Capacity bounds of spectrum sharing networks with no channel state information

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***Abstract*—Recently, substantial attention has been paid to improve the spectral efficiency of communication setups using different spectrum sharing techniques. This paper aims to study the capacity of fading spectrum sharing channels in the case where there is no channel state information available at the transmitters and receivers. The channel capacity bounds are obtained under secondary user input power, different primary user received interference power and also primary user peak and average signal-to-interference-and-noise (SINR) constraints. Simulation results show that there is considerable potential for data transmission of unlicensed users even with no fading channels state information.**

I. Introduction

Spectrum sharing networks are initiated by the apparent lack of spectrum under the current spectrum management policies. Currently, most of frequency bands useful to wireless communication are under control of primary license holders that have exclusive right to transmit over their spectral bands. This is the point that has created the perception of spectrum shortage, leading to ever-growing complains about available spectral resources. On the other hand, recent studies such as [1], [2] show that at any given time, large portions of the licensed bands remain unused and so, it is expected that we can improve the data transmission strategies by better utilizing the licensed resources. Spectrum sharing network is one of the most promising techniques created for this purpose.

Generally, the goal of a spectrum sharing scheme is to better utilize the radio spectrum by allowing the unlicensed secondary users (SU's) to coexist with the licensed primary users (PU's). Along with the standard interference channel [3]–[5], where independent transmitters send independent messages to independent receivers, there are other ways such as interference-avoiding and simultaneous transmission schemes to exploit the idea of spectrum sharing. The interference-avoiding paradigm [6]–[8] refers to an approach where the SU transmitter, provided that it can sense the spatial, temporal or spectral gaps of the PU resources, can adjust its transmission parameters to fill these white spaces. Although this scheme can theoretically lead to significant spectral efficiency improvement, it suffers from some practical drawbacks mainly related to imperfect gap detection. In the simultaneous transmission approach, on the other hand, a secondary user can simultaneously coexist with a primary user as long as it works below a certain interference level imposed by the primary user quality-of-service requirements [9], [10]. In such methods, the limits on the interference level received at the PU receiver, normally denoted interference temperature, can be considered to be long-term average or short-term peak constraints.

Assuming different levels of channel state information (CSI), several results about the performance limits of spectrum sharing networks have been presented recently. For instance, considering different primary or secondary user power constraints, [11]–[14] investigated the secondary user channel capacity under full CSI assumption. These works were later extended by [15]–[17] where the secondary channel performance was analyzed under different SU transmitter knowledge imperfection conditions. The gain estimation of SU-SU and PU-SU channels, however, is not easy for the SU receiver, as there is always unknown interferences created by the primary transmitter. Moreover, even if there is (im)perfect CSI at the SU receiver, it may not be convenient to provide the transmitter with the same information, as it leads to impractical feedback signaling overhead [18]–[21]. Therefore, it is important to study the channel performance when the fading channels are unknown both by the transmitters and the receivers.

In this perspective, this paper presents some bounds of secondary channel capacity in the case where none of the fading channels are known by the secondary user. The results are obtained under SU input power, different PU received interference power and also PU peak and average signal-to-interference-and-noise (SINR) constraints. As seen in the following, there is considerable potential for data transmission of unlicensed users even with no information about the channels quality.

The rest of the paper is organized as follows. System model is illustrated in section II. Then, the theoretical results are presented in section III. Section IV consists of simulation results and finally, the last section concludes the paper.

II. System model

As illustrated in Fig.1, we consider a standard spectrum sharing network where two primary and secondary users share the same narrow-band frequency with bandwidth B. With no loss of generality we set $B = 1$. Let g_{pp}, g_{ps}, g_{sp} and g_{ss} be the instantaneous channel gains of PU-PU, PU-SU, SU-PU and

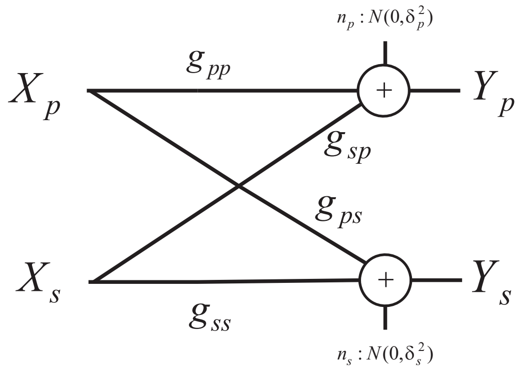

Figure 1. Channel model. The channels share the same narrow-band frequency with bandwidth B.

SU-SU links, respectively, which are assumed to be mutually independent. The white Gaussian noises added at PU and SU receivers, which are denoted by n_p and n_s, are supposed to have distributions $\mathcal{N}(0, \delta_\mathrm{p}^2)$ and $\mathcal{N}(0, \delta_\mathrm{s}^2)$, respectively. In this way, the channel outputs can be stated as

$$\begin{cases} Y_\mathrm{p} = X_\mathrm{p} g_\mathrm{pp} + X_\mathrm{s} g_\mathrm{sp} + n_\mathrm{p} \\ Y_\mathrm{s} = X_\mathrm{s} g_\mathrm{ss} + X_\mathrm{p} g_\mathrm{ps} + n_\mathrm{s} \end{cases}, \; EX_\mathrm{p}^2 = T_\mathrm{p},\; EX_\mathrm{s}^2 = T_\mathrm{s} \quad (1)$$

in which X_p and X_s represent the primary and secondary users input messages having powers T_p and T_s, respectively, and Y_p and Y_s denote their corresponding outputs. Finally, we focus on *Heavy Traffic* systems where there is an infinite amount of information to be transmitted by both users making the communication *continuous* [22].

III. Theoretical results

With no information about the fading channels, the SU-SU channel capacity can be represented as

$$C_\mathrm{s} = \max_{f_{X_\mathrm{s}}(x)} I(X_\mathrm{s}; Y_\mathrm{s}) = \max_{f_{X_\mathrm{s}}(x)} \{h(Y_\mathrm{s}) - h(Y_\mathrm{s}|X_\mathrm{s})\} \quad (2)$$

where the maximization is done with respect to SU input probability density function (pdf) $f_{X_\mathrm{s}}(x)$. Here, $I(U;V)$ denotes the mutual information between two random variables U and V and $h(u) = -\int_{-\infty}^{\infty} f_U(u)\log(f_U(u))\mathrm{d}u$ is the differential entropy of the variable U having pdf $f_U(u)$ [23]. As we know, due to the fading distributions, we can not necessarily obtain the optimal SU input distribution. Also, even if we know the distribution, whether the differential entropies $h(Y_\mathrm{s})$ and $h(Y_\mathrm{s}|X_\mathrm{s})$ can be calculated depends on the input and output pdfs. Therefore, we find a lower bound of secondary channel capacity as follows.

Selecting the secondary input distribution to be zero-mean Gaussian of power T_s (which is not necessarily the optimal one maximizing the mutual information), we can write[1]

$$\begin{aligned} C_\mathrm{s} &= \max_{f_{X_\mathrm{s}}(x)} I(X_\mathrm{s}; Y_\mathrm{s}) = h(X_\mathrm{s}) - h(X_\mathrm{s}|Y_\mathrm{s}) \\ &\overset{(a)}{\geq} \frac{1}{2}\ln 2\pi e T_\mathrm{s} - h(X_\mathrm{s}|Y_\mathrm{s}) \\ &\overset{(b)}{=} \frac{1}{2}\ln 2\pi e T_\mathrm{s} - h(X_\mathrm{s} - \alpha Y_\mathrm{s}|Y_\mathrm{s}) \\ &\overset{(c)}{\geq} \frac{1}{2}\ln 2\pi e T_\mathrm{s} - h(X_\mathrm{s} - \alpha Y_\mathrm{s}) \\ &\overset{(d)}{\geq} \frac{1}{2}\ln 2\pi e T_\mathrm{s} - \frac{1}{2}\ln 2\pi e \delta^2_{X_\mathrm{s} - \alpha Y_\mathrm{s}}. \end{aligned} \quad (3)$$

Here, (a)-(d) follow from the facts that

(a): considering the nonoptimal Gaussian input distribution, we have $h(X_\mathrm{s}) = \frac{1}{2}\ln 2\pi e T_\mathrm{s}$,

(b): adding a known random variable does not change the conditional differential entropy,

(c): conditioning reduces the differential entropy, and

(d): for a fixed power $E\{(X_\mathrm{s} - \alpha Y_\mathrm{s})^2\} = \delta^2_{X_\mathrm{s} - \alpha Y_\mathrm{s}}$, Gaussian distribution maximizes the differential entropy $h(X_\mathrm{s} - \alpha Y_\mathrm{s})$.

Since (3) is valid for any known value of α, we can select it such that αY_s becomes the linear minimum mean square error (LMMSE) estimate of X_s in terms of Y_s. Therefore, since X_s, X_p, g_ss, g_ps and n_s are independent and X_s and n_s are zero mean, we have

$$\begin{aligned} \alpha &= \frac{EX_\mathrm{s}Y_\mathrm{s}}{EY_\mathrm{s}^2} = \frac{E\{X_\mathrm{s}(X_\mathrm{s}g_\mathrm{ss} + X_\mathrm{p}g_\mathrm{ps} + n_\mathrm{s})\}}{E\{(X_\mathrm{s}g_\mathrm{ss} + X_\mathrm{p}g_\mathrm{ps} + n_\mathrm{s})^2\}} \\ &= \frac{T_\mathrm{s}\mu_\mathrm{ss}}{T_\mathrm{s}Eg_\mathrm{ss}^2 + T_\mathrm{p}Eg_\mathrm{ps}^2 + \delta_\mathrm{s}^2}, \; \mu_\mathrm{ss} \doteq Eg_\mathrm{ss} \end{aligned} \quad (4)$$

and so,

$$\begin{aligned} \delta^2_{X_\mathrm{s} - \alpha Y_\mathrm{s}} &= EX_\mathrm{s}^2 + \alpha^2 EY_\mathrm{s}^2 - 2\alpha EX_\mathrm{s}Y_\mathrm{s} \overset{\alpha = \frac{EX_\mathrm{s}Y_\mathrm{s}}{EY_\mathrm{s}^2}}{=} T_\mathrm{s} - \frac{(EX_\mathrm{s}Y_\mathrm{s})^2}{EY_\mathrm{s}^2} \\ &= \frac{T_\mathrm{s}^2\delta_\mathrm{ss}^2 + T_\mathrm{s}T_\mathrm{p}Eg_\mathrm{ps}^2 + T_\mathrm{s}\delta_\mathrm{s}^2}{T_\mathrm{s}Eg_\mathrm{ss}^2 + T_\mathrm{p}Eg_\mathrm{ps}^2 + \delta_\mathrm{s}^2}, \; \delta_\mathrm{ss}^2 \doteq Eg_\mathrm{ss}^2 - \mu_\mathrm{ss}^2. \end{aligned} \quad (5)$$

According to (3) and (5), the secondary channel capacity is lower bounded by

$$C_\mathrm{s} \geq \frac{1}{2}\ln(1 + \frac{T_\mathrm{s}\mu_\mathrm{ss}^2}{T_\mathrm{s}\delta_\mathrm{ss}^2 + T_\mathrm{p}Eg_\mathrm{ps}^2 + \delta_\mathrm{s}^2}). \quad (6)$$

Finally, note that using the LMMSE estimate of αY_s, the right-hand side of (3) has been maximized over α. While (6) represents the lower bound of secondary channel capacity under limited SU input power conditions, it is studied under other constraints in the following.

A. Primary user received interference power constraint

Provided that the secondary user is transmitting at power T_s, the PU instantaneous received interference power is found as

[1] All results are presented in natural logarithm basis.

$\varphi_{\rm p}=g_{\rm sp}^2T_{\rm s}$. Therefore, assuming that the PU average received interference power is limited to β implies that

$$E\varphi_{\rm p}=E\{g_{\rm sp}^2T_{\rm s}\}\le\beta\Rightarrow T_{\rm s}\le\frac{\beta}{Eg_{\rm sp}^2}. \tag{7}$$

As a more realistic constraint, we can consider the case where the PU instantaneous received interference power is with probability P less than some value β. In this case, we have

$$\text{Prob}\{\varphi_{\rm p}\le\beta\}=\text{Prob}\{g_{\rm sp}\le\sqrt{\frac{\beta}{T_{\rm s}}}\}=F_{g_{\rm sp}}(\sqrt{\frac{\beta}{T_{\rm s}}}) \tag{8}$$

where $F_{g_{\rm sp}}(.)$ is the SU-PU channel gain cumulative distribution function (cdf). Therefore, defining $F_{g_{\rm sp}}^{-1}(.)$ as the inverse function of the SU-PU channel gain cdf, the secondary user transmission power is found as

$$T_{\rm s}\le\frac{\beta}{[F_{g_{\rm sp}}^{-1}(P)]^2}. \tag{9}$$

Finally, it is worth noting that assuming Rayleigh SU-PU gain pdf $f_{g_{\rm sp}}(x)=\frac{x}{\lambda_{\rm sp}^2}e^{-\frac{x^2}{2\lambda_{\rm sp}^2}}, x\ge0$, (9) is simplified to

$$T_{\rm s}=-\frac{\beta}{2\lambda_{\rm sp}^2\ln(1-P)}. \tag{10}$$

B. Primary user received SINR constraint

The primary user received SINR is a random variable given by

$$\Omega_{\rm p}=\frac{T_{\rm p}g_{\rm pp}^2}{T_{\rm s}g_{\rm sp}^2+\delta_{\rm p}^2} \tag{11}$$

and so, its cdf is found as

$$\begin{aligned}F_{\Omega_{\rm p}}(x)&=\text{Prob}\{\frac{T_{\rm p}g_{\rm pp}^2}{T_{\rm s}g_{\rm sp}^2+\delta_{\rm p}^2}\le x\}\\&=\int_0^\infty f_{g_{\rm sp}}(y)\text{Prob}\{g_{\rm pp}\le\sqrt{\frac{x}{T_{\rm p}}(T_{\rm s}y^2+\delta_{\rm p}^2)}\}\mathrm{d}y\\&=E_{g_{\rm sp}}\{F_{g_{\rm pp}}(\sqrt{\frac{x}{T_{\rm p}}(T_{\rm s}g_{\rm sp}^2+\delta_{\rm p}^2)})\}\end{aligned} \tag{12}$$

where $F_{g_{\rm pp}}(.)$ is the PU-PU gain cdf and $E_{g_{\rm sp}}$ represents the expectation with respect to SU-PU link fading distribution $f_{g_{\rm sp}}(y)$. Note that, considering Rayleigh PU-PU and SU-PU gain distributions, (12) leads to

$$\begin{aligned}F_{\Omega_{\rm p}}(x)&=\int_0^\infty\frac{y}{\lambda_{\rm sp}^2}e^{-\frac{y^2}{2\lambda_{\rm sp}^2}}(1-e^{-\frac{x}{2\lambda_{\rm pp}^2T_{\rm p}}(T_{\rm s}y^2+\delta_{\rm p}^2)})\mathrm{d}y\\&=1-\frac{e^{-\frac{\delta_{\rm p}^2}{2\lambda_{\rm pp}^2T_{\rm p}}x}}{1+\frac{T_{\rm s}\lambda_{\rm sp}^2}{\lambda_{\rm pp}^2T_P}x}\end{aligned} \tag{13}$$

and

$$\begin{aligned}E\Omega_{\rm p}&=\int_0^\infty xf_{\Omega_{\rm p}}(x)\mathrm{d}x\\&\overset{(a)}{=}\int_0^\infty(1-F_{\Omega_{\rm p}}(x))\mathrm{d}x=\frac{\lambda_{\rm pp}^2T_{\rm p}}{T_{\rm s}\lambda_{\rm sp}^2}e^{\frac{\delta_{\rm p}^2}{2T_{\rm s}\lambda_{\rm sp}^2}}\text{Ei}(-\frac{\delta_{\rm p}^2}{2T_{\rm s}\lambda_{\rm sp}^2}).\end{aligned} \tag{14}$$

Here, (a) is found by partial integration, Ei(.) is the standard exponential integral function and $\lambda_{\rm pp}$ and $\lambda_{\rm sp}$ denote the Rayleigh pdf parameters normally determined by the path loss and shadowing between the terminals.

Consequently, the SU transmission power guaranteeing the primary user average received SINR to be higher than a value θ is found as the numerical solution of equation

$$T_{\rm s}\le\max\{0,\arg_{T_{\rm s}}(\frac{\lambda_{\rm pp}^2T_{\rm p}}{T_{\rm s}\lambda_{\rm sp}^2}e^{\frac{\delta_{\rm p}^2}{2T_{\rm s}\lambda_{\rm sp}^2}}\text{Ei}(-\frac{\delta_{\rm p}^2}{2T_{\rm s}\lambda_{\rm sp}^2})=\theta)\}. \tag{15}$$

Finally, the primary user instantaneous received SINR is another quality-of-service requirement which may be imposed to the secondary transmitter. In this way, constraining that the PU received SINR is with probability P higher than θ, the secondary user input power is found as

$$\begin{aligned}&\text{Prob}\{\Omega\ge\theta\}=P\Rightarrow\frac{e^{-\frac{\delta_{\rm p}^2}{2\lambda_{\rm pp}^2T_{\rm p}}\theta}}{1+\frac{T_{\rm s}\lambda_{\rm sp}^2}{\lambda_{\rm pp}^2T_{\rm p}}\theta}=P\\&\qquad\Rightarrow T_{\rm s}\le\max\{0,\frac{\lambda_{\rm pp}^2T_{\rm p}}{\lambda_{\rm sp}^2\theta P}\{e^{-\frac{\delta_{\rm p}^2}{2\lambda_{\rm pp}^2T_{\rm p}}\theta}-P\}\}.\end{aligned} \tag{16}$$

Implementing different power constraints, the next section studies the SU-SU channel capacity bounds in more details.

IV. Simulation results

In all simulations, the Rayleigh pdf parameters and the AWGN variances are set to 1. Considering different primary user input powers, Fig.2 studies the channel capacity bounds in different primary user received interference or SINR conditions. Here, the probability parameter P is selected to be 0.8. Then, Fig.3a demonstrates the effect of PU input power on the SU-SU channel performance in the case where the PU instantaneous received interference is with probability $P=0.8$ less than β. Note that, with proper scaling, the figure also represents the results obtained under limited SU input power or PU average received interference power conditions. Moreover, the same results are obtained under limited primary user received SINR conditions, as illustrated in Fig.3b. Finally, Fig.4 verifies the effect of primary user tolerability, modeled by parameter P, on the performance of secondary channel under different constraints.

V. Conclusion

This paper studies the performance of spectrum sharing fading channels in the case where there is no information about the fading channels at the transmitters and receivers. A channel capacity lower bound is presented which is verified under different secondary user input power, different primary user received interference power and both peak and average primary user received SINR constraints. Simulation results show that:

- Although there is high potential for unlicensed secondary users data transmission under limited SU input power or

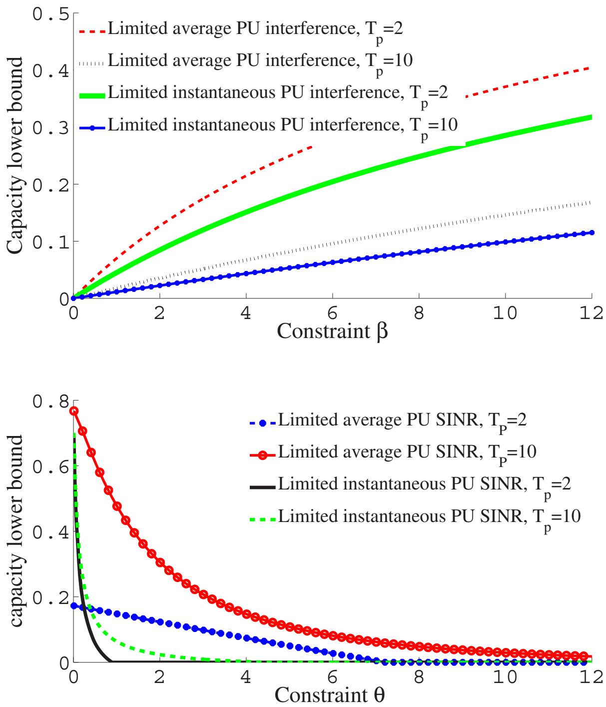

Figure 2. Secondary channel capacity lower bounds under different primary user quality-of-service constraints. (a): limited primary user received interference condition, (b): limited primary user received SINR condition. The probability parameter $P = 0.8$.

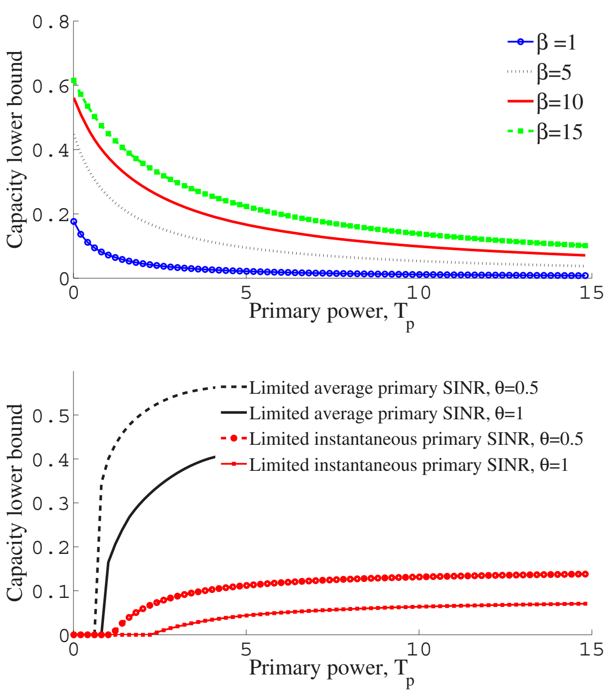

Figure 3. Capacity lower bounds vs primary user input power under (a): limited instantaneous received interference power, (b): limited primary user received SINR constraint. The probability parameter $P = 0.8$.

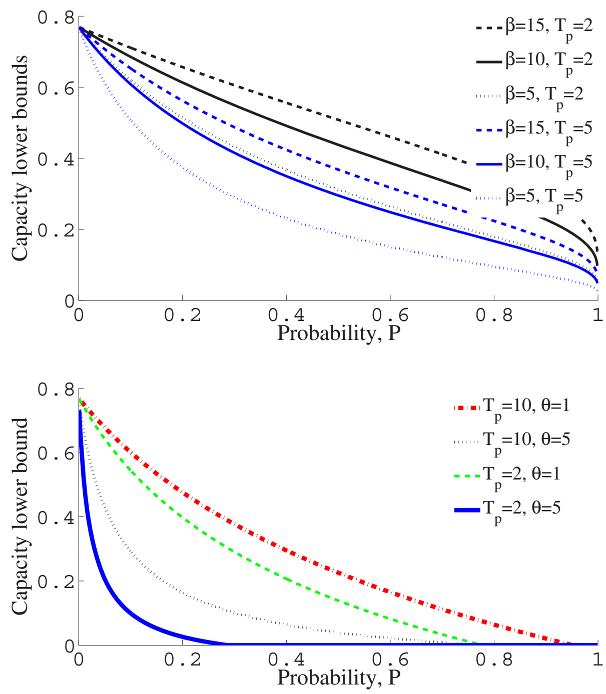

Figure 4. Capacity lower bounds vs probability constraint P. (a): limited primary user instantaneous received interference condition, (b): limited primary user instantaneous received SINR condition.

PU received interference power conditions, the achievable rates decreases drastically as the licensed users input power increases (Fig.2a and 3a).

- The harder the licensed user received SINR constraint is, the less rates can be achieved by the unlicensed users converging to zero (Fig.2b). Moreover, as illustrated by Fig.3b, although spectrum sharing is not permitted by primary users transmitting at low powers, considerable rates are obtained by secondary users as the primary user input power increases.
- Under both limited primary user received interference and SINR conditions, the intolerability of primary user, modeled by probability parameter P, plays a great role in the secondary channel achievable rates. That is, the more secure the primary user instantaneous quality-of-service requirements should be satisfied, the less rate is achieved at the secondary channel, converging to zero (Fig.4).

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