main root cause. Locator 1 is a hole in the front part of the assembly, controlling the assembly in z-direction.

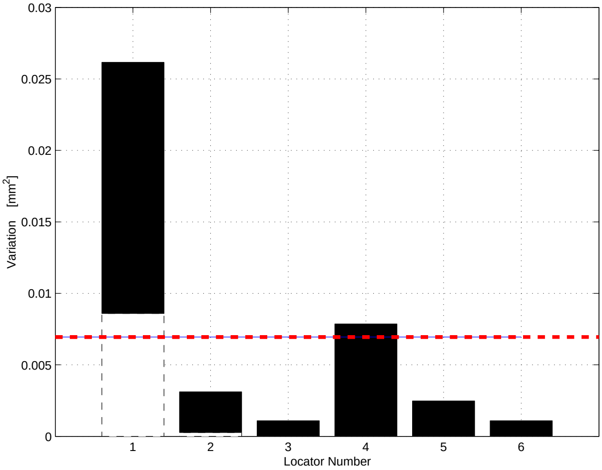


Figure 3.18: *Minimal and maximal variation in the locators due to incomplete diagnosability, side panel assembly. The extension of the dark area is from the minimum variance to the maximum variance.*

The methods are applied on the bumper assembly as well. The assembly is completely diagnosable. Here, both the estimates by Carlson and Söderberg (6) and by Ding et al. (11) are tested. The estimates can be seen in Figure 3.19. The first bar in each pair corresponds to the estimate developed by Carlson and Söderberg and the second one to the estimate by Ding et al.. Locator number six is the one containing most variation according to both estimates. There is also much variation in the first locator according to the second method. The sixth locator is a pin/hole contact controlling translation in y-direction. In this case study there is a key; the

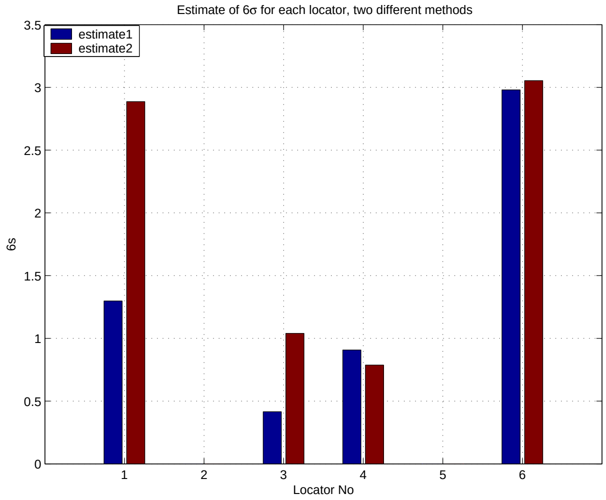


Figure 3.19: *Estimate of* 6σ *for the bumper assembly, measurement 17 to 36. The first bar in each pair corresponds to the estimate by Carlson and Söderberg, while the second one corresponds to the estimate by Ding et al.*

adjustment made was a correction of the pin/hole contact, i.e. the sixth locator in Figure 3.19. This adjustment reduced the variation in the measurements. This locator was pinpointed as the locator with most variation by both methods. However, the method by ding et al. indicated almost as much variation in the first locator.

There is also considerably variation in locator one, three and four. The fixture index indicated that there were fixture related variations in the process in measurement one to 16, i.e. before the variation in locator six occurred. Possibly the variation in these measurements could have been reduced by an adjustment of locator one.

3.4.2 Principal Component Analysis

In Section 3.3.2 PCA was described and utilized as a tool for process control. However, it is also possible to identify the sources of variation using PCA. Hu and Wu (17), propose that the result of a PCA can be interpreted by plotting the elements of each eigenvector at the respective inspection point location. If the normal directions of the inspection points are of opposite signs, it is important to include this information in the analysis. It is convenient to plot the eigenvector times the sign of the normal direction of the respective inspection point. This approach is data based, so when using this method there is no need to calculate the sensitivity matrix used in Section 3.4.1.

The PCA is conducted on the estimated joint covariance matrix, S, for the inspection points evaluated in all directions. This seems to be a more attractive approach than the method conducted by Hu and Wu (17). Their method is based on separate PCAs on the covariance matrix for the inspection points in each direction. The difference is illustrated in the following

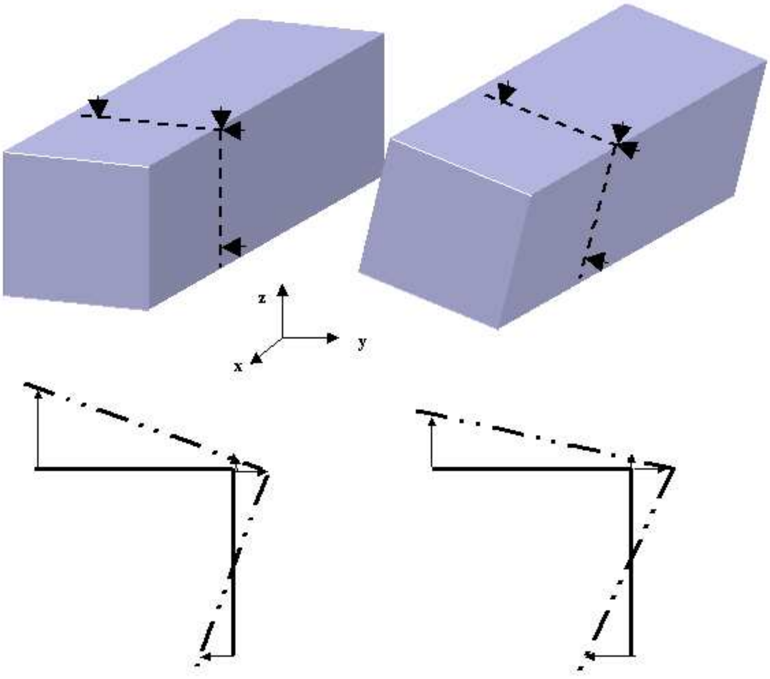


Figure 3.20: *Top: A box with four inspection points. To the right the box is rotated. Bottom: Principal components. To the left, based on the joint covariance matrix and to the right, based on separate covariance matrices. Observe that the right angle between the sides of the rotated box (illustrated by the dotted line) is preserved in the left picture, but not in the right one.*

example. Consider a box with four inspection points, see the upper part of Figure 3.20. Two of the points are evaluated in the y-direction, and two are evaluated in z-direction. The box is rotated around the x-axis as shown in the right-hand upper part of Figure 3.20. In the bottom part of Figure 3.20 the principal components for this rotation are outlined. If the principal components are based on the joint covariance matrix for the four inspection points the geometry of the box is preserved. If the PCA is based on the two separate covariance matrices, the relation between the y-plain and the z-plain of the box is lost, and less information of the deviation can be extracted.

If the separate matrices are considered it is difficult to compare the amount of variation explained by the principal components in the different directions. Therefore, when comparing the length of the eigenvectors of the different covariance matrices, the geometric proportions between the movements in x- and z-directions are not preserved.

Test on data

The analysis is now applied to the case studies. As before, only the within group variation, Σ_ϵ, is considered in the side panel assembly, and the principal component analysis is conducted on this matrix. This results in two principal components that together contain 78% of the total variation. In Figure 3.21 and Figure 3.22 the elements of the eigenvectors are plotted at the respective inspection point location. The first eigenvector, representing 58% of the total variation, corresponds mainly to a translation in z-direction, see Figure 3.21. The second eigenvector corresponds mainly to a translation in x-direction, see Figure 3.22. These translations are though combined with rotations, since the arrows, representing movements in different inspection points, are of unequal length.

The first principal component, explaining much of the variation in the assembly, corresponds mainly to a translation in z-direction. The conclusion must be that the root cause is one of the locators that position the assembly in z-direction. In the previous section was locator number one pointed out as the root cause. This locator was positioning the side panel in z-direction. However, the interpretation of the analysis is not completely obvious, since the translation is combined with a rotation.

The method is also applied to the bumper assembly. Here is the interpretation of the result more clear. In Figure 3.23 is the first principal

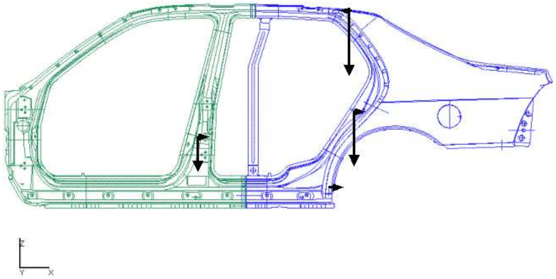


Figure 3.21: *The first eigenvector, representing 58% of the variation, side panel assembly.*

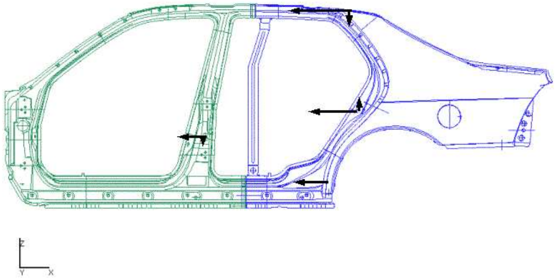


Figure 3.22: *The second eigenvector, representing 20% of the variation, side panel assembly.*

component drawn. This component explains 90% of the variation in data and indicates that there has been a translation in y-direction. This is in agreement with the conclusion drawn in Section 3.4.1.

This method is illustrative and no sensitivity matrix is needed. How-

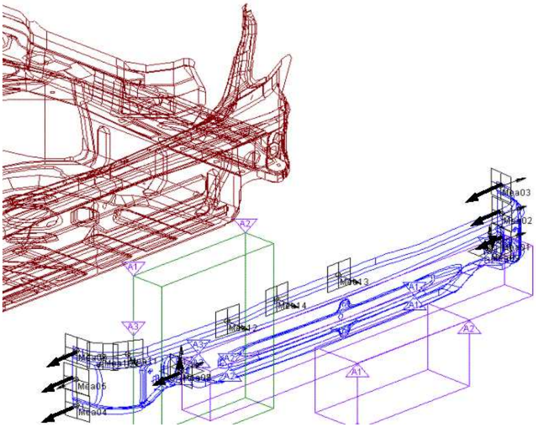


Figure 3.23: *The first principal component. Contains 90% of the variation, bumper assembly.*

ever, the method does not give a result as exact as the methods described in Section 3.4.1. This visual RCA is best suited for detecting single locator fault for small assemblies in one station (to avoid reorientation).

3.4.3 Designated Component Analysis

Designated Component Analysis (DCA) is an approach to fixture fault analysis developed by Camelio and Hu (3). DCA requires knowledge of the sensitivity matrix A. It is aiming to identify multivariate patterns, just like the PCA. This is achieved by defining a set of mutually orthogonal variation patterns with known physical interpretations. In sheet metal assembly processes, the physical interpretations are usually rigid body motion. Hence, the assembly variation can be decomposed in terms of all rigid body motions.

The designated patterns, denoted $\mathbf{d}_i$, $i = 1, ..., p$, span the subspace of the sensitivity matrix A. Their corresponding designated components, $\mathbf{w}_i$, can be calculated from inspection data $\mathbf{X}$ in the following way:

$$\mathbf{w}_i = \mathbf{d}_i^T * \mathbf{X}, \ i = 1, ..., p$$

The inspection data can then be expressed as the sum of rank one matrices:

$$\mathbf{X} = \mathbf{P}_1 + \mathbf{P}_2 + ... + \mathbf{P}_p$$

where

$$\mathbf{P}_i = \mathbf{d}_i * \mathbf{w}_i, \ i = 1, ..., p.$$

Using this decomposition, the multivariate variation contained in $\mathbf{X}$ can be separated into p terms, each corresponding to a designated pattern.

When the designated components are calculated they can be analysed and removed from the original data. The remaining variation, contained in the residuals, $\mathbf{R} = \mathbf{X} - \sum_i \mathbf{P}_i$, can be analysed by applying the principal component analysis described in the previous section to the residual covariance matrix $\mathbf{S}_{\mathbf{R}}$.

Test on data

When applying DCA to the side panel assembly described in Section 3.2 the three first designated patterns, corresponding to rigid body motions, are obtained. These span the subspace of the sensitivity matrix A. The first designated variation pattern contains 42% of the variation; the second 37% and the third one contains 21%. The first designated pattern, see Figure 3.24, seems to correspond to a translation in x-direction, but only in three out of four points. Therefore, this designated pattern cannot be interpreted. The second DC, see Figure 3.25, is also difficult to interpret. The third designated pattern, in Figure 3.26, corresponds to a translation in z-direction in two of the three points evaluated in z-direction.

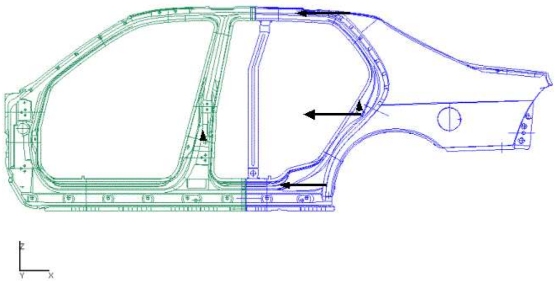


Figure 3.24: *First DC side panel, explains 42% of the variation.*

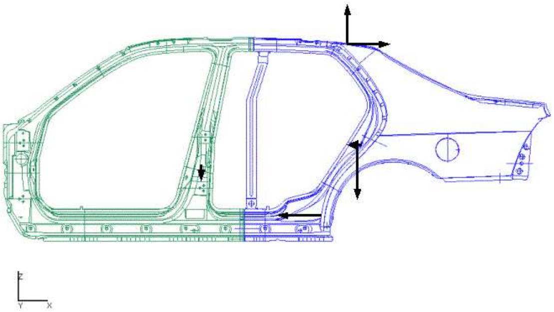


Figure 3.25: *Second DC, side panel assembly, explains 37% of the variation*

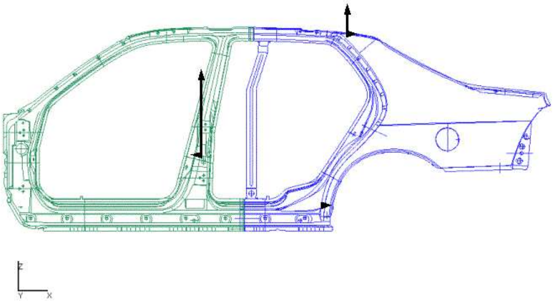


Figure 3.26: *Third DC, side panel assembly, explains 21% of the variation.*

After removing the designated components from data, a principal component analysis of the residuals is carried out. This gives the fourth, fifth and sixth designated components. The fourth DC, see Figure 3.27, corresponds to 75% of the variation in the residuals, and is also difficult to interpret.

In the second case study, the bumper assembly, three designated com-

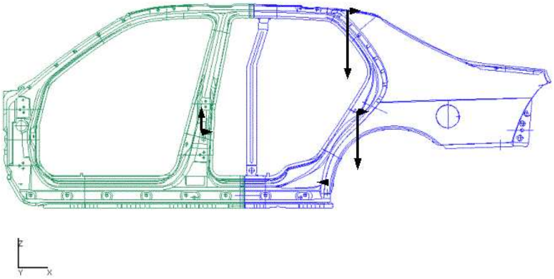


Figure 3.27: *Fourth DC, side panel assembly, explains 75% of the variation in the residuals.*

ponents corresponding to rigid body movements caught by the model were included.

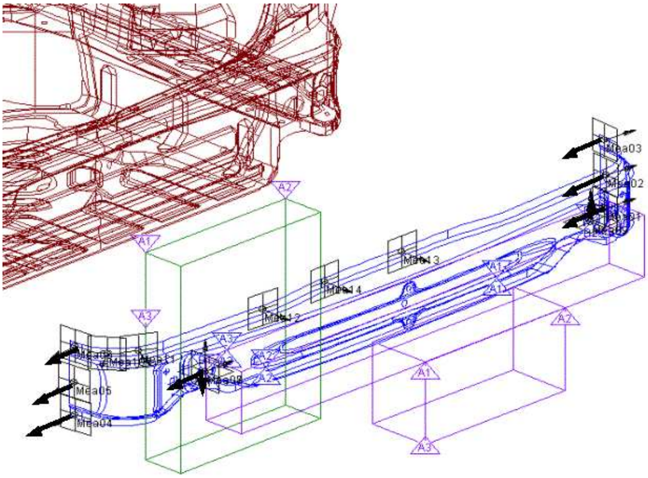


Figure 3.28: *The first designated component, bumper assembly. Contains 38% of the variation.*

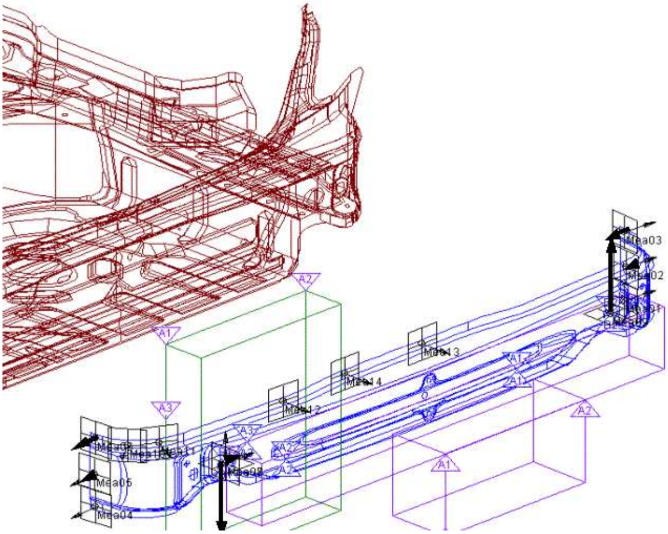


Figure 3.29: *The second designated component, bumper assembly. Contains 37% of the variation.*

The first component shown in Figure 3.28, explains 38% of the variation caught by the model and corresponds mainly to a translation in y-direction. The second component, see Figure 3.29, corresponds to a rotation around the x-axis, just like the third one, Figure 3.30. The second designated component contains 37% of the variation caught by the model and the third one contains 13%.

In the bumper assembly the DCA method points out translation in y-direction as a major root cause, just like the other methods tested. Some kind of rotation around the x-axis is incorrectly pointed out by DCA. When it comes to the side panel assembly there is no obvious interpretation of the results. The first DC seems to mainly correspond to a translation in x-direction, but only in three out of four inspection points. The second DC gives contradictory results, just like the third.

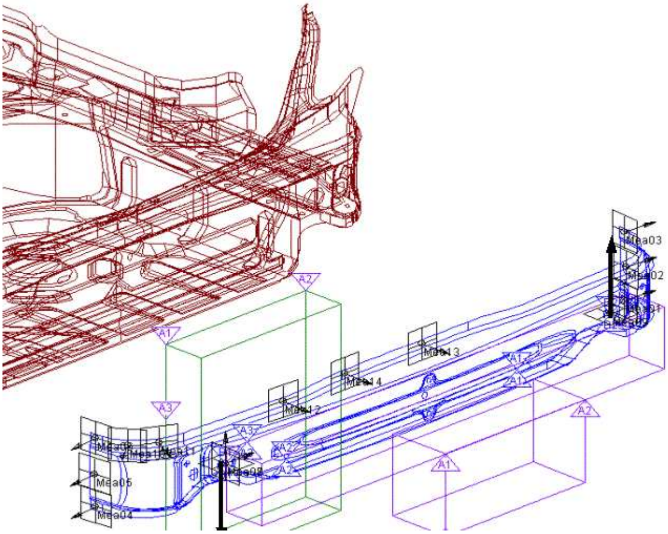


Figure 3.30: *The third designated component, bumper assembly. Contains 13% of the variation.*

3.5 RCA on another case study

Among the methods for diagnosis that was tested, the RCA described by Carlson and Söderberg (6) gives the most easily interpreted result. This method gave also the best agreement with the corrections known to be done in the bumper assembly. On the other hand, there is need of much information about the assembly considered. In this section this method will be further tested on industrial data in order to evaluate its usefulness.

3.5.1 The assembly

The assembly considered is a rear wheelhouse. The wheelhouse consists of five parts and is assembled in two stations. In the first station the wheelhouse panel is positioned and three different reinforcements are assembled to the panel, see left part of Figure 3.31. As shown in the right part of Figure 3.31 this subassembly is then put together with the last part of the rear wheelhouse, namely the support for the parcel shelf. Finally, the complete wheelhouse is measured in an inspection station. It is important to note that the subassembly from station one is positioned in station two using the locators of the wheelhouse panel. This is also the case when the wheelhouse is measured; the locators used can be seen in Figure 3.32. Using those locators results in that a variation in the contact between locator and wheelhouse panel in station one will never be seen as a variation in the

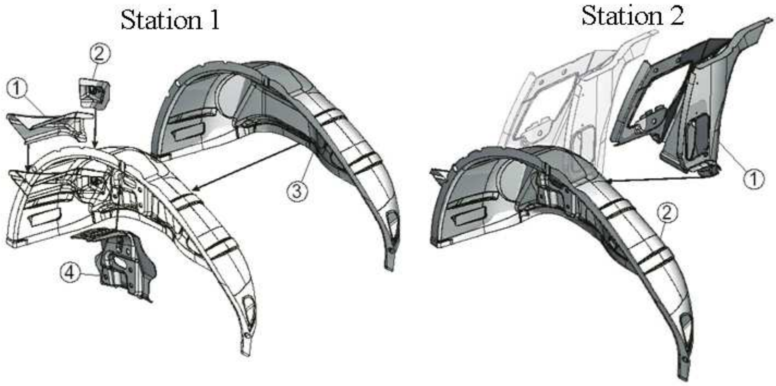


Figure 3.31: *In station one three reinforcements are assembled to the wheelhouse panel. In station two the subassembly from station one is put together with the support for the parcel shelf.*

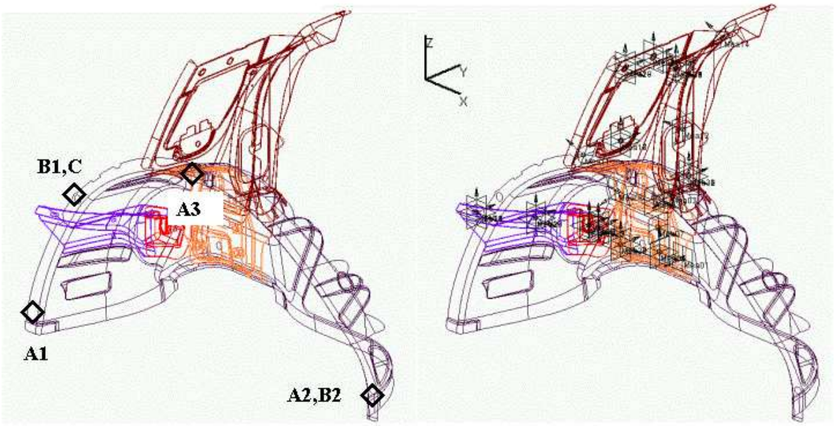


Figure 3.32: *To the left: The locators used to position the subassembly during inspection. To the right: The inspection points are illustrated by arrows.*

inspection points at the wheelhouse panel. Instead, this variation appears in the inspection points situated on the reinforcements that are joined to the panel in the first station. If there is variation in the contacts between the locators and the panel in the second station, this will result in variation in the inspection points on the parcel shelf support. In the right part of Figure 3.32 the 38 inspection points utilized for analysis are illustrated.

RCA is a method that demands knowledge about the sensitivity matrix A, describing the relation between movements in inspection points and movements in the contact between locators and parts. In this case the sensitivity matrix is determined by using simulations in a program called "Robust Design and Tolerancing" (RD&T). It is necessary to describe how every included part is positioned and if the position is completely determined by the fixture or if the mating part positions the part in some direction. The coordinates of the inspection points are also required.

3.5.2 Inspection data

The complete wheelhouse assembly is measured using a coordinate measurement machine. Totally, 38 inspection points are used in the analysis. There are 14 samples of wheelhouses, where each sample consists of three consecutive parts. The inspection data can be seen in Figure 3.33.

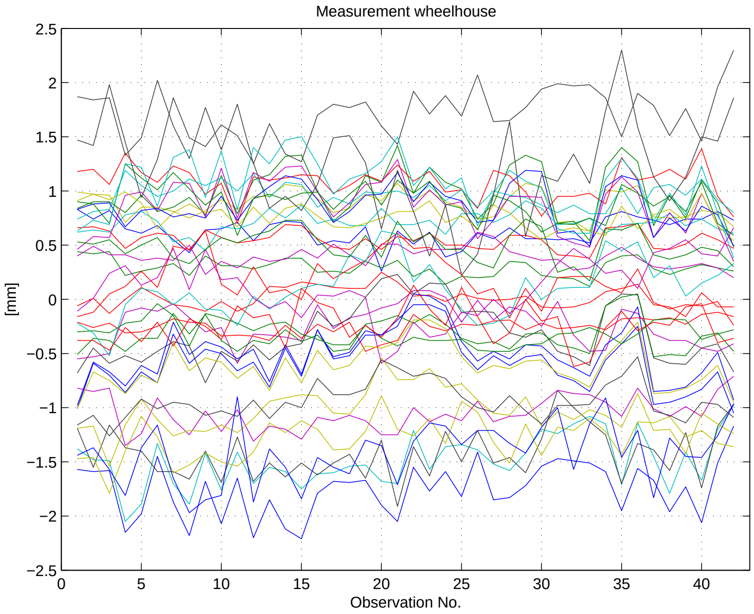


Figure 3.33: *Measurements of 42 wheelhouses in 38 inspection points. Deviations from nominal value are measured.*

The inspection data can be influenced by a lot of different sources of variation. Some of the variations are long-term variations, which slowly change the process over time, for example variations due to changes in raw material or wear in tools. To avoid mixing up these sources of variation with variation caused by the fixtures, only within samples variation is considered. This is logical since if there is fixture related variation, this variation will affect every produced item, and consequently also contribute to the variation within every sample.

In order to decide if the variation in data can be a consequence of variation in the contacts between parts and locators, the fixture fault variation index is determined. The index is calculated for each sample, see Figure 3.34. Since the index is above the line corresponding to the value of Ψ when

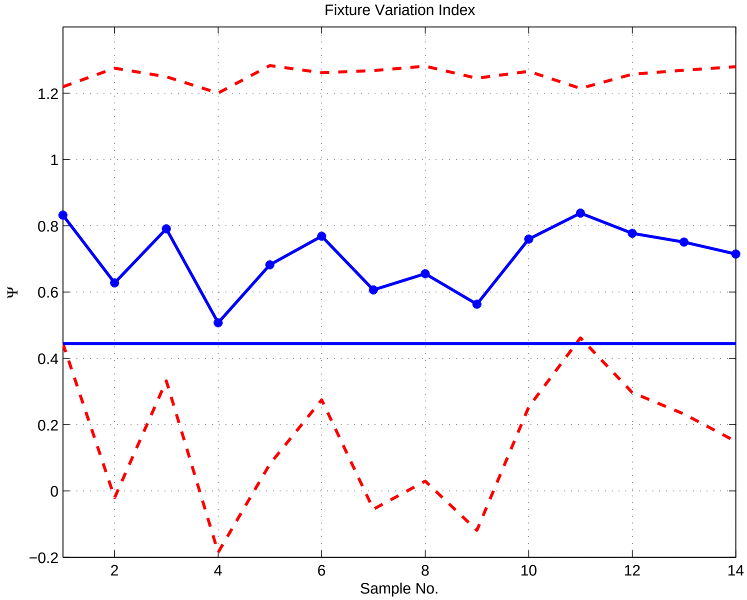


Figure 3.34: *Fixture fault variation index for 14 samples.*

there is no fixture fault, it seems reasonable to continue with the root cause analysis.

3.5.3 Root Cause Analysis

To conclude what fixture or fixtures that caused the variation, the variation in the contact between parts and locators are estimated using inspection data. This is done for most of the locators. Some locators are though excluded. The reason is that it otherwise would be necessary to use more inspection points in order to carry out a complete analysis.

In Figure 3.35 are the estimated variances shown. As seen, the major source of variation is the contact between the locator called B2 and the wheelhouse panel in the first station. The locator B2 consists of a pin in a slot and position the wheelhouse panel in z-direction, see Figure 3.36.

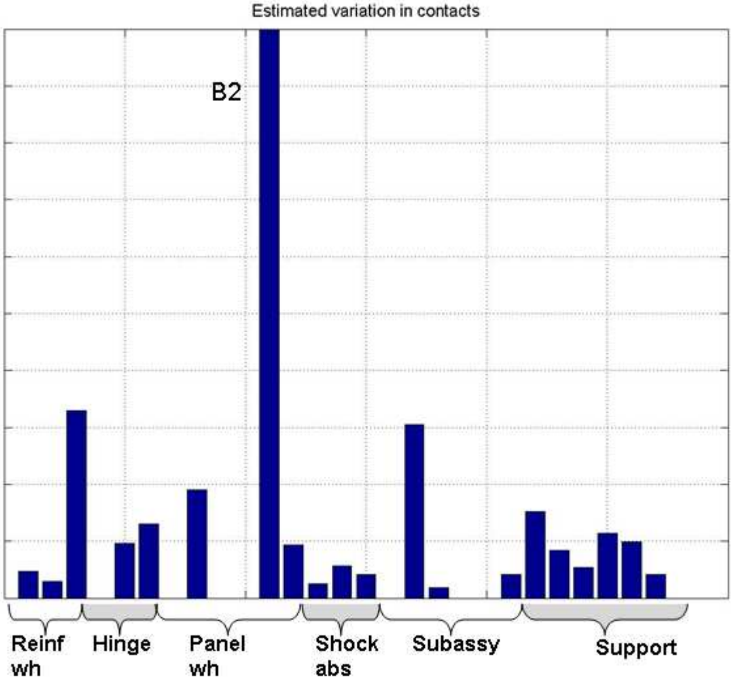


Figure 3.35: *Estimated variance in the contacts between parts and locators.*

3.5.4 Adjustment of the fixture

RCA is a tool for identifying the fixture related sources of variation in a process. When this identifying is done, the result should be translated into an adjustment of the fixtures. It is though important to note that RCA gives no outline for this adjustment. The work of doing the adjustment should be done by someone with good knowledge of the process and a good understanding about how different kinds of locators affect the positioning of parts.

Since the contact between the wheelhouse panel and the locator B2 was pinpointed as a major source of variation this locator is adjusted. The adjustment consists of changing the pin in B2 to an egg-shaped pin corresponding to the shape of the slot. After this modification 24 complete wheelhouses are measured. Unfortunately, this adjustment did not reduce

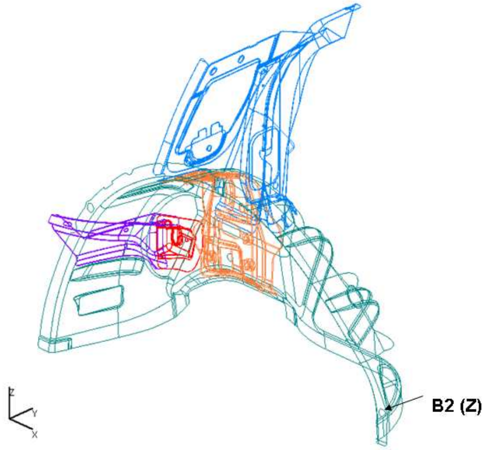


Figure 3.36: *The contact B2 in station 1 is a major source of variation.*

the variation. The variation in the inspection points situated on the reinforcements assembled in station one increased, see Figure 3.37. As mentioned before, variation in the positioning of the wheelhouse panel in station one, will give rise to variation in the inspection points on the reinforcements assembled to the panel in station one. The reason is that the wheelhouse assembly is positioned using the locators of the panel in the inspection station.

The adjustment lead to increased variation in the inspection points, but it is still of interest to analyse the inspection data after the adjustment to estimate the corresponding variation in contacts between parts and locators. In Figure 3.38 is the estimated variation before and after the adjustment shown. Here, the locators in station two are excluded, since they are not involved in the adjustment.

From Figure 3.38 it can be seen that the variation in the contact between the part and the adjusted locator B2 undoubtedly has increased. The variation has also increased in the contact between the panel and the locator A2. This locator is situated just beside B2, and position the panel in y-

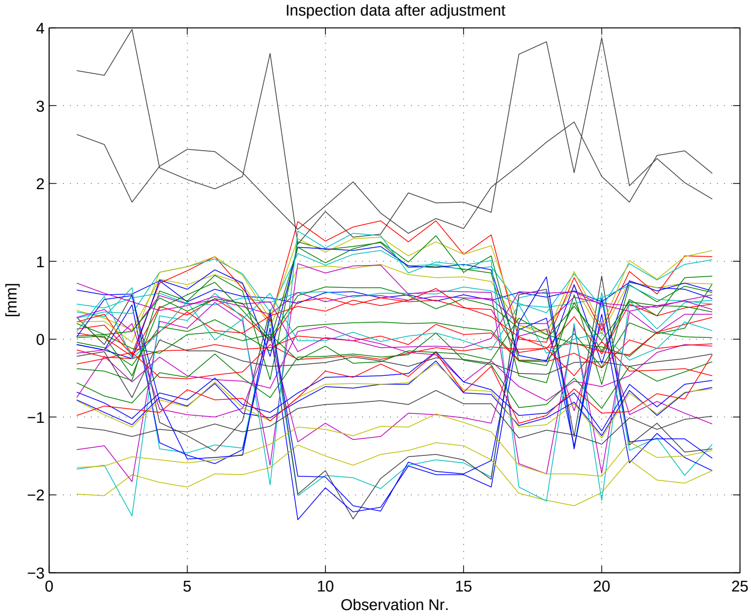


Figure 3.37: *Inspection data after the adjustment.*

direction.

3.5.5 Conclusions of the case study

Since the adjustment of the fixtures is well known it is a very good case for testing the method. In this case the variation increased, but the important thing is that the locator corresponding to these increased variation could be pinpointed by using RCA.

If the case would have been the reversed, i.e. there would have been much variation because of an unsuitable positioning element (like B2 after the adjustment), the method could have been used to pinpoint the source of the variation and is thereby a tool for reducing the variation.

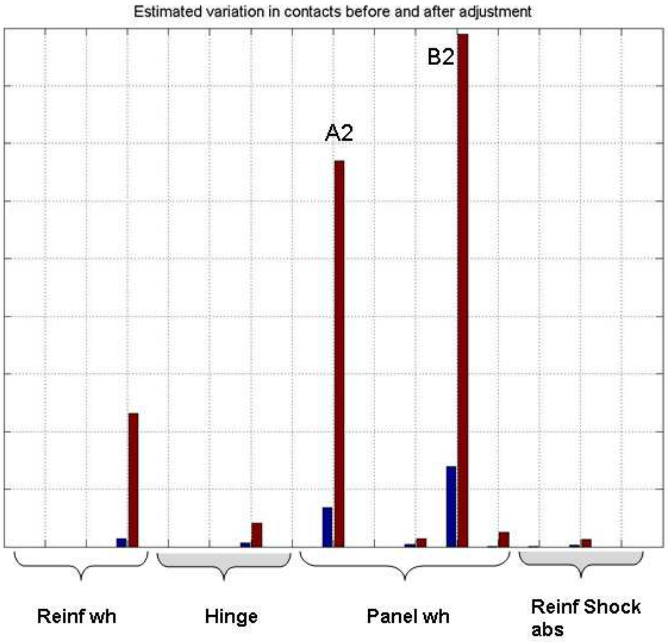


Figure 3.38: *For each locator the left bar corresponds to estimated variation in the contact between part and locator before adjustment, and the right bar corresponds to estimated variation after the adjustment.*

3.6 Discussion and conclusions

Different methods for as well multivariate quality control as diagnosis have been tested on the case studies. Both case studies turned out to be out of statistical control. The second case study, the bumper assembly, was in advance known to be out of control due to a fixture failure occurring after 16 measurements. This fixture failure occurred as a result of a defect locator in y-direction. No prior information of fault or fault types were available when it comes to the side panel assembly, but the result from different methods can still be compared.

3.6.1 Process control

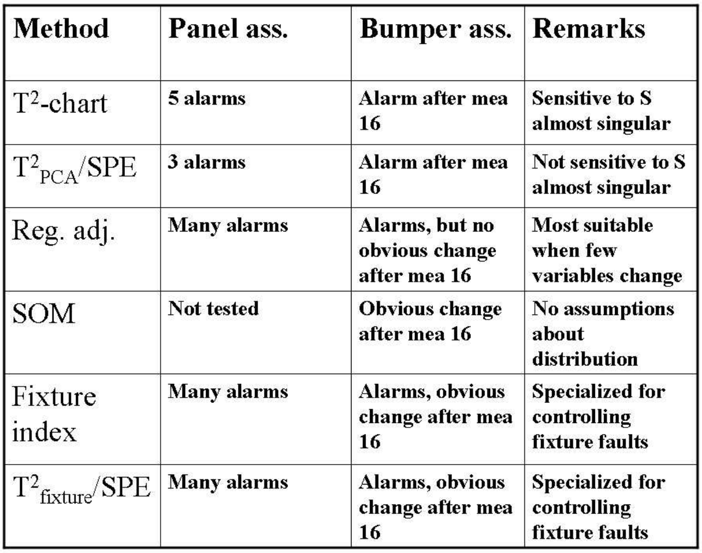
The T^2-chart is one of the most popular multivariate charts and it works satisfyingly on the case studies. It detects quickly the change in the process of the bumper assembly. The covariance matrix for the inspection data is though nearly singular. This affects the T^2-statistic and may in some cases lead to misinterpretations. When it comes to the side panel assembly there are several alarms even though trends in data are eliminated. The T^2-chart requires no advanced calculations and is easy to use. A disadvantage is that the chart gives no indication of which inspection points that caused an alarm. The chart is also based on the assumption that data is normally distributed.

The PCA- and SPE-chart is similar to the T^2-chart but operates in the subspace spanned by the principal components. This means that the dimension of data is reduced, but in the same time some information is lost. In addition there is need of two charts, both the T^2_{pca} and the SPE-chart. The PCA/SPE chart alarms after the change in the bumper assembly process, nevertheless there is a delay compared to the T^2-chart. The PCA-chart are, unlike the usual T^2-chart, not affected by singularity in the covariance matrix.

Regression adjustment differs from the other charts. This method can be used for controlling mean and variance separately, which is an advantage. However, this is also possible to achieve by using a T^2-chart complemented by a multivariate chart for controlling within group variation. However, such a chart can be complicated to use when the data are ungrouped. The regression adjustment chart is suitable when only a few of the variables are expected to change. However, this is usually not the case if the fault is caused by fixture fault. This property makes this chart unsuitable for controlling the process from the case studies. Using this chart it is necessary to generate one chart for each variable, but on the other hand this chart indicates what variable causing an alarm.

To train a SOM there is need of much data, but there is no need of assumptions on distribution of the data. In some cases the SOM helps finding the root cause of an erroneously state.

The fixture fault index and the fixture fault chart are both aimed to detect fixture related faults. For both methods there is need to know the so called A-matrix, the sensitivity matrix relating a movement in the locators to a corresponding movement in the inspection points. The methods are also intended for normally distributed data. The fixture fault index is useful when it comes to find the root cause of a variation. If the index is high a fixture probably caused the fault and the main efforts can be concentrated to examining the fixtures. The methods work well on the case studies.



Method	**Panel ass.**	**Bumper ass.**	**Remarks**
T^2-chart	**5 alarms**	**Alarm after mea 16**	**Sensitive to S almost singular**
T^2_{PCA}/SPE	**3 alarms**	**Alarm after mea 16**	**Not sensitive to S almost singular**
Reg. adj.	**Many alarms**	**Alarms, but no obvious change after mea 16**	**Most suitable when few variables change**
SOM	**Not tested**	**Obvious change after mea 16**	**No assumptions about distribution**
Fixture index	**Many alarms**	**Alarms, obvious change after mea 16**	**Specialized for controlling fixture faults**
$T^2_{fixture}$/SPE	**Many alarms**	**Alarms, obvious change after mea 16**	**Specialized for controlling fixture faults**

Figure 3.39: *A comparison of the different methods used for multivariate quality control.*

In Figure 3.39 the performances of the different charts are tabulated.

3.6.2 Fixture fault diagnosis

When it comes to finding the cause of the fixture fault there are two main methods, namely the RCA and the both visual methods PCA and DCA. The RCA is more complex then the PCA and DCA methods. On the other hand the result is clear and easy to interpret. In the PCA-method there is no need of the sensitivity matrix A, which is demanded in the RCA and DCA methods. For both PCA and DCA the calculation is simple, but the interpretation of the result is not always trivial. The DCA is similar to the PCA, but is specialized to find fixture faults. A drawback of DCA compared to PCA is that the sensitivity matrix is needed.

When applying the methods to the case studies the RCA identified the failing locator in the bumper assembly. This is a locator in y-direction and also the PCA and the DCA show translation in y-direction as a main problem. In this case there is only one locator in y-direction and this locator is consequently pointed out as the main cause of the variation. If there would have been several locators in y-direction, it might have been hard to separate them using the visual methods. The DCA does also incorrectly point out rotation around the x-axis as a root cause.

In the side panel assembly a pin/hole contact in z-direction located in the front of the doorframe is pointed out as the root cause by the RCA. This is confirmed by the PCA, where the first eigenvector corresponds to a translation in z-direction. This translation is though combined with a rotation. The designated components are very difficult to interpret and seem to give contradictory results. The multi-fixture side panel assembly is not completely diagnosable and both the visual methods PCA and DCA are very hard to interpret.

To sum up, PCA and DCA is easy to calculate and when using the PCA there is no need to know the sensitivity matrix A. However, the methods seem to be best suited for identifying single locator fault in small completely diagnosable assemblies. The RCA is a more versatile method that gives more exact results. However, to use this method the sensitivity matrix must be known.

Bibliography

[1] J. Ahola, E. Alhoniemi, and O. Simula. Monitoring industrial processes using the self-organizing map. In *Midnight-Sun Workshop on Soft Computing Methods in Industrial Applications*, Kuusamo, Finland, June 1999. IEEE.

[2] F. B. Alt. Multivariate quality control: State of the art. In *Transactions of the 1982 ASQC Quality Congress*, pages 886–893, 1982.

[3] J. A. Camelio and S. J. Hu. Multiple fault diagnosis for sheet metal fixtures using designated component analysis. *Journal of Manufacturing Science and Engineering*, 126, February 2004.

[4] J. S. Carlson, L. Lindkvist, and R. Söderberg. Multi-fixture assembly system diagnosis based on part and subassembly measurement data. In *Proceedings of the ASME Design Engineering Technical Conferences, Baltimore*, 2000.

[5] J. S. Carlson and R. Söderberg. Locating scheme analysis for robust assembly & fixture design. In *Proceedings of the ASME Design Engineering Technical Conferences, Las Vegas*, 1999.

[6] J. S. Carlson and R. Söderberg. Assembly root cause analysis - a way to reduce dimensional variation in assembled products. *International Journal of Flexible Manufacturing Systems*, 15(2):113–150, April 2003.

[7] J. S. Carlson, R. Söderberg, and L. Lindkvist. Geometrical inspection point reduction based on combined cluster and sensitivity analysis. In *Proceedings of the ASME International Mechanical Engineering Congress & Exposition, Washington D. C.*, 2003.

[8] D. L. Ceglarek and J. Shi. Dimensional variation reduction for automotive body assembly. *Manufacuring Review*, 8(2):139–154, 1995.

[9] T. C. Chang and F. F. Gan. Application of $\bar{x}$ control chart with modified limits in process control. *Quality and Reliability Engineering International*, 15:355–362, 1999.

[10] H. Chen. A multivariate process capability index over a rectangular solid tolerance zone. *Statistica Sinica*, 4:749–758, 1994.

[11] Y. Ding, A. Gupta, and D. Apley. Singularity issues in fixture fault diagnosis for multi-station assembly processes. *ASME Transactions, Journal of Manufacturing Science and Engineering*, 126(1), February 2004.

[12] M. D. Hawkins. Regression adjustment for variables in multivariate quality control. *Journal of Quality Technology*, 25(3), 1993.

[13] S. Haykin. *Neural Networks.* Prentice Hall, 2nd edition, 1999.

[14] A. J. Hayter and K-L. Tsui. Identification and quantification in multivariate quality control problems. *Journal of Quality Technology*, 26(3):197–208, 1994.

[15] S.D. Holmes and A. E Mergen. Exponentially weighted moving average acceptance control chart. *Quality and Reliability Engineering International*, 16:139–142, 2000.

[16] H. H. Hotelling. Multivariate quality control illustrated by the air testing of sample bombsights. *Techniques of Statistical Analysis*, pages 111–184, 1947.

[17] S. J. Hu and S. M. Wu. Identifying sources of variation in automobile body assembly using principal component analysis. In *Transactions of NAMRI/SME*, 1992.

[18] J. E. Jackson. Principal components and factor analysis: Part 1 - principal components. *Journal of Quality Technology*, 12:201–213, 1980.

[19] J. E. Jackson. Multivariate quality control. *Communication in Statistics - Theory and Method*, 14(11), 1985.

[20] J. E. Jackson. *A User's Guide To Principal Components.* John Wiley & Sons, New York, 1991.

[21] R. A. Johnson and D. W. Wichern. *Applied Multivariate Statistical Analysis.* Prentice-Hall, 1998.

[22] K. G Jöreskog. Analysis of covariance structures. *Scandinavian Journal of Statistics*, 8:65–92, 1981.

[23] T. Kohonen. *Self-Organizing Maps.* Springer, New York, 3rd edition, 2000.

[24] L. Lindkvist and R. Söderberg. Computer-aided tolerance chain and stability analysis. *Journal of Engineering Design*, 14(1):17–39, March 2003.

[25] D. C. Montgomery. *Introduction to Statistical Quality Control.* John Wiley and Sons, 4rd edition, 2001.

[26] G. C. Runger, F. B. Alt, and D. C. Montgomery. Contributors to a multivariate statistical process control chart signal. *Communication in Statistics - Theory and Methods*, 25(10):2657–2213, 1996.

[27] Saab standard no. 3546.

[28] Saab standard no. 3581.

[29] H. Shahriari, N. F. Hubele, and F. P. Lawrence. Multivariate process capability vector. In *Proceedings of the 1995 4th Industrial Engineering Research Conference.* IEE, May 1995.

[30] R. Söderberg and L. Lindkvist. Computer aided assembly robustness evaluation. *Journal of Engineering Design*, 10(2):165–181, 1999.

[31] R. Söderberg and L. Lindkvist. Two-step procedure for robust design using cat technology. In *6th CIRP International Seminar on Computer Aided Tolerancing*, Enschede, the Nederlands, March 1999.

[32] W. Taam, P. Subbaiah, and J. W. Liddy. A note on multivariate capability indices. *Journal of Applied Statistics*, pages 339–351, 1993.

[33] G. Taguchi. *Introduction to Quality Engineering.* Asian Productivity Centre, 1986.

[34] A. Ultsch and H. P. Siemon. Kohonen's self organizing feature maps for exploratory data analysis. In *Proc. Int. Neural Network Conf. Dordrecht, The Netherlands*, 1990.

[35] F.K Wang, N. F. Hubele, F. P. Lawrence, J. D. Miskulin, and H. Shahriari. Comparison of three multivariate process capability indices. *Journal of Quality Technology*, 32(3):263–275, 2000.

[36] G. B. Wetherhill and D. W. Brown. *Statistical Process Control, Theory and practice.* Chapman & Hall, 1 edition, 1990.

[37] C. Wickman and R. Söderberg. Comparasion of non-nominal geometry models represented in physical versus virtual environments. In *Proceedings of IMECE'03 ASME International Mechanical Engineering Congress & Exposition*, Washington D.C., November 2003.

[38] W. H Woodall. Controversies and contradictions in statistical process control. *Journal of Quality Technology*, 32(4), 2000.

PAPER B

An investigation of the effect of sample size on geometrical inspection point reduction using cluster analysis

Wärmefjord, K., Carlson, J., Söderberg, R., 2010, CIRP Journal of Manufacturing Science and Technology, vol. 3 pp. 227–235, doi: http://dx.doi.org/10.1016 /j.cirpj.2010.12.001.

PAPER C

A Measure of the Information Loss for Inspection Point Reduction

Wärmefjord, K., Carlson, J., Söderberg, R., 2009, Journal of Manufacturing Science and Engineering, 131 (5), doi:10.1115/1.4000105.

Also published in Proceedings of the ASME 2008 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, August 3-6, 2008, Brooklyn, New York, USA.

PAPER D

Tolerance Simulation of Compliant Sheet Metal Assemblies Using Automatic Node-Based Contact Detection

Wärmefjord, K., Lindkvist, L., Söderberg, R., 2008, Proceedings of ASME International Mechanical Engineering Congress & Exposition, October 31-November 6, 2008, Boston, USA .

PAPER E

Strategies for Optimization of Spot Welding Sequence with Respect to Geometrical Variation in Sheet Metal Assemblies

Wärmefjord, K., Söderberg, R., Lindkvist, L., 2010, Proceedings of ASME International Mechanical Engineering Congress & Exposition, November 12-18, 2010, Vancouver, Canada.

Also submitted to ASME Journal of Manufacturing Science and Engineering.

PAPER F

Variation Simulation of Spot Welding Sequence for Sheet Metal Assemblies

Wärmefjord, K., Söderberg, R., Lindkvist, L., 2010, Proceedings of NordDesign2010 International Conference on Methods and Tools for Product and Production Development, August 25-27, 2010, Gothenburg, Sweden. ISBN 978-91-633-7064-9.

PAPER G

Combining Variation Simulation with Welding Simulation for Prediction of Deformation

Pahkamaa, A., Wärmefjord, K., Karlsson, L., Söderberg, R., Goldak, J., 2010, Proceedings of ASME International Mechanical Engineering Congress & Exposition, November 12-18, 2010, Vancouver, Canada.

Also submitted to ASME Journal of Computing and Information Science in Engineering.

PAPER H

Including Assembly Fixture Repeatability in Variation Simulation

Wärmefjord, K., Söderberg, R., Carlson, J., 2010, Proceedings of ASME International Mechanical Engineering Congress & Exposition, November 12-18, 2010, Vancouver, Canada.