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Interference management using one bit feedback

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Abstract—This paper studies the performance of quasi-static spectrum sharing networks utilizing one bit interference indicator feedback. Assuming no channel state information at the transmitters, the channel average rate is obtained under different power allocation strategies. Simulation results show that interference indicator feedback leads to considerable rate increment even with no transmitter channel state information.

I. INTRODUCTION

Spectrum sharing networks are initiated by the apparently lack of spectrum under the current spectrum management policies. Currently, most of frequency bands useful to wireless communication are under control of primary license holders that have exclusive right to transmit over their spectral bands. This is the point that has created the perception of spectrum shortage, leading to ever-growing complaints about available spectral resources. On the other hand, recent studies such as [1], [2] show that at any given time, large portions of the licensed bands remain unused and so, it is expected that we can improve the data transmission strategies by better utilizing the licensed resources. Spectrum sharing is one of the most promising techniques created for this purpose.

Generally, the goal of a spectrum sharing scheme is to better utilize the radio spectrum by allowing the secondary users (SU’s) to coexist with the primary users (PU’s). Along with the standard interference channel [3]–[5], where independent transmitters send independent messages to independent receivers, there are other ways such as interference-avoiding and simultaneous transmission schemes to exploit the idea of spectrum sharing. The interference-avoiding paradigm [6]–[8] refers to an approach where the PU transmitter, provided that it can sense the spatial, temporal or spectral gaps of the PU resources, can adjust its transmission parameters to fill these white spaces. Although this scheme can theoretically lead to significant spectral efficiency improvement, it suffers from some practical drawbacks mainly related to imperfect gap detection. Moreover, it can not be implemented in delay-sensitive applications, as the SU transmission rate, particularly when the channels experience slow variations. However, even if there is (im)perfect channel state information available at the SU receiver, it may not be convenient to provide the transmitter with the same information, as it may lead to impractical feedback signaling overhead [18]–[21]. Therefore, it is important to study the channel performance when the fading channels are unknown by the transmitters.

To the best of authors knowledge, all developed simultaneous transmission approaches, e.g., [9]–[17], are based on the assumption that the PU transmitter has infinite amount of information continuously transmitted to its receiver. This point, however, is not valid in many occasions [1], [2]. On the other hand, the interference-avoiding methods permit no data transmission within the PU transmission time slots reducing their practicality in delay-sensitive applications.

In this perspective, this paper investigates the secondary channel average rate when there is no channel quality information available at the SU transmitter. Here, we focus on the case where the primary user turns on only for a portion of time slots indicated to the SU transmitter via one bit feedback. Considering exponential fading distributions, the channel average rates are obtained under different, namely, short- and long-term, power constraints. Finally, the results are generalized to the case when arbitrary number of users, experiencing different fading conditions, share the same frequency band for data transmission. Simulation results show that interference indicator feedback leads to considerable rate increment even with no transmitter channel state information.

The rest of the paper is organized as follows. System model is illustrated in section II. Then, the theoretical results are presented in section III. Section IV consists of simulation results and finally, the last section concludes the paper.

II. SYSTEM MODEL

As illustrated in Fig.1, we consider a standard quasi-static fading spectrum sharing network where two primary\(^1\) and secondary users share the same narrow-band frequency with

\(^1\)Primary users considered in the model are not necessarily the license holders but are the users that, while sharing the same spectrum, are out of the control.
bandwidth $B$. With no loss of generality we set $B = 1$. Let $G_{pp}$, $G_{ps}$, $G_{sp}$ and $G_{ss}$ be the instantaneous channel gains of the PU-PU, PU-SU, SU-PU and SU-SU links, respectively, which are assumed to be mutually independent. The channel gains remain constant for a duration of $t_c$, generally determined by the channels coherence times, and then change independently according to their corresponding fading probability density functions (pdf’s) $f_{G_{pp}}$, $f_{G_{ps}}$, $f_{G_{sp}}$ and $f_{G_{ss}}$. $t_c$ is supposed to be much larger than the length of codewords so that multiple packets are sent within one coherent period. It is assumed that the SU receiver has perfect instantaneous knowledge about its corresponding channel gains, which is an acceptable assumption under quasi-static condition [19]–[23]. On the other hand, the transmitters are supposed to have no information about the fading channels quality. A further extension of this work, which evaluates the effect of transmitters imperfect channel state information, will be presented by us soon. Finally, the white Gaussian noises added at the PU and SU receivers, which are denoted by $n_p$ and $n_s$, are supposed to have distributions $\mathcal{N}(0, \sigma_p^2)$ and $\mathcal{N}(0, \sigma_s^2)$, respectively. This is an appropriate model of stationary or slow-moving users such as wireless local area networks (WLANs) [24]. Particularly, since capacity-approaching codes can be implemented in such systems, the results can provide realistic insight about the performance bounds of the channel.

We assume that the secondary user has infinitely many information nats for transmission so that the SU-SU communication link is continuous [25]. On the other hand, the PU transmitter is active only for a portion of time, in harmony with practical investigations reported by, e.g., [1], [2]. One bit feedback is considered for informing the SU transmitter about PU activeness. The feedback can be sent from the PU transmitter, the SU receiver or by means of a band manager which mediates between the two parties [26]. Let $I$ be the PU status indicator in which $I = 1$ ($I = 0$) represents its activeness (inactiveness). In this way, the secondary user received signal can be stated as

\[
Y_s = \begin{cases} 
X_s\sqrt{G_{ss}} + X_p\sqrt{G_{ps}} + n_s & \text{if } I = 1 \\
X_s\sqrt{G_{ss}} + n_s & \text{if } I = 0
\end{cases}
\]  

(1)

in which $X_p$ and $X_s$ are the primary and secondary users’ input powers, respectively.

Under delay-insensitive conditions, the channel capacity is a valid performance measure of the spectrum sharing networks illustrated in, e.g., [11]–[14]. However, Many wireless applications are delay-limited where the codewords span a fixed (and not infinitely many) fading block. In this case, as discussed in the following, other performance evaluation metrics should be considered among which the channel average rate is the most common [19]–[23].

III. THEORETICAL RESULTS

Provided that the primary user is not transmitting, the SU transmitter considers some power $T_1$ and send the data at rate $R_1$. The transmitted codeword is successfully decoded by the SU receiver if the SU-SU channel gain supports the rate, i.e., $R_1 \leq \log(1 + \frac{G_{ss}T_1}{\sigma_s^2})$. Therefore, the expected SU-SU channel rate obtained with no PU interference is

\[
\bar{R}_1 = R_1 \Pr \{R_1 \leq \log(1 + \frac{G_{ss}T_1}{\sigma_s^2})\} = R_1 \left(1 - F_{G_{ss}}\left(\frac{e^{R_1} - 1}{T_1}\right)\right) \tag{2}
\]

in which $F_{G_{ss}}$ is the SU-SU channel gain cumulative distribution function (cdf).

On the other hand, if the SU transmitter is informed about the PU transmitter activeness, it transmits the data with power $T_2$ and rate $R_2$. In this case, based on the fact that the secondary user received signal-to-interference-and-noise-ratio (SINR) is

\[
\Omega_s = \frac{T_2 G_{ss}}{\sigma_s^2 + T_p G_{ps}}, \tag{3}
\]

the data is successfully received at the SU receiver if $R_2 \leq \log(1 + \frac{T_2 G_{ss}}{\sigma_s^2 + T_p G_{ps}})$. Consequently, the channel expected rate in the presence of PU interference signal is obtained by

\[
\bar{R}_2 = R_2 \Pr \{R_2 \leq \log(1 + \frac{T_2 G_{ss}}{\sigma_s^2 + T_p G_{ps}})\} = R_2 \int_0^\infty f_{G_{ps}}(y) \Pr \{G_{ss} \geq \frac{e^{R_2} - 1}{T_2} (\frac{\sigma_s^2 + T_p G_{ps}}{\sigma_s^2})\} dy = R_2 E_{G_{ps}}\left(1 - F_{G_{ss}}\left(\frac{e^{R_2} - 1}{T_2} (\frac{\sigma_s^2 + T_p G_{ps}}{\sigma_s^2})\right)\right) \tag{4}
\]

where $E_{G_{ps}}(.)$ denotes the expectation with respect to PU-SU channel gain random variable. Finally, considering the PU activeness probability to be $\alpha$, we can use (2) and (4) to find the SU-SU channel average rate and the SU total input power as

\[
\bar{R} = (1 - \alpha)\bar{R}_1 \left(1 - F_{G_{ss}}\left(\frac{e^{R_1} - 1}{T_1}\right)\right) + \alpha R_2 E_{G_{ps}}\left(1 - F_{G_{ss}}\left(\frac{e^{R_2} - 1}{T_2} (\frac{\sigma_s^2 + T_p G_{ps}}{\sigma_s^2})\right)\right), \tag{5}
\]

and

\[
\bar{T} = (1 - \alpha)T_1 + \alpha T_2 \tag{6}
\]
respectively. Using (5), (6) and the SU power constraint $\bar{T} \leq T_{s,\text{total}}$, the power-limited SU-SU channel rate optimization problem is formulated as

$$
\hat{R}_{\text{max}} = \max_{R_1,T_1,T_2} \left( 1 - \alpha \right) R_1 \left( 1 - F_{G_m}(e^{R_1 - 1}) \right) + \alpha R_2 E_{G_m} \left[ 1 - F_{G_m}(e^{(e^{R_2 - 1}) + T_2 G_m}) \right]
$$

subject to $(1 - \alpha) T_1 + \alpha T_2 \leq T_{s,\text{total}}$.

Normally, there are two different interpretations of the power constraint. Short-term power allocation [19]-[23] implies that $T_m = T_{s,\text{total}}, m = 1, 2$. Under the more relaxed long-term power constraint, the transmitter can adapt the power based on the channels conditions such that $T \leq T_{s,\text{total}}$. In this way, the optimal powers can be found by numerical analysis of (7). Finally, it is worth noting that assuming exponential gain pdf’s, e.g., $f_{G_m}(y) = \lambda_p e^{-\lambda_p y}, y \geq 0$, (5) is simplified to

$$
\hat{R} = (1 - \alpha) R_1 e^{-\lambda_p (R_1 - 1)} + \frac{\alpha R_2 e^{-\lambda_p (R_2 - 1)}}{1 + \frac{\lambda_p}{\lambda_p T_2} (e^{R_2 - 1})}
$$

where $\lambda_{ss}$ and $\lambda_{ps}$ denote the SU-SU and SU-PU exponential gain pdf parameters normally determined by the path loss and shadowing between the terminals.

A. Extension to multiple primary users case

The results can be generalized to the case where there are $M > 1$ primary users experiencing fading pdf’s $f_{G_{pj}}, j = 1, ..., M$. The primary users activeness status is provided at the secondary transmitter via $M$ bits feedback. Let $J \subset \{1, ..., M\}$ be the set of primary users that are active within the current fading block. Getting the set $J$ the SU transmission rate and power are considered to be $R_J$ and $T_J$, respectively. On the other hand, the SU receiver $J$ active primary users changes to

$$
\Omega = \sum_{J \subset \{1, ..., M\}} T_j G_{ss} \frac{1}{\delta^2 + \sum_{j \in J} T_j G_{pj}}
$$

in which $T_{pj}$ is the $j$-th PU input power. Again, the SU receiver successfully decodes the data if $R_J \leq \log(1 + T_j G_{pj})$ and so the channel expected rate is found as

$$
\hat{R}_J = R_j \Pr\{R_J \leq \log(1 + T_j G_{pj})\}
$$

$$
= \int_{y_j = 0}^{\infty} \prod_{j \in J} f_{G_j}(y_j) \left( \prod_{j \in J} f_{G_m}(y_j) \right) dy_j
$$

$$
= R_j \left( 1 - E_{G_{pj}, \forall j \in J} \left[ F_{G_m}(e^{(R_j - 1) + \sum_{j \in J} T_j G_{pj})} \right] \right)
$$

(10) which considering exponential fading pdf’s $f_{G_{pj}}(x) = \lambda_p e^{-\lambda_p x}, x \geq 0, j = 1, ..., M$, and $f_{G_m}(x) = \lambda_m e^{-\lambda_m x}, x \geq 0$, is found as

$$
\hat{R}_J = R_J \int_{y_j = 0}^{\infty} \left( \prod_{j \in J} \left( \lambda_p e^{-\lambda_p y_j} \right) \right) \left( e^{-\lambda_m \left( \sum_{j \in J} T_j G_{pj} \right)} \right)^{\infty} dy_j
$$

$$
= R_J \left( \frac{1}{\lambda_p \lambda_m + \lambda_p T_{\text{total}}} \right)
$$

Consequently, the channel average rate and total power are obtained by

$$
\hat{R} = \sum_{J \subset \{1, ..., M\}} \left( \prod_{j \in J} \alpha_j \right) \left( \prod_{j \in J} \left( 1 - \alpha_k \right) \right) \hat{R}_J
$$

$$
\hat{T} = \sum_{J \subset \{1, ..., M\}} \left( \prod_{j \in J} \alpha_j \right) \left( \prod_{j \in J} \left( 1 - \alpha_k \right) \right) T_J,
$$

respectively. Here, $\alpha_j$ is the activeness probability of the $j$-th PU and $\bar{J} = \{1, ..., M\} \setminus J$ is the complement set of $J$. Finally, the optimal SU-SU channel average rate is found by numerical solution of

$$
\hat{R}_{\text{max}} = \max_{T_J, R_J, \forall J \subset \{1, ..., M\}} \hat{R}
$$

subject to $\hat{T} \leq T_{s,\text{total}}$. (13)

(10)-(13) are particularly simplified if the primary users have the same characteristics, i.e., the same fading pdf $f_{G_m}(\cdot)$, activeness probability $\alpha$ and transmission powers $T_{pj}$, $T_m$. In this case, independent of primary users indexes, the rate and power $R_m$ and $T_m$ are respectively selected by the SU transmitter if it detects $m$ primary users interference signals. Therefore, with the same arguments as before, the channel average rate and power, i.e., (11) and (12), are rephrased as

$$
\hat{R} = \sum_{m=0}^{M} \left( \frac{M}{m} \right) \alpha^m (1 - \alpha)^{M-m} R_m \times \left( E_{G_{pj}, \forall j} \left[ \left( e^{R_m - 1} \right) + \sum_{j \in J} T_j G_{pj} \right] \right)
$$

and

$$
\hat{T} = \sum_{m=0}^{M} \left( \frac{M}{m} \right) \alpha^m (1 - \alpha)^{M-m} T_m
$$

respectively, where the expectation is taken with respect to random variables $G_{pj}, j = 1, ..., m$ experiencing the same pdf $f_{G_m}(\cdot)$. Finally, it is worth noting that using exponential pdf’s $f_{G_{pj}}(x) = \lambda_p e^{-\lambda_p x}, x \geq 0, j = 1, ..., M$ (14) changes to

$$
\hat{R} = \sum_{m=0}^{M} \left( \frac{M}{m} \right) \alpha^m (1 - \alpha)^{M-m} R_m \times \int_{y_1 = 0}^{\infty} \int_{y_2 = 0}^{\infty} \left( \prod_{k=0}^{k} \left( 1 - \alpha_k \right) \right) \left( e^{-\lambda_m \left( \sum_{j \in J} T_j G_{pj} \right)} \right) dy_1 ... dy_m,
$$

$$
= \sum_{m=0}^{M} \left( \frac{M}{m} \right) \alpha^m (1 - \alpha)^{M-m} R_m \times \left( \frac{1}{\lambda_p \lambda_m + \lambda_p T_{\text{total}}} \right)
$$

...
Note that setting $M = 1$ the results are simplified to the ones obtained in (5-8).

**IV. SIMULATION RESULTS**

In all simulations, the exponential pdf parameters and the AWGN variances are set to 1. Considering different primary user input powers and activation probabilities, Fig.2 studies the effect of interference indicator feedback on the SU-SU channel average rate. Here, the number of primary users is selected to be 1. Under the same conditions, Fig.3 presents some comparisons between the channel average rates obtained under long- and short-term power constraints. Moreover, considering a single primary user, Fig.4 demonstrates the effect of primary user input power and activation probability on the SU-SU channel performance. Finally, Fig.5 investigates the channel average rate in the case where $M > 1$ primary users, having the same properties, are working within the same frequency band.

**V. CONCLUSION**

This paper studies the performance of quasi-static spectrum sharing channels in the presence of one bit interference indicator feedback. The channel average rates are obtained in the case where there is no information about the fading channels at the transmitters. We evaluate the effect of power allocation on the channel data transmission efficiency under different primary user transmission conditions. Moreover, the results are generalized to case when arbitrary number of users experiencing different fading conditions share the same frequency band for data transmission. Simulation results show that:

![Figure 2](image-url)  
**Figure 2.** Secondary channel average rate in the presence or absence of primary user activation indicator feedback. Long-term secondary user power constraint, single primary user scenario.

![Figure 3](image-url)  
**Figure 3.** Comparison between the channel average rates obtained under short- and long-term power constraints. Single primary user scenario.

![Figure 4](image-url)  
**Figure 4.** Average rate vs (a): primary user input power $T_p$, (b): activation probability $\alpha$. Long-term secondary user power constraint, single primary user scenario.
The presence of interference indicator feedback can greatly affect the SU-SU channel average rate. The feedback becomes more effective as the primary user input power or activation probability increases (Fig.2).

Although considerable performance improvement is achieved via optimal power allocation, its influence diminishes by reducing the primary input power or activation probability (Fig.3).

While there is high data transmission potential for secondary users utilizing interference indicator feedback, the achievable rates decrease as the primary users input power (Fig.2 and 4a) or activation probability (Fig.2 and 4b) increases.

Increasing the number of primary users can drastically reduce the SU-SU channel average rate, particularly when the PU transmission period or input power increases (Fig.5).

REFERENCES


