THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Product Configuration from a Mathematical Optimization Perspective

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Abstract

The optimal truck configuration for a certain customer is very specific and depends on for what transport mission and in which operating environment the truck is to be used. In addition, the customers normally specify other feature requirements ranging from visual appearance to advanced driver support systems. For this reason, the truck configurations that are made available for the customers are highly specialized. To achieve this in a cost efficient way, the manufacturer must be able to produce these configurations with a limited set of technical solutions. This is done by the use of a common architecture, in which technology is shared such that the same parts are used in different combinations, leading to a relatively small number of parts but a large number of possible configurations.

This thesis presents an approach to the configuration problem by analyzing it from a mathematical optimization perspective. By assuming that a truck can be described by a number of quality measures, which different customers may appreciate differently, the problem of deciding on a good product offer can naturally be formulated within a multi-objective optimization framework.

The areas within product development in which mathematical optimization can be applied are many and most often fundamentally different. In this thesis several subproblems of the whole product development problem are formulated in mathematical terms, and in the appended papers, the resulting mathematical models are considered in generalized forms. As established in the different papers, not just one optimization discipline is relevant in the product development context. The disciplines considered include multi-objective optimization, clustering, robust optimization, continuous global optimization, and nonlinear integer programming with an extension to a special type of non-sortable discrete variables.

For a large number of objectives—typical in many real-world applications— multi-objective optimization becomes cumbersome; the first appended paper provides a method for problem reduction such that the representation of the Pareto optimal set is kept as accurate as possible. The second paper considers a simplification of the configuration problem of finding appropriate sets of technical solutions that are to be combined into configurations, by assuming that the decision variables are continuous and box-constrained. Clearly, real-world problems often involve uncertainties in models and / or data. In the third appended paper, a new measure of robustness of solutions to multi-objective optimization problem is developed. The preferences of the decision maker are estimated by an utility function on which the robustness measure is based. In the fourth appended paper, optimization with respect to a type of non-sortable discrete variables is addressed. Theoretical and methodological investigations are performed, and by a reformulation of the problems as nonlinear integer programming problems mathematical optimization techniques are shown to be applicable. In the last appended paper, we explore a new principle for continuous global optimization of computationally expensive functions by, for each new function evaluation, minimizing the resulting worst-case optimality gap.

The main contribution of this thesis is the interpretation and formulation of a product development problem, including subproblems, in a mathematical optimization framework.

The thesis has been written in close cooperation with Volvo 3P.

Keywords: optimization, multiple objectives, platform-based products, heavy-duty trucks, configuration management, categorical variables, global optimization, robustness

Dissertation

The thesis consists of an introduction and five appended papers:

Paper I: P. Lindroth, M. Patriksson, and A.-B. Strömberg, *Approximating the Pareto optimal set using a reduced set of objective functions*. European Journal of Operational Research, 207 (3) (2010), pp. 1519–1534.

Paper II: P. Lindroth, M. Patriksson and A.-B. Strömberg, *Multi-objective design of a combinatorial structure*. Preprint 2011:12, Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, 2011.

Paper III: C. Cromvik, P. Lindroth, M. Patriksson and A.-B. Strömberg, *A new robustness index for multi-objective optimization based on a user perspective*. Submitted for publication in Journal of Optimization Theory and Applications.

Paper IV: P. Lindroth and M. Patriksson, *Pure categorical optimization – a global descent approach*. Submitted for publication in Optimization and Engineering.

Paper V: S. Jakobsson, P. Lindroth and A.-B. Strömberg, *A minimax strategy for global optimization*. Submitted for publication in Journal of Optimization Theory and Applications.

Parts of this thesis have been presented at the following international conferences:

- EURO XXII, European Conference on Operational Research, Prague, Czech Republic, July 8–11, 2007.
- SIAM OP08, SIAM Conference on Optimization, Boston, MA, USA, May 10–13, 2008.
- EngOpt 2008, International Conference on Engineering Optimization, Rio de Janeiro, Brazil, June 1–5, 2008.
- EURO XXIII, European Conference on Operational Research, Bonn, Germany, July 5–8, 2009.
- ISMP, International Symposium on Mathematical Programming, Chicago, Il, USA, August 23–28, 2009.

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I have had the pleasure of writing two of the papers in collaborations and I am grateful to my two co-authors Christoffer Cromvik and Stefan Jakobsson. I have enjoyed the teamwork a lot.

Many thanks to thank my colleagues at Volvo 3P, at FCC and at Chalmers for enjoyable working environments and for making it a hard choice to decide on which office to pick each morning. Thank you also for all assistance on various topics.

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Peter Lindroth
Göteborg, April 2011

Contents

1 Introduction

This thesis is the outcome of an industrial PhD project entitled *Product Configuration with respect to Multiple Criteria in a Heterogeneous and Dynamic Environment within an Extended Enterprise* performed at Volvo 3P and in cooperation with the Fraunhofer-Chalmers Research Centre for Industrial Mathematics (FCC) and the Department of Mathematical Sciences at Chalmers University of Technology and University of Gothenburg.

The thesis takes a mathematical optimization perspective on the product development of platform-based products with a common architecture enabling shared technology, specifically trucks, that are developed for heterogeneous markets. In 2008, the licentiate thesis *Product Configurations with respect to multiple Criteria – a Mathematical Programming Approach* [37] was published within the same project and of which this thesis is a continuation. In its introductory part, several aspects of the main problem were presented and analyzed. In the introduction of this thesis, the most important parts of [37] will be reviewed once more; however, the scope is narrowed and focuses on the areas in which actual contributions have been made.

One purpose of the introductory Sections 1–6 is to describe the practical problem at Volvo and to place it in a scientific context. Another purpose is to briefly introduce the mathematical areas in which the appended papers are localized and to place the appended papers in their scientific context. To see a clear connection between the real-world problem and the generalized mathematical problems studied, many cross-references will be given between the different aspects of the actual problem and the contents of the appended papers.

1.1 Background

The ultimate goal of a commercial company is to maximize the long-term dividends to the stakeholders. To enable this it is necessary to offer products to the market which match its demands in a cost-effective way.

Volvo 3P is a business unit within the Volvo group responsible for product planning, product development, and purchasing for the four brands Volvo Trucks, Mack Trucks, Renault Trucks, and UD Trucks. The research work presented in this thesis has been carried out within the product development of Volvo 3P, whose interest in this project is to explore the possibilities of utilizing a mathematical framework to support the development process of a platform-based product. This in order to provide a broad competitive product offer with high platform efficiency on an increasingly complex and global truck market.

Volvo 3P is a global company whose developed products are used in markets with very different characteristics concerning operating environments, legislations, and transport missions. This fact, together with a stiff competition, has led to a high degree of specialization and truck customization for individual customers. Thus, to be able to fulfil the demands of the customers, a great variety of truck configurations must be offered. A number of varying transport missions are illustrated in Figure 1, where the diverse necessary superstructures call for differing requirements on the trucks.

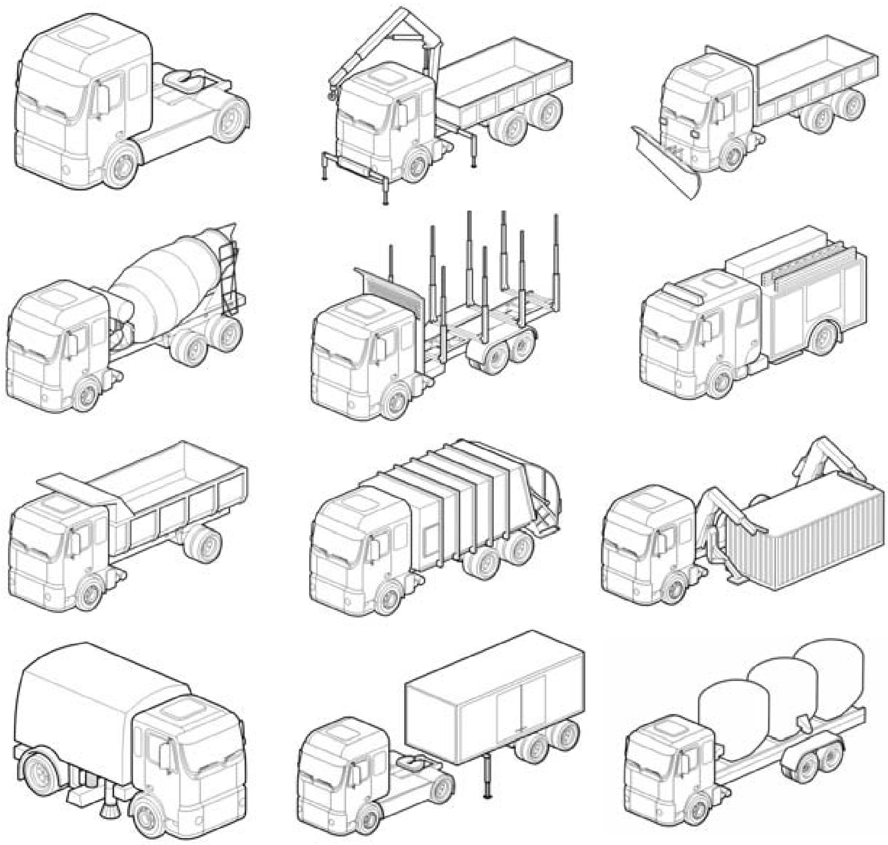


Figure 1: A number of superstructures illustrating various transport missions.

Each individual configuration has a certain perceived quality for each specific customer, given by his / her *utility function*[1]. By assuming that quality can be divided into a number of components, the quality of a configuration can be represented by a polygon in a diagram of the type drawn in Figure 2. (Note that the term “quality” is used for all objectives and should not be confused with the classical “product quality”.)

A first critical assumption on which the theory in this thesis is based is that the quality functions exist. A second critical assumption is that each customer evaluates each individual quality in the same way, or at least such that all configurations

[1]In **Paper III**, utility functions are estimated in order to measure the robustness of solutions with respect to uncertainties in data and / or quality functions.

are sorted equally with respect to this quality function. The different customers are, however, allowed to appreciate the combination of individual quality functions differently depending on transport missions, operating environments, personal preferences, financial strengths etc. It then becomes obvious that customers are not interested in any truck that is worse than some other truck with respect to *all* of the individual quality functions. Formally, this is a concept called *Pareto optimality*; it is mathematically defined in Section 4.1 and is a central concept in three of the appended papers. In Figure 2, a Pareto optimal solution corresponds to a polygon which is not entirely enclosed in any other polygon.

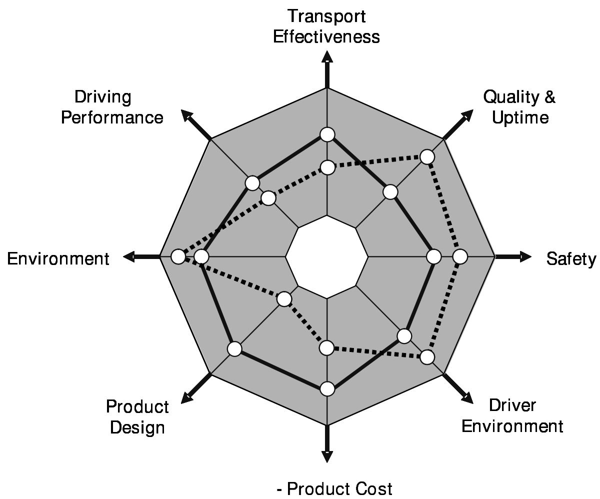


Figure 2: The quality of each configuration (the figure illustrates two configurations) can be represented as a polygon intersecting each axis at the numeric measure of the corresponding quality. The gray region represents all possible values of the quality measures. Good values of a quality measure correspond to values far from the center of the diagram.

We have an intricate connection that the quality function “Product cost” is a function both of the individual configuration but also of the whole set of produced configurations. This is due to “economy-of-scale” effects when costs for development and other common activities are shared by utilizing the same technical solutions. This connection is illustrated in Appendix A of [37], *Discretization is a win-win concept*, in which it is shown that each single customer, for a suitable limitation / discretization of the product offer, may obtain a configuration which is better in all individual quality measures than what he / she would have received if the product offer was not limited. The topic of collective quality is also considered in **Paper II**, where the optimal sets of technical solutions are sought under the above Pareto perspective on the customer requirements.

1.2 Thesis objectives

The vision of the project is to design a procedure that consistently and systematically reduces the size of the assortment of variants used for configuring the population of trucks in such a way that each customer is guided towards a configuration that is at least as good as the one that he / she would have chosen without using the procedure. The aim is essentially to satisfy as many customers (in selected market segments) as possible using the least number of technical solutions, and by this increase the profitability for the company.

To take a step towards fulfilling the vision, the main theme of this thesis is to create and explore mathematical frameworks for different aspects of the product development problem—the configuration problem—and to devise procedures of how to solve some of the identified subproblems. The main problem considered is to:

Create an optimal set of technical solutions resulting in trucks simultaneously fulfilling a great variety of customer feature requirements and business objectives for platform efficiency.

An underlying assumption is that there exist functions measuring the different qualities of a truck. This is a critical assumption, and for the frameworks to be effective tools, it is an important task for industry to learn what the customers seek, and hence how quality should be measured.

The appended papers have their own objectives, inspired by the different aspects of the main problem. These are presented using references from the real-world problem description in the following sections, and also when the papers are summarized in Section 6.

The expected benefit for the company from this thesis, and from subsequent work inspired by it or as a consequence of it, is a reduction of costs related to product development, production, maintenance, and sales of trucks. In addition, one goal is to create an understanding of what a company can gain from the use of mathematical optimization and what this requires from the company in order to function (e.g., numerical measures on quality parameters and clear definitions of what is allowed and what is not). The expected goal for academia is an increased insight into complex product development, and how to adapt mathematical models and methods to such an environment. Finally, the expected benefit for the customers is a guidance towards and within the set of available and verified appropriate vehicle configurations.

1.3 Outline

The introductory sections are organized in the following way: In the next section, we describe the basic facts about Volvo in terms of how a product is defined and how the global market, on which the products are to be sold, is structured. These surrounding facts define the cornerstones of the general framework for our problem formulations. In Section 3, the product development problem is characterized in terms of different complexity measures in order to emphasize the most important challenges of the development of trucks, and to delimit our studies to a well-defined area.

In Section 4 the various optimization disciplines concerned in the appended papers are briefly introduced, and relations to the real-world situation described in Section 2 are presented when appropriate, as well as relations to the appended papers.

In Section 5 the contributions of the thesis are highlighted, and suggestions for future work are given in order to continue along the route towards the project vision. Finally, in Section 6 the appended papers are introduced, summarized, and localized within the product development process.

2 The product configuration framework at Volvo

At Volvo there is a product development process with an associated organization. The process with its methods and tools have, together with the organization, been developed evolutionary over a long time to gradually adapt to a changing environment and to continuously improve their efficiency. Needless to say, one cannot start over from a blank sheet when formulating the mathematical frameworks for the different subproblems identified, but must adapt to some of the standards and constraints imposed by the current conditions. We start the following subsections by describing the product structure at Volvo. Thereafter we describe the market at which the products are to be sold, and present discretization concepts of operating environments and customer feature levels.

2.1 The product structure at Volvo

This subsection begins with a description of the current product structure and how decision spaces for optimization problems given this product structure can be defined. In the following subsection, the concept of *Strategic Vehicle Specifications* is introduced; this is a recent concept for the discretization of the large number of possible configurations to a graspable test set representing the product offer.

2.1.1 Variant families, variants and restrictions

An organized product structure can be utilized in many ways: It makes it possible for different departments to develop different parts of the truck, it enables the company to ensure that the right physical material is produced and assembled, and it also makes it possible to secure that each truck fulfills the legislation that is valid at the market where it is sold.

At Volvo, a truck is specified with its so called *variants*, each of which belong to a certain *variant family*. The variant families describe a vast variety of entities, some of which represent physical alternatives such as which engine type or which frame height the configuration should have, while others describe, for example, on which type of road the truck should be driven. In principle, a truck is completely specified by its list of variants, in which in almost all cases, exactly one variant from each variant family connected to the type of truck must be chosen. Therefore, the truck specification—the list of all variants—can be viewed as the fingerprint or the DNA-profile of the truck. An example of a truck specification is shown in Figure 3 containing a part of such a list of variants. A few of the entries are highlighted to give some examples of actual variants and variant families; these are described in Table 1. Figure 4 contains pictures of different rear axle installations, each corresponding to a variant in one of the variant families.

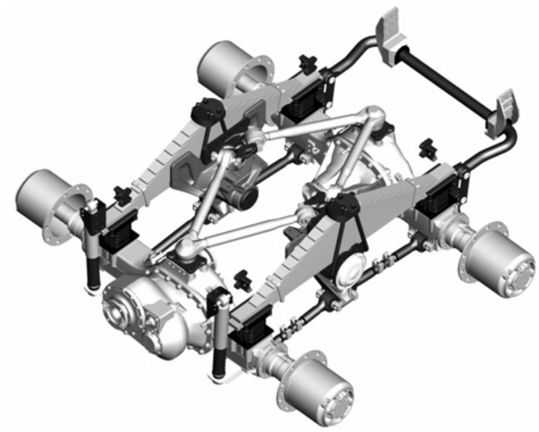
Vehicle Specification

04-TLV	FM 62R B	MAT-GREY	UXFUEL	ULADDER	SPOTP-R	UDECALTP
TMPF166	URUPP	UACMR	U1ACM	U2ACM	FFL1295	USPWCAD
UBULBKIT	ULAMPIN	UGAUGETP	UCHAINS	VERSION2	EMBL-TRP	WSTUD-S
UCROSSRE	RFE-T	UEGR	TYPE-FH	TP-DICON	ULOADIND	6*2
DASH-BLU	CABPT-R1	RIGID	RC-ROUGH	GCW60.0	CONC-BAS	UALARMB
ENG-VE13	VH4.4	WARNVEST	UBUNKBOX	UAIRBAG	EXHP-SAC	SUNV-SMO
APF-ENH	USIGNSWC	UCOMEQ	SUPH-TEX	SEATBRIA	AUXSW-6	WRITEPAD
USIGNS	TELECARD	UFUFIWH	STWPOS-L	RAA21T	DIMPLATE	UPAXLE
VW2600	MATT-PRE	FA-HIGH	FRAX-UNI	FAL9.0	FATYPE90	FAP3040
FAA10	TA-FIXED	RAL21	RADT-AR	RAP7340	LUGOP-BS	INST-MED
AUD-MED2	UADR	VERS-BAS	CHH-STD	BSR1365	URSTS	UCBRKTOW
UCBRKT	D13A440	VOLENG	ETOR2250	CHOSE-RU	EBR-VEB	BUGNET
RADI-NAR	VAL-BAS3	WTD-DUAL	UMRUP	UWCAP	CLUT-BAS	COOLC40
USPEEDDU	SPEED90	CRUISEC	FAN-VISE	UFUELFP	AIRIN-HI	FUELSEC2
FUELTFIL	UAUXFUEL	ULFUEL	RFUEL490	FUELTS71	TNK-SING	FTANK-AL
FCAP-L	MUF-HOR	EXD-VERR	EXSH-ST	EXSTH40	EAS-SCR	EM-ECO6B
NR-80EC	ACL1ST	URPLCR	ENGPROT	UPSCOOL	24V	TRANSF-H
UTRACON	UTABS	UACC	PHONE	LOWB-DAY	UFLASHL	HL-ASYMR
DETECT-S	HORN-ELS	SWS-RP	HORN-JER	HBAT-CF	ELCE-CK	BBOX-L
BBOXC-BA	2BATT225	UHOOKSC	UFMS	UADTL	RAG-ADBL	TWEETER
URTOLL	EST-AID	TL-BAS	MAINSW-M	FOGL-WC	DRIVL2	FUP-X1
UHLADJ	HL-CLEAN	UIMMOBIL	UIDLAMP	MARKL-SR	UALTTR	DOL24S12
RLIGHT-W	UWPREBOC	UWLAMP	BEACONA2	SPG-KM	ESUNV2P	ASCOP85
UPSCAT	SBD-BLAC	SBP-BLAC	EPTT650	CIRCP-F	TEMP-AMB	TEMP-TRA

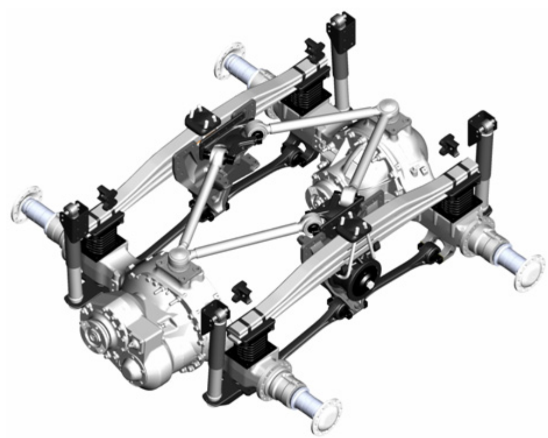
Figure 3: A part of a truck specification. Each entry is a code for a variant in some variant family.

Variant	Variant family	Description of the variant
RC-ROUGH	Road condition	Badly maintained road
6*2	Axle arrangement	6 wheels thereof 2 driving
RFUEL490	Fuel tank at the RHS	490 litre right side fuel tank

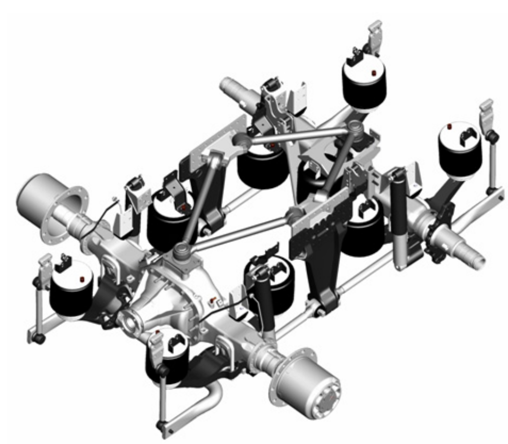
Table 1: Examples of items in the truck specification shown in Figure 3.



(a) RST-MUL



(b) RST-PAR



(c) RST-AIR2

Figure 4: An illustration of different rear axle installation types, defined by one of the variant families. a) has a multi-leaf spring, b) has a parabolic spring and c) has an air spring. The captions are the names of the corresponding variants.

A typical product type (a coarse division of the truck configurations) has about 500 valid variant families, each containing two or more variants. Thus, the number of possible configurations—if we define a unique configuration by the selection of one variant from each variant family—is huge (much larger than $2^{500} \approx 10^{150}$). In reality, however, not all variants can be combined just anyhow, due to geometrical, physical, functional, and legislative reasons. In the product structure this is documented by so called *restrictions*, which represent disallowed combinations of two or more variants. This way of defining feasible and infeasible configurations is sometimes denoted as using a system of *positive variants*. The opposite, *negative variants*, presupposes that all configurations are infeasible except the ones that are explicitly defined as feasible. Which of the systems to use is a fundamental strategic decision for the company. The advantages of using a system of positive variants are that the flexibility increases and that in practice it leads to more configurations and thus more possibilities for customer adaptations. A serious disadvantage is, however, that positive variants impose a large complexity due to the large number of restrictions required. In addition, the set of restrictions is dynamic in the sense that restrictions are added over time whenever an infeasibility is discovered, and it is a huge task to maintain the set of restrictions and to adapt it to newly developed variants and changes in the legislations. Currently, the number of restrictions are in the order of $100{,}000$. Since each restriction represents a prohibited combination of variants, it can cut off a large number of configurations from the feasible set.

2.1.2 Design and configuration spaces

The purpose of this subsection is to introduce notation necessary for explaining the development problem in mathematical terms and to set up relations between the development problem and the problems studied in the appended papers.

We disregard the actual product structure used at Volvo for a while and return to the actual configuration problem to be solved. The business idea is, in the space of all possible truck configurations, to produce trucks that are at least close to Pareto optimal. In addition, the company wants to use common parts for the configurations that are constructed. To enable a well-functioning product development process and to facilitate an efficient specification of the product itself, the product must be partitioned into groups or subsystems. The current product structure represents an example of such a partitioning. Therefore, we identify the different groups with the different variant families in the current product structure and let X^j be the (possibly infinite) set of all technically possible variants in variant family j, $j = 1, \dots, n$. Furthermore, we let the actual variants of today define the sets $\bar{X}^j \subseteq X^j$, $j = 1, \dots, n$.

We define alternative decision spaces, in which the configurations are located. The *configuration space*, or $\bar{X}$, consists of all configurations (feasible or not) that can be defined using the current variants, i.e., $\bar{X} = \bar{X}^1 \times \dots \times \bar{X}^n$. By allowing new variants in each variant family we define the *design space*, or X, as the Cartesian product of all possible variants (feasible or not) in the current variant families, i.e., $X = X^1 \times \dots \times X^n$. We refer to the licentiate thesis [37] for a broader discussion about the different decision spaces. In [37], two modes of the configuration problem, one in which the decision space equals the configuration space $\bar{X}$ and one in which the decision space equals the design space X, are introduced. We denote the former problem as the *mode 1* problem, and the latter as the *mode 2* problem. The *mode 1* problem is more of an operational type, trying to find good configurations using the variants already available in the product structure, while *mode 2* is more of a strategic or development type. One obvious difference between the two modes is that the restrictions / constraints are only formulated for *mode 1* while they are unknown for *mode 2*. Furthermore, the *mode 1* formulation is completely discrete whereas the *mode 2* formulation contains continuous portions.

In **Paper IV** we take a step towards the solution of the *mode 1* problem by proposing a solution method for a single-objective version of the problem. In **Paper II** we take a step towards solving the *mode 2* problem by restricting each variant family to be represented by a (continuous) interval.

2.1.3 Strategic vehicle specifications

We now return to the actual product structure. The number of configurations in the configuration space, i.e., the trucks that can be defined using the current product structure, is much larger than 10^{150}. The number of trucks that are produced every year within the Volvo group is in the order of 200,000. Only a few of these are perfectly identical to another truck. The number of trucks that are evaluated in computer simulations and built and tested at proving grounds during a certain development project is however often only around 10–100. The orders of magnitude of the above numbers are significantly different, and the actual specifications that are considered constitute just a small fraction of those in the configuration space. To be able to control the product offer in the best way, a recent definition is that of *Strategic Vehicle Specifications* (SVSs). These should consist of sufficiently many (possibly 100–1000) specifications in the configuration space and which are thought to be particularly important. The purpose of using SVSs is that when updating the products with modified technical solutions, then by analyzing the SVSs the features for all trucks interesting for the customers should be sufficiently controlled. For this to be achievable, the SVSs must be suitably spread in the configuration space. The SVSs can be used as a test set in order to monitor the quality and the platform efficiency of the product offer over time.

The SVSs can be of different types, depending on why they are seen as particularly important. They can be *volume critical*, i.e., high volume specifications from a sales perspective, they can be *business critical*, i.e., with a high strategic value from a product offer perspective, or they can be *platform critical*, i.e., specifications critical from, e.g., a functional, geometrical, or manufacturing perspective such that nonfulfillment of requirements leads to corrective actions. The concept of strategic vehicle specifications is roughly illustrated in Figure 5.

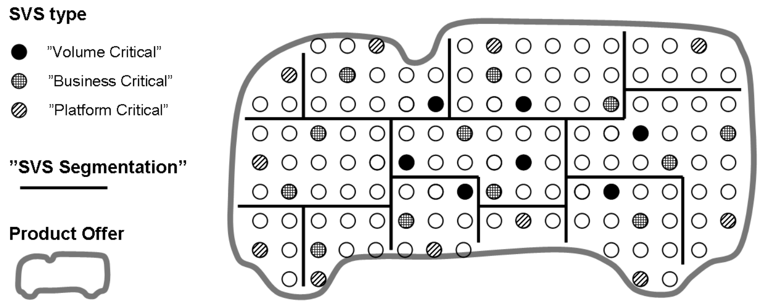


Figure 5: A rough illustration of the SVS concept. All configurations in the product offer are symbolized as circles and the SVSs of the respective types are marked with certain patterns. In each coarse segment of the product offer, which can be partitioned differently depending on the stakeholder, there is at least one SVS.

Each SVS should carry attributes explaining why it is considered. When a development project is initialized, the SVSs that are believed to be affected should be selected and analyzed throughout the project. The hope is that by using SVSs, synergies will arise when constructing test vehicles, including both physical and digital representations, and also that the definitions of the test vehicles used will be more clear, simplifying evaluations and comparisons between different analyses and creating a better historical reference material.

Until now, the SVS definition is in a conceptual phase. Future work should focus on the automatic creation and maintenance of a suitable set of SVSs with the purpose of following their performance and efficiency over time and after design changes. When optimizing a subset of features in a development project, the SVSs should be used in order to evaluate the whole resulting product offer and, in addition, as a test set safeguarding against possible deterioration arising in other features or in the platform efficiency.

2.2 The heterogeneous market

Volvo is a global company whose products are sold world-wide and are to be used in essentially different operating environments for various transport missions. The idea of collecting many different products within the same company is the possibility of creating beneficial synergies since knowledge and technology can be shared. However, it also imposes a difficulty in the sense that the qualities of the product may be appreciated differently in different segments of the market.

The feature structure of a truck is at Volvo divided into 8 *feature areas*. These are the ones illustrated in Figure 2 on page 3. The areas are decomposed into 32 *customer features*, such as e.g., *durability*, *ride comfort*, *exterior noise* and *fuel economy*. The customer features are further divided and translated into *technical features* (and *technical subfeatures*), which should be quantitatively measurable. However, the measurable quantities are dependent of each other, which implies the functional complexity of the product development as discussed in Section 3.

Objective and subjective preferences result in customers not preferring identical configurations. The objective category comprises the transport mission and the geographical location, and is directly related to the operating environment and the legislation to adapt to. Subjective preferences are the feature profile wanted and opinions about the technical solutions (i.e., in the design space X, cf. Section 2.1.2). Intuitively, it seems like the only important thing is how the truck behaves and not how the actual technical solution is constructed (i.e., the location in the design space). However, there may exist underlying requirements such as that the customer may wish to reduce the variety in his / her vehicle fleet, or that an old superstructure should be used also on a new truck. Furthermore, the customer might have strong feelings for a certain design, such as the size of the engine. A reasonable assumption is that the customer presets the transport mission of the truck (including what kind of superstructure that will be used), the cabin type, the engine (or driveline), and the axle configuration. This is in principle equivalent to defining the product type.

Figure 6 illustrates how the vehicle specification is defined. First, objective and subjective design requirements preset some of the main characteristics. Then the customer feature requirements yields the specification.

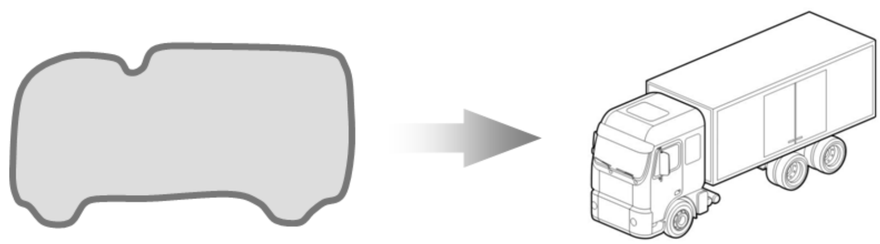


Figure 6: The gray silhouette to the left symbolizes the main characteristics of the truck that are preset by the customer requirements. Together with a preferred feature profile, the vehicle specification, illustrated to the right, should be defined.

2.2.1 Discretization of environments and customers

In order for the company to achieve a reasonable guarantee that the different market segments are covered by the available product offer and that a designed product targets a certain segment, the customers are discretized in different ways. One discretization that is made is the operating environment being partitioned into so called *Global Transport Application* (GTA) (previously Global Truck Application) cells ([14]). Parameters defining a certain cell are, for example, the curve density, the ambient temperature and the yearly usage. Some of the GTAs are currently also considered as their own variant families in the product structure; one example is given by the variant family “road condition” in Table 1. The thesis [35] deals with topics related to discretization of the operating environment. Parametric statistical models are constructed describing lateral loads acting on a truck for customers in different GTA cells. One of the purposes of [35] is to create input signals for computer simulations for fatigue analyses adapted to the specific use of the truck. Utilizing these signals, the aim is to construct trucks that are not overly durable for the intended use, wherefore, e.g., weight could be saved and thereby the transport efficiency being increased.

Another discretization related to the customers that is under implementation at Volvo is to target the desired feature levels in order to construct solutions having sufficiently high feature levels in relation to product cost. The discrete levels *Basic*, *Economic*, and *Demanding*, respectively, for each of the high-level objectives are used to define customer requirement profiles. Analogously to the GTA cells, BED cells (Basic / Economic / Demanding) is being defined. After a strategic choice on which of the combination of GTA and BED cells that should be covered with the product offer, the task for the company is then to create products in an as cost-efficient way as possible in each of these target cells. The BED concept is illustrated in Figure 7.

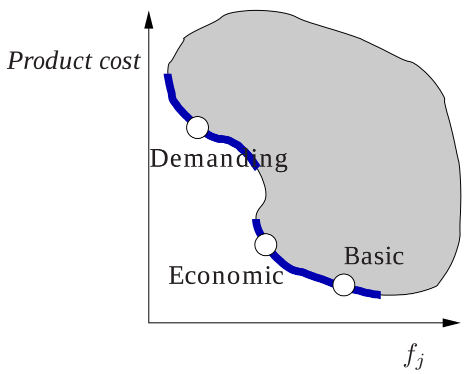


Figure 7: With the BED concept, the idea is, for each high-level feature (represented by f_j), to discretize the set of potential customers with respect to desired feature levels when balancing with product cost. The thick curve segments represent the Pareto front of the feasible set of configurations (the gray area).

In order to define suitable BED levels, it is important to consider what could be defined as *perceived feature steps* and *willingness to pay*. The former represents the minimum size necessary for a change in a continuous measure of a feature such that it is perceivable by a customer. The latter is a measure of a customers local evaluation of a quality increase, which is related to the trade-off, i.e., the slope of the tangent, to the Pareto front, illustrated in Figure 7. For different markets and different BED cells, the willingness to pay will possess different threshold values (at which the feature receives a high enough value for the product to be interesting) and saturation levels (at which the value of the feature is sufficiently high, and the customer not being willing to pay for further improvements). Future research should analyze these concepts further such that the concept of BED discretization is utilized in the best possible way. When defining BED cells, it is important to analyze the preferences of the customers and to evaluate the resulting set of configurations. This is related to utility function estimation which is performed in **Paper III** and to evaluations of approximate Pareto sets (cf. Section 4.1.1), which is considered in **Paper I** and **Paper II**.

3 Complexities in product development

Complexity is a word whose exact meaning is not very clear. In [4], a *complex system* is defined as a collection of interacting parts whose collective behavior cannot be understood by studying its parts separately. When using the daily interpretation of complexity as something being very large and incomprehensible, the product development of trucks can be viewed as a complex environment in at least three dimensions. In [37], the complexity space for product development (from a truck development perspective) was defined, and three main dimensions were identified: *functional complexity*, *combinatorial complexity* and *dynamic complexity*. We defined the *complexity space* as the three dimensional space spanned by the functional, combinatorial, and dynamic dimensions. Below, the complexity dimensions are briefly reviewed; we refer to [37] for a more extensive description.

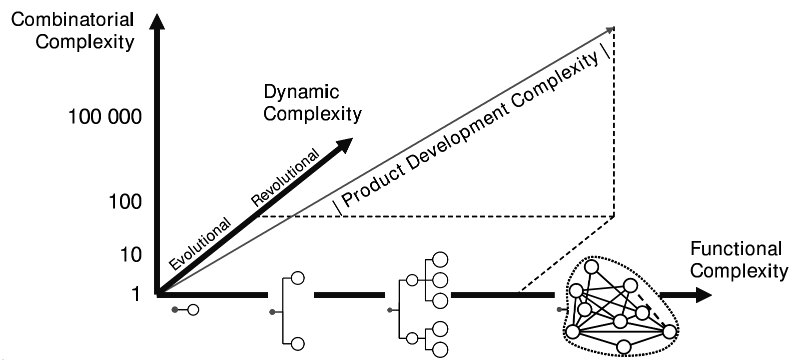


Figure 8: A complex environment. Product development of trucks comprises complexity components in the three dimensions of functional, combinatorial, and dynamic complexity, thereby making the total product development complexity very high.

The meaning of a “large functional complexity” is that the functions or features given by a design are not obvious from the design itself, and furthermore, that functional or feature requirements cannot be partitioned into independent measures, which can be individually controlled, as illustrated in Figure 9.

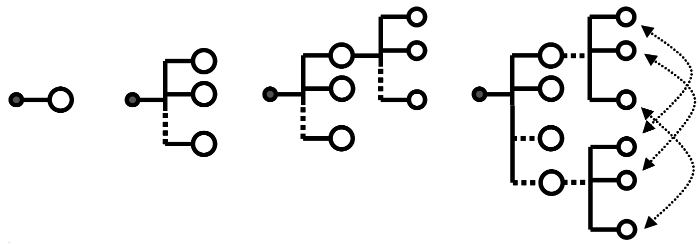


Figure 9: The features of a truck are connected and cannot be broken down into independent entities. The dotted arrows symbolize connections between the subfeatures, where a design change targeting one subfeature inevitably also affects others.

With combinatorial complexity we mean the degree of shared parts in the products developed. To be able to meet customer demands and, at the same time, keeping development and manufacturing costs down, the concepts of *Product family design* and *Platform-based product development* are central in modern product development ([30, 48]). A large number of shared parts that are to be combined to create a great variety of products (or, configurations) results in a high combinatorial complexity. With the notation used in [4], this type of complexity is denoted *complication*. Clearly, this is the case for Volvo (cf. Section 2.1). With the purpose of reducing the combinatorial complexity, a software for automatic packaging of chassis components within the wheel-base area is under development (cf. Section 5.1.3).

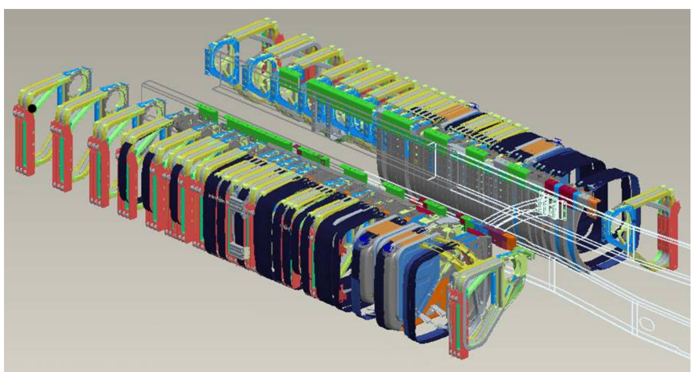


Figure 10: The picture shows a large number of tank bracket positions that can be chosen on the chassis, illustrating one aspect of the large combinatorial complexity.

The dynamic complexity concerns the time axis of the development process, where the scale goes from slow (evolutionary) changes to drastic (revolutionary) changes. Evolutionary changes correspond, for example, to the tuning of current variants to increase the quality of the truck population, while revolutionary changes correspond to topological changes in the form of new variants and variant families representing completely new technologies enabled by innovations or forced by legislation. The truck market is constantly changing with a continuous development of new technology, and with a legislative environment that is regularly updated, which means that the dynamic complexity for the truck development process is high.

The functional complexity is addressed in the thesis [38], in which the functionality retardation is investigated. For an efficient retardation system, a number of subsystems are interconnected, providing functional building blocks which together compose the retardation function.

Another thesis considering aspects of functional complexity is [21]. The thesis investigates both the interconnections between different sound and vibration components for the perceived sound experience in the cabin, and also the problem of quantification of a certain, typically qualitatively described, quality.

In the thesis [19], the focus is instead on the combinatorial complexity. The thesis deals with computer aided design (CAD) and the management of geometrical building blocks for combinatorial products, in which common parts are to be used in a large variety of different configurations. Figure 10 is generated using a software which is an outcome from the project, within which [19] was written.

In this thesis we simultaneously consider the functional and the combinatorial complexities, i.e., multi-objective optimization of combinatorial products. It was decided that the inclusion of dynamic complexity—which is related to the words “dynamic environments” in the project title—in the scope would result in a too ambitious task. Instead, an additional PhD project was initiated focusing on research problems within this area; see [51]. Included in the latter project are questions such as how to design a product structure that is perspicuous, flexible, efficient, and robust with respect to, e.g., new developments and new legislations.

What distinguishes the development of trucks from the development of many other products, is that the complexity contributions in both the functional and the combinatorial dimensions are high; see Figure 11, which roughly illustrates where other types of products are located in two dimensions of the complexity space.

Aspects of functional complexity are the targets in the appended **Paper I**, **Paper III**, and **Paper V**. **Paper IV** targets in a sense the combinatorial complexity, and **Paper II** approaches the combination of functional and combinatorial complexity.

4 Optimization disciplines concerned

Optimization is a vast discipline of applied mathematics containing many subdisciplines, all with their own theory and methods. The common characteristic among problems considered within the area of optimization is that the goal is to find the best alternative with respect to some measure given a set of possible alternatives.

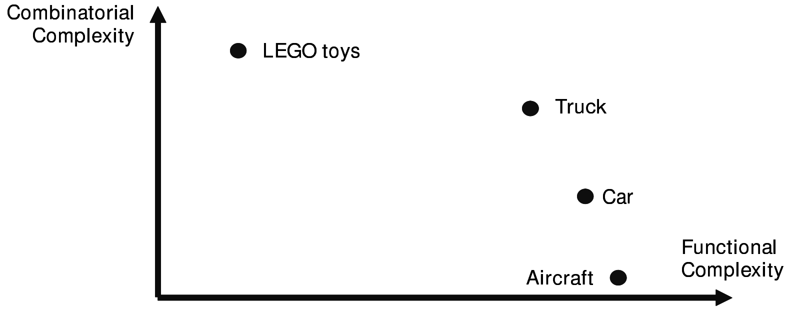


Figure 11: An illustration of the location of some products of different types in the complexity space projected onto the functional and combinatorial dimensions.

Many of the subdisciplines are considered in this thesis, and for the most important disciplines, short introductions are presented in this section. Other subdisciplines that are not directly considered in this thesis are, e.g., *Linear Programming* [41], *Quadratic* and *Convex Programming* [5], *Mixed-Integer Programming* [42], *Semi-Definite Programming* [52] and *Mathematical Programming with Equilibrium Constraints* [39].

The scopes of the following introductions vary greatly, depending on the importance of the respective subdisciplines for the content of this thesis. The introductions are not complete in any sense, and their contents are biased towards the utilization of the respective fields in the appended papers.

4.1 Multi-objective optimization and Pareto optimality

Multi-objective optimization is a discipline which permeates the whole product development problem and therefore this introduction is the far most extensive one.

A single-objective optimization problem posed in a finite dimensional (numerical) space is traditionally written as that to

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}), \\ \text{subject to} & \mathbf{x} \in X, \end{array} \tag{1}$$

where $X \subseteq \mathbb{R}^n$ represents the feasible decision vectors and $f : \mathbb{R}^n \to \mathbb{R}$. The goal is to find an $\mathbf{x} \in X$ that minimizes the objective function f over X.

In a practical optimization problem, there are often several conflicting objectives $\mathbf{f} = \{f_1, \ldots, f_k\}$ that are to be considered simultaneously. Formulating such a situation mathematically leads to a *Multi-objective (Nonlinear) Optimization Problem* (MONP) as that to

$$\begin{array}{ll} \text{minimize} & \{f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})\}, \\ \text{subject to} & \mathbf{x} \in X, \end{array} \tag{2}$$

where $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, k$. The problem (2) is not well-defined in the common sense if there is a conflict between the objectives, i.e., if there exists no vector $\mathbf{x} \in X$ minimizing all f_i, $i = 1, \ldots, k$, over X simultaneously. The reason is that there is no total ordering among vectors; for example, $(1, 1)^{\mathrm{T}} < (2, 2)^{\mathrm{T}}$, but how does one order the vectors $(1, 2)^{\mathrm{T}}$ and $(2, 1)^{\mathrm{T}}$? The goal in multi-objective optimization is to find the *Pareto optimal* subset $\mathcal{P} \subseteq X$, defined according to the following.

DEFINITION 4.1 *Given a set X of feasible vectors and a set $\{f_1, \ldots, f_k\}$ of objective functions to be minimized, a vector $\mathbf{x}^* \in X$ is defined as* Pareto optimal *if there exists no vector $\mathbf{x} \in X$ such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$, $i = 1, \ldots, k$, and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one $j \in \{1, \ldots, k\}$. An objective vector $\mathbf{z}^* = \mathbf{f}(\mathbf{x}^*)$ is called* Pareto optimal *if the corresponding vector $\mathbf{x}^*$ is Pareto optimal. The set of all Pareto optimal vectors $\mathbf{x}^* \in X$ is denoted $\mathcal{P} \subseteq X$.*

We adopt the convention of extending the minimization operator by allowing it to apply also to vectors, meaning that this operator extracts the Pareto optimal subset of X. We define the *Pareto operator* $P : \mathbb{R}^n \times \mathcal{F} \to \mathbb{R}^n$ by $P(X, \mathbf{f}) = \mathcal{P}$, where $\mathcal{F}$ denotes the space of all functions $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^k$.

An illustration of Pareto optimality for a MONP with $k = 2$ is shown in Figure 12. This picture motivates the fact that the set of corresponding objective points $\mathbf{f}(\mathcal{P})$ to a Pareto optimal set $\mathcal{P} \in X$ is often denoted the *Pareto optimal front*.

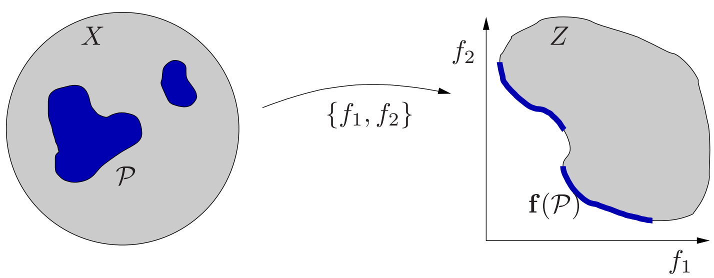


Figure 12: The decision space X with the Pareto optimal set $\mathcal{P} \subseteq X$, the objective functions $\mathbf{f} = \{f_1, f_2\}$, the objective space $Z = \mathbf{f}(X)$, and the image, $\mathbf{f}(\mathcal{P})$, of $\mathcal{P}$.

Historically, the first reference to studies addressing multiple conflicting objectives is usually the book [43] by the Italian economist Vilfredo Pareto, after whom the concept of optimality is named. As for optimization as a whole, there are also subclassifications of multi-objective problems widely differing in theoretical properties and consequently also in solution methods. Examples include *Multi-Objective Linear Programming* ([50]), (continuous) *Multi-Objective Nonlinear Programming* ([15, 40], with *Multi-Objective Convex Optimization* ([36]) as a special case, and *Multi-Objective Combinatorial Optimization* ([16]). As for other optimization disciplines, there is also a distinction between multi-objective problems with analytical expressions for the objective and constraint functions, and problems where these are black-box outcomes from computer simulations or physical experiments. With the introduction of *categorical variables* (cf. Section 4.4), it follows that there is also a distinction between numerical discrete multi-objective problems and categorical multi-objective problems.

When solving MONPs, the goal is not always the same. Often, the wish is to find a single solution $\mathbf{x}^* \in X$ that is optimal for a certain decision maker (DM). This is really a (hidden) single-objective optimization problem, in which the DM's scalar-valued utility (or value) function measuring his / her overall preference of the design points is the single objective function. However, the utility function is often hard to specify, and a common belief is that it is easier to specify the components of the utility and to formulate a multi-objective problem using these components as the objective functions. Then the DM can wait to express his / her preferences until he / she knows more about the trade-offs between the different criteria, i.e., when more information about the problem is revealed. In **Paper III**, the utility function for each DM is estimated in order to evaluate the robustness of solutions from the DM's perspective.

The underlying multi-objective optimization problem in the configuration problem studied in this thesis is nonlinear with simulation-based objective functions that are in general not convex. Some of the variables are discrete and many of these are categorical. There is no obvious structure of the decision space. It is a true multi-objective optimization problem, in which the entire set $\mathcal{P}$ is interesting. Subproblems hereof are studied in the appended papers. In the multi-objective optimization problems studied in **Paper II** and **Paper III**, the variables are assumed to be continuous. For the problem considered in **Paper I**, concerning the reduction of the number of objectives in the multi-objective problem, the variables can be of any type.

The solution methods for multi-objective optimization problems are designed to find the Pareto optimal set, or at least an approximate representation of it. A discussion about approximations of Pareto optimal sets is provided in the Subsection 4.1.1. See [37] for a discussion about various solution methods. In none of the appended papers concerning MONPs, the focus is to develop or analyze solution methods for the MONPs; such methods are only used as tools when needed. A possible continuation of **Paper IV** is, however, to develop a MONP solver for problems involving categorical variables.

4.1.1 Approximations of the Pareto optimal set

The approximation of Pareto optimal sets is a central concept throughout this thesis. For the actual product development problem this concept is important in order to evaluate the configurations available given sets of technical solutions for each component or design option or, in other words, to evaluate the collective quality of a population of configurations. Among the appended papers, approximations of Pareto optimal sets are considered in **Paper I**, **Paper II** and **Paper III**, and it will also be for the natural extension to **Paper IV** to consider multiple objectives.

In practical applications one cannot usually expect to find the whole Pareto optimal set $\mathcal{P}$. For a general nonconvex MONP it is neither possible to find $\mathcal{P}$ in finite time without some further assumptions on the problem properties (such as Lipschitz continuity with known constants). Approximating methods, i.e., methods for creating good approximations of $\mathcal{P}$, are developed for this reason; a survey of such methods is contained in [46]. When an approximating method is used, it is important to obtain a quality measure for the resulting set of solutions. For single-objective problems, such a measurement is straightforward to define since the objective values for two solutions can be compared directly. For MONPs, there is no standard measure for evaluating an approximate solution set. The subject is discussed and propositions are given in, e.g., [8, 9, 44, 47, 53, 54].

When discussing approximate sets, there is a need for two kinds of distance measures. First, a measure describing the distance between two points is needed; the *point measure*. This measure is then a part of the definition of a second measure, the *set measure*, describing the distance between two sets of points.

The distance measures should ideally be defined in both the decision and objective spaces. While the decision space may contain non-numerical variables (cf. Section 4.4), the objective space contains only numerical vectors provided that the objective functions quantify the qualities considered. Therefore, it is typically easier to define measures in the objective space. However, from a practical point of view, the distances in the decision space are also important. How to define these in a meaningful way is strongly problem specific and dependent. In the specific application of the configuration problem at Volvo, the decision space defined by the product structure (cf. Section 2.1.2) is heterogeneous with variants and variant families representing totally different entities, making it hard to define a suitable distance measure in the decision space.

In the objective space we remark that if the objectives measure different entities (such as, e.g., safety and cost, or stress and volume), then it is not obvious how to scale the objectives such that the units (e.g., if measuring cost in Euros or cents) do not affect the results. A means to handle this difficulty is to normalize the objectives such that their respective values vary (approximately) between 0 and 1 when the corresponding decisions vary over $\mathcal{P}$, as suggested in [40]. It is, however, important to note that if two collections of objectives order the vectors in the decision space X equally (i.e., the collections $\{f_1, \dots, f_k\}$ and $\{g_1, \dots, g_k\}$ are such that $f_i(\mathbf{x}) < f_i(\mathbf{y}) \Leftrightarrow g_i(\mathbf{x}) < g_i(\mathbf{y})$, $i \in \{1, \dots, k\}$, $\mathbf{x}, \mathbf{y} \in X$), then the implied Pareto optimal sets are equal, but evaluations of approximate Pareto sets are in general not. Therefore, the actual objective measures matter, and not just their implied orderings of X (this is usually also true for the result when solving MONPs).

For the application we study, the point measure should reasonably be asymmetrically defined. One choice for comparing the point $\mathbf{y} \in X$ with another point $\mathbf{x} \in X$ is to use some norm of the components of $\mathbf{f}(\mathbf{y})$ being larger than the corresponding components of $\mathbf{f}(\mathbf{x})$, i.e.,

$$c(\mathbf{x}, \mathbf{y}) = \rho(\max\{\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x}), \mathbf{0}\}), \tag{3}$$

where the $\max\{\cdot, \cdot\}$ operator is defined element-wise and $\rho(\cdot)$ is a suitable vector norm. Only deteriorated components are selected in order not to punish objectives that are improved. Figure 13 illustrates the indifference curves, i.e., points evaluated as equally good, induced by the point measure (3) where $\rho(\cdot)$ is chosen as the L^2 norm.

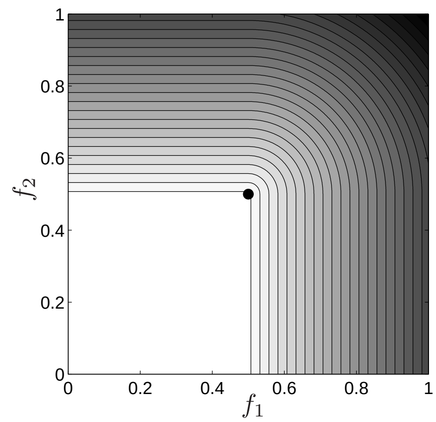


Figure 13: Indifference curves when comparing $\mathbf{x}^0$ with $\mathbf{f}(\mathbf{x}^0) = (0.5, 0.5)$ to all points $\mathbf{x}$ with $\mathbf{f}(\mathbf{x}) \in Z = [0,1]^2$ induced by the point measure $c(\mathbf{x}^0, \mathbf{x}) = \rho(\max\left\{\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{x}^0), \mathbf{0}\right\})$ with $\rho(\cdot)$ being the L^2-norm. A darker color means a larger distance from $\mathbf{x}^0$. White color corresponds to zero distance.

The point measure (3) is open to certain objections. For example, one does not gain anything when the value of an objective is improved if it is already at least as good as for the reference point. In **Paper III** we locally estimate utility functions corresponding to solution points. We suggest another, more sophisticated, point measure that better adapts to real customer behaviour.

The set measure can be either symmetrically or asymmetrically defined depending on the aim of the comparison. If the aim is to compare Pareto optimal sets for two problem formulations (e.g., if one formulation is a computationally less demanding but approximate version of the other), then a symmetric measure is reasonable. Such a measure is utilized in **Paper I** where the distance between two sets is defined as the largest (point measure) distance between any point in any of the sets to its respective nearest point in the other set.

If, instead, the aim is to evaluate the quality of a representation of the Pareto optimal set, then an asymmetric measure is more reasonable. This is common, e.g., when evaluating evolutionary algorithms for MONPs. Such an asymmetric measure is used in **Paper II** as the objective function in a single-objective problem considering the optimal choice of sets of technical solutions, or, in other words, where a good representation of the Pareto optimal set with a limited set of decision vectors that are required to share similarities in the decision space is sought. A good representation $\mathcal{R}$ of a Pareto optimal set $\mathcal{P}$ should contain points which are (at least) near-Pareto optimal as individuals and also well distributed along the Pareto optimal front. One measure capturing both these properties is, with $c(\cdot,\cdot)$ being the point measure defined in (3),

$$d(\mathcal{R},\mathcal{P}) = \max_{\mathbf{x}^1\in\mathcal{P}} \min_{\mathbf{x}^2\in\mathcal{R}} c(\mathbf{x}^1,\mathbf{x}^2), \tag{4}$$

which is illustrated in Figure 14. This is the $Dist2$ measure from [9], which we utilize in **Paper II**.

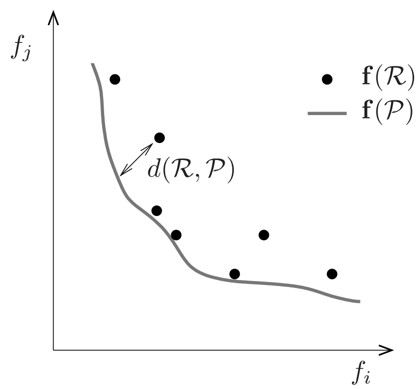


Figure 14: The (set) distance between $\mathcal{R}$ and $\mathcal{P}$ defined by (4) is illustrated by an arrow.

In the actual configuration problem, we search for a discrete subset of the entire set $\mathcal{P}$ of Pareto optimal trucks. Here, it is reasonable to use a point measure that depends on the region of the objective space Z where the points are located. Sales volumes and / or other strategical issues could be incorporated in the measures, e.g., by using region-dependent weights. By using such a measure, it would be possible to steer against representations containing certain designs, e.g., trucks with a high profitability or trucks whose features are carefully validated in simulations and tests (cf. SVSs in Section 2.1.3). When deciding on the feature levels defining the BED cells described in Section 2.2.1, the set of the resulting solutions should be evaluated as a representation of a Pareto optimal set. Here, the point measure should be carefully defined, truly capturing the perceived feature distances for the customers in the different cells. A measure similar to the point measure developed in **Paper III** could be used for this purpose.

4.1.2 Robustness in multi-objective optimization

To be able to utilize optimization for solving a real-world problem, the first step is to formulate a mathematical model of the problem. This modeling step normally includes simplifications, quantifications, and limitations of the real-world problem. Clearly, the model can seldom capture the real-world problem exactly. The next step is to solve the problem represented by the model, where a problem solved to optimality means that the objective function in the model is minimized. Thereafter, the optimal solution found is to be interpreted and implemented in the real-world situation. Obviously, it is not clear whether or not the solution implemented is really the best one.

In the area of *Optimization under uncertainty*, typically with respect to a single objective, the aim is to quantify the uncertainties of the parameters defining the optimization problem and to incorporate these quantities in the solution process. There are two main disciplines within this area: *Stochastic Programming* (SP, [7, 34]) and *Robust Optimization* (RO, [6]).

In SP, whose origin dates back to the 1950s (cf. [10]), one assumes that the uncertainty in the model has a probabilistic description. The aim is then to optimize, e.g., the expected value of the objective function under the constraint that the solution is feasible for all (or almost all) realizations of the uncertain parameters.

In RO, which was introduced during the 1970s (cf. [49]), the uncertainty is not stochastic, but rather deterministic and set-based. The idea is to find a set of solutions that are feasible for all outcomes of the uncertain parameters, and among these, the one that has the best objective value for its worst outcome of the uncertain parameters.

The inclusion of uncertainties in the problem formulation increases the precision of the model; it increases, however, also the difficulty in solving the problem. Possibly, this is the reason why only a few of the papers published on optimization under uncertainty consider optimization problems with multiple objectives. In the papers [11, 12, 13] the authors have made a direct extension of SP to multi-objective problems, by replacing each objective function with its expected value function, in order to define a *robust Pareto optimal set*. In **Paper III**, we suggest a new procedure for measuring the robustness of points in multi-objective problems by estimating the utility function for each DM and compute the average loss with respect to optimality when the uncertain parameters are varied.

4.2 Global and simulation-based optimization

Consider the optimization problem (1). A point $\mathbf{x}^* \in X$ is a *global minimum* of f over X if

$$f(\mathbf{x}^*) \leq f(\mathbf{x}), \quad \mathbf{x} \in X. \tag{5}$$

Further, a point $\mathbf{x}^* \in X$ is a *local minimum* of f over X if there is some *neighborhood* $\mathcal{N}(\mathbf{x}^*)$ of $\mathbf{x}^*$ such that

$$f(\mathbf{x}^*) \leq f(\mathbf{x}), \quad \mathbf{x} \in \mathcal{N}(\mathbf{x}^*) \cap X. \tag{6}$$

The definition of a neighborhood depends on the characteristic of the problem (1). In **Paper IV** we consider optimization problems in categorical variables, and define a certain suitable neighborhood. If the decision variables are continuous, then $\mathcal{N}(\mathbf{x})$ is typically an open Euclidean ball centered at $\mathbf{x}$ with a positive radius.

Now consider the problem (1) when the domain X is convex and when f is a convex function over X, i.e., when

$$\left.\begin{array}{l}\mathbf{x},\mathbf{y}\in X\\ \lambda\in(0,1)\end{array}\right\}\Longrightarrow f(\lambda\mathbf{x}+(1-\lambda)\mathbf{y})\leq\lambda f(\mathbf{x})+(1-\lambda)f(\mathbf{y}).\tag{7}$$

Then, a fundamental theorem in continuous optimization implies that any local minimum of f over X is also a global minimum. Therefore, when a problem fulfills these properties, it is enough to apply a local optimization algorithm in order to find a global optimum of (1). For example, any descent algorithm with mild conditions on the step lengths is guaranteed to converge to a global minimum. For further information, see, e.g., [2].

If the problem under study is not convex, typical for an optimization problem from industry, then the problem can contain local minima that are not global minima. The discipline concerning the problem of finding a global optimum point in this situation is denoted *Global Optimization* (GO). Different aspects of GO is considered in **Paper IV** and **Paper V**. In the former, a new method is developed in order to find a global minimum, or a point which under certain assumptions, in the worst case is as close as possible to a global minimum with respect to function values. In the latter, we consider problems defined in categorical variables. A definition of convexity for such problems is made; for the problems considered this property is, however, not expected to hold, wherefore we cannot be satisfied with solution methods searching for local minima. Methods from GO considering integer variables are extended to the case of categorical variables.

A discipline related to GO is the discipline of *Simulation-based Optimization*. In this discipline, the objective and / or constraint functions are outcomes of computer simulations or physical experiments. Usually it is assumed that the function values are expensive to evaluate, thus putting a practical restriction on the maximum number of function evaluations that can be performed. Furthermore, it is often assumed that there are no analytical derivatives available or, in order words, that the functions are assumed to be so called black-box functions.

The class of so called *Response surface methods* (RSMs) are popular for these types of problems. A review of such methods is given in [31]. In principle, an RSM provides an approximation, i.e., a *surrogate model* of the expensive function. This surrogate may then be used instead of the true function when performing an optimization. The surrogate model is usually iteratively updated with new sample points. Examples of surrogate models used are quadratic approximations ([45]), Kriging-based interpolations ([33]) and radial basis function interpolations ([23, 29]).

Two other popular methods for global optimization of black-box functions, however not when function evaluations are computationally very expensive, are DIRECT (DIviding RECTangles, [32]) and MCS (Multilevel Coordinate Search, [27]). In principle, these methods divide the domain iteratively in the regions that are currently the most promising for containing a global optimum in the sense of balancing large unexplored territories with good function values.

If the problems are not of the black-box type, that is, if analytical expressions are available for the functions involved, then there are Branch-and-Bound based methods available, utilizing an implicit enumeration of the decision space and iteratively cutting off parts of the decision space in which a global minimum cannot be located as certified by, e.g., interval analysis or the minimization of convex underestimates. Also for problems whose involved functions lack analytical expressions, Branch-and-Bound based methods are available if assumptions can be made on the values of Lipschitz constants (that is, bounds on the rate of change of the objective function).

For further information on global optimization, see, e.g., the monographs [24, 25, 26].

In **Paper V** we present and explore a new principle for global optimization of computationally expensive black-box functions. The aim is to find points such that the optimality gap is minimized provided some assumptions on the class of functions that the objective function belongs to. As a special case, we consider Lipschitz continuous functions.

The applications considered throughout this thesis involve functions that are simulation-based. In a natural extension of **Paper II**, surrogate models for multiple objectives should be developed in order to obtain “cheap” approximations of the simulation-based objective functions that are present in the applications. A difficulty in this case is that the surrogates must be accurate in a large portion of the decision space, as compared to for a problem with a single objective.

4.3 Clustering

The scientific area of *clustering*, or *data clustering*, concerns the automatic classification of a set of objects into more or less homogeneous groups, such that the objects assigned to the same group are similar according to some suitable distance measure. The area is widespread with applications in a large number of domains, such as pattern recognition, image analysis, marketing, and machine learning. See [20, 28] for an overview of the subject.

It is clear that the configuration problem at Volvo fits into a clustering context, where all wishes, i.e., wanted configurations or points on the ideal Pareto front, are to be partitioned into a number of groups, each corresponding to a certain truck configuration. Volvo wishes to produce trucks that are good in the sense of Pareto optimality, however the Pareto optimal set may be very large and it is not reasonable to offer all of the corresponding configurations to the customers. Instead, the wish is to reduce the offer to a limited and discrete set of technical solutions. But our application possesses special characteristics leading to aggravating circumstances. First, due to the combinatorial complexity we cannot cluster the designs using information from the objective space Z only. Second, since the quality of the configurations is the central property, neither is it possible to cluster the solutions in a good way using information from the design space X only. Hence, clustering using a combination of the two is necessary.

In [37] we presented a distinction between two clustering frameworks: *explicit clustering* and *implicit clustering*, which is reviewed in the following subsections. The former category contains traditional clustering techniques which possibly can be adapted to fit our application, while for the latter category we present a new framework of clustering design solutions implicitly by using optimization. This procedure by construction takes care of the difficulties of treating the two spaces X and Z simultaneously. The focus of **Paper II** is the implicit clustering.

4.3.1 Explicit clustering

In the configuration problem, assume that all customers possess objective functions that order the configurations identically. Then, only designs mapped onto the Pareto front $\mathbf{f}(\mathcal{P})$ are desired by the customers (without considering product cost, see Section 1.1 and Appendix A of [37]). However, $\mathbf{f}(\mathcal{P})$ is a very large set and for cost reasons, not all of the corresponding configurations should be made available to the customers. Instead, $\mathbf{f}(\mathcal{P})$ should be clustered into a limited set Z_D of points corresponding to the configurations $X_D \subset X$. These should be such that for every customer wish, i.e., a point $\mathbf{z}^* \in \mathbf{f}(\mathcal{P})$, there is an available configuration $\mathbf{x} \in X_D$ at some small enough distance $c(\mathbf{x}^*, \mathbf{x})$ from $\mathbf{x}^*$, where $\mathbf{x}^*$ is such that $\mathbf{f}(\mathbf{x}^*) = \mathbf{z}^*$. Such a clustering is illustrated in Figure 15. This clustering, however, lacks an important property which is crucial for our application: it does not consider how X_D is structured in X.

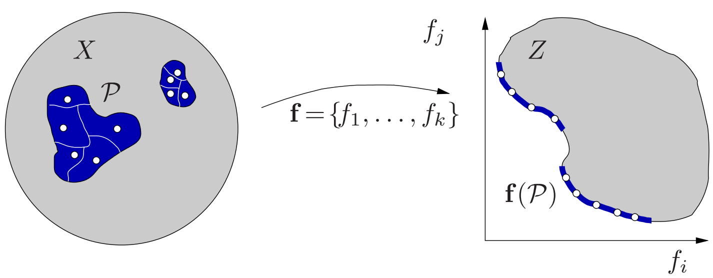


Figure 15: A clustering of the Pareto optimal set $\mathcal{P}$ into the set X_D, such that $Z_D = \mathbf{f}(X_D)$ is "evenly" distributed over $\mathbf{f}(\mathcal{P})$.

The above description can be denoted as an explicit clustering in the Z space. The clustering, which is done with respect to $\mathbf{f}(\mathcal{P})$, being a numerical set, is straightforward since there exist natural distance measures between pairs of numerical points. The textbook [20] describes various algorithms that could be used for this type of clustering. Common to the explicit clustering algorithms is that they try to partition the set into subsets, or clusters, such that the distances within clusters are small, while distances between points in different clusters are large.

It is also possible to define an explicit clustering in the decision space X, possibly with the aim of partitioning $\mathcal{P} \subseteq X$ into groups. Then it is, however, not clear how to control that the set of representative points $Z_D = \mathbf{f}(X_D)$ is a good representation of

$\mathbf{f}(\mathcal{P})$ (cf. Section 4.1.1 for a discussion on approximations of Pareto sets). But, more seriously, it is hard to control the structure of X_D such that the number of contained variants is of reasonable size. A third difficulty is that distance measures between points in the X space needs to be defined, which is hard when the set of variables types is heterogeneous or when the variables are of a categorical type, as discussed in Section 4.1.1.

It is possible that explicit clustering in the composition of the decision and objective spaces, i.e., in $X \times Z$, could be used. How such a clustering should be defined is, however, not known to us. Instead, we propose so called *Implicit clustering*, in which we solve an optimization problem whose solution represents a clustering both with a desired structure in X and which represents a good approximation of $\mathbf{f}(\mathcal{P}) \subseteq Z$. Further, the problem of defining a suitable distance measure in the design space is resolved.

4.3.2 Implicit clustering

Presupposing the existing product structure with X^j representing the set of possible variants in variant family j, $j = 1, \ldots, n$, we can write $X = X^1 \times X^2 \times \cdots \times X^n$ (cf. Section 2.1.2). We have that $X = X_{\text{feas}} \cup X_{\text{infeas}}$, and $X_{\text{feas}} \cap X_{\text{infeas}} = \emptyset$, where X_{feas} (X_{infeas}) is the subset of X (not) fulfilling all of the restrictions. By restricting the cardinality of Z_D to be at most a number N, i.e., $\text{card}(Z_D) \leq N$, an explicit clustering of $\mathbf{f}(\mathcal{P}) \subseteq Z$ may lead to as many as N variants in each variant family, since there is no control over the resulting structure of X_D. To eliminate this problem we instead propose to cluster the whole objective space Z, and not just $\mathbf{f}(\mathcal{P})$, under the constraint that the configurations chosen in the design space constitute a "grid", in which the size restriction is imposed separately on each variant family, i.e., $\text{card}(X^j) \leq m_j, \; j = 1, \ldots, n$. The objective for the "implicit" clustering is now to select limited sets $X_D^j \subseteq X^j$, $j = 1, \ldots, n$, such that the "quality" of the product set $X_D = X_D^1 \times \cdots \times X_D^n$ is as high as possible. Mathematically, this is expressed as to minimize the distance measure (4) between the sets $\mathcal{P}$ and $P(X_D, \mathbf{f})$ (i.e., the non-dominated part of the set of the resulting configurations). The implicit clustering is the focus of **Paper II**. Figure 16 illustrates a clustering where the available configurations belong to the product set of the variants chosen. An arrow in the lower part of the objective space in the picture illustrates the distance for one such vector $\mathbf{x}^*$ ($\mathbf{f}(\mathbf{x}^*)$ is marked as a gray dot) to its nearest vector $\mathbf{x} \in P(X_D, \mathbf{f})$.

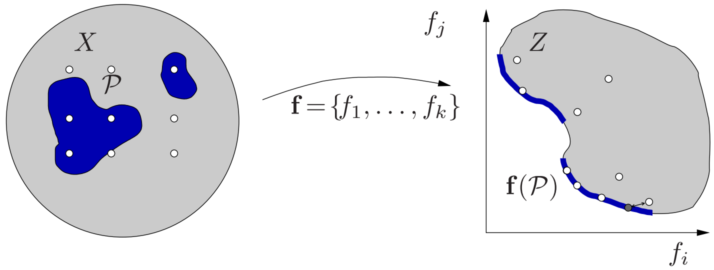


Figure 16: An illustration of an “implicit” clustering of the Pareto optimal set when X_D is required to be a product set. The white dots represent X_D and Z_D in the design and objective space, respectively. The arrow between the gray dot and one of the elements of Z_D represents the distance from the corresponding Pareto optimal vector to its nearest available solution.

Clearly, by using implicit clustering we do not require that $X_D \subseteq \mathcal{P}$, i.e., that the objective values of the clustered decision points are Pareto optimal. Often, e.g., in industrial applications, or when there are large uncertainties in the models, this is, however, not a necessary requirement.

4.4 Optimization over categorical variables

Many optimization models contain only continuous decision variables, i.e., variables whose values lie in intervals on the real line. Other models, e.g., for problems involving fixed costs or different routing problems, require the use of discrete or integer variables, representing on/off decisions or indivisible quantities. However, many real-world applications in, e.g., engineering design, involve decisions between discrete options which cannot be naturally ordered. Such options can be modelled with *categorical* variables, which are thus discrete variables without an intrinsic order.

In a number of recent papers (cf., e.g., [3]) *mixed-variable programs* (MVPs) containing continuous, numerically discrete, and categorical variables are studied. Pattern search methods for problems containing a mixture of continuous and discrete variables are extended to also include (a few) variables of the categorical type.

In [1] the categorical variables of an MVP are removed using integer variables which then allows for relaxation-based solution methods from the area of *mixed-integer nonlinear programming* (MINLP) to be applied to the resulting model. For this method to be reasonable it is, however, required, if not the categorical variables are very few, that the functions involved are defined for fractional points, which is typically not the case if the functions are of a black-box type. In [17, 18] another method is suggested. It is assumed that the categorical variables represent high-level choices of options, each of which corresponding to a set of parameter values in a high-dimensional space (an example is that different engine variants might correspond to a weight, a performance measure, and an environment measure). Further, it is assumed that the transformation from the categorical variables, or *integer choice variables*, as they are denoted in this context, to the parameter space is known. Then, also here Branch-and-Bound methods become available, in which the representation of the integer choice variables in the higher-dimensional space is utilized in order to branch on the categorical variables.

It is clear that the configuration problem contains decisions that can be represented by categorical variables. In the *mode 1* problem introduced in Section 2.1.2, in which an objective is to be optimized with respect to the current variants contained in the current variant families, all decision variables can be viewed as categorical.

In **Paper IV** we introduce and explore what we call the *pure categorical optimization problem*, in which all decision variables are categorical and the (single) objective function may be of a black-box type. A natural question to pose is whether problems in categorical variables are too hard to be solved using optimization techniques, and furthermore, which assumptions that have to be posed in order for such methods to be successful. In **Paper IV** steps towards answering these questions are taken.

4.5 The scientific area

A thesis like this, driven by a concrete industrial application, naturally has to make use of theory from a variety of separate but interconnected scientific areas or domains. In the prerequisites for the PhD project in which this thesis is written, it was already presupposed that the product development problem should be approached from a (multi-objective) optimization perspective. Therefore, the theoretical content is highly biased towards optimization issues, and the assumptions made are possibly not always obvious from a practical standpoint. Scientific areas that are important to study to reach the ultimate goal of designing the best product structure with the right technical solutions such that the company, in the long run, is as competitive as possible, include many other subjects not considered in this thesis. Examples are *Decision theory* (by which one wishes to better understand how customers make trade-offs between costs and features), *Behavioral sciences* (analyzing driver behavior, apprehension of qualities, perceived feature steps etc.), *Mathematical statistics* (in order to analyze uncertainties in models and data) and *Economics* (with the intention of creating a better understanding of how shared technical solutions affect the overall costs and earnings).

5 The contributions of the thesis

In this section we summarize the contributions resulting from the research behind this thesis and suggest suitable future work that would make the frameworks more useful and the solution methods more applicable to real-world problems within the product development process.

5.1 Contributions

5.1.1 Academic contributions

The main contribution within academia from this thesis is the comprehensive view of the product development problem in mathematical terms and in an optimization framework. From the various subproblems considered in this thesis, it is clear that there is an increased need for theory and methods combining different optimization disciplines as well as including aspects from other academic disciplines (cf. Section 4.5).

There are also more specific contributions. These are highlighted in Section 6 where the appended papers are summarized.

5.1.2 Business contributions

The business contributions resulting from this thesis are manifold and diverse.

The mathematical formulations of the main problem and the various subproblems have indicated what has to be considered for an efficient product development process focusing on platform efficiency and products with the right level of features. Clarity is an important issue. If a quantitative approach is desired, then quantitative functions describing objectives and constraints, exact descriptions or possessing a well-defined structure of uncertainties, are essential to define.

The competence built within the organization during the years when this PhD project has been performed is an important contribution. This competence will be used when forming the product development process of the future.

The thesis work has contributed to a clear formalism to use within the company. The organization has learned to distinguish between the individual qualities of single truck configurations, and the quality of a population of trucks as whole. The concept of viewing the product offer as a representation of a Pareto optimal set is now used almost in the everyday business. Fundamental steps towards the vision have been taken.

Specific business contributions, as outcomes of the appended papers are highlighted in the summary of the papers in Section 6.

5.1.3 A software for the automatic packaging of chassis components

As a spin-off to this project, the research centre Fraunhofer ITWM in Kaiserslautern is now performing a software project in cooperation with Volvo 3P. The task is to construct a tool for the automatic computation of the selection and positioning of components in the wheelbase area of a truck provided some higher-level variants and a certain transport mission and operating environment. In the language from Section 3, *Complexities in product development*, one of the purposes of the software is to reduce the combinatorial complexity, i.e., the *complication*. In addition, to compute positions and components, instead of documenting feasible and allowed choices given higher-level design options, enables an increased flexibility. The tool is based on optimization, and one of its use cases is to find an optimal set of technical solutions, or variants, such that the resulting set of admissible configurations is suitably spread in the feature space as defined by, e.g., fuel capacity, weight distribution, and ground clearance.

The software works in two modes. One is operational, in order to automatize work that currently is manually performed, e.g., to compute the largest possible fuel tank for a partially completed specification, and to generate all restrictions resulting from geometric violations of components in the wheel-base area. In a second mode, the idea is to include quality measures for a set of packagings (cf. Section 4.1.1) and to perform what-if studies on how this collective quality is changed if components are added or removed, or if they are allowed to be positioned at new places. In Figure 17 a screenshot of the software is shown.

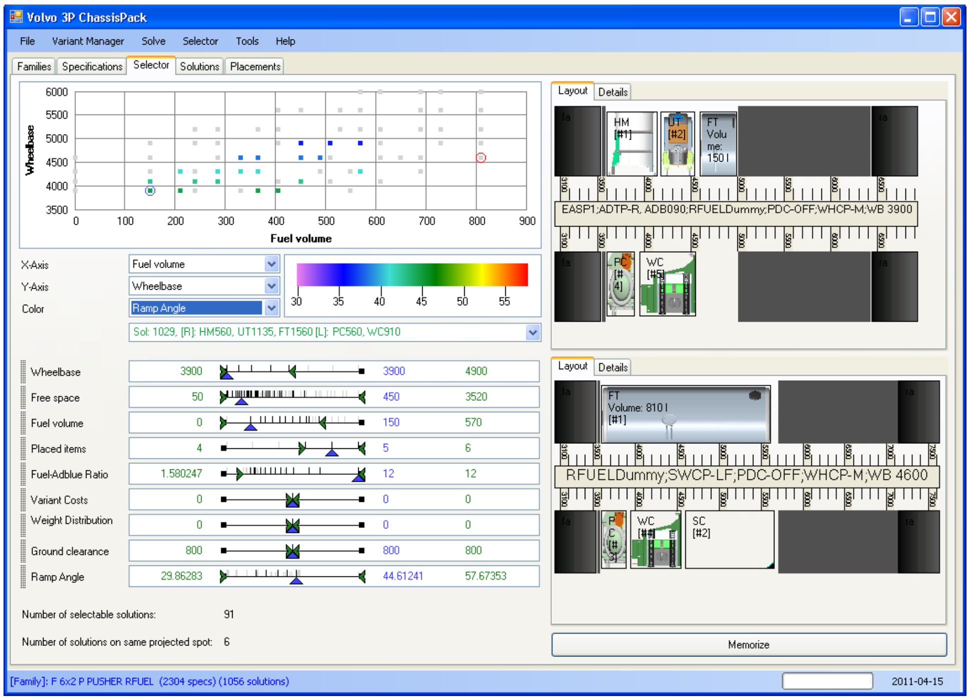


Figure 17: A screenshot of the ChassisPack software for automatic and optimization driven packaging of components in the wheelbase area, developed in order to reduce the combinatorial complexity of the configuration management.

5.2 Future work

To be able to use a Pareto optimality framework in order to guarantee that the right, and the right number of, technical solutions are made available, more quantitative representations of objective functions that evaluate a truck specification defined by its list of variants are needed. More and more such functions are being developed—partly as a result of the research work behind this thesis—and it is important that this development continues.

Furthermore, for a truly quantitative approach for optimizing the sets of technical solutions, it is required that a set of available configurations can be evaluated as a collective. In order to evaluate a set of technical solutions and hence enable an optimization, it must be possible to assign a number representing how well a set of configurations is distributed, given the set of potential customers with their corresponding transport missions, desired feature levels and operating environments. Future work should focus on the development and evaluation of different collective quality functions. These should be analyzed in terms of sensitivity analysis with respect to the different ingredients of the functions. The analyses should also include the density of customers in different regions of the feature space and the earning capacity for the company for trucks in different parts of the configuration space.

Problems containing categorical variables should be further analyzed. There is clearly a need for solution methods for optimization problems involving categorical variables and multiple objectives. In order to take the development of the core problem, described in **Paper II**—to decide on the right set of technical solutions resulting in a good set of configurations—one step further, there is a need for approximate evaluations of the collective quality for a set of solutions. Possibly, enhanced surrogate function modelling for multi-objective problems is the right way to go.

Future work should include complexity analyses of the problems considered for the various aspects for the configuration problem in order to devise suitable solution methods for different special cases. With the purpose of attracting more researchers to the area of product configuration optimization, it would be useful to construct a set of realistic test problems such that concurrent solution methods could be tested and compared. A goal would be to create a subsection within some optimization community focusing on the kind of optimization problems studied in this thesis.

6 Summary of the appended papers

6.1 Paper I — Approximating the Pareto Optimal Set using a Reduced Set of Objective Functions

In this paper, we describe a reduction procedure for multi-objective optimization problems when the number of objective functions is large. Through this method, one creates a new optimization problem with fewer objectives and with a Pareto optimal set that is approximately equal to the Pareto optimal set of the original problem. The smaller number of objectives in the reduced problem makes it, in general, computationally easier to solve.

We utilize the concept of ε-Pareto optimality, which relaxes the concept of Pareto optimality, and introduce the concept of ρ-centrality, which leads to a focus on the (probably most) interesting part of the Pareto optimal set. Our approximation goal is to minimize the distance between the ρ-central part of the original Pareto optimal set and the ρ-central ε-Pareto optimal set of the reduced problem.

Utilizing a new characterization of Pareto optimality, which is valid for finite decision spaces and which yields an explicit formulation of the definition of Pareto optimality, we derive a program whose solution represents an optimal reduction with respect to the approximation objective. We also propose an approximate formulation, which is computationally tractable as opposed to the ideal formulation, and which utilizes correlations between the objectives and separates the derived program into two parts. We demonstrate the method by applying it to a small industrial instance.

The motivation for the method developed is that industrial optimization problems often require computer intensive simulations and that they often possess a large number of objective functions. These characteristics make them computationally hard to solve. In addition, in practice it is not necessary to find the exact Pareto optimal set; it might be well motivated to lose some precision if the problem to solve becomes significantly smaller, and if the size of the error can be estimated.

A main contribution of this paper is the new explicit characterization of Pareto optimality, which may be utilized also in other applications. Another contribution is the actual method developed which can be used as a preprocess for large-scale multi-objective optimization problems. Here, a subset of the decision space consisting of a finite set of points (small enough to enable an exhaustive search for the Pareto optimal set) must be selected. Using this finite subset, a reduced problem can be constructed using the proposed method, and then the reduced problem formulation can be applied to the original problem.

Throughout the product development process at Volvo, the analysts keep track of different key reference signals in order to control that the features of the trucks are not altered in unwanted directions. In this paper we have shown that it is possible to reduce multi-objective optimization problems in terms of number of objectives, and we present a procedure to accomplish this. As a result, it is concluded that it should be possible reduce the number of necessary signals, and as a consequence, the work load in the organization. Especially in early phases of a large development project when different design concepts are decided upon, it is not that important to consider all possible features of the design. Such a detailed level is not necessary to describe since the level-of-detail of the designs that are to be evaluated in these phases is not very high.

6.2 Paper II — Multi-Objective Design of a Combinatorial Structure

In this paper, we approach the problem of incorporating the combinatorial complexity of product development (cf. Section 3) into a multi-objective optimization context and study the “implicit clustering” concept defined in Section 4.3.2.

In the *mode 2* configuration problem of Section 2.1.2, variants are to be selected within variant families such that the resulting set of feasible configurations approximates well the Pareto optimal set of the underlying multi-objective optimization problem. In this paper, we consider a simplification of this configuration problem, in which the design variables are assumed to be continuous and subject to box constraints only.

A single-objective optimization problem, the *Multi-Objective Combinatorial Design Problem* (MOCDP), is introduced. In MOCDP, an underlying multi-objective optimization problem (MONP) is considered and for the MONP a population of solutions, approximating its Pareto optimal set $\mathcal{P}$, is searched (cf. Section 4.1.1 for a discussion on approximations of Pareto optimal sets). The real-world interpretation of MOCDP is to decide on optimal sets of technical solutions such that the configurations given by their combinations are good in a Pareto sense. The decision space is assumed to be combinatorial, i.e., it is a product set for which a solution is composed by one component in each dimension. Thereby, by letting m_j denote the number of variants (i.e., decision variables) in dimension j, $j = 1, \ldots, n$, $\sum_{j=1}^{n} m_j$ decision variables are used to characterize the (much larger) number $\prod_{j=1}^{n} m_j$ of possible configurations (i.e., solutions).

The problem MOCDP is nonconvex and nondifferentiable even under strong assumptions on the underlying MONP. A two-step solution method is proposed for solving MOCDP. In the first step a representation of $\mathcal{P}$ is computed and in the second step, global and local optimization algorithms are combined to find a good approximation of the representation from the first step. The method is demonstrated on instances constructed from a standard test problem from the literature. Suggestions are also made for how to adapt the methodology to problems with expensive function evaluations.

The main contribution of the paper is the methodology of implicit clustering, which yields desired outcomes both in the decision space (a certain structure) and in the objective space (a population of solutions which approximates $\mathcal{P}$).

Two main issues that have to be addressed for the current methodology to apply to general practical configuration problems are how to incorporate other variable types (i.e., integer and categorical), and how to handle more general constraints (or restrictions).

Related to the product development process, **Paper II** follows naturally after **Paper I** in a chronological order. From **Paper I** we know which objectives that should be considered when searching for the right technical solutions; this yields a well-defined problem to solve with the framework of **Paper II**. The problem considered in **Paper II** is a simplified version of the core problem of the product development problem. However, the framework is possible to apply to some actual problems within the product development, given that there are quantified objective functions and continuous variables at hand. Especially in design optimization of components this should often be the case.

6.3 Paper III — A New Robustness Index for Multi-Objective Optimization based on a User Perspective

The concept of optimality fundamentally differs between optimization problems considering a single objective and multiple objective functions, respectively. While all (rational) decision makers (DMs) consider the same points as optimal in a single objective context, in a multi-objective they might not. True is only that they all consider some point(s) in the Pareto optimal set as the best one(s). For this reason, the concept of robustness, considering the DM's perspective, should be characterized differently from that in the single objective case.

A natural definition of robustness in a multi-objective context is that a certain Pareto optimal point is robust with respect to uncertainties if it stays (near-)Pareto optimal for all realizations of the uncertain elements. Such a definition is reasonable for many applications, partly depending on when the realizations of the uncertain elements are revealed. Consider the problem of deciding which variants of a product to make available. Suppose that the objective functions perfectly describe the qualities of a product, but that there are uncertainties regarding the implementation of potential solutions, for example that machines for the manufacturing process have to be constructed, and that there are uncertainties regarding the dimensions of the resulting products manufactured by these machines. In this case the robustness definition described is reasonable since a DM will make his / her choice after the uncertainty is revealed, and what is important for the company is that the product offer as a set is close to the Pareto optimal set no matter the realization of the uncertain parameters.

However, if the uncertainty is not revealed until the DM has made his / her choice, then the above robustness definition is questionable. The DM has chosen the specific point for a certain reason, e.g., that the local trade-offs between the objectives are perfect at this point, meaning that the price for improving one objective is too high with respect to the change in the other objectives. Consider now that uncertainties in the underlying problem contribute to changes in the objective values as compared to the non-perturbed—or ideal—situation. Certainly, it is important how much the objective changes, but the DM would argue that changes in the character, i.e., the local trade-offs, are also important.

The above example serves as the motivation for this paper. We consider robustness from the DM's perspective by estimating utility functions corresponding to potential solution points. Using the utility function as a single objective, we quantify the robustness of solution points to multi-objective problems using traditional measures of robustness for single-objective problems.

In the configuration problem considered at Volvo, the quality measures are many and quantifying these into objective functions cannot be made with certainty. Therefore, the contents of this paper fit well into the actual problem framework. If a certain configuration is assigned to a customer of a certain type, then we want the customer to appreciate the configuration also if the data and / or objective functions used when analyzing the problem and deciding on the actual configuration differ to some degree from reality.

The main contribution of this paper is in the means of estimating and introducing the utility function into the evaluation of robustness for multi-objective problems. We also contribute with a new definition of so called proper Pareto optimal points, which can be seen as reasonable choices of solution points for rational DMs. This definition is shown to be more strict than the standard definition of proper Pareto optimal points given in [22].

The utility functions estimated can be used not only for robustness evaluations, but also for evaluations of approximate Pareto optimal sets (cf. Section 4.1.1) by letting the change in utility value between a pair of points represent the point measure. The reason for a certain Pareto optimal point being selected will then be incorporated in the point measure, which therefore better reflects the perceived difference between the pair of points. Similarly, when deciding on the feature levels for the BED cells (cf. Section 2.2.1), estimations of utility functions can be utilized.

Chronologically in the development process, the use of **Paper III** follows naturally after, or in parallel with, **Paper II**. In **Paper II**, technical solutions are found, provided a subset of all features, as decided from the framework in **Paper I**. With the framework of **Paper III**, we can now include all the features and evaluate the robustness with respect to the increased set of features and to uncertainties in the objectives used.

The ideas developed in this paper can be further developed in various ways. Other definitions of robustness for the resulting single-objective problem could be utilized, and other parameterized expressions for the utility functions may be assumed. In order to find a good set of robust solutions included in the solution process of the multi-objective problem, it would also be interesting to investigate whether utility functions can be estimated in a meaningful way already before the ideal Pareto front is resolved.

6.4 Paper IV — Pure Categorical Optimization – a Global Descent Approach

In this paper, the *mode 1* configuration problem described in Section 2.1.2 is approached. Here, the decision space is composed by the current variants and variant families. A reasonable description of the decision variables is then made by utilizing the concept of categorical variables (cf. Section 4.4). The generalized version of this problem is introduced as the *pure categorical optimization problem*. The developments in the paper are restricted to single-objective optimization problems. A practical problem which fits into the framework of this paper is the search for an optimal truck configuration for a certain customer, given his / her utility function.

In the paper, mathematical properties of the pure categorical optimization problem are established and it is shown that the categorical problem is, in a sense, equivalent to a family of nonlinear integer programming problems. A recent solution method for such problems is then extended in order to be adapted to a categorical problem making use of the family of equivalents. The method suggested is of a heuristic nature; however, it is guaranteed to converge at least to a local categorical minimum.

The main contribution of the paper is that we show that pure categorical problems can be approached using mathematical programming techniques by suggesting a particular method as a proof of concept. We have not seen any such approaches earlier in the literature, possibly because pure categorical problems might be seen as too hard to be solved. In general they are too hard; but under some assumptions on the properties of the problems, which are reasonable to hold for many practical problems, solution schemes can be developed.

In the paper, a penalty approach is used to remove infeasible points. In future extensions, more sophisticated constraint handling techniques should be explored. For example, in our configuration problem an infeasible point is a point which violates one or more restrictions (cf. Section 2.1.2). In addition, the restrictions are defined in low-dimensional subspaces of the domain. This information can be exploited. The infeasibility of a point can be graded by the number of restrictions violated, and "resolving" techniques for an infeasible, but otherwise good, point can be defined; points in the low-dimensional space defined by the restriction are then evaluated, in order to resolve it and find a "close" but feasible point.

Another natural extension is to include more than one objective in the problem formulation. If multi-objective pure categorical problems can be solved, then the vision of the project of reducing the assortment in a controlled and systematic way can be approached. Solving the multi-objective pure categorical problem using the current variants will provide the Pareto optimal configurations given the product structure of today. By removing variants (possibly in the framework of **Paper II**, in which good variants of the available ones are sought) and combine this with evaluations of approximate Pareto sets (possibly by the use of utility function estimations as described in **Paper III**) we can certify that, for a certain cost level, any potential customer will always get a product which is as least as good as the product that would be chosen from the assortment of today.

Chronologically in the development process, **Paper IV** comes last. The framework can be used when finding the right truck for a certain customer given the already decided product offer. That is, the results of **Paper IV** can be applied within the sales process.

6.5 Paper V — A Minimax Strategy for Global Optimization

When considering optimization problems in which the objective function values for some reason are expensive to compute, such as in the case when they are outcomes of time-consuming computer simulations, or involving some physical experiment, the number of function evaluations performed during the optimization must be kept low. After each evaluation of the objective function, one goal is that the best evaluated point so far is as good as possible. Another closely related goal is that the optimality gap, i.e., the difference between the best point evaluated so far and the best possible point in the domain, is minimized.

In this paper we consider the latter goal, which is to minimize the optimality gap; this can be seen as a minimax strategy for global optimization. The objectives considered are of a black-box type, but for the optimality gap to be finite, assumptions on the function class in which the objective function is contained are necessary. The optimality gap, or the *maximum loss* as it is denoted in the paper, is then the *maximum* optimality gap over the class of functions assumed. We begin with letting the function class be a general set of functions, but throughout the paper we frequently consider the special case when the function class is the family of Lipschitz continuous functions bounded by some norm, when we can develop efficient solution algorithms. Concepts from statistical decision theory is utilized so as to classify global optimization algorithms, and as far as we know, the concept to which our proposed method belongs has not been applied to global optimization before.

The method is not straightforward to implement. One difficulty is that the domain for the minimax problem must be reduced from the domain of the original problem in order to guarantee uniqueness of the resulting minimax problem, and in order to make sure that the optimal minimax solution corresponds to a desired evaluation point. In future work, both theory and algorithms for finding a good domain should be developed further. Also, methods searching for approximate solutions to the minimax problems should be analyzed. The complexity of the method does not grow too fast with the dimension of the domain. Furthermore, the method has the advantage that it avoids excessive sampling of points at the boundary of the domain, which is a disadvantage of many global optimization strategies. The method may be further developed into a strategy for finding good sample points with the purpose of updating surrogate models used in simulation-based methods (cf. Section 4.2) for problems in medium-dimensions.

Of the appended papers, **Paper V** is the one which may have the least evident connection to the actual configuration problem at Volvo, and which is the hardest to place in the chronological order. The method considered is however applicable for any design problem in continuous variables; and new truck components are constantly being developed, with or without the use of optimization. Since a typical continuous design problem is nonconvex, global optimization is the correct tool to use. Furthermore, the framework fits well into the project vision that a customer should be guided towards a design solution that is at least as good as the one he / she would have chosen without the procedure of systematic assortment reduction. Then, guarantees on the near-optimality of design solutions are more important than the hope for very good function values. In addition, the definition of maximum loss provided in the paper is general, and the framework can therefore be utilized for many applications. For example, it is possible to replace the above function class with the set of utility functions of potential customers, to define the domain as the set of all possible trucks (the design space of Section 2.1.2), and to define the current evaluated points as the trucks made available. Then the evaluation set should be chosen in the domain such that the maximum loss is minimized, meaning that the maximum difference in utility between a desired and an available configuration is minimized. This also constitutes a clear connection to **Paper II** and to the project vision.

References

[1] K. Abhishek, S. Leyffer, and J. T. Linderoth, *Modeling without categorical variables: a mixed-integer nonlinear program for the optimization of thermal insulation systems*, Optimization and Engineering, 11 (2010), pp. 185–212.

[2] N. Andréasson, A. Evgrafov, and M. Patriksson, *An Introduction to Continuous Optimization*, Studentlitteratur, Lund, Sweden, 2005.

[3] C. Audet and J. E. Dennis Jr., *Pattern search algorithms for mixed variable programming*, SIAM Journal on Optimization, 11 (2001), pp. 573–594.

[4] Y. Bar-Yam, *Dynamics of Complex Systems*, Studies in Nonlinearity, Addison-Wesley, Reading, MA, USA, 1997.

[5] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, *Nonlinear programming Theory and Algorithms*, John Wiley & Sons, Hoboken, NJ, USA, third ed., 2006.

[6] A. Ben-Tal and A. Nemirovski, *Robust optimization – methodology and applications*, Mathematical Programming, 92 (2002), pp. 453–480.

[7] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*, Springer Series in Operations Research, Springer-Verlag, New York, NY, USA, 1997.

[8] W. M. Carlyle, J. W. Fowler, E. S. Gel, and B. Kim, *Quantitative comparison of approximate solution sets for bi-criteria optimization problems*, Decision Sciences, 34 (1) (2003), pp. 63–82.

[9] P. Czyżak and A. Jaszkiewicz, *Pareto simulated annealing – a metaheuristic technique for multiple-objective combinatorial optimization*, Journal of Multi-Criteria Decision Analysis, 7 (1998), pp. 34–47.

[10] G. B. Dantzig, *Linear programming under uncertainty*, Management Science, 1 (1955), pp. 197–206.

[11] K. Deb and H. Gupta, *Handling constraints in robust multi-objective optimization*, in Congress on Evolutionary Computation, vol. 1, 2005, pp. 25–32.

[12] ———, *Searching for robust Pareto-optimal solutions in multi-objective optimization*, in Evolutionary Multi-Criterion Optimization, 2005, pp. 150–164.

[13] ———, *Introducing robustness in multi-objective optimization*, Evolutionary Computation, 14 (2006), pp. 463–494.

[14] S. Edlund and P.-O. Fryk, *The right truck for the job with global truck application descriptions*, in SAE technical paper 2004-01-2645, New York, NY, USA, 2004.

[15] M. Ehrgott, *Multicriteria Optimization*, Springer-Verlag, Heidelberg, Germany, 2005.

[16] M. Ehrgott and X. Gandibleux, *A survey and annotated bibliography of multiobjective combinatorial optimization*, OR Spektrum, 22 (2000), pp. 425–460.

[17] M. Fuchs and A. Neumaier, *Discrete search in design optimization*, in Proceedings of the First International Conference on Complex Systems Design & Management (CSDM), Paris, France, 2010, pp. 113–122.

[18] ———, *A splitting technique for discrete search based on convex relaxation*, Journal of Uncertain Systems, 4 (2010), pp. 14–21.

[19] F. Fuxin, *Evolution and Communication of Geometry Based Product Information within an Extended Enterprise*, PhD thesis, Division of Computer Aided Design, The Polhem Laboratory, Luleå University of Technology, Luleå, Sweden, 2005.

[20] G. Gan, C. Ma, and J. Wu, *Data Clustering: Theory, Algorithms, and Applications (ASA-SIAM Series on Statistics and Applied Probability)*, SIAM, Alexandria, VA, USA, 2007.

[21] A. Genell, *Perception of Sound and Vibration in Heavy Trucks*, PhD thesis, Division of Applied Acoustics, Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg, Sweden, 2008.

[22] A. Geoffrion, *Proper efficiency and the theory of vector maximization*, Journal of Mathematical Analysis and Applications, 22 (1968), pp. 618–630.

[23] H. M. Gutmann, *A radial basis function method for global optimization*, Journal of Global Optimization, 19 (2001), pp. 201–227.

[24] E. M. T. Hendrix and B. G. Toth, *Introduction to Nonlinear and Global Optimization*, Springer-Verlag, New York, NY, USA, 2010.

[25] R. Horst, P. M. Pardalos, and N. V. Thoai, *Introduction to Global Optimization*, Kluwer Academic, London, UK, 1995.

[26] R. Horst and H. Tuy, *Global Optimization: Deterministic Approaches*, Springer-Verlag, Berlin, Germany, 1996.

[27] W. Huyer and A. Neumaier, *Global optimization by multilevel coordinate search*, Journal of Global Optimization, 14 (1999), pp. 331–355.

[28] A. K. Jain, M. N. Murty, and P. J. Flynn, *Data clustering: a review*, ACM Computing Surveys, 31 (1999), pp. 264–323.

[29] S. Jakobsson, M. Patriksson, J. Rudholm, and A. Wojciechowski, *A method for simulation based optimization using radial basis functions*, Optimization and Engineering, 11 (2010), pp. 501–532.

[30] J. Jiao, T. W. Simpson, and Z. Siddique, *Product family design and platform-based product development: a state-of-the-art review*, Journal of Intelligent Manufacturing, 18 (2007), pp. 5–29.

[31] D. R. Jones, *A taxonomy of global optimization methods based on response surfaces*, Journal of Global Optimization, 21 (2001), pp. 345–383.

[32] D. R. Jones, C. D. Perttunen, and B. E. Stuckman, *Lipschitzian optimization without the Lipschitz constant*, Journal of Optimization Theory and Applications, 79 (1993), pp. 157–181.

[33] D. R. Jones, M. Schonlau, and W. J. Welch, *Efficient global optimization of expensive black-box functions*, Journal of Global Optimization, 13 (1998), pp. 455–492.

[34] P. Kall and S. W. Wallace, *Stochastic Programming*, John Wiley & Sons, Chichester, UK, 1994.

[35] M. Karlsson, *Load Modelling for Fatigue Assessment of Vehicles — a Statistical Approach*, PhD thesis, Department of Mathematical Sciences, Chalmers University of Technology and Department of Mathematical Sciences, University of Gothenburg, Göteborg, Sweden, 2007.

[36] K.-H. Küfer, M. Monz, A. Scherrer, P. Süss, F. Alonso, A. S. A. Sultan, T. Bortfeld, D. Craft, and C. Thieke, *Multicriteria optimization in intensity modulated radiotherapy planning*, ITWM Report 77, Fraunhofer-Institut für Techno- und Wirtschaftsmathematik, Kaiserslautern, Germany, 2005.

[37] P. LINDROTH, *Product Configuration with respect to Multiple Criteria — a Mathematical Programming Approach*, Licentiate thesis, Department of Mathematical Sciences, Chalmers University of Technology and Department of Mathematical Sciences, University of Gothenburg, Göteborg, Sweden, 2008.

[38] P. LINGMAN, *Integrated Brake Control: Downhill Driving Strategies*, PhD thesis, Department of Applied Mechanics, Chalmers University of Technology, Göteborg, Sweden, 2005.

[39] Z. Q. LUO, J. S. PANG, AND D. RALPH, *Mathematical Programs with Equilibrium Constraints*, Cambridge University Press, Cambridge, UK, 1994.

[40] K. MIETTINEN, *Nonlinear Multiobjective Optimization*, Kluwer, Norwell, MA, USA, 1998.

[41] K. G. MURTY, *Linear Programming*, John Wiley & Sons, New York, NY, USA, 1983.

[42] G. L. NEMHAUSER AND L. A. WOLSEY, *Integer and Combinatorial Optimization*, John Wiley & Sons, New York, NY, USA, 1999.

[43] V. PARETO, *Cours d'Économie Politique*, Libraire Droz, Genève, Switzerland, 1964. (the first edition in 1896).

[44] M. PILEGAARD HANSEN AND A. JASZKIEWICZ, *Evaluating the quality of approximations to the non-dominated set*, Technical report IMM-REP-1998-7, Institute of Mathematical Modelling, Technical University of Denmark, Lyngby, Denmark, 1998.

[45] M. J. D. POWELL, *The NEWUOA software for unconstrained optimization without derivatives*, in Large-scale Nonlinear Optimization, P. Pardalos, G. Pillo, and M. Roma, eds., vol. 83 of Nonconvex Optimization and its Application, Springer-Verlag, New York, NY, USA, 2006, pp. 255–297.

[46] S. RUZIKA AND M. M. WIECEK, *Approximation methods in multiobjective programming*, Journal of Optimization Theory and Applications, 126 (2005), pp. 473–501.

[47] S. SAYIN, *Measuring the quality of discrete representations of efficient sets in multiple objective mathematical programming*, Mathematical Programming, 87 (2000), pp. 543–560.

[48] T. W. SIMPSON, *Product platform design and customization: Status and promise*, Artificial Intelligence for Engineering Design, Analysis and Manufacturing, 18 (2004), pp. 3–20.

[49] A. L. SOYSTER, *Convex programming with set-inclusive constraints and applications to inexact linear programming*, Operations Research, 21 (1973), pp. 1154–1157.

[50] R. STEUER, *Multiple Criteria Optimization: Theory, Computation and Application*, John Wiley & Sons Inc, New York, NY, USA, 1986.

[51] A. TIDSTAM AND J. MALMQVIST, *Information modelling for automotive configuration*, in Proceedings of NordDesign 2010, Gothenburg, Sweden, 2010.

[52] L. VANDENBERGHE AND S. BOYD, *Semidefinite programming*, SIAM Review, 38 (1996), pp. 49–95.

[53] A. VIANA AND J. PINHO DE SOUSA, *Using metaheuristics in multiobjective resource constrained project scheduling*, European Journal of Operational Research, 120 (2000), pp. 359–374.

[54] E. ZITZLER, L. THIELE, M. LAUMANNS, C. FONSECA, AND V. FONSECA, *Performance assessment of multiobjective optimizers: an analysis and review*, IEEE Transactions on Evolutionary Computation, 7 (2003), pp. 117–132.