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Spatial Correlations of Incremental Sources in Isotropic Environment Such as Reverberation Chamber

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***Abstract*— Spatial correlations of incremental electric and magnetic currents, and Huygen's sources with different arrangements (parallel and collinear) are studied in this paper; showing computed results as well as some analytical closed-form expression. It is found that the correlation depends both on the isolated far field function and the arrangements, and of course the spacing. These correlations are compared with that of two imaginary isotropic source with the same spacing. It is found that two Huygen's sources for certain arrangements have a spatial correlation that is almost the same as that of two isotropic sources. Spatial correlation of an electrically small monopole is also measured in a reverberation chamber to verify some results.**

I. Introduction

Multi-input multi-output (MIMO) systems have drawn considerable interest for improving wireless communications. For compact MIMO terminal (multi-port antenna), correlations between different antenna ports (branches) are inevitable. Because correlations have adverse effects on diversity gain [1] and capacity [2], it is of interest to determine it. In this paper, we always use complex correlation (correlation between complex voltages at different antenna ports). In early literature, mutual coupling in the multi-port antenna was neglected, and the resulting correlation could be correctly called spatial correlation [2], [3]. For minimum scattering antennas [4], spatial correlation is actually equivalent to open-circuit correlation. In [5] and [6], it is claimed that the correlation between the ports of real-life multi-port antenna can be obtained using equivalent impedance circuit model based on a priori knowledge of spatial correlation.

The purpose of this paper is therefore to study correlation between two incremental electric sources in an isotropic environment. Due to the interesting properties of the incremental electric sources (ideal Hertzian), they had been used in [7] to study vanishing antenna spacing effects on MIMO capacity with covariance information. For completeness, we also include an analytic/numerical study of correlation between parallel incremental magnetic sources, and Huygen's sources (see section 4.4 in [8]). The spatial correlations are studied for parallel and collinear arrangements, and compared with the spatial correlation of a pair of imaginary isotropic sources. Spatial correlations measured in reverberation chamber will also be included. The reverberation chamber environment is known to emulate the fading appearing in an isotropic rich multipath environment [9], and this can be repeated deterministically by using stepwise stirring. The latter allows us also to measure correlation by simply shifting the dipole spatially, and repeating the channel measurements, providing we only move the dipole and not its supports and cable. The latter will only be approximately correct. The corresponding correlation measured will be spatial correlation.

II. Spatial Correlation

Assume a pair of isotropic sources is located along x-axis. The spatial correlation between them is given by [3]

$$\rho = E\{\exp(jkd\sin\theta\cos\phi)\} \tag{1}$$

where k is wave number, d is the separation distance between the elements. Equation (1) can be rewritten as

$$\rho = \int_0^{2\pi}\int_0^{\pi} \exp(jkd\sin\theta\cos\phi)\,p(\theta,\phi)\,d\theta\,d\phi \tag{2}$$

where $p(\theta,\phi)$ is the probability density function (PDF) of AOA of incident signals. In three-dimensional (3-D) isotropic environments [10],

$$p(\theta,\phi) = \frac{\sin\theta}{4\pi} \quad (0 \le \theta < \pi, \quad 0 \le \phi < 2\pi) \tag{3}$$

Substituting (3) into (2), the spatial correlation of isotropic source pair reduces to

$$\rho = \frac{\sin(kd)}{kd} \tag{4}$$

Similarly, for incremental sources the spatial correlation also depends on the isolated radiation pattern of the source, i.e.

$$\rho = \int_0^{2\pi}\int_0^{\pi} |\vec{G}(\theta,\phi)|^2 \exp(jkd\sin\theta\cos\phi)\,p(\theta,\phi)\,d\theta\,d\phi \tag{5}$$

where $\vec{G}(\theta,\phi)$ is the complex far field function of the incremental source in free space. In this paper, the PDF of AOA is always assumed to be isotropic.

III. Incremental sources

The far field function of the incremental electric source is [8]

$$\vec{G}_{ie}\left(\hat{r}\right) = C_k \eta I_0 l [\hat{l} - \left(\hat{l}\cdot\hat{r}\right)\hat{r}] \tag{6}$$

where $\hat{r}$ and $\hat{l}$ are unit vector of radial direction and current orientation respectively, η is free space wave impedance, I_0 is electric current amplitude, l is the length of the source and $C_k = -jk/4\pi$. The far field function of the incremental magnetic source is [8]

$$\vec{G}_{im}\left(\hat{r}\right) = C_k \eta M_0 l \left(\hat{l}\times\hat{r}\right) \tag{7}$$

The Huygen's source consists of orthogonal incremental electric and magnetic sources with $M_0 = \eta I_0$. Its far field function is [8]

$$\vec{G}_{H}\left(\hat{r}\right) = C_k \eta J_0 dS' [\hat{l} - \left(\hat{l}\cdot\hat{r}\right)\hat{r} - \hat{l}\times\hat{n}\times\hat{r}] \tag{8}$$

where $\hat{n}$ is the normal to the incremental surface area dS' in the direction of radiation. Without loss of generality normalizations of $\eta I_0 l = 1$, $M_0 l = 1$ and $\eta I_0 dS' = 1$ are assumed.

Provided that the incremental source pair is located along x-axis, they are parallel if the currents are oriented along z-axis; they are collinear if the current are oriented along x-axis. For parallel arrangement, the far field functions of each incremental electric and magnetic source is respectively

$$\vec{G}_{ie}\left(\hat{r}\right) = -C_k \sin\theta\hat{\theta} \tag{9}$$

$$\vec{G}_{im}\left(\hat{r}\right) = C_k \sin\theta\hat{\phi} \tag{10}$$

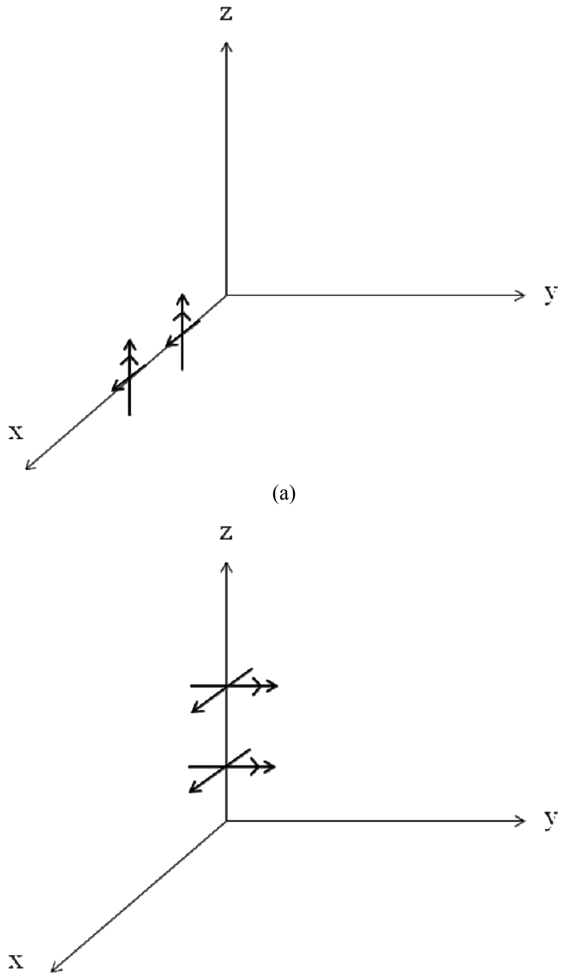
For collinear arrangement, the far field functions of each incremental electric and magnetic sources is respectively

$$\vec{G}_{ie}\left(\theta,\phi\right) = C_k \left(\cos\theta\cos\phi\hat{\theta} - \sin\phi\hat{\phi}\right) \tag{11}$$

$$\vec{G}_{im}\left(\theta,\phi\right) = C_k \left(\sin\phi\hat{\theta} - \cos\theta\cos\phi\hat{\phi}\right) \tag{12}$$

Substituting (9)-(12) into (5), it can be found that, with the same arrangement, the spatial correlation of incremental electric and magnetic sources are identical. From now on, we will only study correlation for incremental electric source pair, knowing the correlation for incremental magnetic source pair will be identical.

The parallel and collinear arrangements do not apply to Huygen's source, because it consists of two orthogonal incremental currents. We define two arrangements for a pair of Huygen's sources: parallel-collinear arrangement, where the incremental electric (magnetic) currents are parallel while magnetic (electric) currents are collinear; double-parallel arrangement, where both incremental currents are parallel. Fig. 1 illustrates these arrangements, where single arrow denotes incremental electric current, and double arrow represents incremental magnetic current.



(a)

(b)

Fig. 1 Huygen's source pair: (a) parallel-collinear arrangement; (b) double parallel arrangement

IV. Theoretical results

The normalized correlation for incremental electric source pair with spacing d in collinear arrangement has been derived in [11],

$$\rho_{co} = \frac{3}{(kd)^2}[\frac{\sin(kd)}{kd} - \cos(kd)] \tag{13}$$

The correlation for Huygen's source pair in double-parallel arrangement is derived here. For the sake of simplification and without loss of generality, we assume the Huygen's source is y-polarized (incremental electric current in y-direction and incremental magnetic current in x-direction), and that the Huygen's source pair is located along z-axis (double-parallel). From (8) the far field function of the y-polarized Huygen's source can be obtained as

$$\vec{G}_{H}\left(\theta,\phi\right) = C_k \cos^2\frac{\theta}{2}\left(\sin\phi\hat{\theta} + \cos\phi\hat{\phi}\right) \tag{14}$$

The correlation then is

$$\rho = \int_0^{2\pi}\int_0^{\pi} |\vec{G}(\theta,\phi)|^2 \exp(jkd\cos\theta)p(\theta,\phi)d\theta d\phi \qquad (15)$$

Substitute (14) into (15), the correlation reduces to

$$\rho_{dp} = \frac{|C_k|^2}{4\pi}\int_0^{2\pi}\int_0^{\pi}\cos^4\frac{\theta}{2}\exp(jkd\cos\theta)\sin\theta d\theta d\phi \qquad (16)$$

The integral in (16) is evaluated in Appendix A. The correlation for double-parallel Huygen's source pair is

$$\rho_{dp} = \frac{|C_k|^2}{2}\{[\frac{1}{kd}-\frac{1}{j(kd)^2}-\frac{1}{(kd)^3}]\sin(kd) -[\frac{1}{jkd}+\frac{1}{(kd)^2}]\cos(kd)\} \qquad (17)$$

The correlation for incremental electric (magnetic) source pair in parallel arrangement and Huygen's source pair in parallel-collinear arrangement, directly using (5), do not result in closed-form formula. However, since incremental sources can be well approximated by minimum scattering antennas, one can derive their spatial (open-circuit) correlation using [4]

$$\rho = \frac{\mathrm{Re}\{Z_{ij}\}}{\sqrt{\mathrm{Re}\{Z_{ii}\}\mathrm{Re}\{Z_{jj}\}}} \qquad (18)$$

where Z_{ij} is mutual impedance between ith and jth incremental sources, and Z_{ii} is the input impedance of ith incremental source. Using (18), one can easily derive that the normalized spatial correlation of parallel incremental electric source is [12]

$$\rho_{pa} = \frac{3}{2}[\frac{\sin(kd)}{kd}+\frac{\cos(kd)}{(kd)^2}-\frac{\sin(kd)}{(kd)^3}] \qquad (19)$$

Fig. 2 compares the normalized correlations of incremental electric (magnetic) source pair in collinear arrangement and Huygen's source pair in double-parallel arrangement (see Fig. 1b) calculated numerically and analytically using (13) and (17). For numerical calculation, we generate 10000 uniformly distributed AOA samples. As expected, the numerical method can predict spatial correlations well. These two correlations are plotted against that of isotropic source (4).

Fig. 3 shows the normalized spatial correlation for parallel incremental electric source, using both numerical and analytical calculation, and parallel-collinear Huygen's sources with only numerical calculation, together with that of isotropic sources (4). It is again, as expected, seen that the numerical calculation agrees with the analytical formula of (19) for parallel incremental electric source. An interesting phenomenon to see is that Huygen's sources in parallel-colinear arrangement (see Fig. 1a) have a spatial correlation almost the same as that of isotropic source.

From both Fig. 2 and 3, it can be seen that the spatial correlation depends on both the far field functions (i.e. type of source) and the arrangement of the source pair. The correlation for parallel arrangement is most similar to that of the isotropic source.

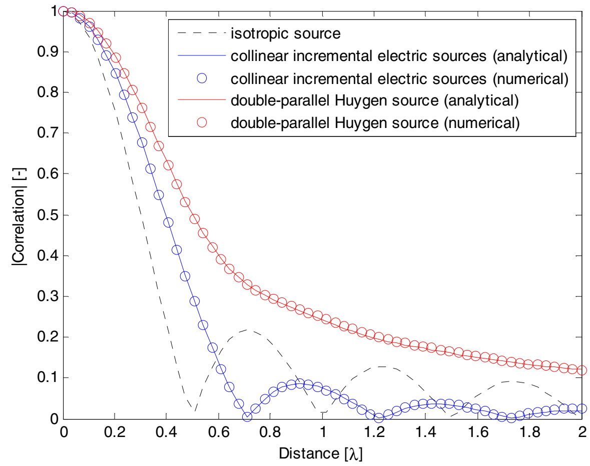


Fig. 2 Comparison of the normalized correlations of incremental electric source pair in collinear arrangement and Huygen's source pair in double-parallel arrangement, calculated numerically and analytically, together with that of isotropic source.

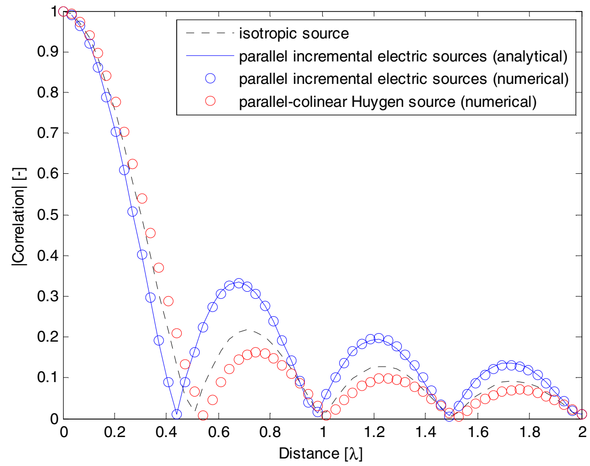


Fig. 3 Spatial correlation for parallel incremental electric source and parallel-collinear Huygen's sources, together with that of isotropic source.

V. REVERBERATION CHAMBER MEASUREMENTS

Apart from the above theoretical study, we also tried to observe the spatial correlation by measurements in reverberation chamber. The chamber in use is Bluetest HP reverberation chamber (see Fig. 4), with a size of 1.75 m × 1.25 m × 1.8 m. In the Bluetest chamber, there are three wall antennas mounted at three orthogonal walls used for polarization stirring [9]. The antenna under test (AUT) is placed on a platform, which will rotate during measurements, referred to as platform stirring [9]. Two metal plates are used as mechanical plate stirrers. In the measurements, the platform is moved to 20 positions spaced by 18°, and for each platform position each of the two plates move to 10 positions simultaneously, evenly along the total distance they can move. At each stirrer position and for each of the three wall antennas, a full frequency sweep is performed by the vector network analyzer (VNA). The chamber emulates Rayleigh fading environment with 3-D isotropic angular distribution [9].

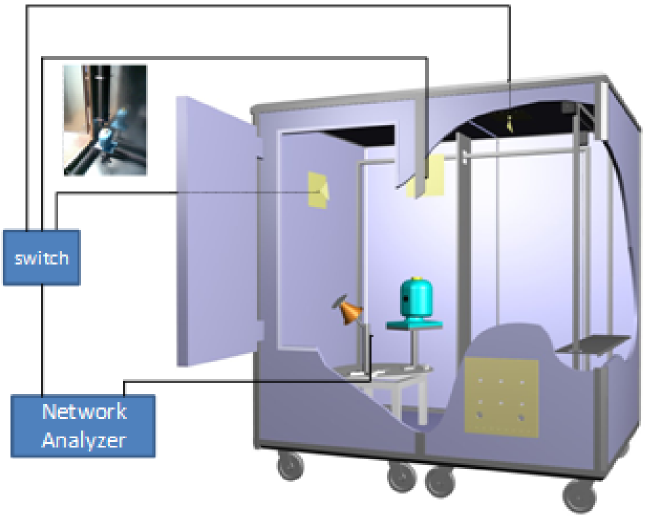


Fig. 4 Drawing of Bluetest RC with two mechanical plate stirrers, one platform and three wall antennas.

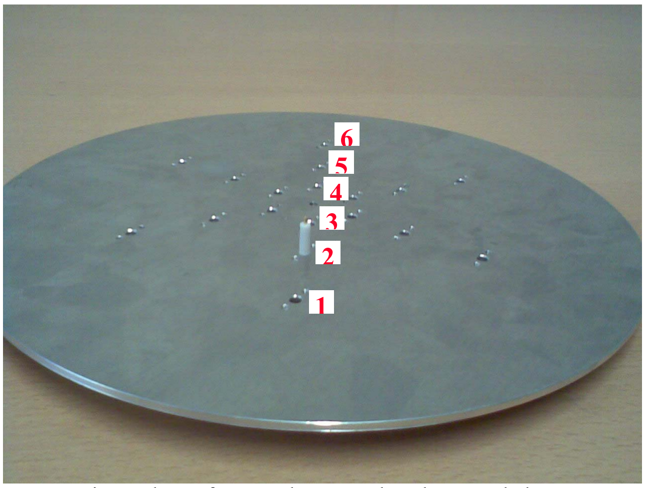


Fig. 5 Photo of monopole mounted on the ground plane.

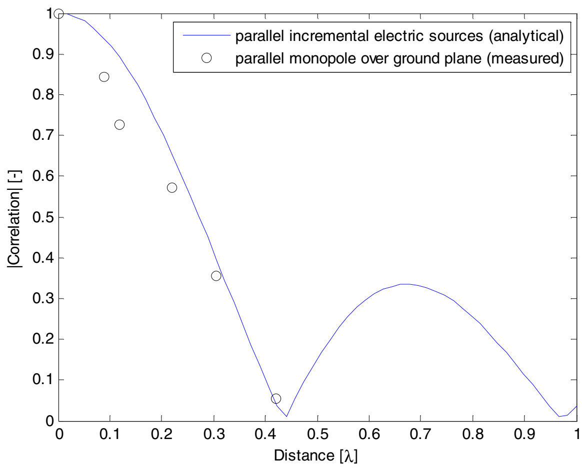


Fig. 6 Measured spatial correlation of parallel monopole in reverberation chamber against that of theoretical parallel incremental electric source.

In this paper, we choose a 17 mm long short monopole above a large circular ground plane with a radius of 140 mm. On the ground plane, there are many fixed holes to support the monopole. Fig. 5 shows a photo of the monopole mounted on position 2 of the ground plane. As marked in Fig. 5, we measure the monopole each time at one of the 6 positions in 6 independent measurements at 1 GHz with 10 MHz frequency stirring bandwidth [13]. The physical distances between consecutive points from position 1 to 6 are 35 mm, 26 mm, 40 mm, 26 mm and 35 mm respectively. Spatial correlations are calculated between these 6 measurements using

$$\rho_{meas} = \frac{E[V_1 V_2^*]}{\sqrt{E[|V_1|^2]E[|V_2|^2]}} \tag{20}$$

where V_i (i = 1, 2) is the complex signal received at ith element port during measurements in reverberation chamber. The measured spatial correlation is plotted against that of parallel incremental electric source, as shown in Fig. 6. As expected, since the vertical electrically small monopole over a large ground plane can be regarded approximately as an electrically small dipole without ground plane, the measured spatial correlation of the monopole is close to that of incremental electric source. Note that, in order to measure spatial correlation exactly, we should be able to repeat the measurement exactly. Although reverberation chamber has a good repeatability, it is impossible to produce two identical scattering environments even from statistic point of view of practical reasons, e.g. due to changes in the position of the feeding cable associated with the monopole. Here we have assumed that the large ground plane has minimized the effects of small changes of the cable location between the measurements for different monopole locations.

VI. Conclusions

Spatial correlations of different incremental sources with collinear and parallel arrangements have been studied. We derived the correlation for double-parallel Huygen's source pair analytically. The derivation is verified by numerical simulation. It is shown that the spatial correlation depends on the far field functions and arrangement of the incremental sources. Among all the incremental sources, the spatial correlation of Huygen's sources with parallel-colinear arrangement (see Fig. 1a) resembles that of isotropic source the most. In the end, spatial correlation of electrically small monopoles over a large ground plane is observed based on reverberation chamber measurements. As expected, it is close to that of parallel incremental electric sources.

Appendix A

Using trigonometric theory, the half-angle cosine in (16) can be written as

$$\cos^4\left(\frac{\theta}{2}\right) = \left(\frac{1+\cos\theta}{2}\right)^2 \tag{A.1}$$

Substitute (A.1) into (16) and integrate over ϕ,

$$\rho_{dp} = \frac{|C_k|^2}{8}\int_0^{\pi}(1+\cos\theta)^2 \exp(jkd\cos\theta)\sin\theta d\theta \tag{A.2}$$

Equation (A.2) can be expressed as

$$\rho_{dp} = \frac{|C_k|^2}{8}(I_1 + I_2 + I_3) \tag{A.3}$$

where

$$I_1 = \int_0^{\pi} \exp(jkd\cos\theta)\sin\theta d\theta \tag{A.3a}$$

$$I_2 = 2\int_0^{\pi} \cos\theta \exp(jkd\cos\theta)\sin\theta d\theta \tag{A.3b}$$

$$I_3 = \int_0^{\pi} \cos^2\theta \exp(jkd\cos\theta)\sin\theta d\theta \tag{A.3c}$$

The integration in (A.3.a) can be evaluated easily as

$$I_1 = \frac{2}{kd}\sin(kd) \tag{A.4}$$

The integrations in (A.3b) and (A.3c) are not as straightforward as that in (A.3a). We denote $t = \cos\theta$, (A.3b) can be rewritten respectively as

$$\begin{aligned} I_2 &= 2\int_{-1}^{1} t\exp(jtkd)dt \\ &= \frac{2}{jtkd}\int_{-1}^{1} td\exp(jtkd) \\ &= \frac{2}{jkd}\{[t\exp(jtkd)]_{-1}^{1} - \int_{-1}^{1}\exp(jtkd)dt\} \\ &= \frac{4}{jkd}[\cos(kd) - \frac{1}{kd}\sin(kd)] \end{aligned} \tag{A.5}$$

Similarly, using integration by parts (A.3c) is

$$I_3 = \frac{2}{kd}[\sin(kd) + \frac{2}{kd}\cos(kd) - \frac{2}{(kd)^2}\sin(kd)] \tag{A.6}$$

Substitute (A.4)-(A.6) into (A.3), the correlation of Huygen's source pair in double-parallel arrangement can be obtained as

$$\begin{aligned} \rho_{dp} = \frac{|C_k|^2}{2}\{&[\frac{1}{kd} - \frac{1}{j(kd)^2} - \frac{1}{(kd)^3}]\sin(kd) \\ -&[\frac{1}{jkd} + \frac{1}{(kd)^2}]\cos(kd)\} \end{aligned} \tag{A.7}$$

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