Configuration Requirements for Log-Periodic Array Antennas

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Abstract—Log-periodic array antennas can provide a constant radiation performance and good reflection coefficient performance for a wide frequency band, which can be considered as an important characteristic of this type of antenna. In this paper, the configuration requirement for general log-periodic array antennas is presented in order to have constant radiation function and periodicity of the reflection coefficient. Therefore, a guide rule for designing a wideband log-periodic array is obtained and a more efficient numerical simulation method can be applied for purely log-periodic array antennas.

I. INTRODUCTION

Log-periodic array antenna is widely used in wideband applications since it was introduced 50 years ago [1]-[3]. One of the recent developments of log-periodic array antenna is the Eleven antenna [4]-[17]. The Eleven antenna is a log-periodic folded-dipole pair array which has two unique radiation characteristics: constant beamwidth and fixed phase center location over decade bandwidth. Therefore, the Eleven antenna is very suitable as a feed for reflector antennas for radio telescopes (SKA and VLBI2010). Although log-periodic array antennas have been investigated thoroughly, some new questions still arise. For example, if a log-periodic array with a scaling factor \( k \) is infinite, it is obvious that the performance of both radiation and reflection coefficient will be the same at frequencies \( k^i \cdot f_0, i = -\infty, \cdot \cdot \cdot , 0, 1, \cdot \cdot \cdot , \infty \), where \( f_0 \) is any frequency point, which means the log-periodic array has periodic performance over the frequency. However, for practical antennas, the log-periodic array has to be truncated at the both ends, the high frequency end and the low frequency end. Then, one question is: in which conditions, a log-periodic array antenna has minimum size for a constant performance over a certain frequency band?

In this paper, some basic questions about log-periodic array antennas are discussed, and some new results are presented.

II. CONDITIONS FOR CONSTANT RADIATION CHARACTERISTICS

For N-element log-periodic array with a scaling factor \( k \) (dipole \( i \) is a dipole scaled from dipole 1 with a scaling factor \( k^{i-1} \)), we can use the equivalent circuit shown in Fig. 1. Assume that the resonant frequency for each dipole is \( f_1, f_2 = f_1/k, f_N = f_1/k^{N-1} \). It is obvious that the antenna has minimum size for a constant performance at \( f_i, (i = 1, \cdot \cdot \cdot , N) \) if at each frequency \( f_i \), only dipole \( i \) is operating while all rest dipoles are not operating. In other words, at \( f_i = f_1/k^{i-1} \), currents on all dipoles except for dipole \( i \) should vanish. Therefore, the condition for the minimum size log-periodic array antenna with constant radiation characteristics can be written as

\[
V_i(f_i) = \frac{Z_{ij}(f_i)}{Z_{ii}(f_i)} j = 1, \cdot \cdot \cdot , i, i + 1, \cdot \cdot \cdot , N. \tag{1}
\]

where \( V_i(f_i) \) is the voltage excitation on dipole \( i \) at frequency \( f_i \), \( Z_{ij}(f_i) \) the mutual impedance between dipole \( i \) and \( j \) at frequency \( f_i \). For detail of the derivation, please refer to [11]. This means that when the above condition is satisfied, the voltage source from the feeding network and the voltage source due to the mutual couplings on all dipoles except dipole \( i \) are in the same amplitude but 180° out of phase at frequency \( f_i(i = 1, 2, \cdot \cdot \cdot , N) \), which leads to currents on all dipoles except dipole \( i \) to vanish. Therefore, the radiation function of this array will be constant as \( G(\theta, \phi) \) at frequency \( f_i(i = 1, 2, \cdot \cdot \cdot , N) \) with the minimum size.

![Fig. 1. Equivalent circuit of N-element log-periodic array.](image)

III. CASCADED LOG-PERIODIC ARRAY

For cascaded log-periodic array, shown as in Fig. 2, we can use the equivalent circuit shown in Fig. 3, where \( Z_{ii} \) is the input impedance of radiating element \( i \) when the output port
is shorted \( (V_{i+1} = 0) \) and all other radiating elements do not exist \( (Z_{ij} = 0 \text{ for all } j \neq i) \), and \( Z_{ij} \) is the mutual impedance among the radiating elements. Then, the condition for the minimum size for cascaded log-periodic array antenna with constant radiation characteristics can be written as

\[
\frac{[1 - S_{21,i+1}(f_i)]^2}{2S_{21,i+1}(f_i)} = \frac{Z_{i+1,i}(f_i)}{Z_{ii}(f_i)}
\]

when \( S_{11,i} = S_{22,i} = 0 \). Here \( S_{21,i} \) means S parameter from port 1 to port 2 in dipole \( i \). For detail, refer to [11].

IV. PERIODICITY OF THE INPUT IMPEDANCE

If a log-periodic array is infinite at both ends (infinitely small at one end and extended to infinitely large at other end), it is obvious that the input impedance of the array is periodic over the frequency. However, for practical antennas, log-periodic arrays have to be truncated in some way. Therefore, the purely periodic performance will not hold any more. A more detailed analysis and derivation of the periodicity of the input impedance will be presented in a journal article, and a brief is given here.

Here we assume that the excitation (feeding) port is at the input port of the smallest radiation element in the array, which is also the case in the most log-periodic array in practice. If we use \( T_i(f) \) to present the ABCD matrix of the first element in the array at frequency \( f \), we have the following periodicity theorem for a finite log-periodic array as

\[
Z_{in}(f) = T \otimes Z_{in}(f/k^n)
\]

where

\[
T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=0}^{n-1} T_i(f/k^{n-i}),
\]

and \( k \) is the scaling factor for the array. From the above, we can calculate the input impedance at high frequency by the value of the impedance at low frequency, which leads to a very efficient computation for numerical simulation.

V. EXAMPLES

We take the Eleven antenna as an example to verify the analysis in Sect. III and Sect. IV.

The basic configuration of the Eleven antenna is two parallel folded dipoles separated by about half the resonant wavelength of the folded dipole pair and located above a ground plane. Then, the basic configuration is scaled log-periodically to a number of folded dipole pairs and these folded dipoles are cascaded one after another in each side of the pair. Fig. 4 shows a photo of the dual polarized Eleven antenna for 2-13 GHz [13].

A. Constant Radiation Beam

Here a 7-folded-dipole-pair Eleven antenna is simulated by using a commercial solver - CST Microwave Studio [22] for verifying the analysis in Sect. III. The folded dipoles are modeled by infinitely-thin PEC strips, shown in Fig. 5. Two ports are defined for each folded dipole for the analysis, as for example \( P_{16} \) and \( P_{26} \) in the figure represent port 1 and port 2 for dipole 6. The ground plane lies on the \( x-y \) plane. The Eleven antenna is excited from the innermost pair of dipoles (the first pair of dipoles) at the differential port \( P_{11} \).
with the same amplitude and phase for both the dipoles in the pair. Therefore, we set the \( y-z \) plane as a PMC (perfect magnetic conductor) symmetry plane in the model. The first pair of folded dipoles are approximately halfwave dipoles at their working (resonant) frequency \( f_1 \). The next pair of dipoles work at frequency \( f_2 = f_1/k \), and so on.

We take dipoles 4 and 5 in the Eleven antenna as an example for analysis of the condition for constant radiation characteristics. The resonant frequency for folded dipole 4 is \( f_4 \approx 2.8 \) GHz.

The impedances of the dipoles were simulated by using CST. Fig. 6 shows the equivalent dipoles of the antenna mode of dipole 4 and dipole 5 in CST, and the simulated antenna mode self impedance \( Z_{D,4} \) and mutual impedance \( Z_{D,45} \) versus frequency. By using [10] [18]

\[
Z_{ii} \approx 4Z_{D,ii},
\]

\( Z_{44} \) can be calculated. It is known from [10] that there is no radiation from the transmission-line mode of the folded dipole. Therefore, the total mutual impedance between folded dipoles is equal to the antenna mode mutual impedance, i.e., \( Z_{45} = Z_{D,45} \). Thus, the values of the right side of (2) for dipoles 4 and 5, i.e., \( -Z_{45}/Z_{44} \), can be calculated and are shown by the solid lines in Fig. 8.

Then, let’s look at the left side of (2) for dipoles 4 and 5. The port definition for each folded dipole can be seen in Fig. 5. Using CST, we can obtain the S parameters for each folded dipole. Fig. 7 shows the amplitudes of \( S_{11,4} \), \( S_{22,4} \) and \( S_{21,4} \). We can see that \( S_{11,4} \) and \( S_{22,4} \) are below -15 dB from 2.7 to 2.9 GHz. Therefore, it is acceptable to use (2) to judge if the folded dipole array satisfies the condition for constant radiation characteristics. The calculated values of the left side of (2) are shown by the dash lines in Fig. 8.

Now it can be seen that the amplitudes and the phases of the left side and the right side of (2) for dipoles 4 and 5 are quite close to each other from 2.7 to 2.9 GHz. Thus, the condition for constant radiation characteristics is satisfied for the Eleven antenna. From the physical viewpoint, it can be observed that a folded dipole makes \( Z_{ii} \) about four times of \( Z_{D,ii} \) and the phase of \( S_{21,4} \) about 180 degrees. Therefore, the current on dipole 5 due to the transmission of the cascaded network from dipole 4 and the current induced from dipole 4 by mutual coupling have similar amplitudes, but are about 180\(^\circ\) out of phase. This leads that, from 2.7 to 2.9 GHz, dipole 4 radiates significantly but not dipole 5. The radiation function is therefore kept constant.

Fig. 9 shows the measured radiation patterns in the \( \varphi = 45^\circ \) plane of the Eleven feed for 1 - 13 GHz in [6]. It can be observed that the radiation patterns are constant over a more-than-one-decade bandwidth.

**B. Input Impedance**

Fig. 10 shows a 14-folded-dipole-pair Eleven antenna. It should be noted that in order to verify (3), we use an Eleven antenna of randomly chosen dimensions. Fig. 11 shows comparison of the input reflection coefficients obtained through the simulation over 2 - 18 GHz by using CST, and obtained first through the simulation over 2 - 5 GHz by using CST, then by using (3) to calculate the input impedance values at high frequency up to 18 GHz. From the figure, it can be seen that the agreement between the two methods is quite good, and the computation time for the present method is less than one third of the full simulation up to 18 GHz by CST. Therefore, by using the present more accurate formula of the periodicity for the input impedance for a finite log-periodic array in (3), a more efficient calculation of the reflection coefficient is obtained.

**VI. CONCLUSIONS**

Analysis and discussions on the condition for constant radiation performance for the minimized size of a log-periodic array antenna, and a more accurate formula for the input impedance of a finite log-periodic array are presented in...
the paper. These analysis and discussions help us to design compact, wideband log-periodic array antennas with constant radiation performance in a very efficient way.

ACKNOWLEDGMENT

This work has been supported in part by The Swedish Foundation for Strategic Research (SSF) within the Strategic Research Center Charmant.

REFERENCES


