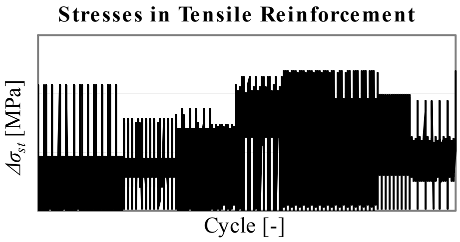


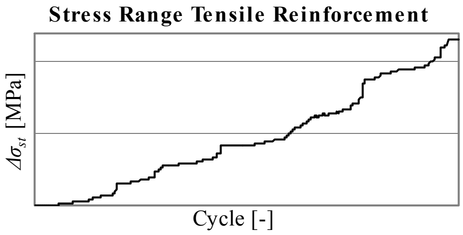


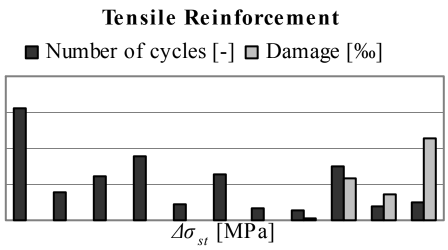


# Fatigue Assessment Methods for Reinforced Concrete Bridges in Eurocode

## Comparative study of design methods for railway bridges

*Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design*

KARIN OLSSON, JOSEF PETTERSSON

Department of Civil and Environmental Engineering
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*Concrete Structures*
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2010
Master's Thesis 2010:100

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Cover:
The figures shows a serie of stress range obtained due to loading with fatigue load models in three different ways. The series represent the loading during one day.

Chalmers Reproservice / Department of Civil and Environmental Engineering
Göteborg, Sweden 2010

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ABSTRACT

At the present day the European Standards, Eurocode, are introduced as the new reference design codes in the field of construction. One issue which is treated by Eurocode is the assessment of the fatigue life of structures. Fatigue failure is characterized by a fracture in a local area of a structure which is subjected to varying cyclic loading. This loading can be caused by traffic, wind, ocean waves or likewise. The fatigue life of a reinforced concrete structure depends as much on the stress levels as on the stress range and the number of loading cycles and their importance is related to which material that is considered.

The purpose of this study is to compare the methods for fatigue assessment available in Eurocode. The aim is to see how the methods correspond to each other and how the results are affected by different parameters. This is done by performing parametric studies on reinforced concrete bridges and evaluating the results.

In Eurocode there are two alternative methods by which fatigue in reinforced concrete can be calculated, the Cumulative Damage Method, and the $\lambda$-Coefficient Method. Both methods consider the loading during the lifetime of a structure but in different manners. The Cumulative Damage Method calculates a fatigue damage factor which expresses the actual damage occurred in the structure in relation to the design fatigue life. The $\lambda$-Coefficient Method simply checks if the structure fulfils the demands for a given service life.

In order to use the two methods a large amount of input data is needed such as the bridge geometry, material properties and the loading on the bridge. The loading includes both permanent loads, long term parts of variable loads and short term traffic loads inducing fatigue.

Parametric studies mainly regarding the bridge span with its influencing factors has been performed and the behavior of the bridges analyzed. Some conclusions regarding the comparability of the methods and the outcome of the results are made. One conclusion is that the number of load cycles is not a leading factor governing the result. Another is that the Cumulative Damage Method is very sensitive to small adjustments in the sectional design.

Key Words: Eurocode, reinforced concrete, fatigue assessment, fatigue load models, Cumulative Damage Method, $\lambda$-Coefficient Method.

Beräkningsmetoder gällande utmattning i armerade betongbroar enligt Eurocode
Jämförande studie av beräkningsmetoder
Examensarbete inom Structural Engineering and Building Performance Design
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SAMMANFATTNING

För närvarande är de gemensamma europeiska beräkningsstandarderna, Eurokod, introducerade som gällande normverk inom byggbranschen. Ett område som behandlas inom ramen för koderna är utvärderingen av en konstruktions livslängd med avseende på utmattning. Utmattningsbrott kan beskrivas som ett lokalt brott i en byggnadsdel utsatt för cyklisk last. En cyklisk last kan orsakas av väg- och järnvägstrafik, vind, vågor och liknande. Livslängden med avseende på utmattning i en armerad betongkonstruktion beror lika mycket på medelnivån för spänningarna som på amplituden och antalet lastcykler.

Syftet med denna studie är att jämföra de två metoder för beräkning av utmattningsbrott som finns till förfogande i Eurokoderna. Detta är uppnått genom att ett antal parameterstudier är genomförda och utvärderade på olika modeller av armerade betongbroar. I Eurokoderna finns det som tidigare nämnts två metoder för att beräkna utmattning i armerade betongkonstruktioner och dessa är Delskademetoden och $\lambda$-Koefficientmetoden. Båda metoderna tar hänsyn till belastningen under brons livstid varvid det antingen beräknas en delskada som beskriver kvarvarande livslängd på konstruktionen (Delskademetoden), eller där det helt enkelt kontrolleras om konstruktionen uppfyller kraven för utmattningsbrott ($\lambda$-Koefficientmetoden).

I beräkningarna för de båda metoderna så krävs en mängd indata såsom brons geometri, materialparametrar och belastningen av bron. Lastkombinationerna inkluderar både lång- och korttidslast samt cyklisk trafiklast. Snittkrafterna som orsakas av dessa laster måste kombineras och beräknas om till spänningar. Dessa spänningar används sedan för att beräkna utmattningen i kritiska snitt enligt de båda metoderna.

Parameterstudier gällande broarnas spannlängd och av dessa beroende faktorer har genomförts och konstruktionernas respons har analyserats. Av resultaten från studierna har ett antal slutsatser kunnat dras. Ett exempel är att antalet lastcykler inte är en av de direkt avgörande faktorerna när det gäller utmattning enligt Delskademetoden. En annan är att känsligheten för förändringar i tvärsnittsdimensioneringen hos Delskademetoden är väldigt stor.

Nyckelord: Eurokod, armerad betong, Delskademetoden, $\lambda$-Koefficientmetoden utmattningsanalys, utmattningslaster.

# Contents

# Preface

The first calculations performed with the fatigue assessment methods according to Eurocode showed some inconsistent or unexpected results. Therefore Trafikverket initiated this study in order to deepen the knowledge about the characteristics of the methods. This master’s thesis project was carried out in cooperation with NCC Teknik, Sweco Infrastructure and the Division of Structural Engineering, Department of Civil and Environmental Engineering, Chalmers University of Technology during the spring 2010.

Our first thanks are aimed for our supervisor at NCC Teknik, Ph.D. Jonas Magnusson. He has been helpful during the entire project with providing information and knowledge regarding the subject and acting as a sounding board with ideas and theories.

We would also direct our gratitude to the group consisting of experienced officials from Trafikverket, Elisabeth Helsing, Ebbe Rosell and Lennart Askling, our supervisor at Sweco Infrastructure, bridge consultant Helmer Palmgren, our supervisor and examiner at Chalmers University of Technology, Senior lecturer Rasmus Rempling and Professor Björn Engström respectively, for their participation and guidance how to handle different issues throughout the project.

Furthermore, we would like to thank the developer of the program AFB, Thomas Petersson, for giving us access to, and also giving us help handling the software.

At last we would like to thank our opponents, Per Lindberg and Jonas Nilsson for their helpful comments and feedback during the project.

Göteborg August 2010

Karin Olsson & Josef Pettersson

# Notations

**Roman upper case letters**

| | |
|---|---|
| $A_{cc}$ | is the cross-sectional area of compressive concrete zone |
| $A_s$ | is the cross-sectional area of the compressive reinforcement |
| $A_s^{'}$ | is the cross-sectional area of the tensile reinforcement |
| $A_{II,ef}$ | is the effective transformed concrete area, in state II |
| $D$ | is the damage obtained for stress range |
| $"D_c"$ | is the value of the design criteria for compressed concrete obtained by the $\lambda$ -Coefficient Method |
| $"D_\lambda"$ | is the value of the design criteria for reinforcing steel obtained by the λ-Coefficient Method |
| $E_{cd,\max,equ}$ | is the damage equivalent stress spectrum upper stress level |
| $E_{cd,\min,equ}$ | is the damage equivalent stress spectrum lower stress level |
| $E_{cd,\max,i}$ | is the maximum compressive stress level “i” |
| $E_{cd,\min,j}$ | is the minimum compressive stress level “j” |
| $E_{cm}$ | is the mean Young’s modulus of elasticity for concrete |
| $E_s$ | is the Young’s modulus of elasticity for steel |
| $F_{cs}$ | is the shrinkage force |
| $G_{k,j}$ | is the characteristic value for the permanent load “j” |
| $H$ | is the height of the cross section of the simply supported bridge model |
| $I_{cc}$ | is the moment of inertia |
| $I_{II,ef}$ | is the moment of inertia, in state II’ |
| $L$ | is the span |
| $L$ | is the determinant length $L_\Phi$ |
| $L_\Phi$ | is the determinant length |
| $N^*$ | is a reference amount of cycles until failure depending on which type of reinforcing steel which is verified |
| $N_i$ | is the ultimate number of constant amplitude cycles in interval “i” that can be carried before failure |
| $N(\Delta\sigma_i)$ | is the resisting number of cycles for a stress range $\Delta\sigma_i$ |
| $M$ | is the sectional bending moment |
| $M_{Ed.f}$ | is the sectional bending moment for field section in ULS |
| $M_{Ed.s}$ | is the sectional bending moment for support section in ULS |
| $M_f$ | is the maximum sectional bending moment for field section induced by traffic load, LM71 |
| $M_{i,j}$ | is the sectional force in the j:th calculation section at the i:th load movement |
| $M_{Perm.f}$ | is the sectional bending moment field section induced by permanent loads |

| | |
|---|---|
| $M_{Perm.s}$ | is the sectional bending moment for support section induced by permanent loads |
| $M_s$ | is the maximum sectional bending moment for support section induced by traffic load, LM71 |
| $M_{0Ed}$ | is the first order bending moment in design load combination (ULS) |
| $M_{0,Eqp}$ | is the first order bending moment in quasi-permanent load combination |
| $P$ | is the relevant value of the prestressing force |
| $P_k$ | is the characteristic value of the prestressing force |
| $P_n$ | is the n:th load acting on the beam in the current load configuration |
| $Q_{k,1}$ | is the characteristic value for the variable main load 1 |
| $Q_{fat}$ | is the relevant fatigue load |
| $Q_{k,j}$ | is the characteristic value for the variable main load ”j” |
| $R_{equ}$ | is the damage equivalent stress spectrum ratio |
| $R_i$ | is the stress ratio “i” |

**Roman lower case letters**

| | |
|---|---|
| $d$ | is the distance from outermost compressed concrete fibre to the level of analysis in the concrete cross-section |
| $d'$ | is the distance from outermost tensile concrete fibre to the level of analysis in the concrete cross-section |
| $f_{cd}$ | is the design compressive concrete strength in [MPa] |
| $f_{cd,fat}$ | is the design concrete fatigue strength |
| $f_{ck}$ | is the characteristic compressive concrete strength in [MPa] |
| $h_l$ | is the height of the simply supported support of the Degerfors Bridge and the continuous bridge model |
| $h_r$ | is the height of the fully fixed support of the Degerfors Bridge and the continuous bridge model |
| $k_n$ | is the n:th influence value obtained from influence line at current position on the load $P_n$ |
| $k_1$ | is the slope of the S-N-line or while assessing reinforcement steel with the Cumulative Damage Method, a the slope of the S-N relation until $N^*$ |
| $k_1$ | is the a coefficient affecting the fatigue strength |
| $k_2$ | is the slope of the S-N-line or while assessing reinforcement steel with the Cumulative Damage Method, a the slope of the S-N relation until $N^*$ |
| $n_i$ | is the actual number of constant amplitude ”i” |
| $n(\Delta\sigma_i)$ | is the applied number of cycles for a stress range $\Delta\sigma_i$ |
| $s$ | is a coefficient which depend on the type of cement |
| $t_0$ | is the time of first cyclic load application in days |
| $v$ | is the maximum permitted train speed |
| $x_1$ | is the length to the beginning of the haunch of the Degerfors Bridge |

| | |
|---|---|
| $x_2$ | is the length of the haunch of the Degerfors Bridge |
| $\bar{x}_{cc}$ | is the distance from the compressive edge to the gravity centre of the compressive zone |
| $\bar{x}_{II,ef}$ | is the distance from the compressive edge to gravity centre of the transformed effective concrete section |
| $z$ | is the distance to neutral axis |

**Greek upper case letters**

| | |
|---|---|
| $\Delta\sigma_i$ | is the reference normal stress range |
| $\Delta\sigma_{s,71}$ | is the steel stress range due to load model 71 |
| $\Delta\sigma_{s,equ}$ | is the equivalent stress range in the reinforcement corresponding to $n$ cycles |
| $\Delta\sigma_{s,equ}(N^*)$ | is the equivalent stress range obtained according the λ-Coefficient Method |
| $\Delta\sigma_{Rsk}$ | is the ,while assessing reinforcement steel with the Cumulative Damage Method, the reference resisting stress depending on which type of steel which is verified |
| $\Delta\sigma_{Rsk}(N^*)$ | is the resisting stress range at $N^*$ cycles |
| $\Phi_2$ | is the dynamic factor |

**Greek lower case letters**

| | |
|---|---|
| $\alpha_s$ | is the ratio between Young's modulus of reinforcement steel and concrete |
| $\alpha_{s,ef}$ | is the ratio between Young's modulus of reinforcement steel and concrete (sustained loading) |
| $\beta_{cc}(t_0)$ | is a coefficient for concrete strength at first load application |
| $\varepsilon_{cs}(t)$ | is the final shrinkage strain, including drying shrinkage and autogenous shrinkage strain at time t |
| $\gamma_d$ | is the partial coefficient taking the risk of injuries into account |
| $\gamma_{F,fat}$ | is the partial factor taking material uncertainties into account |
| $\gamma_G$ | is the partial coefficient multiplied with the self-weight |
| $\gamma_{S,fat}$ | is the partial factor taking the uncertainties in the fatigue load model |
| $\gamma_Q$ | is the partial coefficient multiplied with the variable load |
| $\lambda_c$ | is the correction factor to establish the upper and lower compressive stress from the damage equivalent stress spectrum caused by application of load model 71 |

| | |
|---|---|
| $\lambda_{c,0}$ | is a factor who takes into account of permanent stress |
| $\lambda_{c,1}$ | is a factor accounting for element type that take into account the damaging effect of traffic depending on the critical length of the influence line or area |
| $\lambda_{c,2,3}$ | is a factor to take account of the traffic volume and design life |
| $\lambda_{c,4}$ | is a factor to be applied when the structural element is loaded by more than one track |
| $\lambda_{s}$ | is the correction factor to establish the stress from the damage equivalent stress spectrum caused by application of load model 71 |
| $\lambda_{s,1}$ | is a factor witch is a function of critical length of influence line and traffic |
| $\lambda_{s,2}$ | is a factor witch value denotes the influence of the annual traffic volume |
| $\lambda_{s,3}$ | is a factor witch denotes the influence of service life |
| $\lambda_{s,4}$ | is a factor witch values denotes the effect of loading from more than one track |
| $\sigma_{cc}$ | is the concrete stress at level of the compressive steel |
| $\sigma_{cd,\max,equ}$ | is the upper stress in the damage equivalent stress spectrum |
| $\sigma_{cd,\min,equ}$ | is the lower stress in the damage equivalent stress spectrum |
| $\sigma_{cd,\max,i}$ | is the upper stress in a cycle “i” |
| $\sigma_{cd,\min,j}$ | is the lower stress in a cycle “j” |
| $\sigma_{ct}$ | is the concrete stress at the level of the tensile reinforcement |
| $\sigma_{c,perm}$ | is the compressive concrete stress caused by characteristic load combination without the variable loads |
| $\sigma_{c,\min,71}$ | is the minimum compressive stress caused by the characteristic load combination including load model 71 |
| $\sigma_{c,\max,71}$ | is the maximum compressive stress caused by the characteristic load combination including load model 71 |
| $\sigma_{c,II,ef}$ | is the concrete stress in the effective transformed section |
| $\sigma_{sc,II}$ | is the stress in the reinforcement in the compressive level |
| $\sigma_{st,II}$ | is the stress in the reinforcement in the tensile level |
| $\varphi_{ef}$ | is the effective creep coefficient |
| $\varphi(\infty,t_0)$ | is the final creep coefficient (quasi-permanent load) |
| $\psi_{0,i}$ | is the combination factor for the variable load ”i” |
| $\psi_{1,1}$ | is the factor for the frequent value for the main variable load 1 |
| $\psi_{2,i}$ | is the factor for the quasi-permanent value of the secondary variable load 1 |

# 1 Introduction

At the present day the European Standards, Eurocode, are introduced as the new reference design codes in the field of construction. The purpose of Eurocode is to harmonize the technical rules for European engineers and contractors in order to simplify the cooperation within the construction sector. It also aims to widen the knowledge among engineers and thereby increase the quality of structural design.

One issue which is treated by Eurocode is the assessment of the fatigue life of structures. Fatigue failure is characterized by a fracture in a local spot of a structure which is subjected to varying cyclic loading. This loading can be caused by traffic, wind, ocean waves or likewise. The fatigue life of a reinforced concrete structure depends as much on the stress levels as on the stress range and the number of loading cycles and their importance is related to which material that is considered. A fatigue failure can occur at stresses well below the critical stress level in the Ultimate Limit State (ULS).

## 1.1 Problem description

In Eurocode there are two alternative methods by which fatigue can be calculated for bridges, the $\lambda$-Coefficient Method and the Cumulative Damage Method. Both methods consider the loading during the lifetime of a structure. The

$\lambda$-Coefficient Method is a simplified method with a single load model amplified with a number of coefficients. The Cumulative Damage Method is a complex model which considers the load history more deeply. The $\lambda$-Coefficient Method simply checks if the structure fulfils the demands given in the codes while the Cumulative Damage Method calculates a fatigue damage factor which expresses the actual damage occurred in the structure in relation to the design fatigue life.

In order for the different actors in the field of construction to learn how the Eurocodes are to be used, and to determine which problems that can occur during the design process; Banverket[1] and Vägverket[2] initiated a study in which a number of companies were given a bridge to design according to Eurocodes. Their work was then presented together with their experiences from the design process. These reports showed on a number of issues which were troublesome during the process. One of those issues was the method of evaluating the fatigue life.

As a result of the study, a master's thesis, Fall and Petersson (2009), was performed which treated the fatigue assessment methods for bridges according to Eurodode. This thesis mainly handled fatigue in steel, both for road and railway bridges. It also treated fatigue in reinforced concrete, but in a simplified way and only for road traffic. An analysis was made for both the tensile and compressive reinforcement. The compressed concrete however, could only be assessed by the Cumulative Damage Method and not by the $\lambda$-Coefficient Method. The evaluation of the results in the study showed that the methods gave contradictory results regarding the fatigue damage. This was valid for both steel and concrete bridges. A parameter that seemed to be decisive was the bridge spans and especially there were inconsistencies observed between the two design methods for short spans.

[1] The Swedish Rail Administration
[2] The Swedish Road Administration

Since two thirds of the existing bridges has a span of less than 10 meters, where the majority is concrete bridges, Trafikverket[3] whishes to further examine the fatigue assessment methods in Eurocode. In the previously mentioned thesis, the assessment of steel bridges was performed thoroughly and that part of the investigation is considered to be completed. For reinforced concrete bridges a more deepened study was still desired which was the reason for the present thesis project.

## 1.2 Purpose

The purpose of the thesis is:

- to study the methods of fatigue assessment for reinforced concrete bridges according to Eurocode, the Cumulative Damage Method and the $\lambda$-Coefficient Method.
- to perform parametric studies of bridges regarding fatigue assessment.
- to present, explain and evaluate the results from the studies in order to see if any conclusions can be made, or if any recommendations can be given regarding the usage of the methods in design or assessment situations.

## 1.3 Method

In order to get the proper understanding of how concrete, reinforcing steel, and finally reinforced concrete structures, behave under cyclic loading, studies of existing literature was carried out. Also a thorough review of the fatigue assessment methods available in Eurocode was performed.

From the theoretical studies a method to select design sectional forces using existing software's was developed. These forces were used to design the considered critical sections in the ULS and to calculate the sectional stresses due to loading from certain fatigue load models. The obtained stresses were used to assess the fatigue life of the actual structure according to the methods presented in Eurocode.

The entire calculation procedure was then used to perform parametric studies on a number of fictitious bridges. The properties of these fictitious bridges were developed with an existing bridge as a reference. The results from the parametric studies were evaluated regarding the influence of certain parameters.

## 1.4 Scope and limitations

The project was limited to treat fatigue failure in reinforced concrete bridges. The bridges should be solely horizontal and straight slab bridges which could be calculated as a 4.5 meter wide strip. Structures with prestressed reinforcement were not considered.

Further the studies were limited to handle railway bridges in case of pure bending. Some studies have been performed regarding the assessment of fatigue due to shear; these are presented as informative material in Appendix J-K.

In order to simplify and emphasize the fatigue assessment, the load combinations used in this project only includes, except the fatigue traffic loads, the self-weight of the structure, ballast and rail.

[3] The Swedish Transport Administration

# 2 Fatigue failure in reinforced concrete structures

Due to the fact that structures are becoming more slender, the traffic volume is increasing, the axel loads are larger, and the traffic speed limits are higher; the interest of fatigue in concrete structures has increased during the last few years. Concrete fatigue is mainly a problem of offshore structures, railway sleepers and bridges because these types of structures are often exposed to repeated loading. This project is focused on reinforced concrete railway bridges. With increased axel loads the condition for the bridges has changed and many existing bridges are nowadays required to carry larger loads than what they where originally designed for.

This chapter begins with a summary of the fatigue phenomenons for reinforced concrete, plain concrete and reinforcing steel. It will treat the behaviour under fatigue loading and the failure characteristics in tension, compression and shear with regard to fatigue.

## 2.1 Basis of fatigue

Fatigue is a phenomenon where a material loses its original strength due to cyclic loading with successive damage development. Fatigue in concrete depends on the load amplitude and the number of cycles as well as on the stress level. For steel, the amplitude and number of cycles cause the fatigue. The failure can occur even if the maximum stress is below the ordinary strength of the material. Some materials have a certain stress limit e.g. steel, which means that the stress variation below a certain level can be repeated infinitely many times without fatigue failure. Fatigue failure is characterized by fracture in a localized area of a structure which is exposed to cyclic loading.

When a structure fails due to fatigue loading, the structure has reached its fatigue life. There are two types of fatigue loading that can result in different failure characteristics. They are called Low-cycle fatigue and High-cycle fatigue. Low-cycle fatigue means that the load is applied at high stress levels for a relatively low number of cycles, while the High-cycles fatigue corresponds to a large number cycles at lower stresses.

## 2.2 Fatigue in reinforced concrete

Since reinforced concrete is a composite material, a structure built in reinforced concrete can fail from fatigue in several different ways. Failure is often a consequence of many factors and the failure modes can have significantly different characteristics. Local failure can occur in the concrete, in the reinforcement and in the bond between the materials. Compressive fatigue failure in reinforced concrete can be described as ductile, since cracks in the concrete can develop considerably before the structure fails. The tensile fatigue failure in reinforced concrete has a more brittle behaviour since the crack propagation rate in the reinforcement at the end is rather rapid, Elfgren and Gylltoft (1977). The different modes of failure in reinforced concrete structures can be divided into sub-groups depending on their appearance and they are described in the following sections.

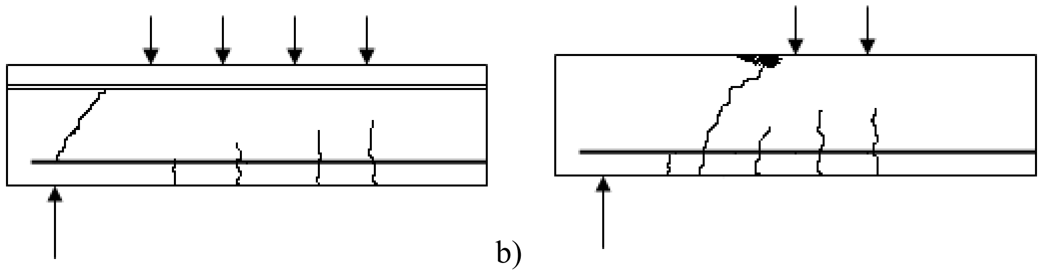
### 2.2.1 Compression and bending failures

One group of fatigue failures is constituted by compression and/or bending failures. Tensile failure due to bending occurs in the reinforcement and this is valid especially for an under-reinforced cracked cross-section. For a normal- or over-reinforced section the situation is much more complex. The compressive failure might take place in the concrete, but it can also be influenced by effects between the compressive reinforcement and the concrete. The latter due to different deformations in the steel and concrete at the same load level, causing transverse tensile stresses in the concrete which leads to unfavourable cracking in the compressed zone, Elfgren and Gylltoft (1993). The pure compressive and tensile failures in the respective materials are further dealt with later in this chapter.

### 2.2.2 Shear and bond failures

The next group of failures is shear and bond failures. The fatigue resistance for these cases is, relatively to the static resistance, sometimes very low, about 40-60%; and therefore it is very important to consider this in design. The shear fatigue failure is highly dependent on if the beam is provided with shear reinforcement, or not. In total the fatigue shear resistance is higher with shear reinforcement than without, but it is dependent on the different types of fatigue loading explained in Section 2.1.

Beams without shear reinforcement have two different modes in which shear fatigue failure can occur. Either the beam can fail when a diagonal crack has propagated across the entire section, or by crushing in the concrete in the compressive zone above the shear crack, Figure 2.1. When the beam is provided with shear reinforcement it can fail in four different ways. They are fatigue in the shear reinforcement, fatigue in the longitudinal reinforcement where it is crossed by a shear crack, fatigue in the compressed concrete above the shear crack and fatigue in the compressed concrete in the web, see Figure 2.2.



a) b)

*Figure 2.1 Possible shear fatigue failure modes in beams without stirrups: a) excessive development of diagonal cracking b) fatigue of concrete in compression above the shear crack.*

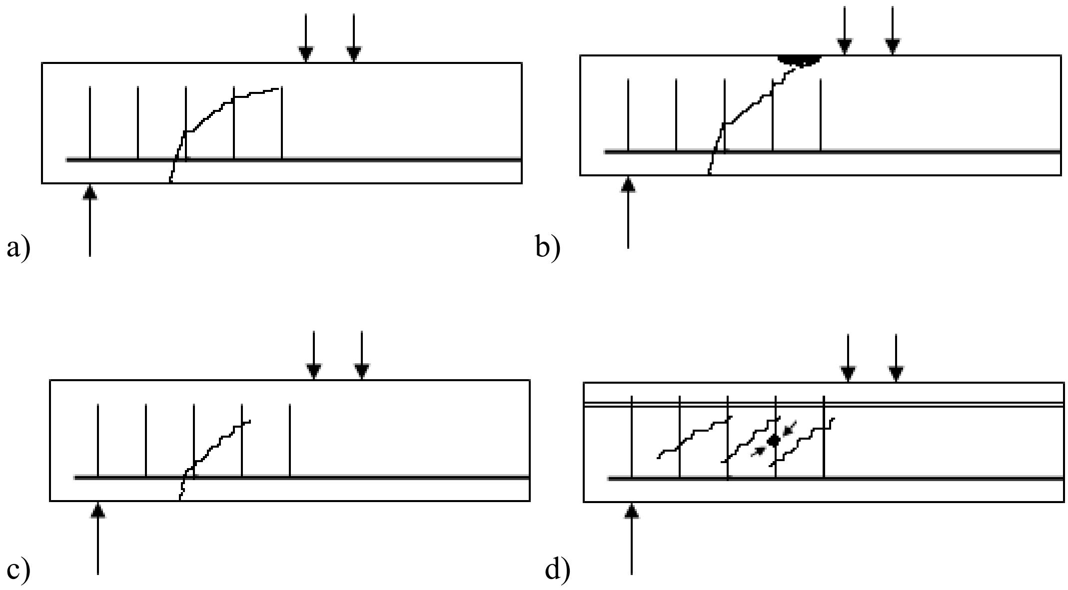



*Figure 2.2 Possible shear fatigue failure modes in beams with shear reinforcement: a) fatigue of the stirrups, b) fatigue of the concrete in compression above the shear crack, c) fatigue of the longitudinal reinforcement crossing the shear crack, d) fatigue of the concrete in compression in the web.*

The bond strength between concrete and reinforcement is dependent on several different factors e.g. type of reinforcement bars, concrete cover, bar spacing, and the state of stresses in the section. One failure mode is break down of pullout bond resistance of the bar; another is caused by splitting of the surrounding concrete, CEB (1988).

## 2.3 Fatigue of concrete

Fatigue in concrete was recognized rather late, in comparison to steel. Concrete is a non-homogenous material and its fatigue resistance is influenced by many different factors e.g. moisture content, cement/water ratio and load effects such as load frequency and maximum load level. During the hardening period air bubbles and micro-cracks are formed. The micro-cracks appear due to thermal strain, which is caused by temperature variations. When the micro-cracks propagate the fatigue process starts, which is a progressive process. At the beginning of the loading the propagation of the micro-cracks is rather slow. As loading continues the micro-cracks will proceed propagate and lead to macro-cracks, which may grow further. The macro-cracks determine the remaining fatigue life caused by stress until failure occurs.

## 2.4 Fatigue of reinforcement

Reinforcement bars can have many different types of surfaces; they can be plain, have ribs or be indented. The purpose of having ribbed or indented bars is to increase the mechanical interaction between the steel and the concrete. Ribbed and indented bars give increased stress concentrations in comparison to plain bars; these stress concentrations reduce the resistance to fatigue and therefore the shaping of the ribs is important, e.g. the transition between the bars and the rib has to be smooth.

Differently from concrete the reinforcing steel has a stress limit; this means that the stress variation below a certain level can be repeated infinitely many times without causing any fatigue damage. This is only possible if the material shows plastic behaviour.

There are many different parameters that determine the fatigue life of the reinforcement. Some parameters that affect the fatigue life are e.g. the stress variation, the surface of the bar and the nominal area/dimension of the bar. Which one that is governing is hard to determine and researchers have come to diverging conclusions regarding which parameter that affects the fatigue strength more than others. In general it is likely to be a combination of several factors.

# 3 Design procedure for a concrete slab bridge

In this chapter the design procedure for a concrete slab bridge will be presented. The calculation programs used in order to simplify the design procedure are presented together with the calculation of sectional stresses considering both short and long term loading.

## 3.1 Preliminary design

Preliminary design of a concrete slab bridge is made to estimate the needed amount of reinforcement. The reinforcement amount is then used for the fatigue assessment calculation when the parametric study is carried out. The preliminary design is based on an existing bridge called Degerfors Bridge which was chosen after discussion with Palmgren[4]. It is a railway bridge and it was considered to be a case simple enough to design and assess within the scope of this project. The Degerfors Bridge was originally designed in the 50's and from drawings of the bridge, the geometrical properties including the reinforcement amount where determined. These where used for verification of the calculations and as guidance when the principles of design where defined for the bridge models used in the parametric study.

The width of the slab was set to 4.5 meters, which is a standard choice with regard to the load spread from one track. To determine a more accurate load spread an additional advanced analysis would have been needed. This was however omitted in this project. The span was initially set to seven meters and the slab was modelled as one end fully fixed and the other end simply supported. By this it was possible to simulate a continuous bridge with an intermediate support region. Further, it was designed with a haunch on the fully fixed side in order to increase the moment capacity here, see Figure 3.1. The effect on the linear elastic moment distribution of the haunch compared to a slab with constant height can be seen in Figure 3.2.

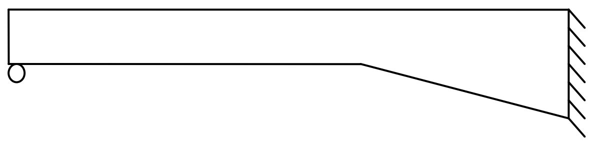


*Figure 3.1 The assumed model of the Degerfors Bridge.*

The material used for concrete and reinforcement in the original design of the bridge, had to be translated into current standards in order to achieve representative values of the material properties. This was done after discussion with Palmgren. The material properties which are commonly used for bridge design today should however be used in the design of the bridge models used for the fatigue assessment.

[4] Helmer Palmgren, supervisor, Sweco Infrastructure, meeting 2010-03-11

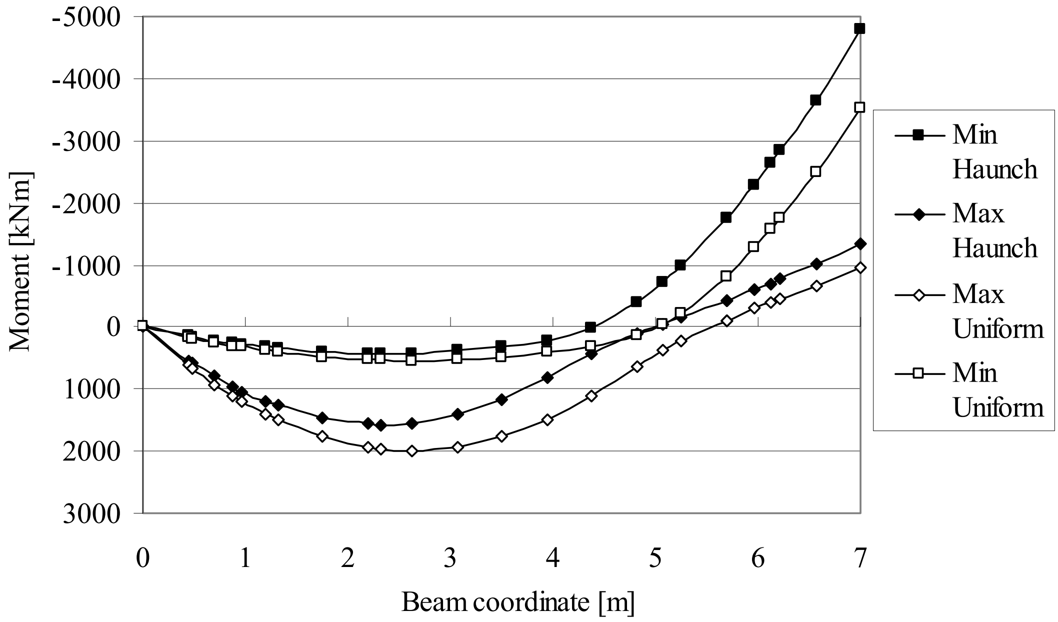



*Figure 3.2* *Change of the moment distribution due the effect of the haunch at the right side of the slab according to linear elastic analysis.*

The sectional forces were determined according to linear elastic analysis using the calculation program StripStep2 aimed for structural analysis. The geometrical and material properties and design load characteristics were inserted as input data into the program. The use of StripStep2 is further described in Section 3.2. The sectional forces are needed to determine the amount of longitudinal, transversal and shear reinforcement.

The bridge design used in the project should be done according to Eurocode and the aim of this was to calculate the amount of reinforcement needed in the considered sections. The design calculations of the bridge were verified in order to be able to use the results in the continued work. The verification was made by comparing the calculated moments and reaction forces from StripStep2 with simple hand calculations presented in Table 3.1. The load applied on the bridge while performing the hand calculations was solely the self-weight; this in order to perform simple hand calculation and to verify the design calculations. Loads applied on the bridge used in design calculations are the self-weight of the structure, including the permanent installations, and the traffic load for ULS design (LM71). Calculations used in reinforcement design with the geometrical properties and load characteristics can be seen in Appendix G.

When performing hand calculations it is complicated to take a haunch into consideration. The calculations were therefore based on a beam with constant stiffness and the results will consequently differ in a certain manner from the results achieved by StripStep2. The support moment in the hand calculations is somewhat smaller than the corresponding moment achieved in StripStep2. Correspondingly the field moment is higher. The difference is related to the assumptions regarding the stiffness distribution, as shown in Figure 3.1, a stiffer region at the fixed end due to the haunch. As the calculations shows, higher stiffness attracts more moment. In the same way the support reaction forces differ since the load divider is moved slightly towards the simply supported end. The sum of the reaction forces is however the same regardless of the support conditions.

*Table 3.1 Results from the verification of the structural analysis, Degerfors Bridge, 7 meter span.*

| *Verification of structural analysis* | | |
|---|---|---|
| | *Hand Calculation* | *StripStep2* |
| | *Moment [kNm]* | |
| *Field section* | 400 | 314 |
| *Support section* | 910 | 984 |
| | *Reaction Force [kN]* | |
| *Right support* | 577 | 582 |
| *Left support* | 276 | 270 |
| *Σ Reaction forces* | 852 | 852 |

## 3.2 Structural analysis with StripStep2

StripStep2 is a program used when performing linear elastic structural analysis of plane beam, frame and truss structures. In this project the program was used to obtain sectional forces, which were caused by permanent and traffic loads. It also determines influence lines, which were used to perform the fatigue calculations.

### 3.2.1 Sectional forces

Sectional forces were calculated in StripStep2 for the permanent load and load combinations in the Serviceability Limit State (SLS) and the Ultimate Limit State (ULS). The load combinations are further explained in Chapter 4. When performing calculations in the ULS both the permanent and traffic loads are taken into account in a design load combination. The traffic load is modelled as a moving load which is placed in different positions on the carriageway. These results in a force envelope with the maximum and minimum value of the sectional force in the considered structural part, see Figure 3.3. This force envelope was used to calculate the reinforcement required in the maximum moment sections.

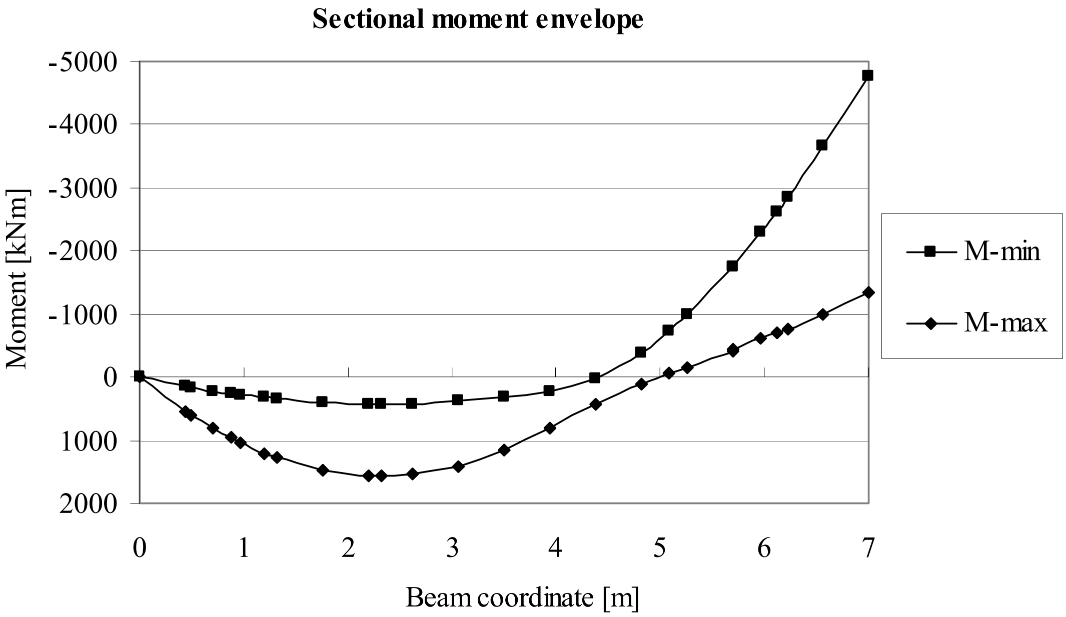



*Figure 3.3 Sectional moment envelopes for ULS design load combination used in the sectional design calculations.*

### 3.2.2 Influence lines

Influence lines can be obtained for bending moments, shear forces, normal forces and reaction forces. In this project the influence lines for bending moments are treated. Influence lines calculated depend on the length of the beam, boundary conditions and stiffness distribution along the beam. In order to continue with the fatigue assessment, an influence line was established by moving a concentrated force in steps across the considered carriageway. The influence line consists of a length coordinate dependent value, called k-value. When it is multiplied with the applied load it gives the force in the considered section. In Figure 3.4 an example of influence lines obtained for a bridge used in this thesis is presented.

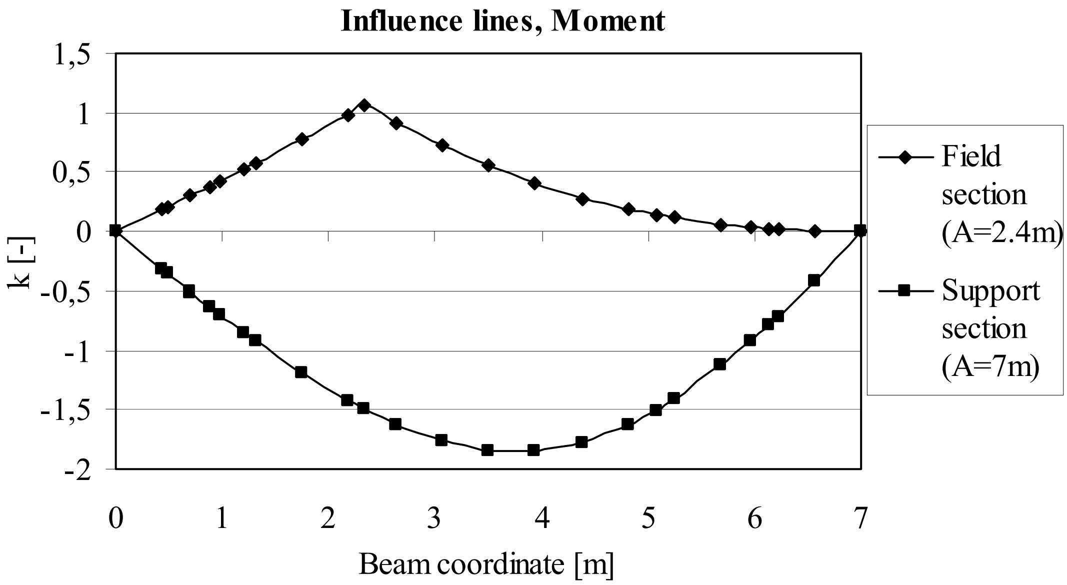



*Figure 3.4 Influence lines for maximum moment sections for a continuous bridge model, span 7 meters.*

To be able to describe this method when establishing influence lines, a simple example will be used. A simply supported single span beam is considered and the beam has a constant stiffness along the entire length. In order to obtain the moment influence line for section *A*, see Figure 3.5, a concentrated force *P* is moved from one end to the other on the beam. The figure shows how the moment in point *A* varies when the load is moved along the beam.

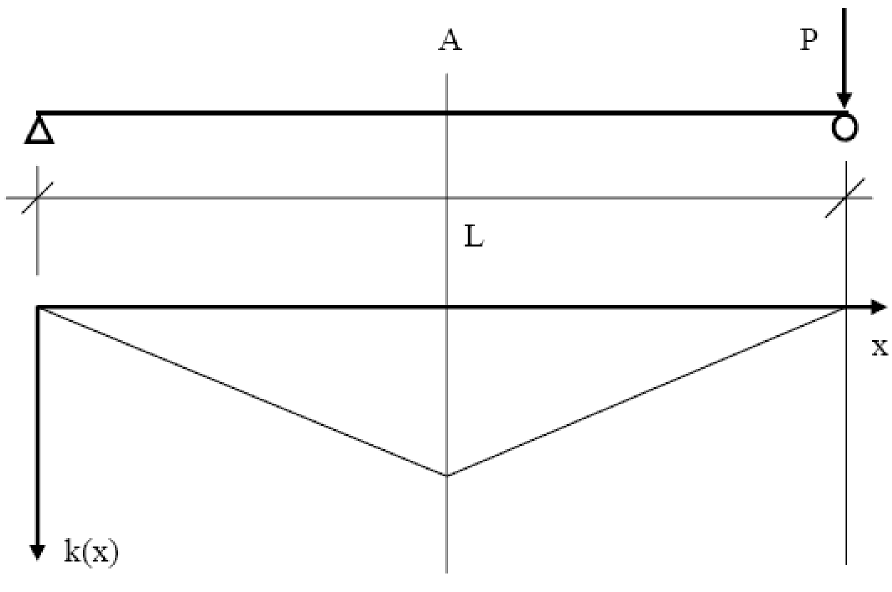



*Figure 3.5 Moment influence line for a simply supported beam*

When a load is moving along the considered carriageway, the sectional moment can be calculated by means of the influence line obtained from StripStep2. The value *k* from the influence line is multiplied with the considered load *P* from a certain coordinate according to equation (3.1). If there are several loads on the structure a summation can be performed, with the forces calculated with same methodology as described before.

$$M_{i,j} = \sum_{n=1}^{m} P_n \cdot k_n \tag{3.1}$$

where $M_{i,j}$ is the sectional moment in the j:th section at the i:th load position,

$P_n$ is the n:th load acting on the beam in the current load configuration,

$k_n$ is the n:th influence value obtained from the influence line at the current position of the $P_n$ load,

$m$ is the total number of loads acting on the beam at the i:th load position.

Furthermore for the $\lambda$-Coefficient Method is the critical length of the influence line needed. The critical length of the influence line is set to the span of the bridge model considered according to EN 1993-2: SIS (2006).

## 3.3 Calculation of sectional forces due to fatigue loading with the program AFB

AFB, which is an acronym for Assessment of Fatigue for Bridges, is a calculation program developed by Fall and Petersson (2009). It was developed in order to simplify their work with fatigue assessment of bridges. With AFB, sectional forces for both the Cumulative Damage Method and the $\lambda$-Coefficient Method can be calculated.

For the Cumulative Damage Method the value of the mean sectional force and the force amplitude is determined for a certain load case in a specific section. Sections of interest can be maximum moment sections and sections that should be checked for shear. The entire series of sectional forces is determined for the time period of one day, by running a number of different train types across the carriageway. An example of such series of forces is shown in Figure 3.6 and consists of approximately 27 000 load steps. Another example is shown in Figure 3.7 which shows a passage of a single train and is the same as the first 260 load steps in Figure 3.6. The series are recalculated within the program into loading cycles and corresponding sectional forces by the Rainflow Cycle Counting Method as seen in Figure 3.6 and are later used in the fatigue calculations.

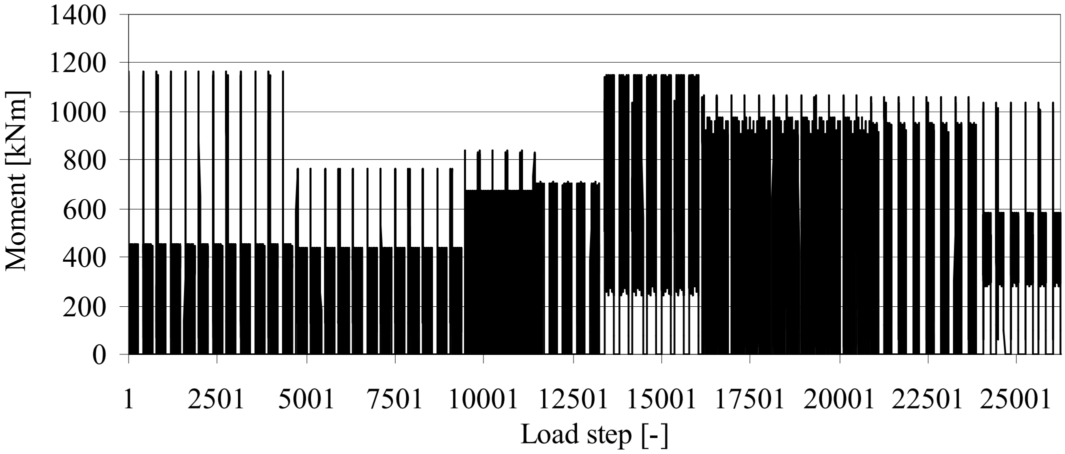



*Figure 3.6* *Result from AFB for the Cumulative Damage Method in a specific section. The sectional moments obtained are from a time period of one day.*

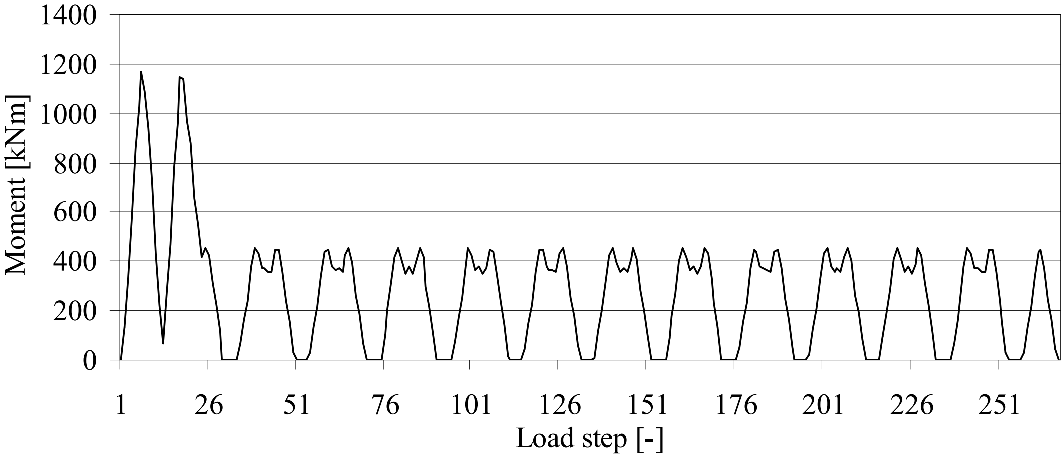



*Figure 3.7* *Result from AFB for the Cumulative Damage Method in a specific section. The sectional moments are obtained for one train passage.*

An example of the obtained sectional moment in the field section for the $\lambda$-Coefficient Method from AFB is shown in Figure 3.8. For the $\lambda$-Coefficient Method solely one load model, LM71, see Section 4.1.1, is applied on the bridge with a single passage. When observing Figure 3.8 at load step 1 the bridge is unloaded. At load step 9 the entire bridge is loaded with a uniformly distributed load. The concentrated forces are evenly distributed in the middle of the bridge and the uniformed distributed loads are placed in the beginning and end of the bridge at load step 15, how this loading appears can be seen in Figure 3.8.

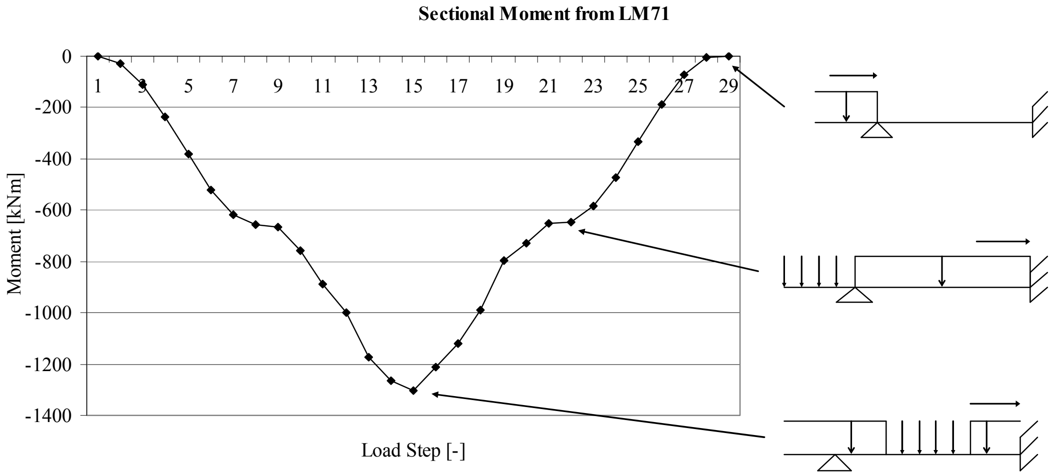



*Figure 3.8 Sectional moment in the field section from AFB for the λ-Coefficient Method.*

In order to run calculations with AFB some choices needs to be made; e.g. type of traffic on the bridge, load model as presented in Chapter 4 and effective span. Further, in order to run this program an influence line is needed. The influence line is used in AFB and depends on which type of result that is requested. This means that if the moment in a certain section is wanted, an influence line for this particular section is required. Further information about the calculations performed by AFB can be found in Fall and Petersson (2009).

## 3.4 Sectional stresses

This section treats the method for calculating the sectional stresses that develop due to the different types of loading on the structure. In order to calculate the stresses in a section, the forces due to both long term and short term loads need to be distinguished. In this project is the long term loads set as the permanent loads and the short term loads as the traffic loads. This is due to the creep effects caused by the sustained loads and not by short term loads. If the creep would be disregarded, the forces could simply be super imposed.

However, when combining the different loads an effective creep coefficient can be calculated according to EN 1992-1-1: SIS (2005). This method is aimed for determining second order effects in structural members subjected to axial loads. On the other hand, according to Engström (2008), this method of combining the loads is said to be reasonable also in case of pure bending in general. An investigation of the method of calculating the effective creep coefficient was performed and is presented in Appendix F. The purpose was to investigate which influence different approaches in calculating the effective creep coefficient had on the results. As an example, by equation (3.2), the effective creep coefficient is calculated for the moments in a section.

$$\varphi_{ef} = \frac{\varphi(\infty, t_0) \cdot M_{perm}}{M_{perm} + M_{fatigue}} \tag{3.2}$$

where $\varphi(\infty, t_0)$ is the final creep coefficient for the permanent load,

$M_{perm}$ is the first order bending moment of the permanent load,

$M_{fatigue}$ is the first order bending moment of the fatigue load.

When the neutral axis, moment of inertia and finally the stresses in a section is to be calculated, a modular ratio is needed. The ratio, or further on called the $\alpha_{s,ef}$-factor, determines the distribution of stresses between the concrete and the reinforcing steel. The $\alpha_{s,ef}$-factor is calculated with regard to the $\varphi_{ef}$-coefficient and the Young's modulus of the materials:

$$\alpha_{s,ef} = \frac{E_s}{E_{cm}}(1 + \varphi_{ef}) \tag{3.3}$$

where $E_s$ is the characteristic modulus of elasticity for steel,

$E_{cm}$ is the mean modulus of elasticity for concrete.

The centroid of the effective transformed concrete section in the cracked state II can be found with area balance and is calculated according to:

$$\bar{x}_{II,ef} = \frac{A_{cc}\bar{x}_{cc} + (\alpha_{s,ef} - 1)A_s^{'}d^{'} + \alpha_{s,ef}A_s d}{A_{II,ef}} \tag{3.4}$$

where $A_{cc}$ is the area of the compressive zone,

$\bar{x}_{cc}$ is the distance from the compressive edge to the centroid of the compressive zone,

$A_s'$ is the area of the tensile reinforcement,

$A_s$ is the area of the compressive reinforcement,

$d'$ is the distance from the compressed edge to the tensile reinforcement,

$d$ is the distance from the compressed edge to the compressive reinforcement,

$A_{II,ef}$ is the area of the effective transformed concrete section in state II.

From the obtained sectional centroid and the area of the transformed concrete section, the moment of inertia can be calculated as:

$$I_{II,ef} = I_{cc} + A_{cc}(\bar{x}_{II,ef} - \bar{x}_{cc})^2 + (\alpha_{s,ef} - 1)A_s^{'}(\bar{x}_{II,ef} - d^{'})^2 + \alpha_{s,ef}A_s(d - \bar{x}_{II,ef})^2 \tag{3.5}$$

where $I_{cc}$ is the moment of inertia of the compressive zone,

The steel and concrete are interacting fully and Navier's formula can be used to calculate the stresses in the effective transformed concrete section. When the stress in a section is determined, the moment caused by the permanent load and the moments caused by the fatigue load are combined and the stress is calculated as:

$$\sigma_{c,II,ef}(z) = \frac{M}{I_{II,ef}} z \tag{3.6}$$

where $M$ is the bending moment caused by the permanent load combined with the current traffic load,

$z$ is the sectional coordinate from the centre of gravity.

In order to determine the steel stresses in the considered section, the concrete stress at the same level is multiplied with the effective modular ratio.

$$\sigma_{sc,II} = \alpha_{s,ef} \sigma_{cc} \tag{3.7}$$

$$\sigma_{st,II} = \alpha_{s,ef} \sigma_{ct} \tag{3.8}$$

where $\sigma_{cc}$ is the concrete stress at the level of the compressive reinforcement,

$\sigma_{ct}$ is the concrete stress at the level of the tensile reinforcement.

### 3.4.1 Shrinkage

When considering the long term loading, sustained loading, the creep deformation is normally associated with shrinkage of the concrete. Reinforced concrete is a composite material. Before the two materials are cast together the steel and concrete are acting separately. The concrete is free for deformation without any restraint from the steel. When the concrete is newly cast the concrete and the steel are both unloaded. With time the concrete hardens and it will start to shrink. The reinforcement will become loaded in compression and an internal restraint force, the shrinkage force, develops which can be determined according to equation (3.9).

$$F_{cs} = E_s \varepsilon_{cs}(t) A_s \tag{3.9}$$

$\varepsilon_{cs}(t)$ is the shrinkage strain, including drying shrinkage and autogenous shrinkage strain, at time $t$, according to EN 1992-1-1: SIS (2005)

The steel struggles to return to its original length and the concrete become loaded in tension. At his stage are the steel and concrete are interacting fully. However, the contribution from shrinkage forces compared to the permanent load is relatively small. Furthermore, if the shrinkage force can be assumed to act symmetrically on the cross section the effect of these additional loading only results in a higher, or lower depending on which material that is considered, level of stress. The stress amplitudes from the fatigue loading will not be affected. The higher stress level could cause additional damage in the fatigue calculations for concrete but consideration of shrinkage was omitted in this project.

# 4 Fatigue load models, load effect and fatigue load combinations

When a structural engineer deigns a new bridge he or she primarily takes into consideration the behaviour in Ultimate Limit State (ULS) and Serviceability Limit State (SLS). As a part of the verification in ULS the bridge should be checked for the possibility of fatigue failure. In order to do this the loading history that can be expected during the structure's lifetime must be simulated. This is done by applying the different fatigue load models that are available in Eurocode. If the purpose is to assess the remaining service life of an existing structure the actual load history may be used if it is known.

In this chapter the different actions used for fatigue verification are described. The fatigue load models and the traffic mixes used by the two fatigue verification methods available in Eurocode are presented. Furthermore the permanent loads and other variable loads, and how they are combined in the different parts of fatigue verification are described. Also the combination factors used in the load combinations are presented.

## 4.1 Train load models for fatigue verification

The train load models are selected in order to represent the effects from the actual traffic as well as possible. In order to do this the models used for fatigue verification are in some cases different from the ones used for the structural verification.

### 4.1.1 Train load model used by the λ-Coefficient Method

When fatigue verification is performed according to the $\lambda$-Coefficient Method, described in EN 1992-2:2005 SIS (2005), only one load model is used. The model is LM71, presented in EN 1991-2:2003 SIS (2007), which represents the static effect of normal rail traffic. The load model is presented in Figure 4.1, and it includes composed of one segment that should characterize the vertical loading from a locomotive, which consists of four concentrated axle loads. The remaining part of the load is represented in the model by a uniformly distributed load. The segments, denoted by *(1)* in the figure, are considered to be infinite in their extension.

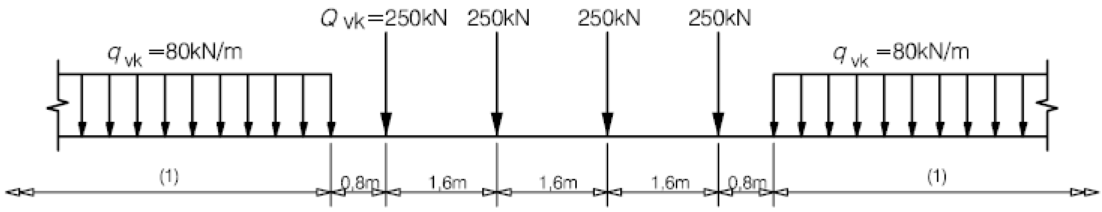



*Figure 4.1 Train load model, LM71.*

When the model is used by a structural calculation program, e.g. StripStep2, the calculation should start by applying the distributed traffic load, $q_{vk}$, over the entire length of the bridge. Then the locomotive segment, the middle part of Figure 4.1, is moved across the bridge in order to generate an envelope with the maximum and minimum values of the sectional forces.

### 4.1.2 Train load model used by the Cumulative Damage Method

As stated in the previous sections the Cumulative Damage Method is a complex method to handle and that is partly caused by the numerous train models and traffic mixes needed when using this. There are 12 train types described in EN 1991-2:2003 SIS (2007) and these train types are supposed to represent the different configurations of the actual trains running on regular railway lines. Examples of train types are: Locomotive-hauled passenger train, Locomotive-hauled freight train, High speed passenger train and Suburban multiple unit train. Two examples of train type configurations are shown in Figure 4.2 and Figure 4.3.

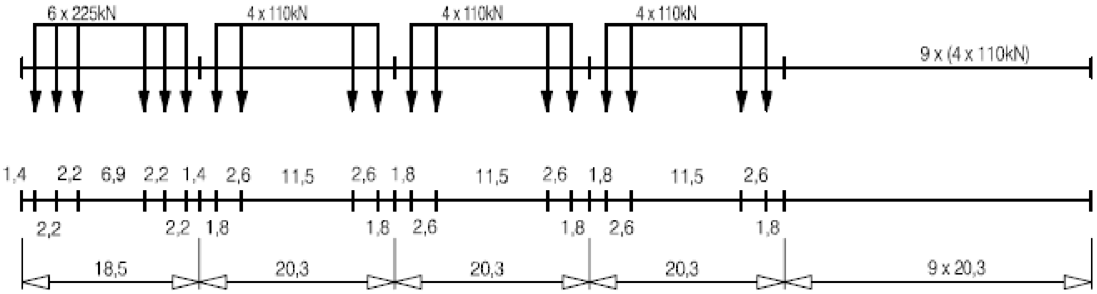



*Figure 4.2 Fatigue train type 1 – Locomotive-hauled passenger train.*

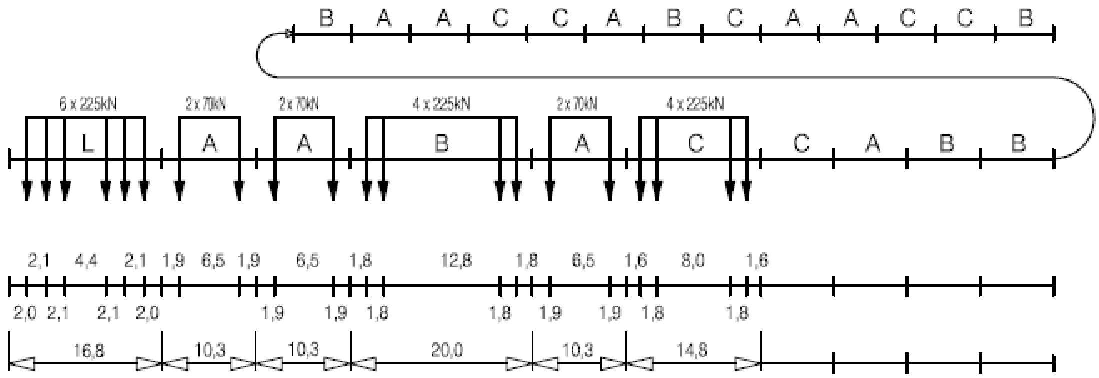



*Figure 4.3 Fatigue train type 6 – Locomotive-hauled freight train.*

The train types are assembled into three traffic mixes. These mixes correspond to the expected railway traffic on the considered line. The mixes are: Standard, Heavy, and Light traffic mix, defined in EN 1991-2:2003 SIS (2007). The traffic mix which was assumed in this project is the standard traffic mix which is shown in Table 4.1. The mixes are included as a part of the calculation program AFB.

*Table 4.1 Standard traffic mix with axle-loads ≤ 22.5 tons (225kN)*

| *Train type* | *Number of trains/day* | *Mass of train [tons]* | *Traffic volume [$10^6$ tons/year]* |
|---|---|---|---|
| 1 | 12 | 663 | 2.90 |
| 2 | 12 | 530 | 2.32 |
| 3 | 5 | 940 | 1.72 |
| 4 | 5 | 510 | 0.93 |
| 5 | 7 | 2160 | 5.52 |
| 6 | 12 | 1431 | 6.27 |
| 7 | 8 | 1035 | 3.02 |
| 8 | 6 | 1035 | 2.27 |
| | 67 | | 24.95 |

## 4.2 Other loads acting on the structure

There can be several different loads acting on a bridge at the same time. The most important and the ones affecting the fatigue verification, except the cyclic traffic load, are the permanent loads. Other loads such as snow, wind, water, temperature, and earth pressure should also be considered.

### 4.2.1 Permanent loads

The permanent loads are more or less constant during the service life of the bridge and do not affect the magnitude of the stress cycles in the fatigue calculation. However, the permanent load determines the persistent stress level in the structure, partly due to their long term effects.

Three permanent loads have been considered in this project. The first and main permanent load is the dead weight of the structure itself i.e. the reinforced concrete. The density of the concrete is set to 25 kN/m$^3$. The second load is the ballast that supports the track. The ballast consists of crushed rock and its thickness is prescribed to be at least 600 mm and the density is prescribed to 20 kN/m$^3$. The load part is the weight of the track running along the bridge. The weight of the rail is set to 0.6 kN/m.

Other permanent loads on the bridge are e.g. the weight of the posts and wires for the electricity, footbridges for inspection and railings. All these extra loads are relatively small and are therefore neglected in the calculations in the present work.

### 4.2.2 Other variable loads

There are other variable loads which must be considered in the design of a bridge e.g. snow and wind. However, in fatigue calculations, the non-cyclic, variable loads are relatively small and are in some cases not included at all. Also due to the fact that secondary loads often are markedly reduced in the design load combination with their combination factors, $\psi$ , these loads are ignored in the calculations in this project.

## 4.3 Additional combination factors for train loads

When the different actions on a structure are to be combined into a design load combination, there are a number of factors by which the actions should be multiplied. There are separate values for permanent, variable and fatigue loads.

### 4.3.1 Partial factors for Ultimate Limit State design and fatigue loads

The values of the partial factors are stated in BFS 2009:16, Boverket (2009). The partial factors for the loads take into account the uncertainties in the load models and the safety class of the structure. According to TK Bro (2009), Safety class 3 should be used for the verification of railway bridges in the ULS.

Partial factors for ULS, BFS 2009:16 Table A1.2(B)(S):

$\gamma_G = 1{,}35$ For permanent actions where $G$ represents self-weight, ballast, soil, removable loads etc.

$\gamma_Q = 1{,}5$ For variable actions where $Q$ represents unfavourable actions due to rail traffic.

Partial factor for fatigue loads are stated in EN 1992-1-1:2005 - 2.4.2.3 (1):

$\gamma_{F,fat} = 1{,}0$ Where $F$ represents fatigue actions.

Partial factor for Safety class 3, BFS 2009:16 - A §10-14:

$\gamma_d = 1{,}0$ Is value represents a high risk of serious injuries.

### 4.3.2 The classification factor

On tracks with traffic lighter or heavier than the standard case the load model needs to be modified with a classification factor denoted $\alpha$. When the load is multiplied with a classification factor it is called "a classified vertical load". In Sweden the value of the $\alpha$-factor is taken as 1.33 for all lines except for tracks with heavy freight traffic BFS 2009:16, Boverket (2009).

According to TK Bro (2009) some small adjustments should be made compared to the traffic mix used when the $\alpha$-factor is taken as 1.33.

### 4.3.3 Dynamic amplification factor for railway loads

The load used for fatigue verification should be amplified with a dynamic factor Φ. This is done in order to catch the vertical dynamic effects in a bridge structure created by a running train. The factor Φ is intended to be used for static load models such as LM71 and can be calculated in two different ways according to Eurocode. The choice is governed by the level of maintenance of the track, which can be either normally maintained or carefully maintained. According to BFS 6.4.5.2(3)P the track should be considered as carefully maintained and therefore the factor is taken as $\Phi_2$ according to equation (4.1) . The dynamic factor is used both for verification in the ULS and by the fatigue assessment methods.

$$1{,}0 \le \Phi_2 = \frac{1{,}44}{\sqrt{L_\Phi} - 0{,}2} + 0{,}82 \le 1{,}67 \tag{4.1}$$

where $L_\Phi$ is the "determinant" length (length associated with dynamic factor Φ) according to table 6.2 of EN 1991-2:2003

However, it is stated in EN 1991-2:2005 Annex D that this will give an excessive unrealistic amplification of the loads used when fatigue verification by the Cumulative Damage Method. The method uses real train models and Eurocode therefore suggests that a reduced dynamic factor should be used. This is done in order to consider the average effects on the structure over the entire service life. The reduction is calculated according to equation (4.2) and should be multiplied with the ordinary dynamic factor. In this project the reduced dynamic factor is considered in the program AFB when calculating the sectional forces. The program AFB is described in Section 3.3.

$$1+\tfrac{1}{2}(\varphi'+\tfrac{1}{2}\varphi'') \tag{4.2}$$

with $\varphi'=\dfrac{K}{1-K+K^4}$

$K=\dfrac{v}{160}$ for $L \le 20\,\text{m}$

$K=\dfrac{v}{47.16L^{0.408}}$ for $L > 20\,\text{m}$

and $\varphi''=0.56e^{\frac{L^2}{100}}$

where $v$ is the speed limit of the train.

$L$ is the "determinant" length $L_\Phi$ .

## 4.4 Combination of actions for fatigue verification

In Eurocode it is stated which load combinations to use while assessing a structure with regard to fatigue. It says that in order to calculate the stress ranges, the actions should be divided into non-cycling and fatigue-inducing cyclic actions. When this is done the basic fatigue load combination is expressed according to equation (4.3). This combination is more or less the frequent combination, normally used for verification in SLS, to which the cyclic actions are added. From here on is combination called the frequent combination.

This combination is used for all fatigue verifications with the Cumulative Damage Method and for road traffic with the $\lambda$-Coefficient Method. However, for railway traffic, with the $\lambda$-Coefficient Method, there are different combinations to use. For verification of the compressed concrete the characteristic combination, equation (4.4), should be used with the traffic load LM71. For verification of the reinforcement only load model LM71 together with the dynamic factor is used. In this case however, the $\alpha$ -factor should be excluded.

Frequent load combination: EN 1992-1-1:2005 – 6.8.3 (6.69)

$$\left(\sum_{j\ge1} G_{k,i} + P + \psi_{1,1}Q_{k,1} + \sum_{i>1}\psi_{2,i}Q_{k,i}\right) + Q_{fat} \tag{4.3}$$

Characteristic load combination: EN 1990:2002 – 6.5.3 (6.14b)

$$\left(\sum_{j\ge1} G_{k,i} + P_k + Q_{k,1} + \sum_{i>1}\psi_{0,i}Q_{k,i}\right) + Q_{fat} \tag{4.4}$$

where $G_{k,j}$ is the characteristic value for the permanent load $j$,

| | |
|---|---|
| $P$ | is the relevant value of the prestressing force, |
| $P_k$ | is the characteristic value the prestressing force, |
| $Q_{k,1}$ | is the characteristic value for the variable main load 1, |
| $Q_{k,j}$ | is the characteristic value for the variable load *j,* |
| $\psi_{0,i}$ | is the combination factor for the variable load *i,* |
| $\psi_{1,1}$ | is the factor for the frequent value for the main variable load 1, |
| $\psi_{2,i}$ | is the factor for the quasi-permanent value of the secondary variable loads *i,* |
| $Q_{fat}$ | is the relevant fatigue load. |

When the $\lambda$-Coefficient Method is used to verify the compressed concrete it includes two different cases of the Characteristic combination. The first case only takes the permanent loads into account in order to find the persistent stress level. The second case includes both the permanent load and LM71 which will give a lower and an upper value of the stresses when the variable load is moved along the bridge.

# 5 Fatigue verification calculations according to Eurocode

In this chapter is the method of verifying a concrete bridge loaded with train traffic for fatigue is presented. The calculation procedures for both reinforcing steel and compressed concrete are presented according to the methods available in Eurocode, the Cumulative Damage Method and the $\lambda$-Coefficient Method.

## 5.1 Methodology of the Cumulative Damage Method

The Cumulative Damage Method is a complex method which rigorously considers the load history of a bridge. This method can be used for reinforced concrete structures subjected to compression, bending and/or shear. It includes models for calculating the damage on compressed concrete, tensile and compressive reinforcement as well as on prestressing steel. For verification in the design phase the fatigue load models presented in Chapter 4 are to be used. The loads are applied on the bridge giving stresses calculated in the appropriate critical sections and from which the cumulative damage can be calculated.

### 5.1.1 Damage calculation procedure for compressed concrete

The Cumulative Damage Method, which is presented in EN1992-2: SIS (2005), uses the Palmgren-Miner's rule to calculate the damage on the structure. The rule, which is a damage summation, should fulfil the requirement defined as:

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \leq 1 \tag{5.1}$$

where $n_i$ is the actual number of constant amplitude cycles in interval *"i"*,

$N_i$ is the ultimate number of constant amplitude cycles in interval *"i"* that can be carried before failure.

$n_i$ is calculated in AFB, see Chapter 3.3. The ultimate number of constant amplitude cycles is determined as:

$$N_i = 10^{14 \cdot \frac{1-E_{cd,\max,i}}{\sqrt{1-R_i}}} \tag{5.2}$$

where $E_{cd,\max,i}$ is the maximum compressive stress level as defined in equation (5.5),

$R_i$ is the stress ratio as defined as:

$$R_i = \frac{E_{cd,\min,i}}{E_{cd,\max,i}} \tag{5.3}$$

where $E_{cd,\min,i}$ is the minimum compressive stress level as defined in equation (5.4).

$$E_{cd,\min,i} = \frac{\sigma_{cd,\min,i}}{f_{cd,fat}} \tag{5.4}$$

$$E_{cd,\max,i} = \frac{\sigma_{cd,\max,i}}{f_{cd,fat}} \tag{5.5}$$

where $\sigma_{cd,\min,i}$ is the lower stress in a cycle, calculated according to section 3.4,

$\sigma_{cd,\max,i}$ is the upper stress in a cycle, calculated according to section 3.4,

$f_{cd,fat}$ is the design fatigue compressive strength of concrete according to equation (5.6).

$$f_{cd,fat} = k_1 \cdot \beta_{cc}(t_0) \cdot f_{cd} \cdot \left(1 - \frac{f_{ck}}{250}\right) \tag{5.6}$$

where $k_1$ is a coefficient depending on reference number of cycles until failure for the damage equivalent stress spectrum with a recommended value of 1.0, which is accepted for use in Sweden by BFS 2009:16. The coefficient is set to 1.0 in this project,

$\beta_{cc}(t_o)$ is a coefficient for concrete compressive strength at first load application as defined in equation (5.7),

$f_{cd}$ is the design compressive concrete strength in [MPa],

$f_{ck}$ is the characteristic compressive concrete strength in [MPa].

The coefficient for concrete strength at first load application is taken according to 3.1.2 (6) of EN 1992-1-1:2005:

$$\beta_{cc} = e^{s \cdot \left(1 - \sqrt{\frac{28}{t_0}}\right)} \tag{5.7}$$

where $s$ is a coefficient which depends on the type of cement,

$t_0$ is the time of the start of the cyclic loading on concrete in days.

The time of the first cyclic load application is set to 28 days in this project. This is the usual time of demoulding and therefore the first time of load application. The cyclic load may not be applied at the same time but it is a fair assumption, also because of stricter schedules on construction sites which enforces earlier load application. The choice of cement type are CEM 32.5 R or CEM 42.5 N. Cement 42.5 N is preferred in more gross constructions with requirement on cautious heat development, possibility for alkali silicon acid reactions and with requirement on higher sulphate resistance.

### 5.1.2 Damage calculation procedure for reinforcement in tension and compression

For the Cumulative Damage Method, the method of calculating the damage on the reinforcement is presented in EN 1992-1-1: SIS (2005). As for concrete, the

Palmgren-Miner's rule is used to calculate the total damage on the reinforcement bars as defined as:

$$D = \sum_{i=1}^{m} \frac{n(\Delta\sigma_i)}{N(\Delta\sigma_i)} \leq 1 \tag{5.8}$$

where $n(\Delta\sigma_i)$ is the applied number of cycles for a stress range $\Delta\sigma_i$,

$N(\Delta\sigma_i)$ is the ultimate number of cycles for a stress range $\Delta\sigma_i$.

When determining the ultimate number of cycles, equations (5.9) and (5.10), a condition is checked in order to decide which slope in the S-N relation, Figure 5.1 that should be used for the current stress range.

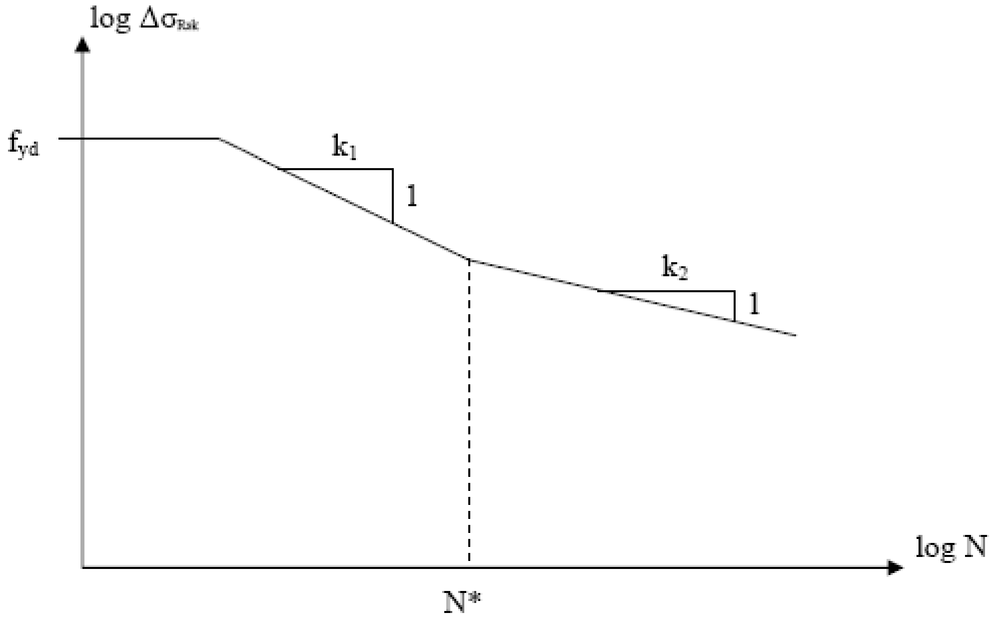



*Figure 5.1 Shape of the characteristic fatigue strength curve (S-N relation) for reinforcing and prestressing steel.*

$$N(\Delta\sigma_i) = N^* \left( \frac{\Delta\sigma_{Rsk} / \gamma_{S.fat}}{\gamma_{F.fat}\Delta\sigma_i} \right)^{k_1} \quad \text{if} \quad \gamma_{F.fat}\Delta\sigma_i \geq \frac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \tag{5.9}$$

$$N(\Delta\sigma_i) = N^* \left( \frac{\Delta\sigma_{Rsk} / \gamma_{S.fat}}{\gamma_{F.fat}\Delta\sigma_i} \right)^{k_2} \quad \text{if} \quad \gamma_{F.fat}\Delta\sigma_i < \frac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \tag{5.10}$$

where $N^*$ is a reference value of number of cycles until failure, depending on which type of reinforcement that is verified, table 6.3N of EN 1992-1-1: 2005,

$\gamma_{S,fat}$ is the partial factor taking the material uncertainties into account, 2.4.2.4 (1) of EN 1992-1-1: 2005,

$\gamma_{F,fat}$ is the partial factor taking the fatigue load model uncertainties into account, defined in section 4.3,

$k_1$ is the exponent defining the first slope of the S-N relation, table 6.3N of EN 1992-1-1: 2005,

$k_2$ is the exponent defining the second slope of the S-N relation, table 6.3N of EN 1992-1-1: 2005,

$\Delta\sigma_{Rsk}$ is the resisting stress range at $N^*$ cycles, depending on which type of reinforcement that is verified, table 6.3N of EN 1992-1-1: 2005

## 5.2 Methodology of the λ-Coefficient Method

The $\lambda$-Coefficient Method, presented in Annex NN of EN 1992-2:2005, is a simplified method compared to the Cumulative Damage Method. In order verify a bridge structure the $\lambda$-Coefficient Method uses a single stress range amplified with a number of $\lambda$-coefficients. The assessment method is applicable to reinforcement and prestressing steel for road- and railway bridges. For concrete subjected to compression the method is only valid for railway bridges.

As mentioned, the single stress range, which is obtained by a passage of a single train model, is amplified with a number of $\lambda$-coefficients. The values of these $\lambda$-coefficients are governed by different parameters such as span, annual traffic volume, design service life, critical length of influence line, and effects of loading if there is more than one track. For the assessment of railway bridges the dynamic factor is also a parameter to consider. The dynamic factor increases the load effects from the static load model LM71, see also Chapter 4.3.

The $\lambda$-factor, which takes into account the structural element type, e.g. continuous beam, and the damaging effect of traffic, should according to TK Bro, be modified. The factors $\lambda_{c,1}$ and $\lambda_{s,1}$ should be multiplied with a factor $\alpha$, according to D.2.1 (g) of TK Bro, when using a heavy traffic mix. The $\alpha$-factor is a load classification factor and the general value is specified in Chapter 4.3. The value of the modified $\alpha$-factor used in the $\lambda$-Coefficient Method is shown in Figure 5.2.

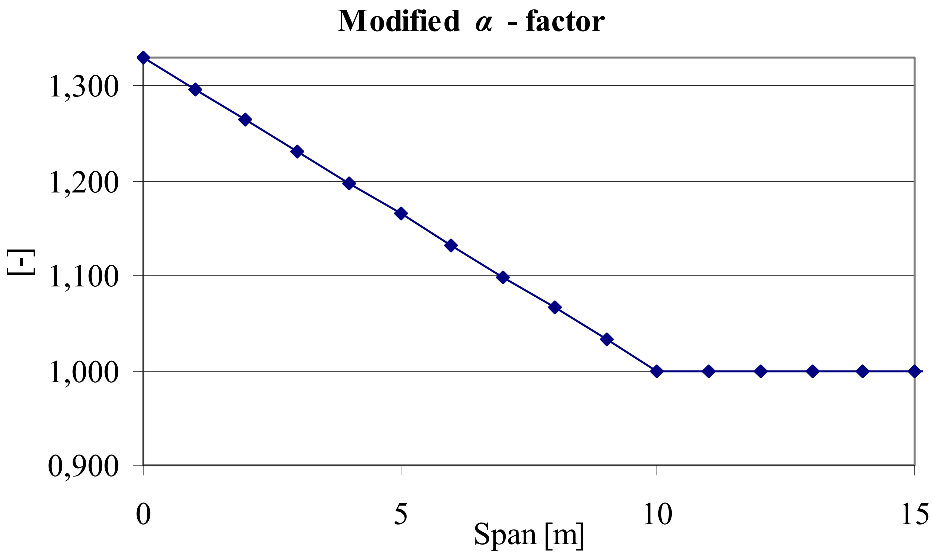



*Figure 5.2 The α-factor which varies linearly from 1.33 to 1.00 for spans between 0 and 10 meters. If the span is larger than 10 meters the factor is equal to one.*

A certain $\lambda$-factor should be applied if the structure is loaded by more than one track. Since the bridge models used in the present work are single track bridges the $\lambda_{c,4}$ and $\lambda_{s,4}$-factors should be set to 1. This was primarily adopted in order to avoid complex load combinations.

### 5.2.1 Fatigue verification procedure for compressed concrete

To verify a bridge structure for fatigue in compressed concrete with the $\lambda$-Coefficient Method, the requirement, expressed by equation (5.11), according to EN 1992-2: SIS (2005), needs to be fulfilled.

$$14 \cdot \frac{1 - E_{cd,\max,equ}}{\sqrt{1 - R_{equ}}} \geq 6 \tag{5.11}$$

where $E_{cd,\max,equ}$ is the damage equivalent stress spectrum upper stress level as defined in equation (5.15),

$R_{equ}$ is the damage equivalent stress spectrum ratio as defined in equation (5.13).

In Eurocode there is no method described of how to calculate damage for the $\lambda$-Coefficient Method in the similar manner as for the Cumulative Damage Method. Therefore it was in the present work necessary to rewrite equation (5.12) in order to deliver the result as a number instead of a requirement. This number, $D_{\lambda,c,}$ is however not comparable with the damage achieved in the Cumulative Damage Method but can bee seen as a degree of utilization of the concrete when assessed for fatigue by the $\lambda$-Coefficient Method. In both cases though, a value exceeding 1 means that is requirement on fatigue resistance is not met.

$$D_{\lambda,c} = E_{cd,\max,equ} + \frac{\log(N)}{14} \cdot \sqrt{1 - R_{equ}} \leq 1 \tag{5.12}$$

where $N$ is the reference number of cycles until failure for the damage equivalent stress spectrum.

$$R_{equ} = \frac{E_{cd,\min,equ}}{E_{cd,\max,equ}} \tag{5.13}$$

where $E_{cd,\min,equ}$ is the damage equivalent stress spectrum lower stress level as defined in equation (5.14).

$$E_{cd,\min,equ} = \gamma_{sd} \cdot \frac{\sigma_{cd,\min,equ}}{f_{cd,fat}} \tag{5.14}$$

$$E_{cd,\max,equ} = \gamma_{sd} \cdot \frac{\sigma_{cd,\max,equ}}{f_{cd,fat}} \tag{5.15}$$

where $\sigma_{cd,\min,equ}$ is the lower stress in the damage equivalent stress spectrum, as defined in equation (5.17),

$\sigma_{cd,\max,equ}$ is the upper stress in the damage equivalent stress spectrum, as defined in equation (5.18),

$\gamma_{sd}$ is a partial factor for model uncertainty for action/action effort.

The lower and upper stresses of the damage equivalent stress spectrum take into account stresses induced by permanent and traffic loads.

$$\sigma_{cd,\min,equ} = \sigma_{c,perm} - \lambda_c(\sigma_{c,perm} - \sigma_{c,\min,71}) \tag{5.16}$$

$$\sigma_{cd,\max,equ} = \sigma_{c,perm} + \lambda_c(\sigma_{c,\max,71} - \sigma_{c,perm}) \tag{5.17}$$

where $\sigma_{c,perm}$ is the compressive stress caused by the characteristic combination of actions without LM71, calculated according to section 3.4,

$\sigma_{c,\min,71}$ is the minimum compressive stress under the characteristic combination of actions including LM71 and latter amplified with the dynamic factor, calculated according to section 4.3,

$\sigma_{cd,\max,i}$ is the maximum compressive stress under the characteristic combination of actions including LM71 and latter amplified with the dynamic factor, calculated according to section 4.3,

$\lambda_c$ is the correction factor to calculate the upper and lower stresses of the damage equivalent stress spectrum, as defined in equation (5.18).

The correction factor takes into account permanent stress, span, annual traffic volume, design service life and multiple tracks; see also equation (5.18) according to EN 1992-2: SIS (2005).

$$\lambda_c = \lambda_{c,0} \cdot \lambda_{c,1} \cdot \lambda_{c,2,3} \cdot \lambda_{c,4} \tag{5.18}$$

where $\lambda_{c,0}$ is a factor which takes into account the permanent stress,

$\lambda_{c,1}$ is a factor accounting for element type and takes into account the damaging effect of traffic depending on the critical length of the influence line or area,

$\lambda_{c,2,3}$ is a factor that takes account of the traffic volume and the design service life of the bridge,

$\lambda_{c,4}$ is a factor to be applied when the structural element is loaded by more than one track.

### 5.2.2 Fatigue verification procedure for reinforcement in tension and compression

To verify the reinforcement in a railway bridge for fatigue with the $\lambda$-Coefficient Method the requirement expressed by equation (5.19) needs to be fulfilled according to EN 1992-1-1: SIS (2005).

$$\gamma_{F,fat} \Delta\sigma_{s,equ}(N^*) \leq \frac{\Delta\sigma_{Rsk}(N^*)}{\gamma_{S,fat}} \tag{5.19}$$

where $\Delta\sigma_{s,equ}(N^*)$ is the damage equivalent stress range considering the number of loading cycles $N^*$ as defined in equation (5.9),

$\Delta\sigma_{Rsk}(N^*)$ is the resisting stress range at $N^*$ cycles. This value depends on which type of reinforcement that is used, i.e. bent or straight bars.

As for concrete there is no method presented in the code to achieve a value of the damage inflicted on the structure. Hence, instead of simply checking the requirement equation (5.20) was rewritten in the present work to equation (5.21) in order to achieve a number, $D_{\lambda,s,}$ which can bee seen as a degree of utilization of the reinforcement when assessed for fatigue by the $\lambda$-Coefficient Method. This number is not comparable to the damage deliver in the Cumulative Damage Method. In both cases though, a value exceeding 1 means that the requirement on fatigue resistance is not met.

$$D_{\lambda,s} = \frac{\gamma_{F,fat} \Delta\sigma_{s,equ}(N^*)}{\frac{\Delta\sigma_{Rsk}(N^*)}{\gamma_{S.fat}}} \leq 1 \tag{5.20}$$

The damage equivalent stress range for reinforcing and prestressing steel, according to Annex NN.3 of EN 1992-2: 2005:

$$\Delta\sigma_{s,equ} = \lambda_s \cdot \Phi \cdot \Delta\sigma_{s,71} \tag{5.21}$$

where $\lambda_s$ is a correction factor to calculate the damage equivalent stress range, as defined in equation (5.22),

| | | |
|---|---|---|
| | $\Phi$ | is the dynamic factor, as defined in Section 4.3, |
| | $\Delta\sigma_{s,71}$ | is the steel stress range due to load model 71, calculated according to Section 3.4. |

$$\lambda_s = \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} \tag{5.22}$$

| | | |
|---|---|---|
| where | $\lambda_{s,1}$ | is a factor that takes into account the critical length of the influence line and traffic, |
| | $\lambda_{s,2}$ | is a factor that depends on the annual traffic volume, |
| | $\lambda_{s,3}$ | is a factor that takes into account the design service life of the bridge, |
| | $\lambda_{s,4}$ | is a factor that considers loading from more than one track. |

# 6 Parametric study of models of reinforced concrete bridges subjected to bending due to railway traffic

To be able to compare the fatigue assessment methods of railway concrete bridges available in Eurocode, two hypothetic bridge models were developed. The purpose was to perform parametric studies of these models and evaluate the results. The cases studied were two simplified concrete bridges and their design was based on the design of an existing bridge. The existing bridge, Degerfors Bridge, was adjusted to simplify the analysis as explained in Chapter 3.

The first case studied, Figure 6.1, simulated a continuous two-span bridge which was simplified into a single span slab. The two supports of the slab were modelled as simply supported at one end and fully fixed at the other end to simulate the intermediate support. The second bridge model, Figure 6.2, was a simply supported single span slab and was developed in order to analyse shorter spans with realistic responses; this since continuous bridges are seldom shorter than 10 meters.

The study was performed by adjusting the geometry of the bridge with the varying span. The adjustment was done by keeping certain ratios between the span and a number of geometrical properties constant. The results for the different sections and models are presented in tables and figures describing the damage, or the value of a design criterion, for the fatigue assessment methods. A section with analysis follows each study.

In the first study the continuous bridge model was used and the results for both field and support sections are presented in Section 6.2. It also includes some deepened inquiries in order to achieve a better understanding of the results. The second study was performed in the same manner but using the simply supported bridge model instead. The study and the results are presented in Section 6.3. The third study was a comparison of the results from the two previous studies; this was done in order to see if any similarities between the models and sections could bee detected, Section 6.4. The last study was an analysis of the sensitivity within the methods and a comparison of the different design criteria that might be governing in the design of a bridge. The study is seen in Section 6.5.

## 6.1 Variation of span and cross-section

After discussion with Helsing[5] it was decided to vary the span between 2 and 15 meters. The span range was chosen to represent the most common bridges built in Sweden. After discussing with Palmgren[6], the bridges were decided to have constant relations regarding the cross-sectional height, equation (6.1) and equation (6.2), and length of the haunched section, equation (6.3) when the span varied. The cross-sectional ratios tht was used were adapted from the reference bridge, the Degerfors Bridge, see Table 6.1.

[5] Elisabeth Helsing, Trafikverket, mail contact 2010-05-19
[6] Helmer Palmgren, SWECO Infrastructure, mail contact 2010-05-19

*Table 6.1 Summary of the geometrical properties of the reference bridge, the Degerfors Bridge.*

| $L$ [m] | $h_l$ [m] | $h_r$ [m] | $x_1$ [m] | $x_2$ [m] |
|---|---|---|---|---|
| 7.00 | 0.54 | 0.83 | 4.50 | 2.50 |

$$\frac{L}{h_l} = \frac{7.00}{0.54} = 12.96 \tag{6.1}$$

$$\frac{L}{h_r} = \frac{7.00}{0.83} = 8.43 \tag{6.2}$$

$$\frac{L}{x_1} = \frac{7.00}{4.50} = 1.56 \tag{6.3}$$

where $L$ is the span of the Degerfors Bridge [m],

$h_l$ is the height of the beam at the simply supported end of the Degerfors Bridge [m],

$h_r$ is the height of the beam at the intermediate support of the Degerfors Bridge [m],

$x_1$ is the length coordinate to the first section of the haunch from the left end [m],

$x_2$ is the length coordinate to the section of the haunch from the left end [m].

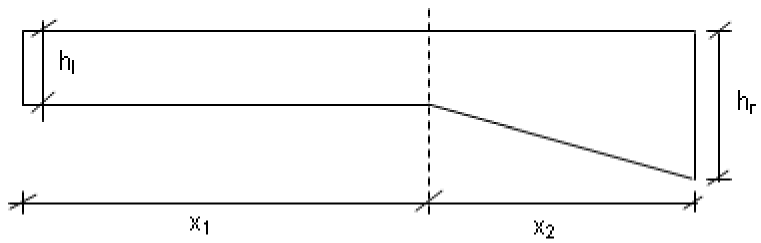



*Figure 6.1 Geometrical parameters used to simulate continuous bridges.*

The geometrical parameters shown in Figure 6.1 were used to simulate continuous bridges where the span varied between 4 and 15 meters. For simply supported bridges the geometrical parameters in Figure 6.2 were used and the span varied between 2 and 10 meters. According to Swedish praxis for railway bridges, a bridge slab should not have a cross-sectional height less than 200 millimetres. If the value calculated according to equation (6.1) was below 200 millimetres the cross-sectional height was set equal to the limitation instead.

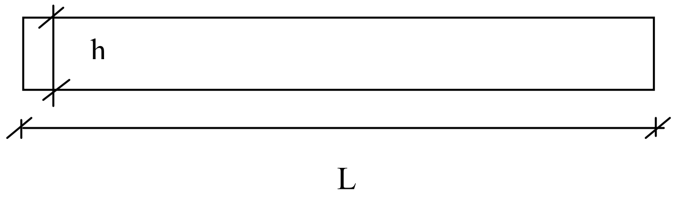



*Figure 6.2 Geometrical parameters used to simulate simply supported bridges.*

## 6.2 Parametric study regarding the span with a model of a continuous bridge

The variation of the cross-sectional properties and bending moments for the model of a continuous bridge is presented in Table 6.2. The index *s* and *f* denotes which section that is considered i.e. support or field section.

*Table 6.2 Geometrical properties and bending moments in maximum moment sections for the various cases of the parametric study. The bending moments were determined by StripStep2.*

| | *Sectional height* | | *Length of segment* | | *Moment ULS* | | *Moment due to permanent load* | |
|---|---|---|---|---|---|---|---|---|
| *L [m]* | $h_l$ *[m]* | $h_r$ *[m]* | $x_1$ *[m]* | $x_2$ *[m]* | $M_{Ed.s}$ | $M_{Ed.f}$ | $M_{Perm.s}$ | $M_{Perm.f}$ |
| 4.0 | 0.31 | 0.47 | 2.57 | 1.43 | (-)1 625 | 534 | (-)247 | 80 |
| 5.0 | 0.39 | 0.59 | 3.21 | 1.79 | (-)2 469 | 819 | (-)423 | 137 |
| 6.25 | 0.48 | 0.74 | 4.02 | 2.23 | (-)3 833 | 1256 | (-)739 | 237 |
| 7.5 | 0.58 | 0.89 | 4.82 | 2.68 | (-)5 451 | 1802 | (-)1 171 | 375 |
| 10.0 | 0.77 | 1.19 | 6.43 | 3.57 | (-)9 426 | 3183 | (-)2 470 | 787 |
| 11.5 | 0.89 | 1.36 | 7.39 | 4.11 | (-)12 356 | 4207 | (-)3 572 | 1 137 |
| 15.0 | 1.16 | 1.78 | 9.64 | 5.36 | (-)21 129 | 7231 | (-)7 292 | 2 315 |

The number of reinforcement bars used in the fatigue assessment is presented in Table 6.3 and Table 6.4. The amount is estimated in the preliminary design and the values are used with two decimal signs instead of entire bars in order to achieve more accurate results. The bars used have a diameter of 25 millimetres.

*Table 6.3 The number of reinforcement bars used for the calculations in the field section*

| *Number of reinforcement bars* | | | | | | | |
|---|---|---|---|---|---|---|---|
| *L [m]* | 4.0 | 5.0 | 6.25 | 7.5 | 10.0 | 11.5 | 15.0 |
| *In tension* | 11.25 | 12.99 | 15.24 | 17.60 | 22.44 | 25.38 | 32.60 |
| *In Compression* | 11.25 | 11.25 | 11.25 | 12.80 | 17.33 | 20.04 | 26.38 |

*Table 6.4 The number of reinforcement bars used for the calculations in the support section*

| *Number of reinforcement bars* | | | | | | | |
|---|---|---|---|---|---|---|---|
| *L [m]* | 4.0 | 5.0 | 6.25 | 7.5 | 10.0 | 11.5 | 15.0 |
| *In tension* | 20.46 | 23.87 | 28.72 | 33.32 | 41.96 | 47.19 | 60.93 |
| *In Compression* | 11.25 | 11.25 | 11.25 | 11.25 | 11.25 | 12.77 | 16.89 |

### 6.2.1 Results for the $\lambda$-Coefficient Method

For the $\lambda$-Coefficient Method, the maximum and minimum sectional moments caused by the traffic load LM71, should be used in the fatigue verification. These maximum moments for different spans are presented in Table 6.5. For the simplified model used in this study the minimum moments are always zero and are therefore not presented. With a more refined structural model the minimum value may differ from zero if e.g. rotation at the intermediate support is possible. The results from the $\lambda$-Coefficient Method are presented in  as values of the design criterion according to Section 5.2.

*Table 6.5 The maximum moments in the critical sections caused by the traffic load LM71 for the $\lambda$-Coefficient Method.*

| *L [m]* | 4.0 | 5.0 | 6.25 | 7.5 | 10.0 | 11.5 | 15.0 |
|---|---|---|---|---|---|---|---|
| *$M_s$ [kNm]* | 433.20 | 677.30 | 1031.90 | 1 467.30 | 2 432.40 | 3 037.70 | 4 821.90 |
| *$M_f$ [kNm]* | 165.50 | 228.00 | 347.30 | 525.40 | 849.70 | 1 104.00 | 1 778.10 |

*Table 6.6 Value of the design criterion in field and support sections for the λ-Coefficient Method*

| *L [m]* | 4.0 | 5.0 | 6.25 | 7.5 | 10.0 | 11.5 | 15.0 |
|---|---|---|---|---|---|---|---|
| | | | *Field Section* | | | | |
| *Tensile Reinforcement* | 1.57 | 1.26 | 1.11 | 1.05 | 0.80 | 0.75 | 0.64 |
| *Compressed Concrete* | 0.88 | 0.77 | 0.72 | 0.70 | 0.63 | 0.62 | 0.60 |
| | | | *Support Section* | | | | |
| *Tensile Reinforcement* | 1.25 | 1.18 | 1.06 | 0.98 | 0.81 | 0.75 | 0.66 |
| *Compressed Concrete* | 0.85 | 0.82 | 0.78 | 0.76 | 0.71 | 0.70 | 0.68 |

The calculated results from the λ-Coefficient Method in field and support sections are also presented in Figure 6.3 and Figure 6.4. Figure 6.3 shows the value of the design criterion for the tensile reinforcement and how it varies with the span. Figure 6.4 displays the corresponding value for the compressed concrete. When the value of the design criteria is above 1 the fatigue life of the structure has been exceeded. For additional results see Appendix A.

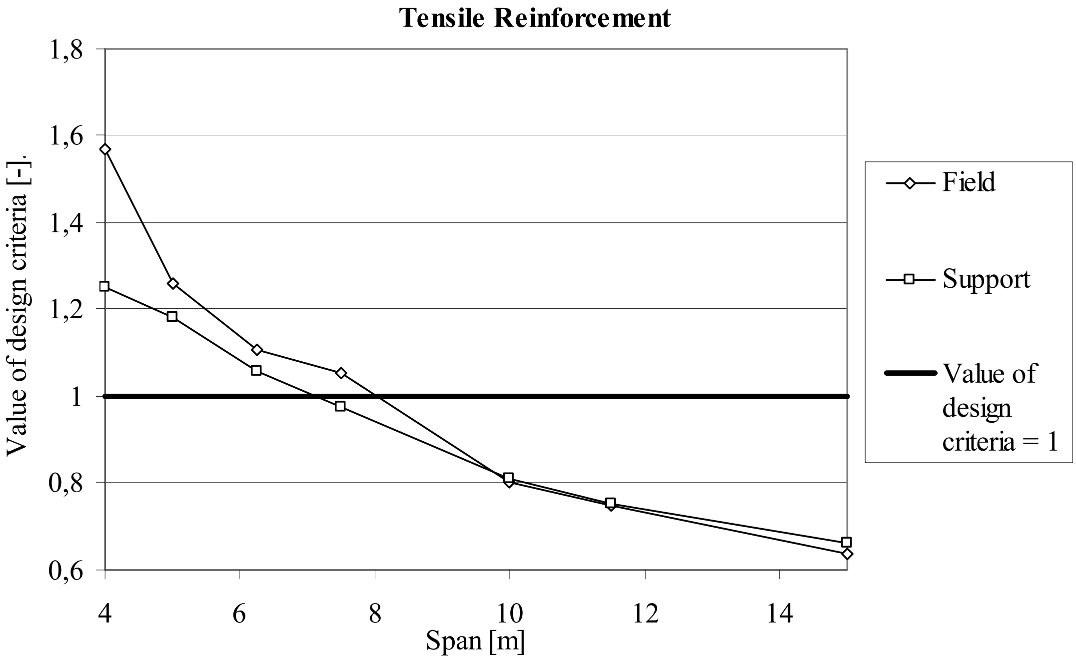



*Figure 6.3 Results from the fatigue assessment according to the λ-Coefficient Method for reinforcement in tension, field and support sections.*

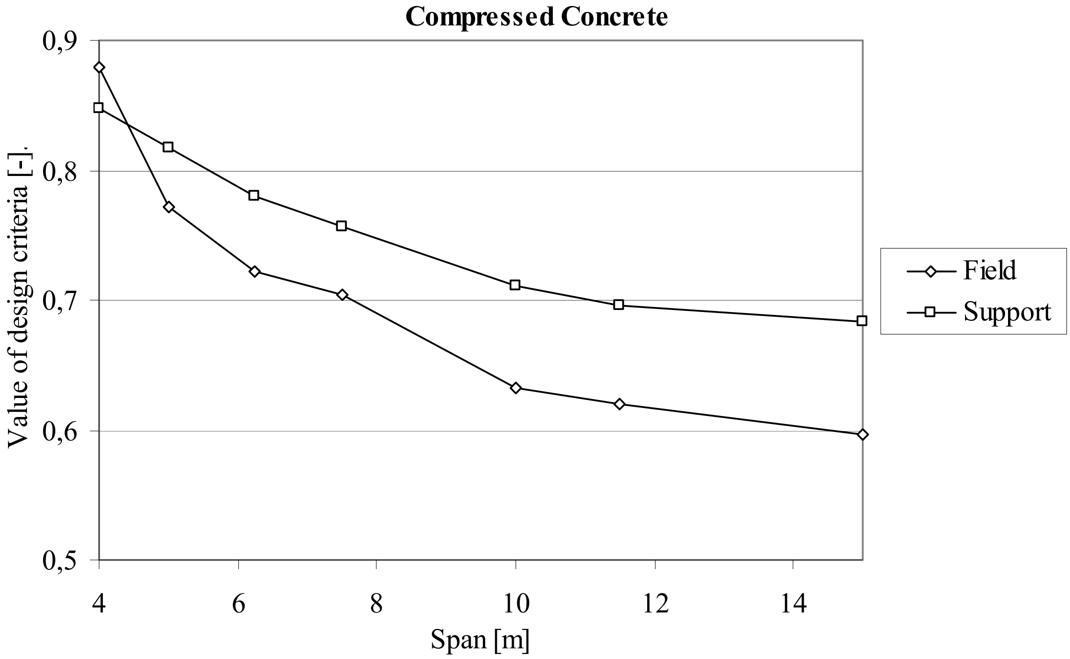



*Figure 6.4 Results from the fatigue assessment according to the λ-Coefficient Method for compressed concrete, field and support sections.*

### 6.2.2 Results for the Cumulative Damage Method

The results for the Cumulative Damage Method in the field and support sections are presented in Table 6.7. When the values are above 1 the fatigue life of the structure has been exceeded. As an example, for the tensile reinforcement in the field section, the damage is just above 1.0 for a span of 7.5 meter. This means that the fatigue life will be reached with regard to this particular case.

*Table 6.7 The value of the damage in field and support section for the Cumulative Damage Method*

| *L [m]* | 4.0 | 5.0 | 6.25 | 7.5 | 10.0 | 11.5 | 15.0 |
|---|---|---|---|---|---|---|---|
| | | | *Field Section* | | | | |
| *Tensile Reinforcement* | 24.31 | 9.64 | 3.51 | 1.36 | 0.24 | 0.09 | 0.02 |
| *Compressive Concrete* | 0.01 | $5.79 \cdot 10^{-4}$ | $2.78 \cdot 10^{-5}$ | $3.60 \cdot 10^{-6}$ | $1.08 \cdot 10^{-7}$ | $1.63 \cdot 10^{-8}$ | $9.53 \cdot 10^{-10}$ |
| | | | *Support Section* | | | | |
| *Tensile Reinforcement* | 5.49 | 2.09 | 0.57 | 0.19 | 0.09 | 0.05 | 0.01 |
| *Compressive Concrete* | $5.26 \cdot 10^{-4}$ | $4.73 \cdot 10^{-5}$ | $4.28 \cdot 10^{-6}$ | $6.63 \cdot 10^{-7}$ | $1.12 \cdot 10^{-7}$ | $2.83 \cdot 10^{-8}$ | $2.73 \cdot 10^{-9}$ |

Figure 6.5 and Figure 6.6 also presents the results obtained for the Cumulative Damage Method. The figures visualises how the damages is influenced by varying span. For additional results see Appendix A.

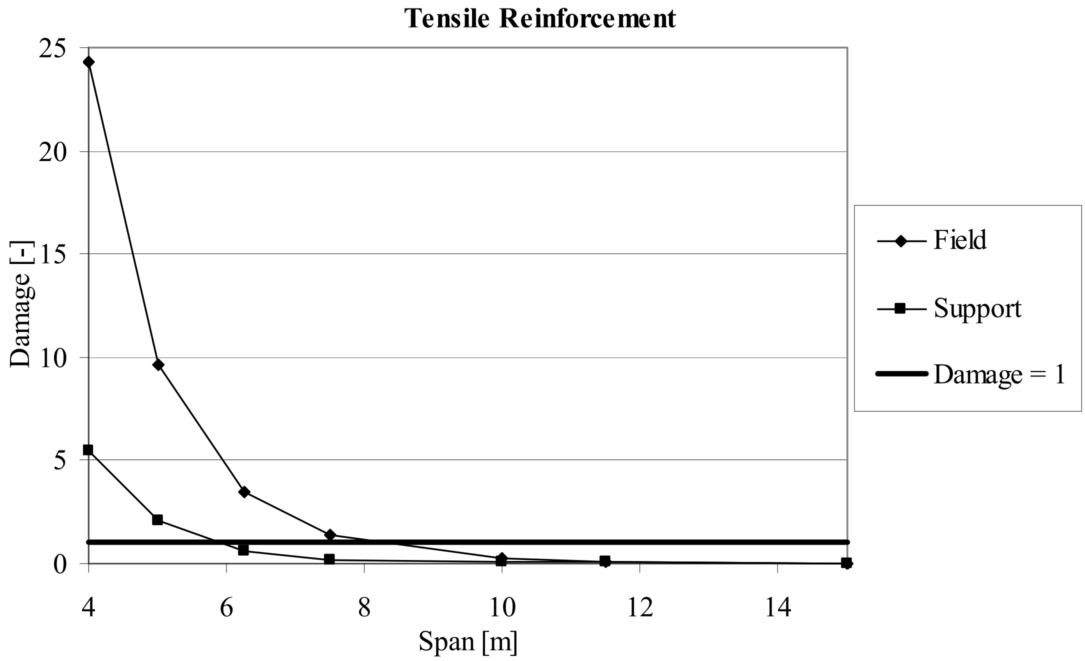



*Figure 6.5 Result from fatigue assessment calculations according to the Cumulative Damage Method for reinforcement in tension, field and support section.*

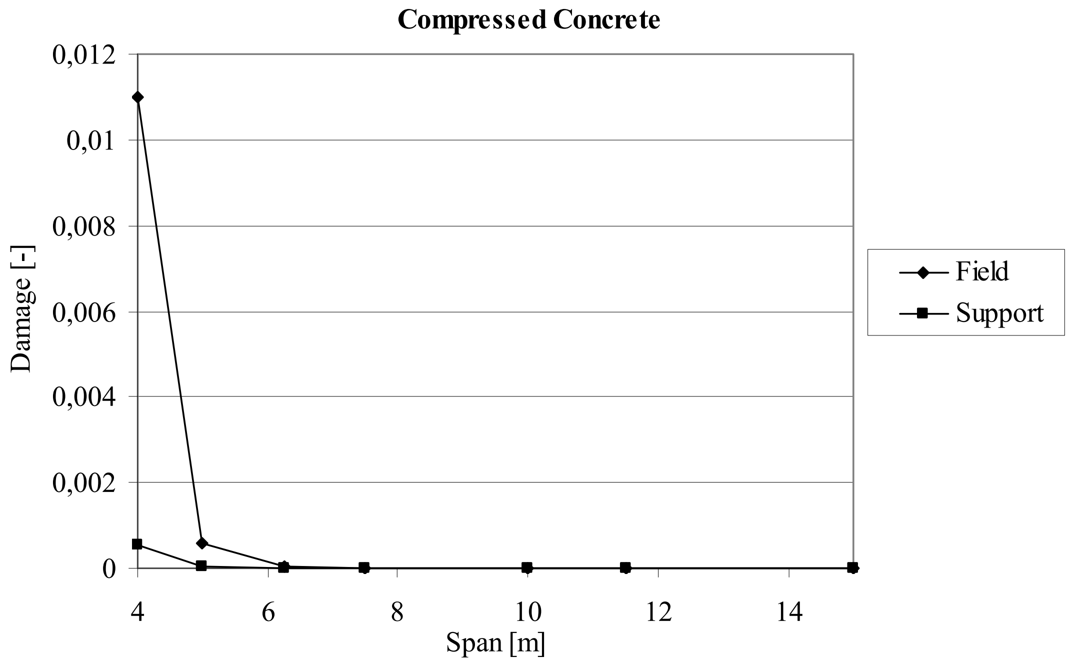



*Figure 6.6 Result from fatigue assessment calculations according to the Cumulative Damage Method for compressed concrete, field and support section.*

### 6.2.3 Analysis of the results with regard to the tensile reinforcement

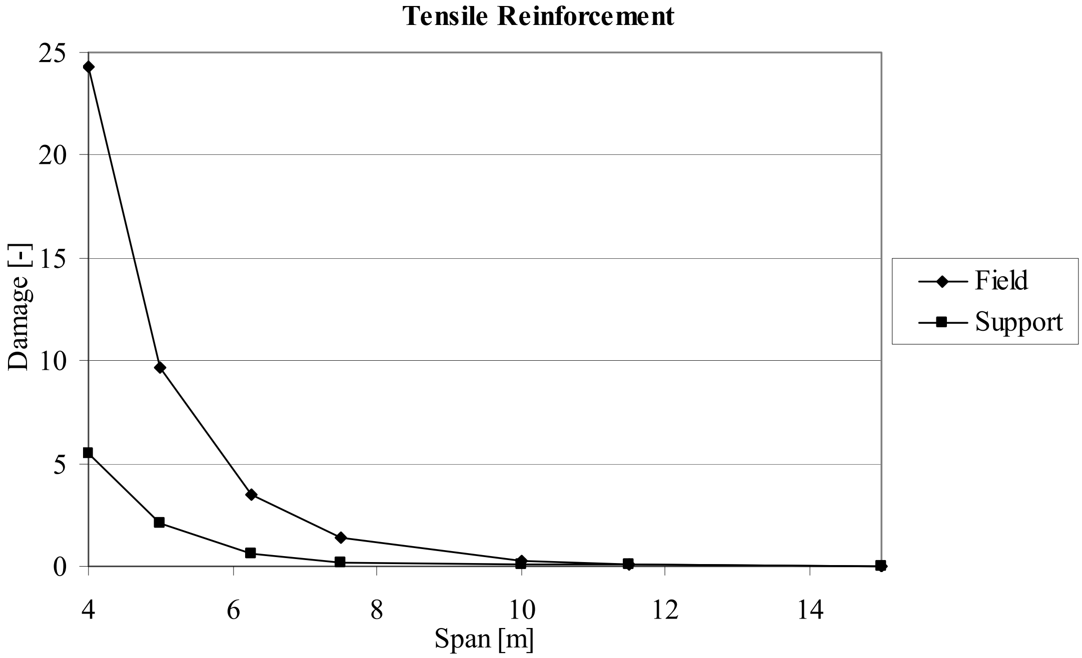
While analysing the results according to the Cumulative Damage Method it can be found that the damage development depends on which section that is considered, see Figure 6.7 a). When looking at the field section, it is observed that the damage progress is slow with decreasing span down to approximately 10 meter where the damage starts to increase rapidly. The damage at the support section follows the same pattern although the rapid increase in damage with decreasing span occurs at approximately 7.5 meter instead.

The explanation to the different behaviour could be the various shape of the influence lines used in the calculations. The influence line concerning the field section has a rather sharp peak located exactly at the considered section. This peak gives a large value of the field moment when a concentrated load is positioned directly above this point. The influence line for the moment in the support section does not have a similar peak and is therefore not as dependent on the position of the loads.

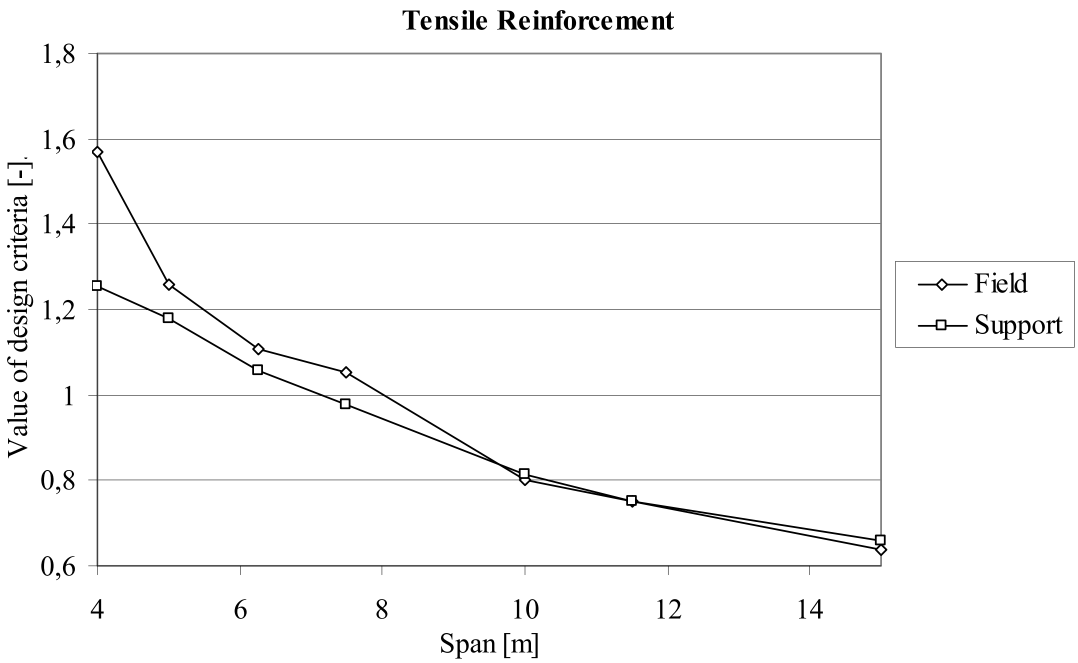
When observing the results from the parametric study with regard to the span for the $\lambda$-Coefficient Method, it can be seen that there is no significant difference depending on which section that is considered, see Figure 6.7 b). It can also be seen that for both the field and support sections, the $\lambda$-Coefficient Method is more likely to exceed the allowable limit for shorter spans, approximately less than 8 meters.

To be able to more easily compare the results from the two fatigue assessment methods, the values of the damage and design criterion combined into one figure, Figure 6.7 c). The figure present both fatigue assessments method and both considered sections in order to see how the methods develop in comparison to each other. Note that the calculated values from the fatigue assessment methods do not have the same definition, but they can still be compared to a certain extent. It is e.g.

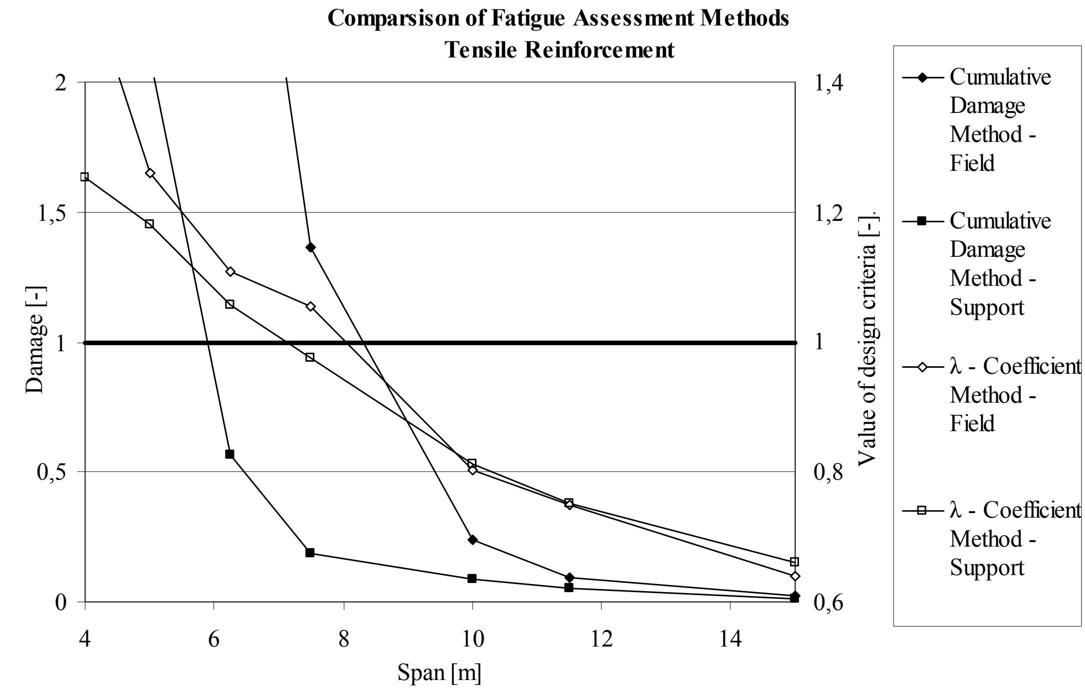
possible to see when the methods become decisive in design, i.e. when the value is above one.




a)




b)



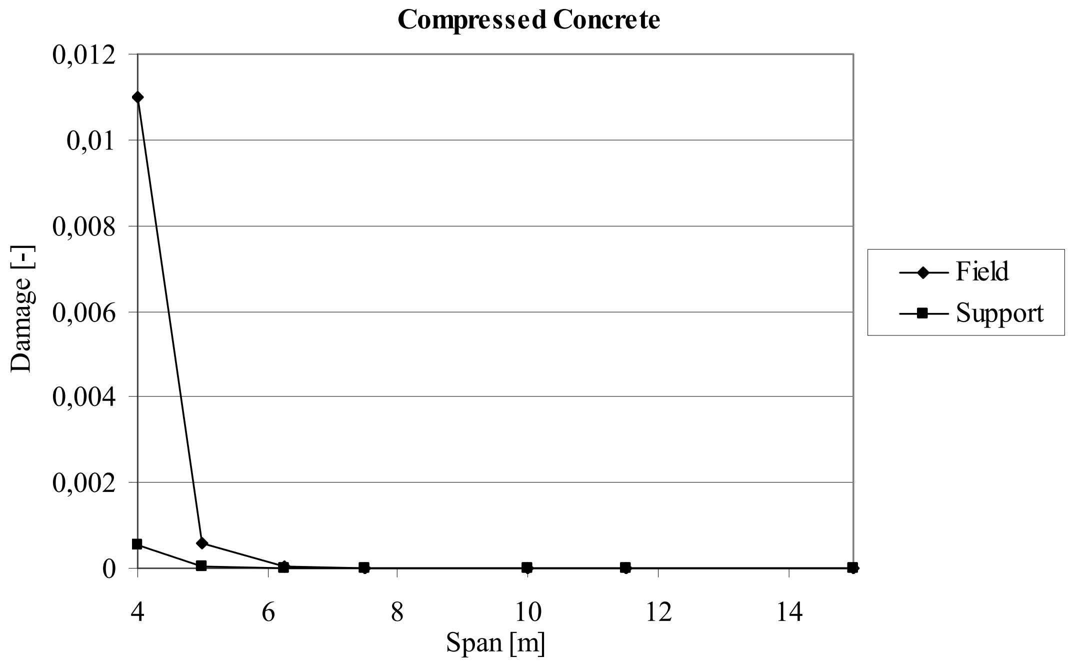

c)

*Figure 6.7 Comparison of design parameters from both design methods with regard to the tensile reinforcement in critical sections. a) The damage for the Cumulative Damage Method, b) The value of the design criteria for the λ-Coefficient Method, c) The results from both fatigue assessments methods. The figure is focused at the area around the design criteria (1). Note that the Cumulative Damage Method corresponds to the left y-axis and the λ-Coefficient Method to the right.*

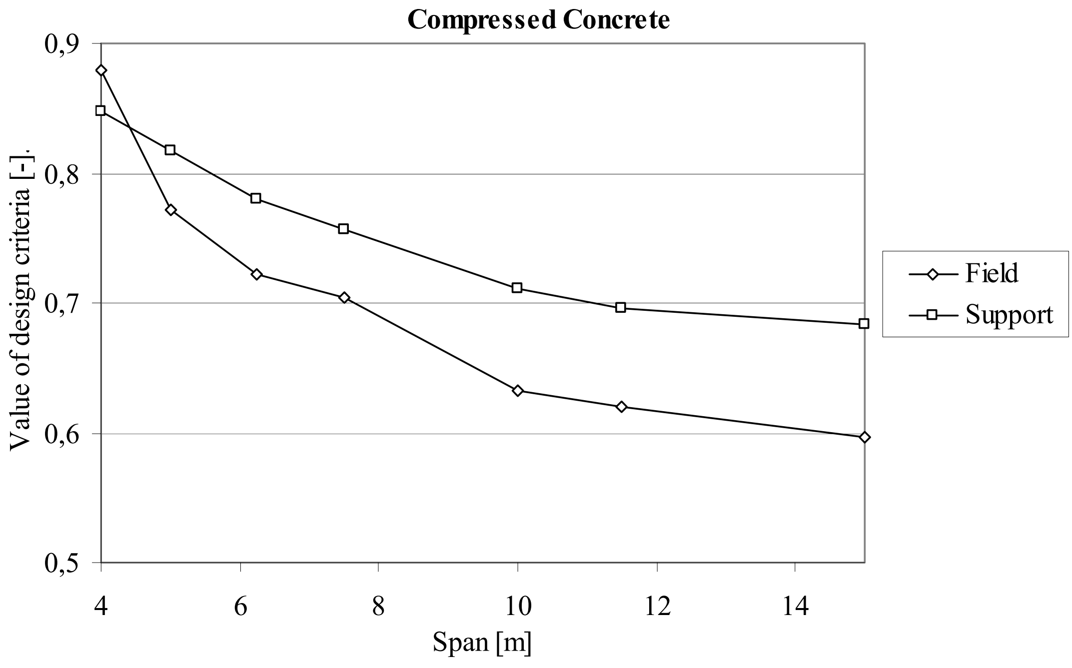
In order to achieve a greater understanding of when and why the rapid increase in damage occurs, a closer study of the results from the Cumulative Damage Method was initiated. This study is presented in Section 6.2.5.

### 6.2.4 Analysis of the results with regard to the compressed concrete

When observing the results with regard to the compressed concrete, Figure 6.8, both the $\lambda$-Coefficient Method and the Cumulative Damage Method show values of the damage/design parameters that are well below the allowable limit. Therefore it is not possible to compare the methods in the same way as for the tensile reinforcement. Although, according to the Cumulative Damage Method, it seems as the span where the damage starts to rapidly increase is close to 4 meters. Also for the $\lambda$-Coefficient Method the value of the design criterion is approaching the allowed limit.




a)




b)

*Figure 6.8* *Comparison of design parameters from both fatigue design methods with regard to the compressed concrete in critical sections. a) The damage for the Cumulative Damage Method, b) The value of the design criteria for the λ-Coefficient Method.*

For the $\lambda$-Coefficient Method the result from the support section is consistently higher than for the field section; except for the shortest span. From this it might follow that the compressed concrete will become decisive in design earlier for the support section than for the field section. This is in case of a more slender cross-section which gives a higher mean level of compressive stress in the concrete. This is in contrast to the tensile reinforcement where the field section is constantly higher. It can be added that for this particular case, the tensile reinforcement has become decisive for the design long before the concrete starts to show high values of fatigue.

### 6.2.5 Study of the damage development in the tensile reinforcement for the Cumulative Damage Method

With regard to the obtained results from the study about the influence of the span for the Cumulative Damage Method, a closer study was performed regarding how the stress and damage vary with the number of cycles. The analysis was performed on the tensile reinforcement in the field section using the model for continuous bridges. At first some general cases was studied, then the analysis was performed on spans where the value of the design criterion was close to the allowed limit. In order to distinguish any similarities in stress, number of cycles and damage.

While performing fatigue assessment of a bridge, a series of stress ranges is obtained. An example of such a series is shown in Figure 6.9 and this series corresponds to the train passages of one day. In order to get a picture about the differences between the results for different spans, the values were sorted by size instead of when they appear in time. This was done for four different spans with the model of continuous bridges and the series is shown in Figure 6.10. Here it can be seen that the development of the series are rather similar; especially for 7.5 and 10 meter.

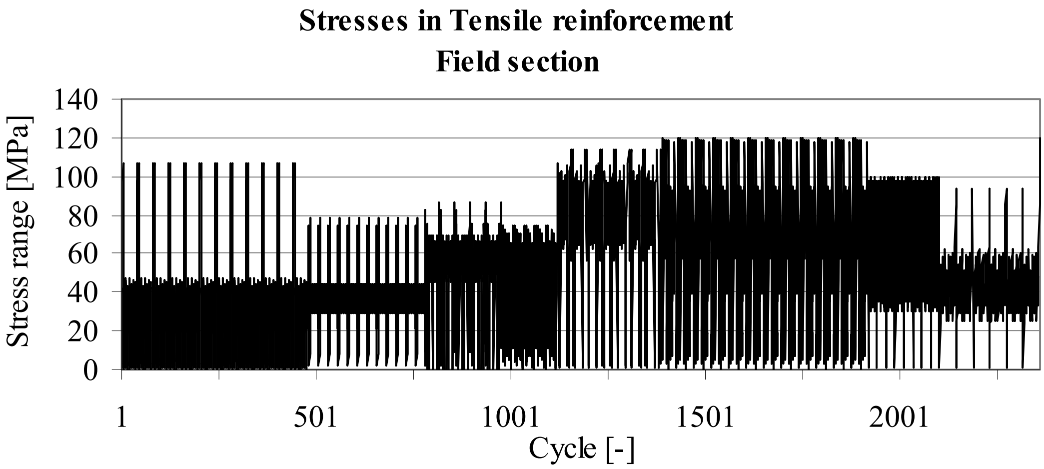



*Figure 6.9 Example of a stress range series achieved for one day as the loads with the fatigue load models appear in time, span 7.5m.*

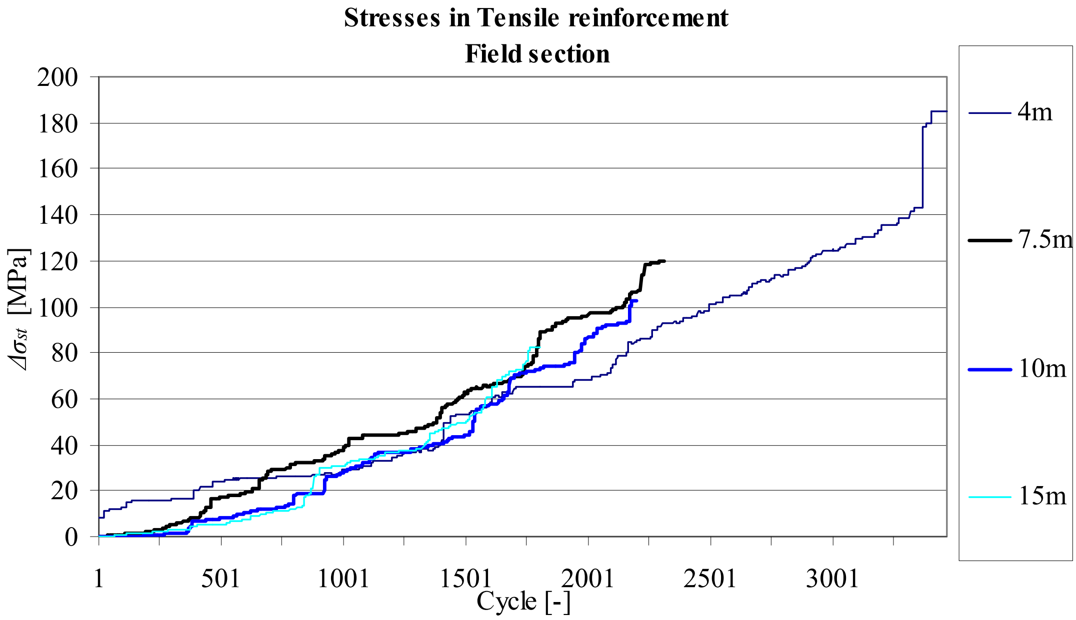



*Figure 6.10 The stress ranges versus the number of cycles for different spans. The stress ranges are sorted by size, not when they appear in time.*

In Table 6.8, some numbers of the different the series are presented. They are the mean stress range over the entire series, the total number of cycles per day and the total damage for the requested service life of 120 years. Despite the fact that the difference in the number of cycles is rather small, for the two middle cases, there is a markedly difference in damage.

*Table 6.8 The mean stress, total number of cycles and the total inflicted damage on the structure for different spans.*

| *Span* | *Mean stress range in the tensile reinforcement* | *Total number of cycles per day* | *Total damage over 120 years* |
|---|---|---|---|
| *L [m]* | *Mean $\Delta\sigma_{st}$ [MPa]* | *n [-]* | *D [-]* |
| 4 | 68.5 | 3 465 | 24.312 |
| 7.5 | 48.7 | 2 361 | 1.363 |
| 10 | 36.2 | 2 210 | 0.241 |
| 15 | 26.9 | 1 792 | 0.027 |

Therefore, to visualise some of the results in Figure 6.10 in a different way, cases with spans 7.5 meters and 10 meters were arranged into histograms instead. They shows the stress range versus the number of cycles together with the development of damage, see Figure 6.11 and Figure 6.12. The series of stress ranges are divided into 10 equally large intervals, and the number of cycles and the corresponding damage in each interval are plotted beside each other. The figures clearly show that most of the damage develops at the end of the series with a high stress amplitude.

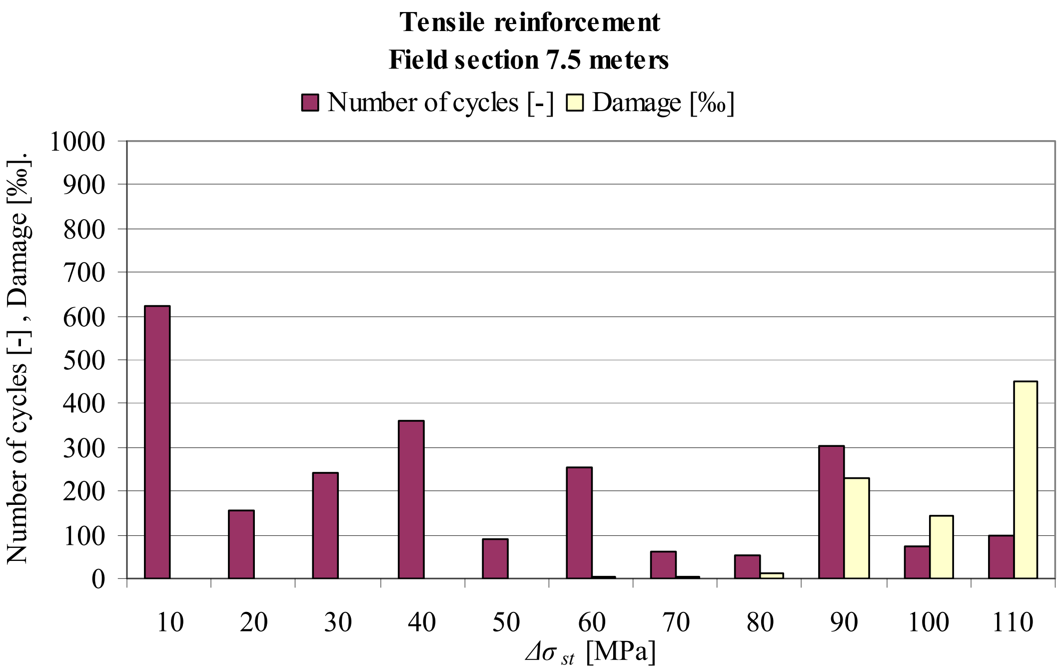



*Figure 6.11 The number of cycles versus the stress range for span 7.5 meters, extracted from Figure 6.10 .*

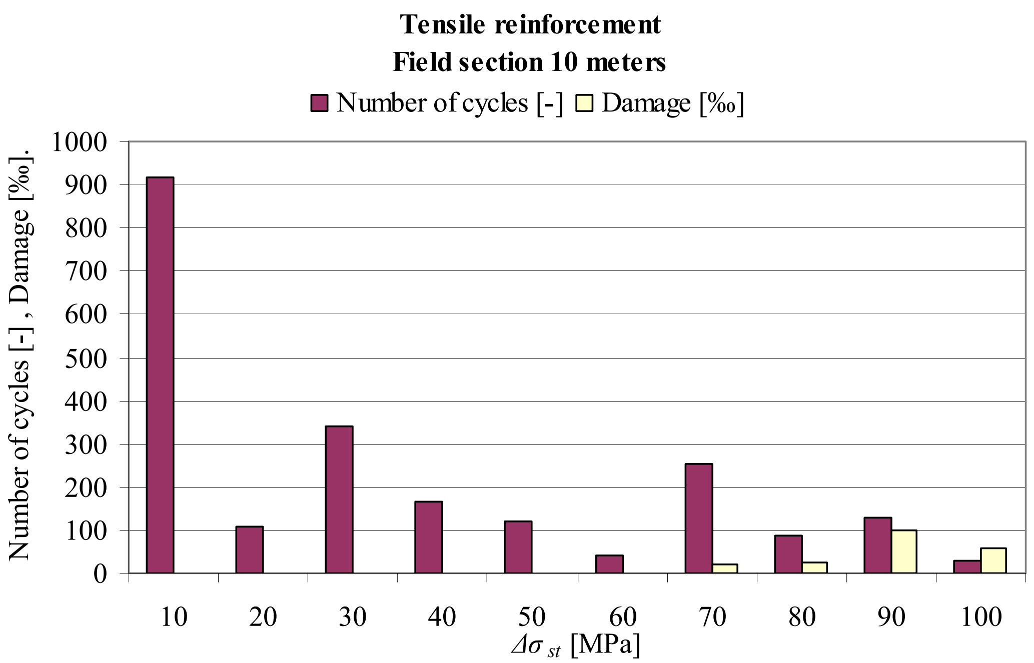



*Figure 6.12 The number of cycles versus the stress range for span 10 meters, extracted from Figure 6.10 .*

Figure 6.10 and Table 6.8 shows those bridges with longer spans experience less number of cycles as well as lower damage than bridges with shorter spans. It can also be seen that the total number of cycles do not affect the damage significantly sincemost of the damage develops during the larger stress amplitudes.

## 6.3 Parametric study regarding the span with a model of a simply supported bridge

The study of the simply supported bridge model was performed on shorter spans to study how shorter concrete bridges respond to railway traffic. The different spans, cross sectional properties, design bending moments in the Ultimate Limit State (ULS) and bending moments induced by the permanent loads are presented in Table 6.9. The calculations were performed in the same manner as with the continuous bridge model.

*Table 6.9 Geometrical properties, amount of reinforcement bars and bending moments extracted from StripStep2 for the maximum moment section.*

| | *Sectional height* | *Moment ULS* | *Moment due to permanent load* | *Amount of reinforcement bars per meter* | |
|---|---|---|---|---|---|
| *L [m]* | *h [m]* | $M_{Ed.f}$ *[kNm]* | $M_{Perm.f}$ *[kNm]* | *Tension* | *Compression* |
| 2.0 | 0.20 | 308 | 39 | 12.81 | 11.25 |
| 2.5 | 0.20 | 489 | 61 | 22.66 | 11.25 |
| 3.5 | 0.27 | 974 | 131 | 28.75 | 11.25 |
| 5.0 | 0.39 | 1 904 | 308 | 33.91 | 11.25 |
| 7.5 | 0.58 | 4 151 | 846 | 44.34 | 11.25 |
| 10.0 | 0.77 | 7 164 | 1 775 | 54.55 | 11.25 |

### 6.3.1 Results for the λ-Coefficient Method

In the λ-Coefficient Method the maximum and minimum moment caused by the traffic load LM71 should be used. The maximum moments for different spans are presented in Table 6.10. Due to the simply supported model the minimum moment is always zero and is therefore not presented.

*Table 6.10 The maximum moment caused by the traffic load LM71 for the λ-Coefficient Method.*

| *L [m]* | 2.0 | 2.5 | 3.5 | 5.0 | 7.5 | 10.0 |
|---|---|---|---|---|---|---|
| $M_{max}$ *[kNm]* | 125.0 | 137.4 | 244.8 | 482.4 | 1 099.5 | 1 855.6 |

The results for the tensile reinforcement and for the compressed concrete are presented in Table 6.11, Figure 6.13 and Figure 6.14.

*Table 6.11 The value of the design criteria for the λ-Coefficient Method, simply supported bridge model.*

| *λ-Coefficient Method* | | | | | | |
|---|---|---|---|---|---|---|
| *L [m]* | 2.0 | 2.5 | 3.5 | 5.0 | 7.5 | 10.0 |
| *Tensile Reinforcement* | 2.28 | 1.42 | 1.24 | 1.10 | 0.96 | 0.80 |
| *Compressive Concrete* | 1.27 | 1.21 | 1.09 | 0.97 | 0.91 | 0.85 |

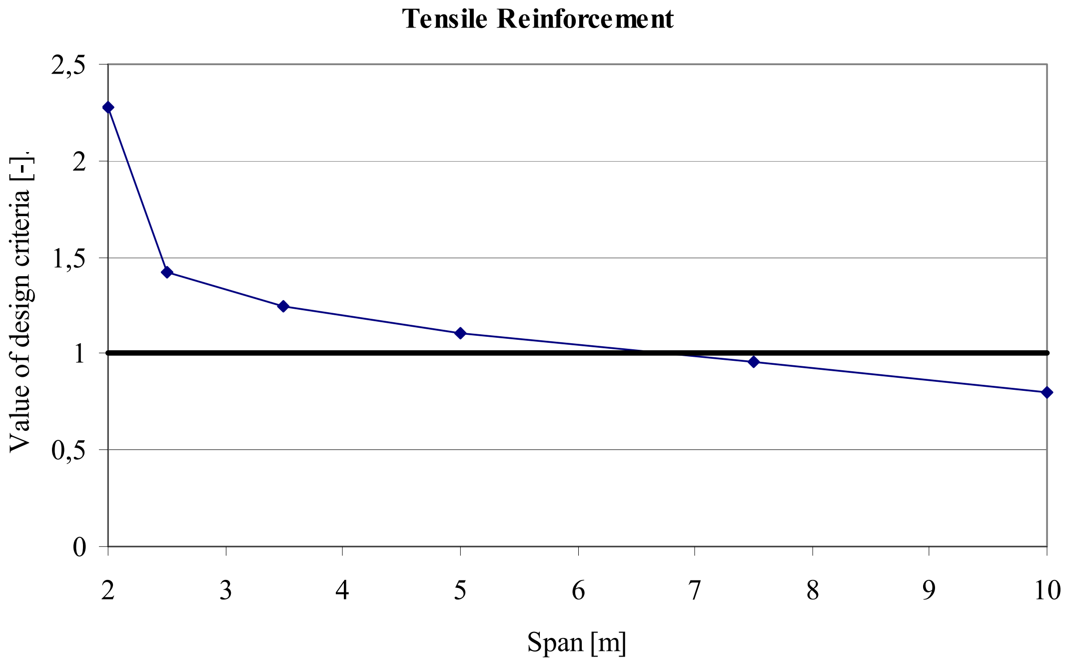



*Figure 6.13 Result for the tensile reinforcement with the λ-Coefficient Method, simply supported bridge model.*

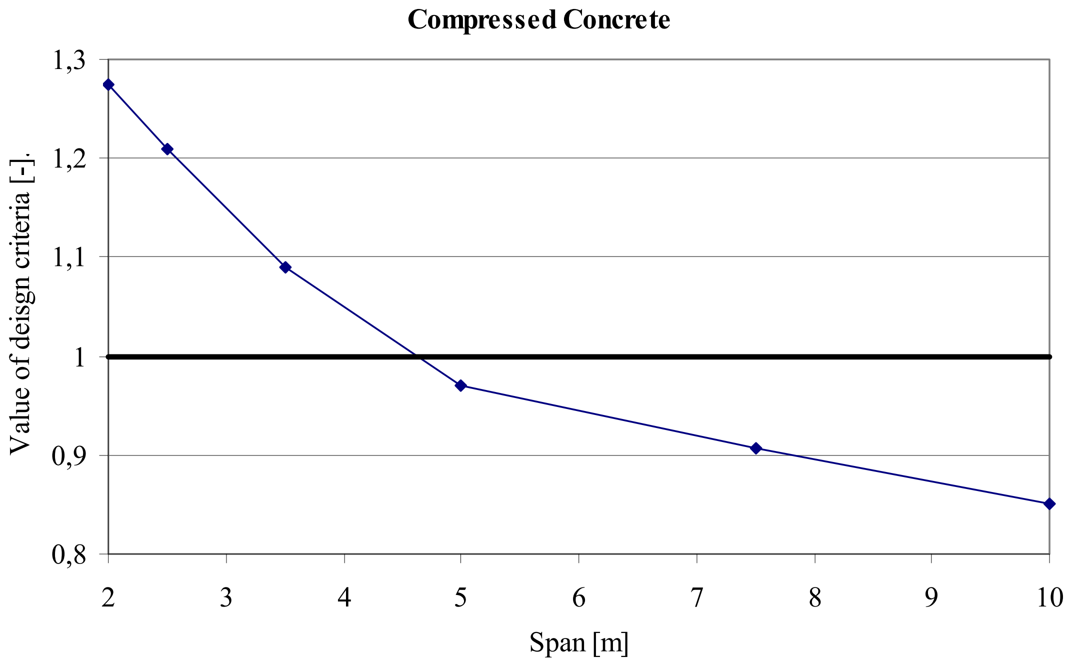



*Figure 6.14 Result for the λ-Coefficient Method for the compressed concrete, simply supported bridge model.*

### 6.3.2 Results for the Cumulative Damage Method

The results from the study for the tensile reinforcement and the compressed concrete are presented in Table 6.12. Figure 6.15 and Figure 6.16 visualises the same results and they show how the damage change with the varying span. When the value of the damage is above 1 the fatigue life of the structure has been reached. For additional results see Appendix B.

*Table 6.12 The value of the damage for the Cumulative Damage Method, simply supported bridge model.*

| *Cumulative Damage Method* | | | | | | |
|---|---|---|---|---|---|---|
| *L [m]* | 2.0 | 2.5 | 3.5 | 5.0 | 7.5 | 10.0 |
| *Tensile Reinforcement* | 92.32 | 18.50 | 4.34 | 1.42 | 0.18 | 0.05 |
| *Compressive Concrete* | 698.76 | $2.96\cdot10^{4}$ | 29.83 | 0.06 | $3.75\cdot10^{-4}$ | $3.50\cdot10^{-5}$ |

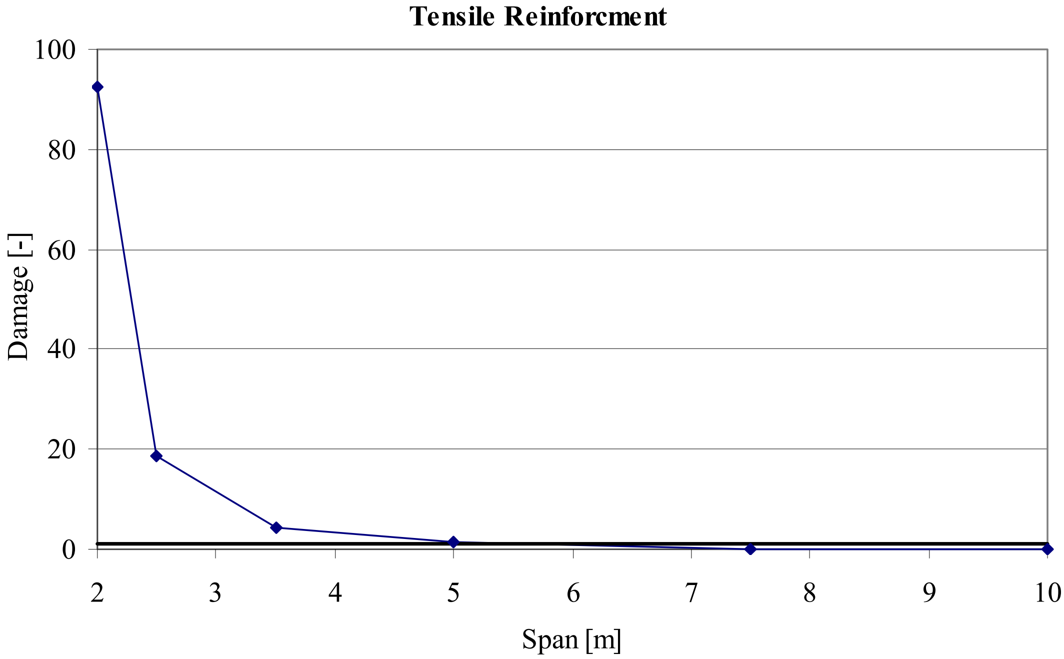



*Figure 6.15 Result from fatigue assessment calculations for the Cumulative Damage Method for tensile reinforcement, simply supported bridge model.*

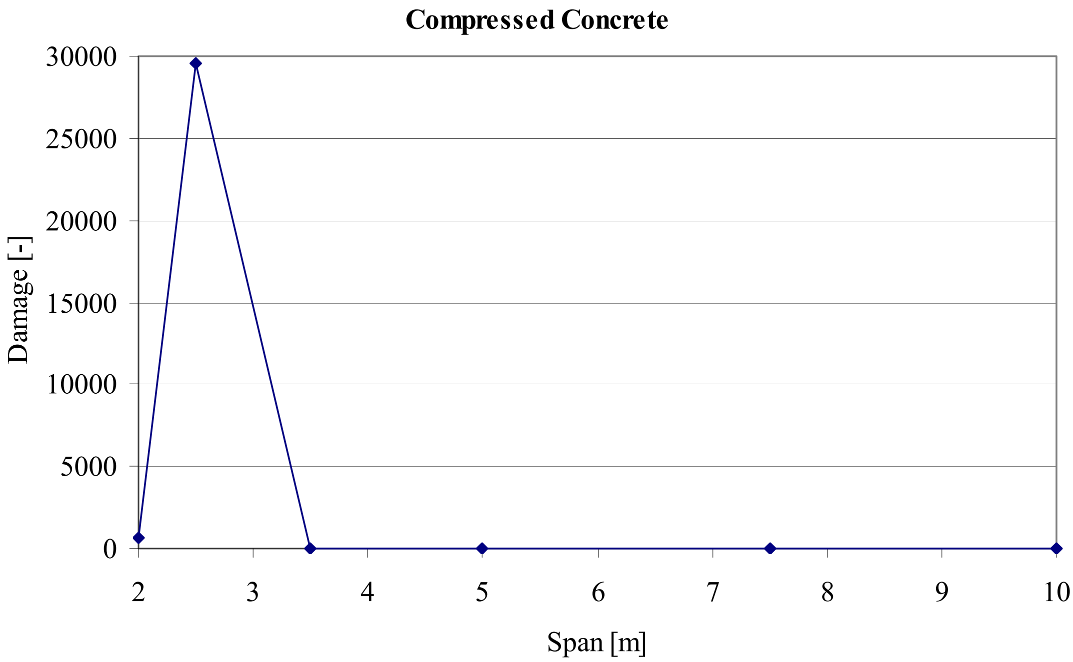



*Figure 6.16 Result from fatigue assessment calculations for the Cumulative Damage Method for compressed concrete, simply supported bridge model.*

### 6.3.3 Analysis of the results with regard to the tensile reinforcement

To be able to more easily compare the results from the two fatigue assessment methods, the results are combined into one figure, Figure 6.17. The figure present both fatigue assessments method and both considered sections in order to see how the methods develop in comparison to each other. Note that the calculated values from the fatigue assessment methods are not directly comparable. However, it is possible to see when the methods become decisive in design, i.e. when the value is above one.

While observing the results for the $\lambda$-Coefficient Method it can be seen that the tensile reinforcement exceeds the allowed limit at spans shorter than 7 meters. When looking at the Cumulative Damage Method the allowed limit will be exceeded at spans shorter than 6 meters.

Therefore it can be said that the methods correspond rather well to each other for this particular case.

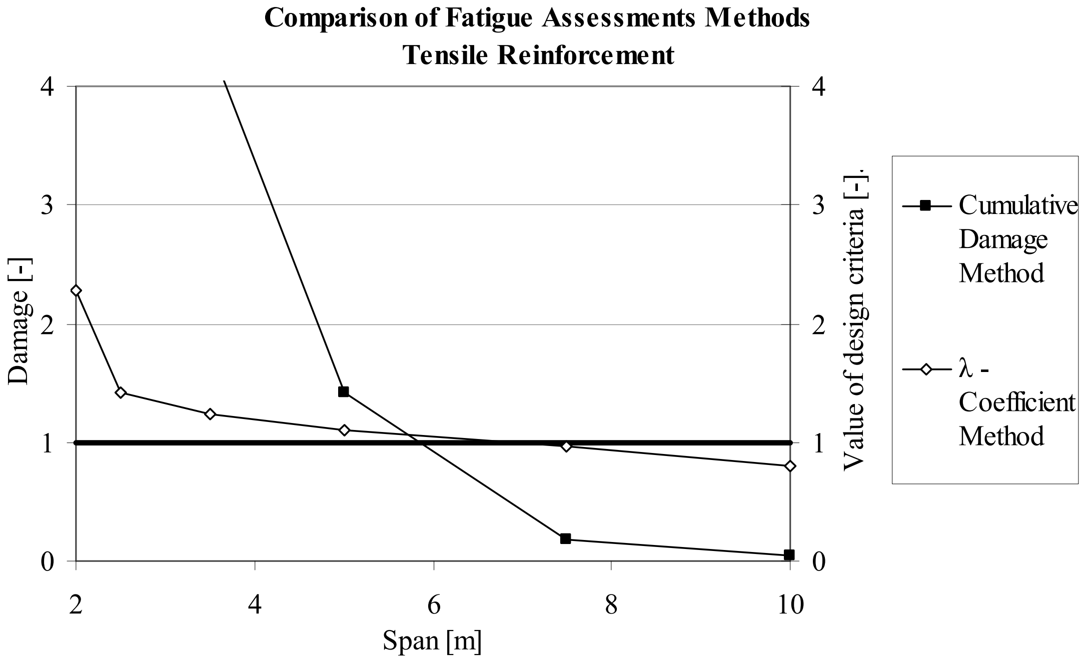



*Figure 6.17 The results with regard to the tensile reinforcement from both the Cumulative Damage Method and the $\lambda$-Coefficient Method. Note that the left y-axis corresponds to the Cumulative Damage Method and the right to the $\lambda$-Coefficient Method.*

### 6.3.4 Analysis of the results for the compressed concrete

When observing the response of the compressed concrete, Figure 6.16, it can be seen that for spans longer than 5 meters the damage established with the Cumulative Damage Method is found to be close to zero. However, for spans shorter than 5 meter the damage is well above the allowed limit. It can also bee seen that the damage drastically decreases for the span of 2 meters. This behaviour can be derived to the demand of minimum thickness for reinforced concrete slabs, explained in Section 6.1. The demand is governing for the sectional height for both 2 and 2.5 meters although the impact on the damage calculations is most obvious for the shorter span. The adjustment of the height for 2.5 meter is only marginal.

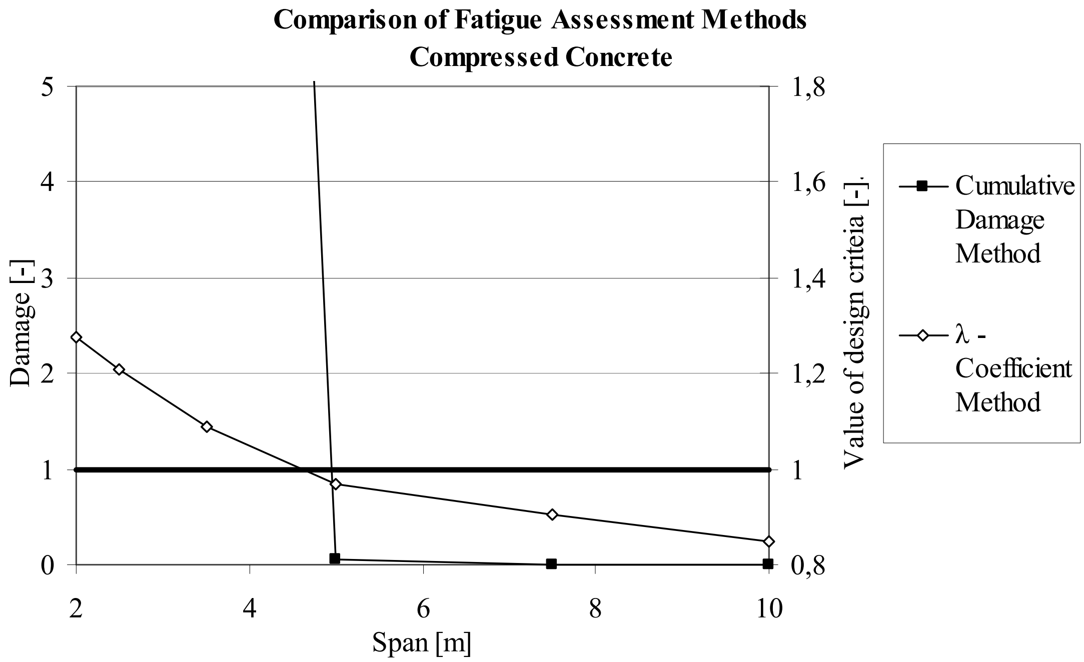



*Figure 6.18 Visualises the results from both methods with regard to the compressed concrete. Note that the left y-axis corresponds to the Cumulative Damage Method and the right to the λ-Coefficient Method.*

## 6.4 Comparison of the different bridge models

To be able to compare the results from the two bridge models the values where combined into two figures, one for each assessment method. The figures visualises results from both field and support sections from the continuous bridge model together with the simply supported bridge model. Figure 6.20 represents the λ-Coefficient Method and Figure 6.19 the Cumulative Damage Method.

While observing the results for the Cumulative Damage Method it can be seen that the simply supported bridge model and the support section for the continuous bridge model approximately have the same behaviour. The allowed limit is exceeded around a span of 6 meters. It could also be said that the field section has the same behaviour although the rapid increase in damage occurs for a bit longer spans than for the other two.

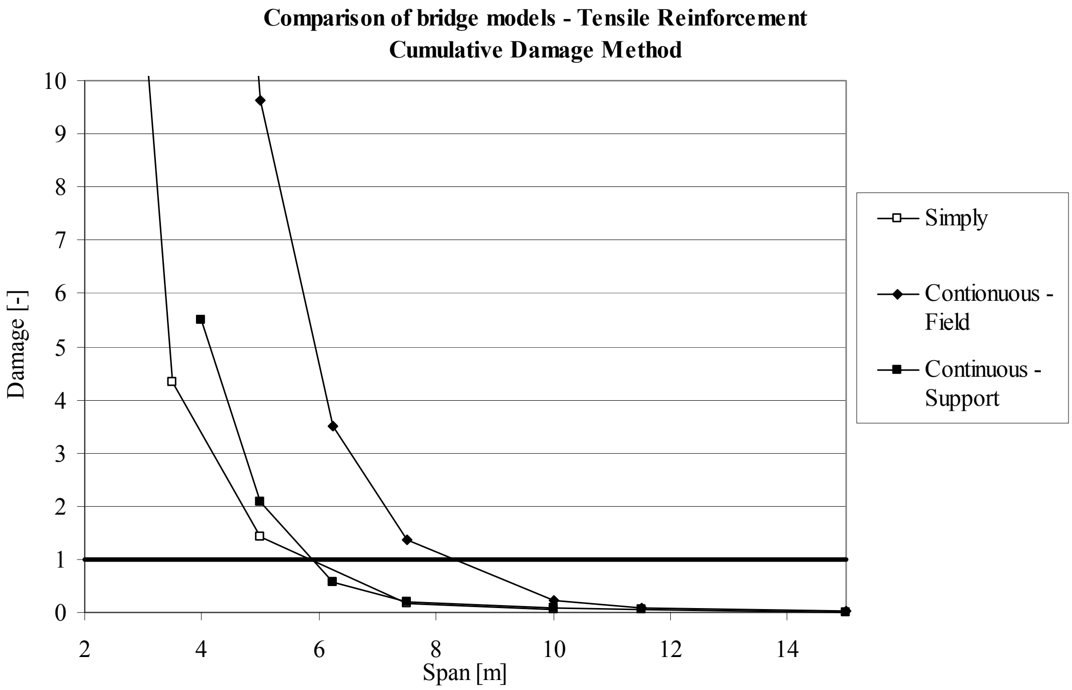



*Figure 6.19 Visualises the results from the Cumulative Damage Method for the simply supported model, and field and support section for the continuous model.*

For the $\lambda$-Coefficient Method, the development is the same for both bridge models down to a span of approximately 8 meter; then all three considered sections starts to diverge from each other. The allowed limit is exceeded for a bit longer spans than for the Cumulative Damage Method, 7 meter for both the simply supported model and the support section for the continuous model.

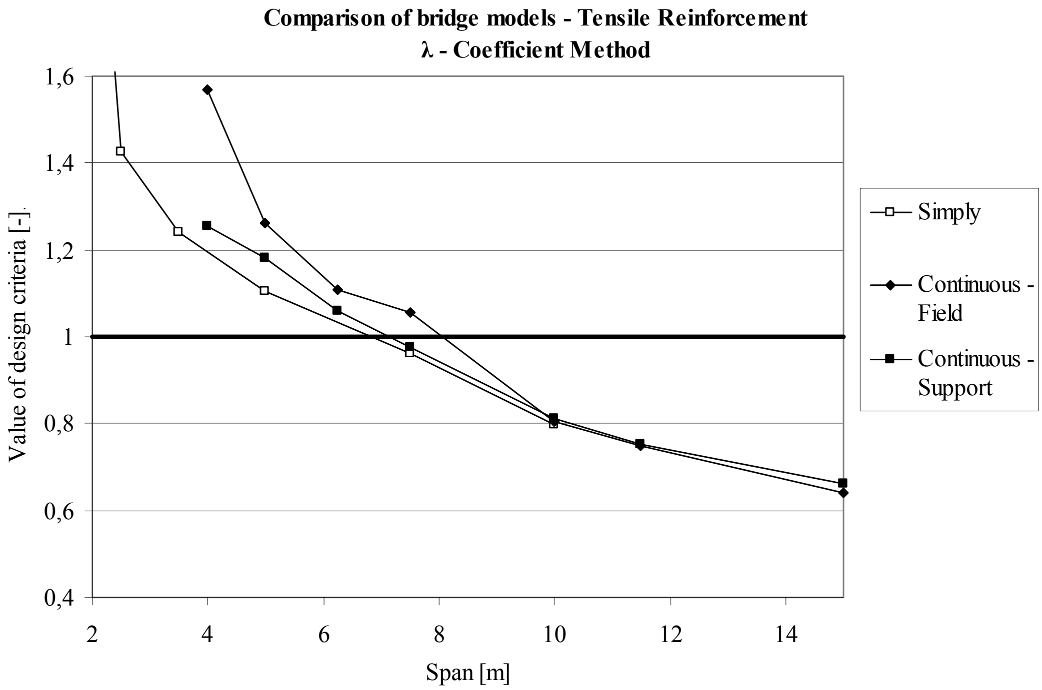



*Figure 6.20 Visualises the results from the λ-Coefficient Method for the simply supported model, and field and support section for the continuous model.*

In order to observe some kind of relation between the bridge models, the results for the Cumulative Damage Method are more thoroughly evaluated. The stress ranges for the tensile reinforcement are compared with the number of cycles in the same manner as for the deepened analysis of the continuous bridge model. The stress is calculated for the spans where the damage was found to be closest to the design limit.

Spans considered for the different models was 5 meters for the simply supported bridge model, 7.75 meters for the field section and 5.625 meters for the support section for the continuous bridge model, see Table 6.13 and Figure 6.21. While analysing the results for the stress range it can be seen that the stress range in the tensile reinforcement is approximately the same for the considered spans, 50 MPa. Also the number of cycles is almost the same for the considered sections, around 2200 cycles.

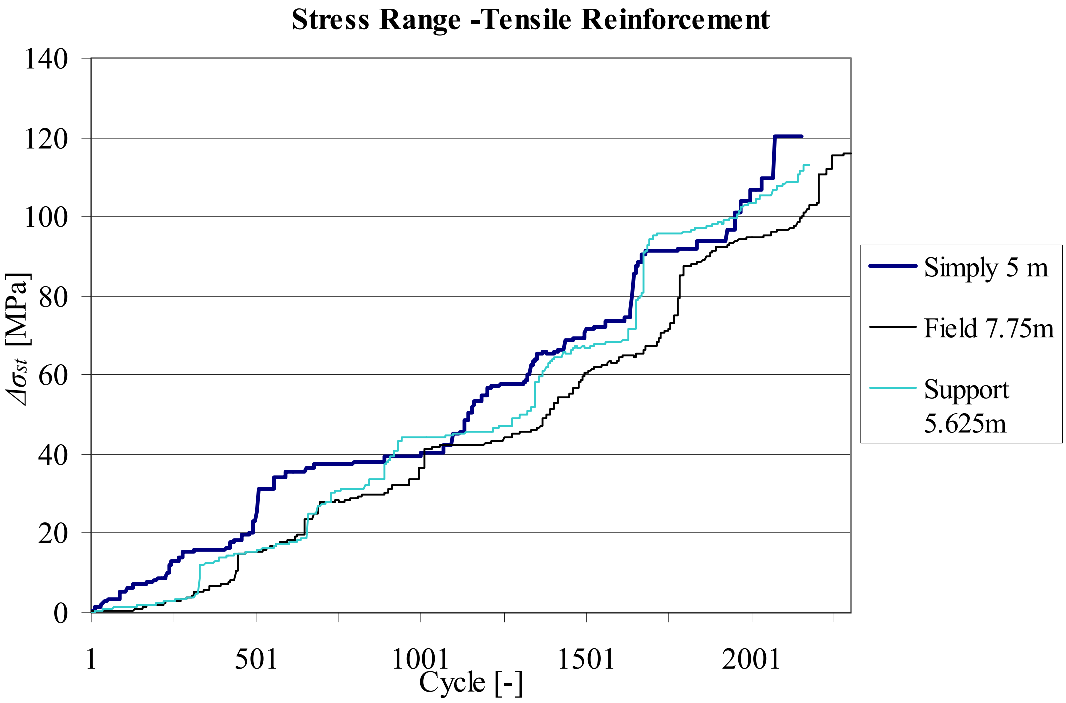



*Figure 6.21 Visualises the stress ranges, which are sorted by size, versus the number of cycles for the Cumulative Damage Method for the considered sections.*

*Table 6.13 The mean stress range in the tensile reinforcement, the total number of cycles and the total calculated damage for the considered sections.*

| *Span* | *Mean stress range in the tensile reinforcement* | *Number of cycles per day* | *Total damage over 120 years* |
|---|---|---|---|
| *L [m]* | *Mean $\Delta\sigma_{st}$ [MPa]* | *n [-]* | *D [-]* |
| 5.0 | 52.77 | 2 152 | 1.42 |
| 5.625 | 48.96 | 2 176 | 1.15 |
| 7.75 | 46.91 | 2 301 | 1.01 |

## 6.5 Study of adjusted continuous bridge models regarding the fatigue design criteria's

This study compares the two fatigue assessment methods for the cases when the design criterion becomes decisive for the bridge design. This means that e.g. the criteria given in the Cumulative Damage Method, equation 5.8, is equal to 1. In order to do this the tensile reinforcement in a number of bridges is adjusted until the required value is obtained. The study is focused on the spans when any of the methods shows a value which gives failure. In general this implies lengths shorter than 8 meters.

### 6.5.1 Results for the λ-Coefficient Method

In Figure 6.22 and Figure 6.23 the values of the design criterion for the tensile reinforcement shown. To be able to compare the results, the values from the rewritten design criterion according to equation 5.20 is used. In the figures there are three different values in display. The first column is the same value as was presented in Section 6.2 and corresponds to the value obtained for the ULS design. The second column is the value of the design criterion when the amount of reinforcement is adjusted according to the Cumulative Damage Method, i.e. the value of the damage is equal to 1. The amount of reinforcement is used in the $\lambda$-Coefficient Method in order to obtain the corresponding value of the design criterion. The transparent column displays the level of the $\lambda$-Coefficient Method design i.e. the value of the design criterion is 1.

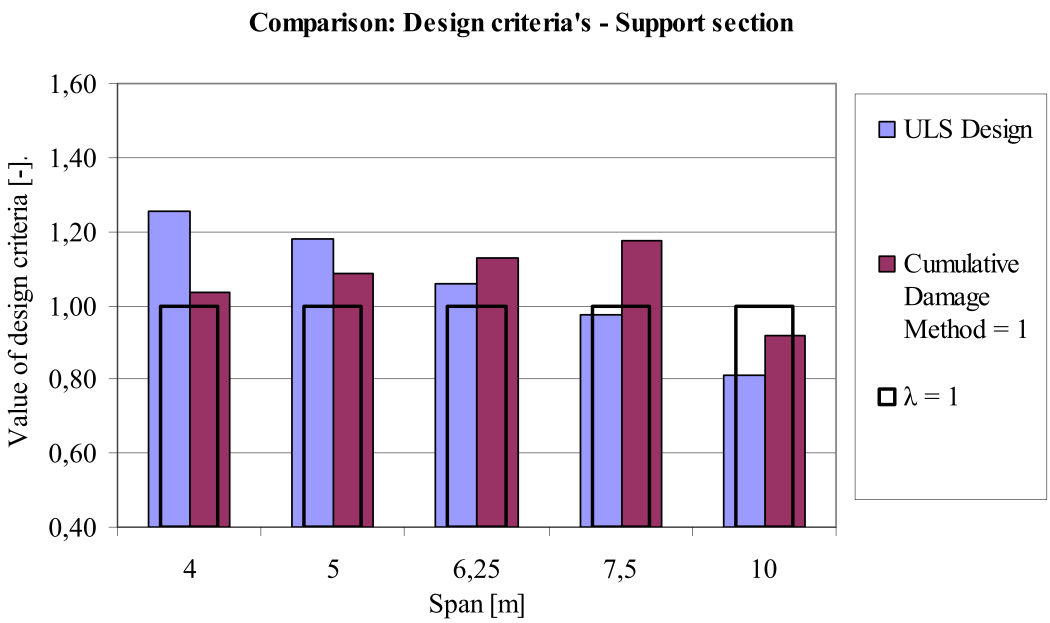



*Figure 6.22 Comparison of the results for the λ-Coefficient Method when different design criteria's govern the design of the support section.*

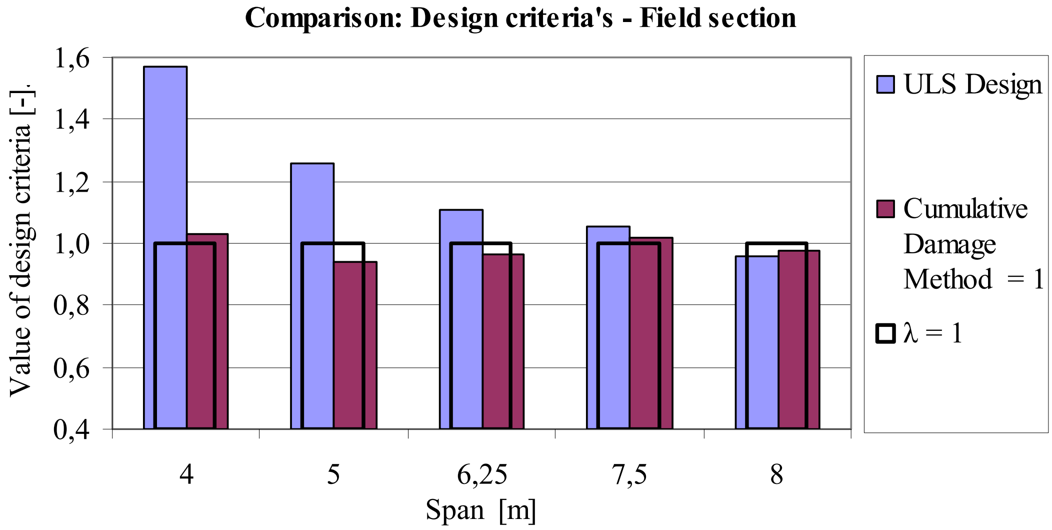



*Figure 6.23 Comparison of the results for the λ-Coefficient Method when different design criteria's govern the design of the field section.*

### 6.5.2 Results for the Cumulative Damage Method

In Figure 6.24 and Figure 6.25 the results from the study regarding the Cumulative Damage Method is shown. The first column is the same as was presented in Section 6.2 and corresponds to the ULS design. The second column describes the value for the damage for the Cumulative Damage Method when the amount of reinforcement is adjusted according to the criteria in the $\lambda$-Coefficient Method. The value of the design criterion for the $\lambda$-Coefficient Method is set equal to 1. The amount of reinforcement is then used in the Cumulative Damage Method in order to obtain a value of damage The transparent column displays the level of the Cumulative Damage Method design i.e. the value of the damage is 1.

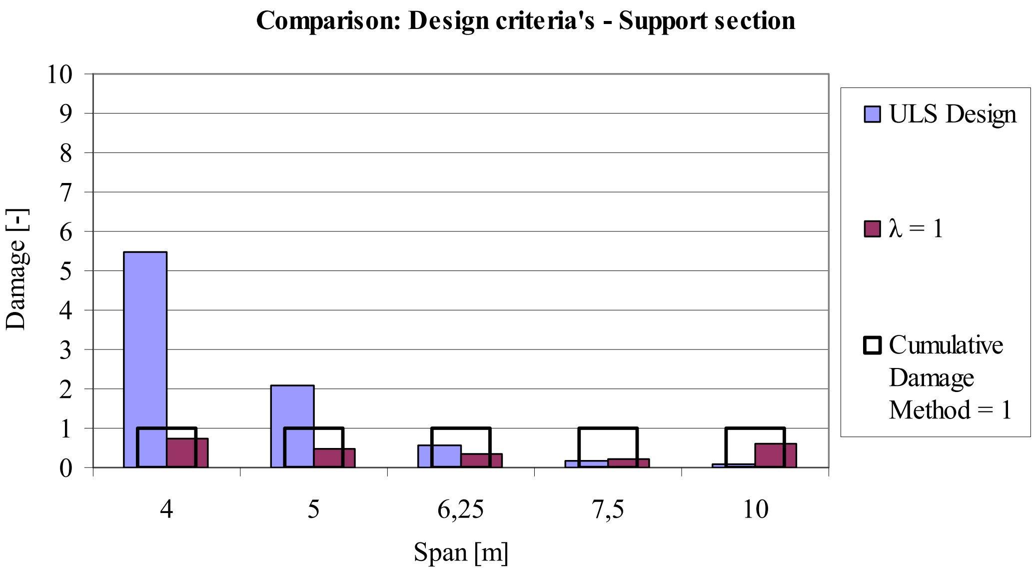



*Figure 6.24 Comparison of the results for the Cumulative Damage Method when different design criteria's govern the design of the support section.*

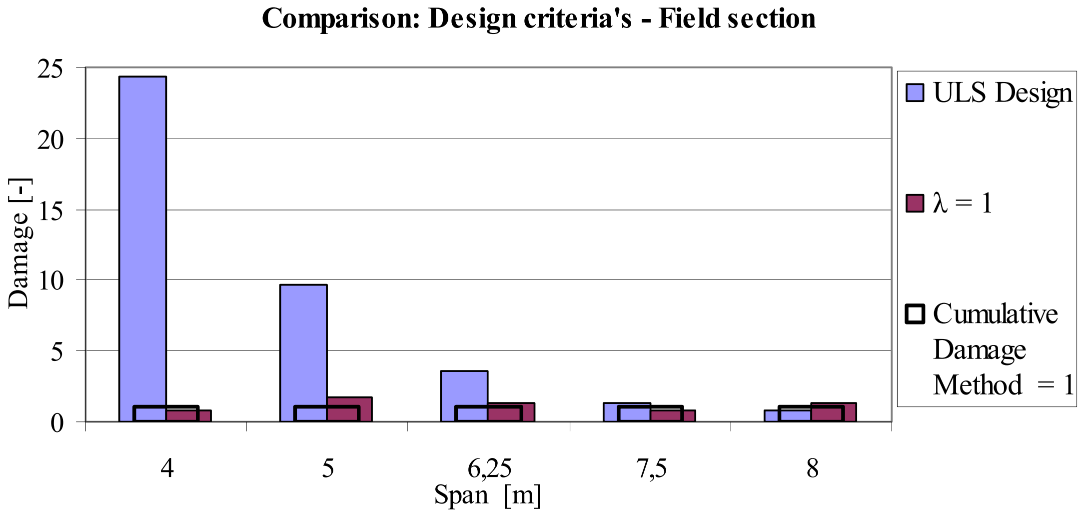



*Figure 6.25 Comparison of the results for the Cumulative Damage Method when different design criteria's govern the design of the field section.*

The amount of tensile reinforcement obtained for the different cases is showed in Figure 6.26 and Figure 6.27. All values are presented in Appendix E.

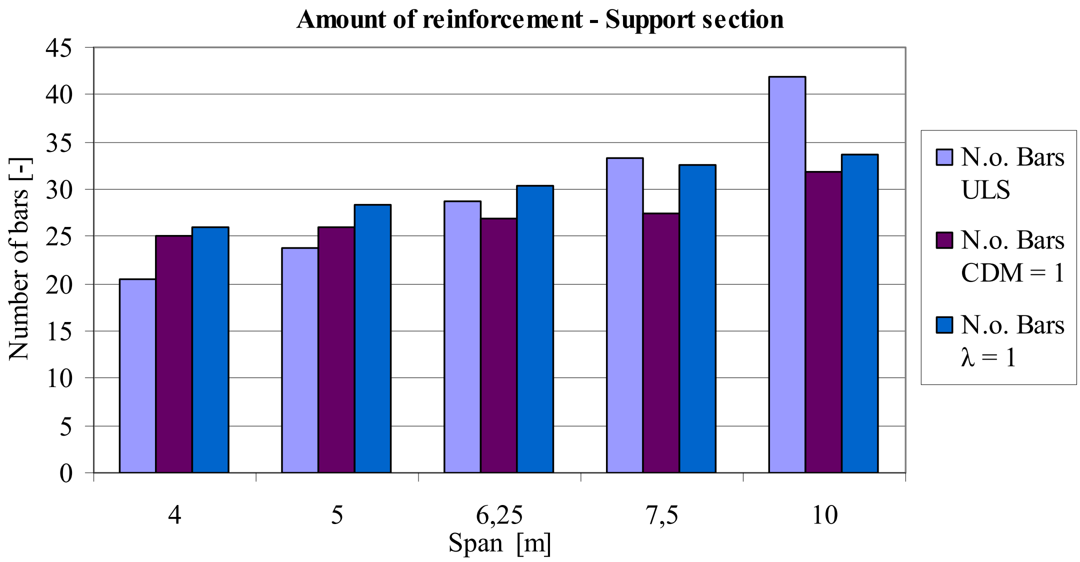



*Figure 6.26 Amount of reinforcement for the different cases in the support section.*

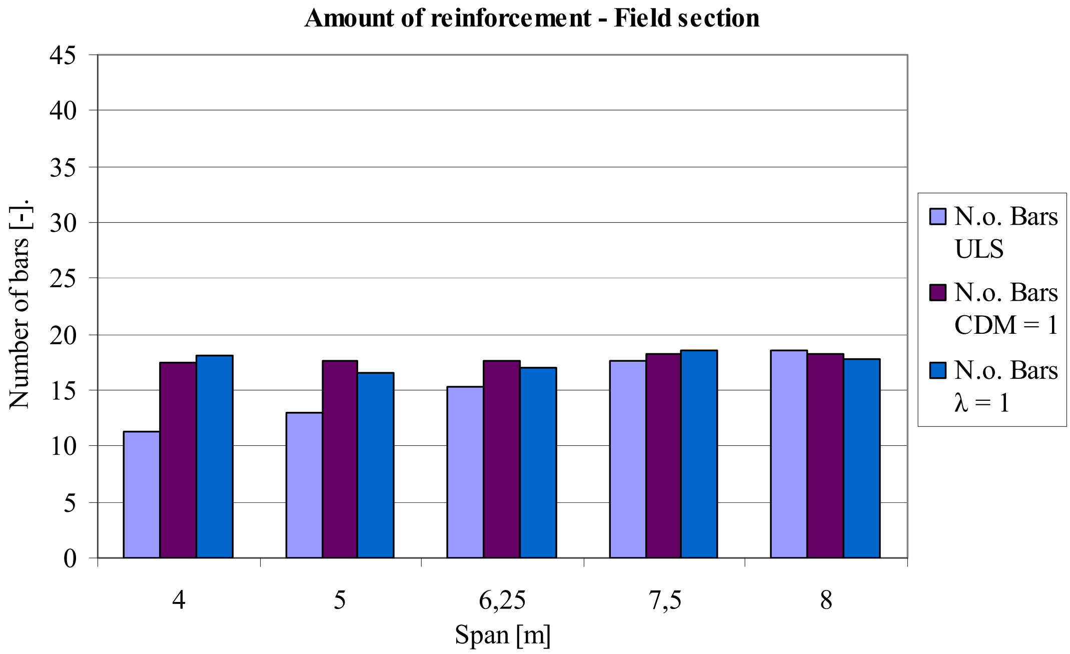



*Figure 6.27 Amount of reinforcement for the different cases in the field section.*

### 6.5.3 Study of the sensitivity of the fatigue calculations

In this study the sensitivity of the fatigue calculations for the Cumulative Damage Method regarding the reinforcement amount was further investigated. The purpose was to follow the development of the damage in order to see what impact a relatively small adjustment of the reinforcement amount might have. The cases chosen for this study was the span's that had the value closest to 1 for the ULS-design in the considered sections. The reinforcement amount was adjusted up or down ten percentages from the amount achieved in the ULS-design. The result is presented in Figure 6.28.

*Table 6.14 The result of the damage from the sensitivity analysis, with the adjusted reinforcement amount in ULS-design.*

| | *Damage [-]* | | | *Amount of reinforcement bars* | | |
|---|---|---|---|---|---|---|
| *L [m]* | *ULS -10 %* | *ULS Design* | *ULS +10 %* | *ULS -10 %* | *ULS Design* | *ULS +10 %* |
| 5 | 3.49 | 1.42 | 0.63 | 30.52 | 33.91 | 37.30 |
| 7.75 | 2.53 | 1.01 | 0.44 | 16.29 | 18.10 | 19.91 |
| 5.625 | 2.87 | 1.15 | 0.51 | 23.57 | 26.19 | 28.81 |

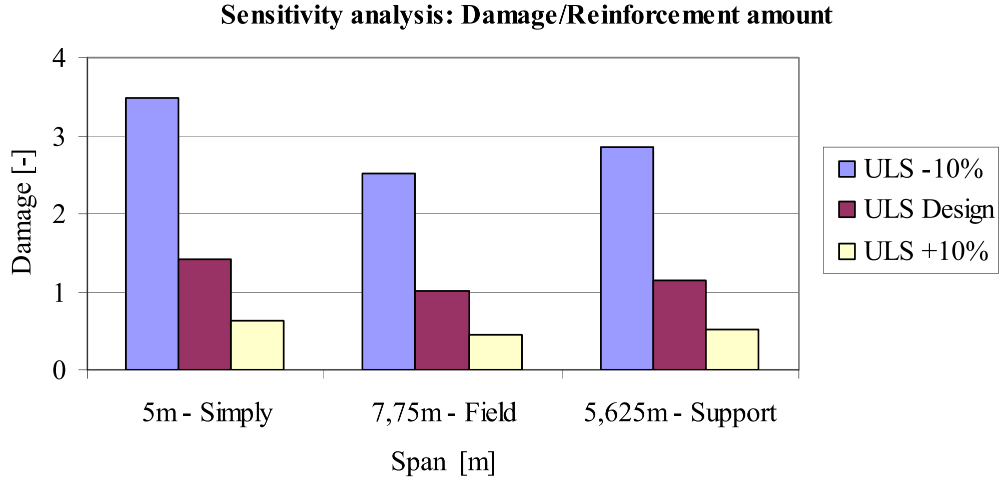



*Figure 6.28 Sensitivity analysis of the Damage development due to the reinforcement amount. The amount is adjusted up or down ten percentages from the amount in ULS-design.*

### 6.5.4 Analysis of the study regarding the Design criteria's with the adjusted continuous bridge model

In this study the amount of tensile reinforcement where adjusted in order to achieve values that is comparable when considering the two assessment methods. Another purpose was to monitor the behaviour of the development of the results in relation to the amount of reinforcement for certain spans. The results are displayed in a number of figures which shows the different calculated values or damages.

First, in the comparison of the damages obtained for the Cumulative Damage Method, Figure 6.24 and Figure 6.25, it can be noted that the damage rapidly increases for the ULS-design with decreasing span. This is valid for both support and field sections although the increase rate is significantly higher in the field section.

Further, it seems that the Cumulative Damage Method gives a safer value compared to the $\lambda$-Coefficient Method consistently in the results for the support section. This is however not the case in the field section where the design criterion varies between the two methods with the changing span i.e. the methods correspond rather well to eachother.

It could also be noted that the ULS-design seldom are the governing criteria for shorter spans, especially in the field section. By this it means that the damage calculated for the ULS-design is well above the allowed limit so that the fatigue criteria's rules the design instead.

In the sensitivity analysis, the tensile reinforcement amount is adjusted up, or down, ten percentages in order to see how sensitive the damage development are for the Cumulative Damage Method. Figure 6.28 shows that the adjustment causes a difference in damage of close to 2.0.

The conclusion is that the method is very sensitive in the area around the design criteria. The damage can be seen as rather low for a certain case, although a small decrease in the reinforcement amount could alter the conditions markedly.

# 7 Concluding remarks

The purpose of this project was to investigate and compare the two fatigue assessment methods for reinforced concrete bridges available in Eurocode; the Cumulative Damage Method and the $\lambda$-Coefficient Method. This has been achieved by parametric studies of models of simply supported and continuous railway bridges subjected to fatigue loading. The results from the two methods have been analysed, both by identifying the differences in outcome of the methods and by studying how the result is influenced by different key parameters within each of the methods.

From the analysis in Chapter 6, the following conclusions regarding the comparison of the fatigue assessment methods can be made:

- The results from the fatigue assessment with the Cumulative Damage Method have implied to be very sensitive around the area where the allowed limit is reached. A small adjustment in the sectional properties changes the conditions markedly.
- The geometry and boundary conditions for the different bridge models and sections does not give any large impact on the results for the $\lambda$-Coefficient Method. For the Cumulative Damage Method on the other hand the effects are clearly shown.
- The largest stress ranges in combination with the number of cycles by which these ranges occur is clearly governing for the total damage obtained for the Cumulative Damage Method. This is independent on which section or model that is considered.
- For the Cumulative Damage Method: The total amount of cycles is not the leading parameter in predicting the damage. It is the amount of high amplitude cycles.
- The results for the $\lambda$-Coefficient Method shows a more linear behaviour and will not be affected as much by the variation in span or by changing the amount of tensile reinforcement.
- It can also be seen that the results for the $\lambda$-Coefficient Method for both sections and for the field section for the Cumulative Damage Method will exceed the allowed limit around the same span for the tensile reinforcement.
- In cases studied, when the results are close to the allowed limit, both the values of the stress ranges and the number of cycles correspond rather well to each other.

## 7.1 Suggested future research

While working with this project a large number of suggested topics on further research have been identified. In the area of reinforced concrete railway bridges subjected to bending there are some issues that need attention. One issue is the combination of different sectional forces. The bridges treated in this thesis are subjected to pure bending and therefore the concrete stress due to the fatigue loading becomes relatively low. With a prestressing force or with a combination of bending moment and normal force, e.g. in a leg of a frame bridge, the situation could be quite different.

An obvious complementary study would be to investigate similar bridges but subjected to road traffic.

Further studies using models of actual continuous bridges instead of the simplified model used in this thesis project could be of interest. This is mainly due to the fact that the total sectional forces might be lower than the permanent ones causing higher amplitudes for both the Cumulative Damage Method and the λ-Coefficient Method.

A similar study as the previous but focusing on the effect of shear forces is also proposed. At this stage models for calculating compressive concrete stresses due to shear exist for both methods. Methods to calculate the stresses in the shear reinforcement under fatigue loading are however not yet fully developed. Issues that might be of interest are e.g. the influence of the inclination of the compressive strut, the reduction of loads close to the supports and how to consider the haunch in the calculations.

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# Appendix A. Results from the study with varying span for the Continuous Bridge model

Before a fatigue assessment for the continuous bridge model can be performed needs e.g. geometrical properties and bending moments in maximum moment section be known, these parameters are presented in Table A.1.

*Table A.1 Geometrical properties and bending moments extracted from StripStep2 in maximum moment sections*

| | *Height of cross section* | | *Length of segment* | | *Moment in the ULS* | | *Permanent moment* | |
|---|---|---|---|---|---|---|---|---|
| *L* *[m]* | $h_0$ *[m]* | $h_{10}$ *[m]* | $x_1$ *[m]* | $x_2$ *[m]* | $M_{Ed.s}$ *[kNm]* | $M_{Ed.f}$ *[kNm]* | $M_{Ed.Perm.s}$ *[kNm]* | $M_{Ed.Perm.f}$ *[kNm]* |
| 4 | 0.31 | 0.47 | 2.57 | 1.43 | (-) 1 625 | 534 | (-) 247 | 80 |
| 5 | 0.39 | 0.59 | 3.21 | 1.79 | (-) 2 469 | 819 | (-) 423 | 137 |
| 5.625 | 0.43 | 0.67 | 3.62 | 2.01 | (-) 3 107 | 1 026 | (-) 566 | 183 |
| 6.25 | 0.48 | 0.74 | 4.02 | 2.23 | (-) 3 833 | 1 256 | (-) 739 | 237 |
| 6.875 | 0.53 | 0.82 | 4.42 | 2.46 | (-) 4 614 | 1 517 | (-) 939 | 301 |
| 7.50 | 0.58 | 0.89 | 4.82 | 2.68 | (-) 5 451 | 1 802 | (-) 1 171 | 375 |
| 7.75 | 0.60 | 0.92 | 4.98 | 2.77 | (-) 5 808 | 1 921 | (-) 1 276 | 408 |
| 8 | 0.62 | 0.95 | 5.14 | 2.86 | (-) 6 184 | 2 050 | (-) 1 395 | 446 |
| 8.75 | 0.68 | 1.04 | 5.63 | 3.13 | (-) 7 314 | 2 442 | (-) 1 745 | 556 |
| 9.375 | 0.72 | 1.11 | 6.03 | 3.35 | (-) 8 330 | 2 802 | (-) 2 085 | 666 |
| 10 | 0.77 | 1.19 | 6.43 | 3.57 | (-) 9 426 | 3 183 | (-) 2 470 | 787 |
| 11.50 | 0.89 | 1.36 | 7.39 | 4.11 | (-) 12 356 | 4 207 | (-) 3 572 | 1 137 |
| 15 | 1.16 | 1.78 | 9.64 | 5.36 | (-) 21 129 | 7 231 | (-) 7 292 | 2 315 |

The amount of reinforcement bars which are used in fatigue assessment for field and support section is presented in Table A.2 and Table A.3. The amount of reinforcement bars are taken from ULS design.

*Table A.2 Amount of reinforcement bars in field section for the continuous bridge model*

| *Field* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.50 | 7.75 | 8 | 8.75 | 9.375 | 10 | 11.50 | 15 |
| *Amount of reinforcement bars* | | | | | | | | | | | |
| *Tension* | 11.24 | 12.99 | 15.24 | 17.60 | 18.10 | 18.61 | 19.90 | 21.25 | 22.44 | 25.38 | 32.60 |
| *Compression* | 11.25 | 11.25 | 11.25 | 12.80 | 13.26 | 13.71 | 15.07 | 16.20 | 17.33 | 20.04 | 26.37 |

*Table A.3 Amount of reinforcement bars in support section for the continuous bridge model*

| *Support* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 5.625 | 6.25 | 6.875 | 7.5 | 10 | 11.5 | 15 |
| *Amount of reinforcement bars* | | | | | | | | | |
| *Tension* | 20.46 | 23.86 | 26.19 | 28.72 | 30.98 | 33.32 | 41.96 | 47.19 | 60.93 |
| *Compression* | 11.25 | 11.25 | 11.25 | 11.25 | 11.25 | 11.25 | 11.25 | 12.77 | 16.89 |

The $\lambda$ - Coefficient Method depends on different parameters e.g. $\lambda$ – factors, traffic load induced moments. Some of theses parameters are presented in Table A.4. The maximum and minimum moments induced by traffic load, LM71, are only governing for the $\lambda$ - Coefficient Method. The minimum moment is not presented, because it is set to zero for all cases. The total $\lambda$ - factors for steel and concrete, including the modified $\alpha$ – factor, are presented in Table A.4 for considered sections.

The dynamic factor and the final creep factor are governing for both fatigue assessments methods, which are varying with chancing span, see Table A.4. Why there is some value that are marked with an “-“, this mean that this span is not considered for that specific section.

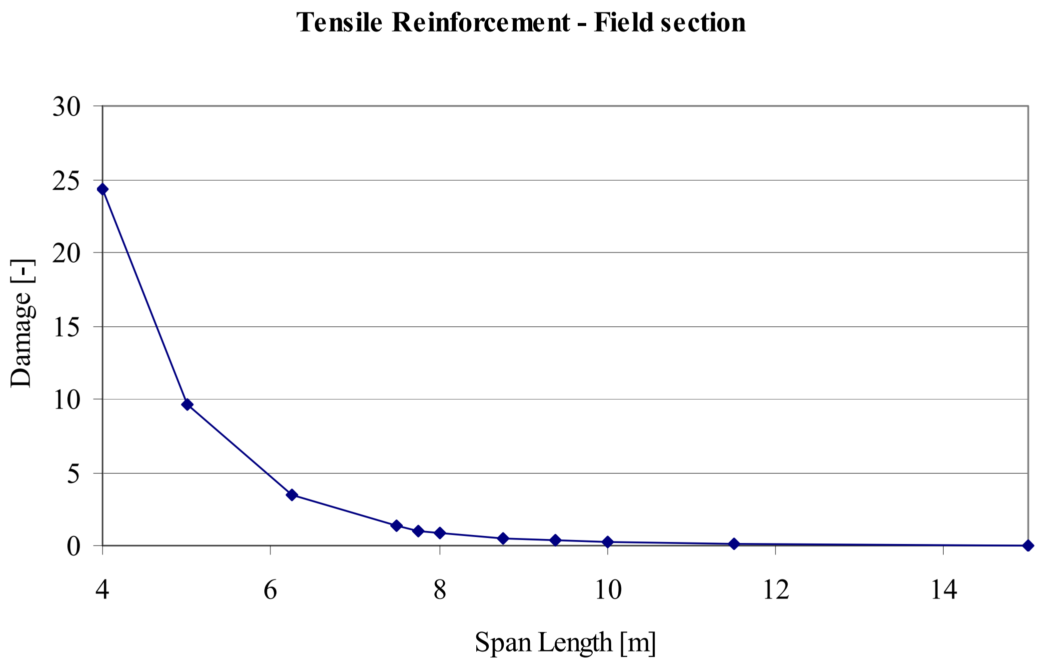
*Table A.4 Factors which are influenced by the span.*

| | *Dynamic factor* | *Final creep factor* | *Maximum Moments induced by traffic load, LM71* | | *Total Lambda factor in Field section* | | *Total Lambda factor in Support section* | |
|---|---|---|---|---|---|---|---|---|
| *L [m]* | $\Phi_2$ | $\varphi_{cr}$ | $M_s$ *[kNm]* | $M_f$ *[kNm]* | $\lambda_s$ | $\lambda_c$ | $\lambda_s$ | $\lambda_c$ |
| 4 | 1.54 | 1.50 | 433.2 | 165.5 | 1.09 | 0.93 | 1.00 | 0.94 |
| 5 | 1.46 | 1.48 | 677.3 | 228.0 | 1.02 | 0.89 | 0.93 | 0.93 |
| 5.625 | 1.42 | 1.46 | 826.8 | - | - | - | 0.94 | 0.92 |
| 6.25 | 1.39 | 1.45 | 1031.9 | 347.3 | 0.95 | 0.85 | 0.87 | 0.91 |
| 6.875 | 1.36 | 1.45 | 1247.9 | - | - | - | 0.90 | 0.89 |
| 7.50 | 1.33 | 1.44 | 1467.3 | 525.4 | 0.88 | 0.81 | 0.87 | 0.88 |
| 7.75 | 1.33 | 1.44 | - | 564.1 | 0.87 | 0.80 | - | - |
| 8 | 1.32 | 1.43 | - | 656.7 | 0.86 | 0.80 | - | - |
| 8.75 | 1.29 | 1.43 | - | 743.7 | 0.82 | 0.77 | - | - |
| 9.375 | 1.20 | 1.42 | - | 849.7 | 0.80 | 0.75 | - | - |
| 10 | 1.26 | 1.42 | 3037.7 | 1104.0 | 0.77 | 0.74 | 0.80 | 0.83 |
| 11.50 | 1.23 | 1.41 | 3037.7 | 1104.0 | 0.75 | 0.73 | 0.79 | 0.83 |
| 15 | 1.18 | 1.39 | 4821.9 | 1778.1 | 0.71 | 0.72 | 0.78 | 0.85 |

The results are considered for the tensile reinforcement and concrete. The results from the $\lambda$ -Coefficient Method in field section and support section for the Continuous Bridge Model are presented in Table A.5 and Table A.6, visualised in Figur A.1 to *Figure A.4*.

*Table A.5 Results for the λ -Coefficient Method in field section for the Continuous Bridge Model*

| *Field* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.50 | 7.75 | 8 | 8.75 | 9.375 | 10 | 11.50 | 15 |
| *λ - Coefficient Method* | | | | | | | | | | | |
| *Reinforcement* | [redacted] | | | | | | | | | | |
| *Tension* | 1.56 | 1.26 | 1.10 | 1.05 | 0.96 | 0.95 | 0.88 | 0.83 | 0.80 | 0.75 | 0.63 |
| *Concrete* | 0.87 | 0.77 | 0.72 | 0.70 | 0.67 | 0.67 | 0.65 | 0.64 | 0.63 | 0.62 | 0.59 |




*Figur A.1 Result from fatigue assessments calculation for the λ - Coefficient Method for reinforcement in tension in field section*

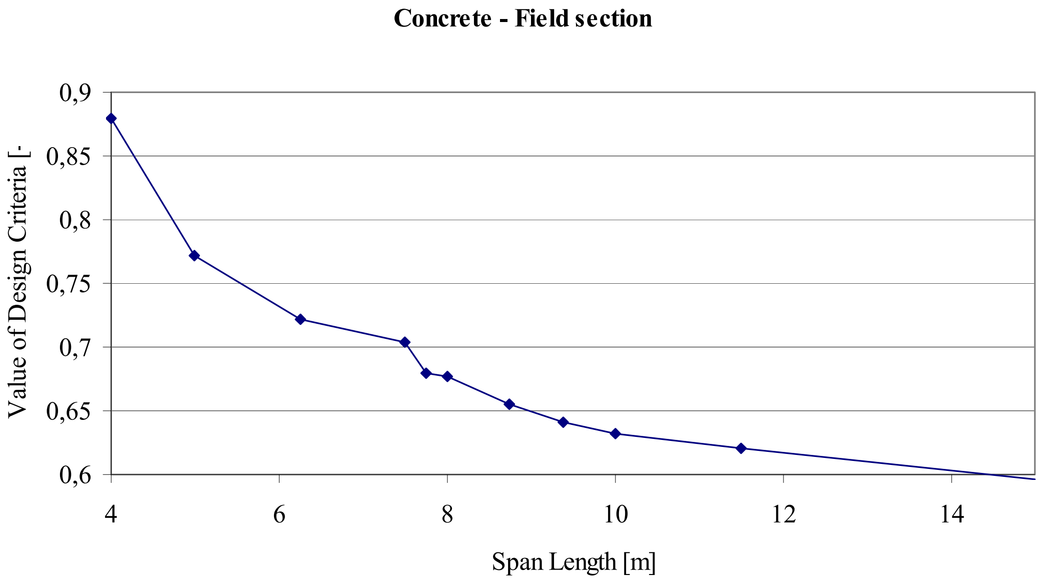



*Figure A.2* *Result from fatigue assessments calculation for the λ - Coefficient Method for concrete in field section*

*Table A.6* *Results for the λ -* Coefficient Method *in support section for the Continuous Bridge Model*

| *Support* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 5.625 | 6.25 | 6.875 | 7.50 | 10 | 11.50 | 15 |
| *λ - Coefficient Method* | | | | | | | | | |
| *Reinforcement* | [redacted] | | | | | | | | |
| *Tension* | 1.25 | 1.18 | 1.09 | 1.06 | 1.02 | 0.98 | 0.81 | 0.75 | 0.66 |
| *Concrete* | 0.85 | 0.82 | 0.79 | 0.78 | 0.77 | 0.76 | 0.71 | 0.70 | 0.68 |

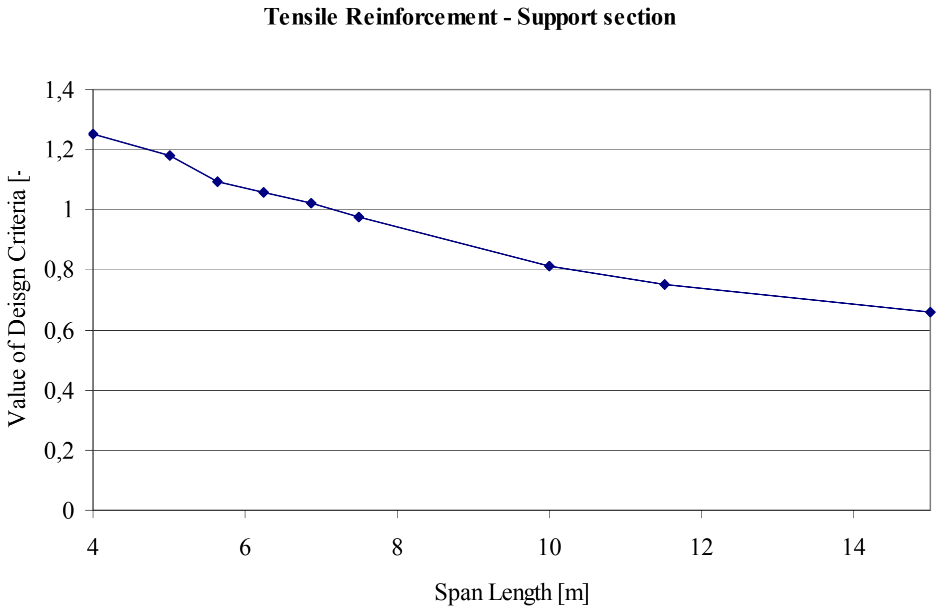



*Figure A.3 Result from fatigue assessments calculation for the* λ - Coefficient Method *for reinforcement in tension in support section*

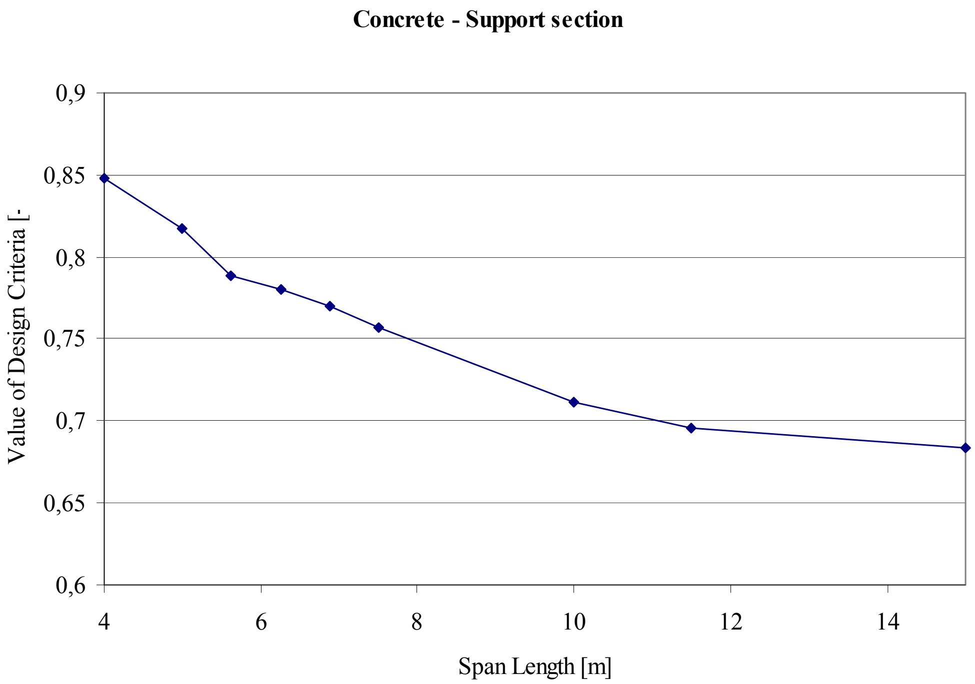



*Figure A.4 Result from fatigue assessments calculation for the λ - Coefficient Method for concrete in support section*

The results for the tensile reinforcement and concrete are presented for the Cumulative Damage Method for field and support section in Table A.7 and Table A.8. The results are also visualised in Figure A.5 to Figure A.8.

*Table A.7 Presenting the damage in field section for the Cumulative Damage Method*

| *Field* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.50 | 7.75 | 8 | 8.75 | 9.375 | 10 | 11.50 | 15 |
| *Cumulative Damage Method* | | | | | | | | | | | |
| *Reinforcement* | [redacted] | | | | | | | | | | |
| *Tension* | 24.31 | 9.64 | 3.50 | 1.36 | 1.01 | 0.85 | 0.54 | 0.34 | 0.24 | 0.09 | 0.02 |
| *Concrete* | 0.01 | $5.79 \cdot 10^{-4}$ | $2.78 \cdot 10^{-5}$ | $3.60 \cdot 10^{-6}$ | $2.04 \cdot 10^{-6}$ | $1.44 \cdot 10^{-6}$ | $5.10 \cdot 10^{-7}$ | $2.16 \cdot 10^{-7}$ | $1.07 \cdot 10^{-7}$ | $1.62 \cdot 10^{-8}$ | $9.53 \cdot 10^{-10}$ |

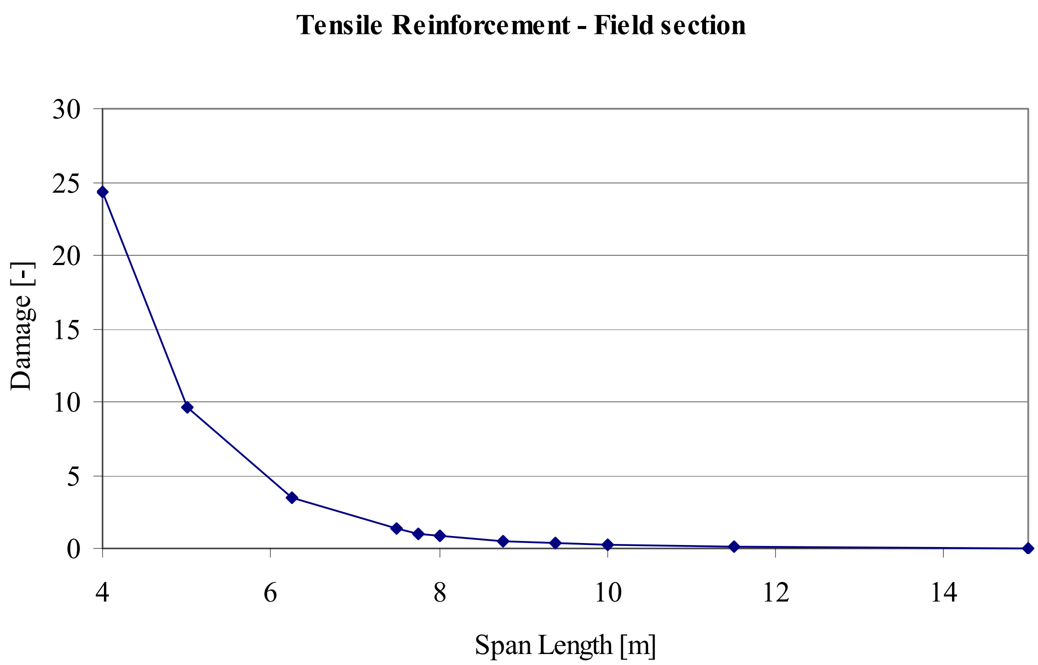



*Figure A.5 Result from fatigue assessments calculation for the Cumulative Damage Method for reinforcement in tension in field section*

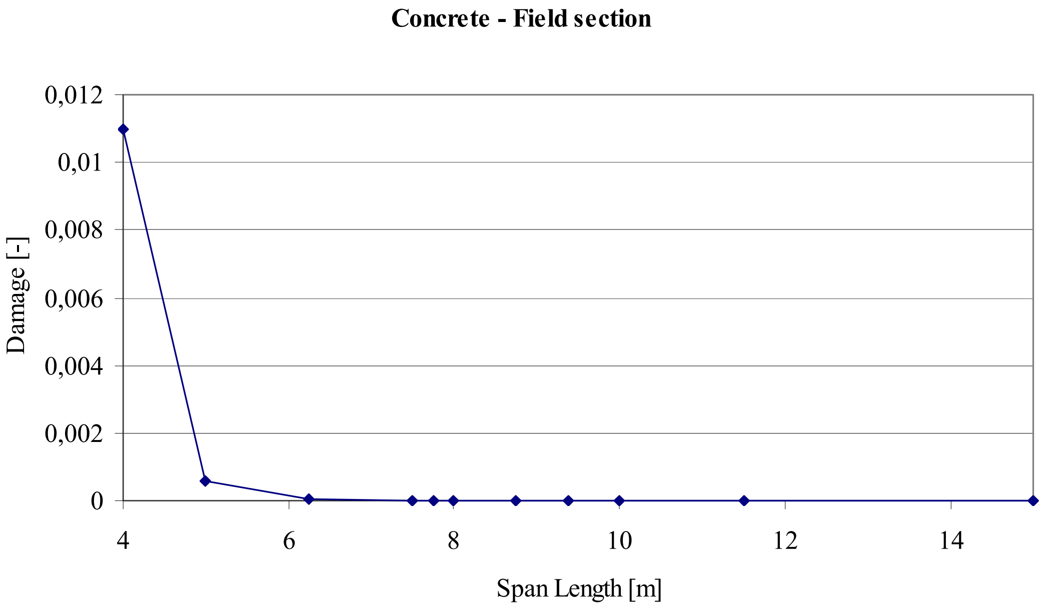



*Figure A.6* *Result from fatigue assessments calculation for the Cumulative Damage Method for concrete in field section*

*Table A.8* *Presenting the damage in support section for the Cumulative Damage Method*

| *Support* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 5.625 | 6.25 | 6.875 | 7.50 | 10 | 11.50 | 15 |
| *Cumulative Damage Method* | | | | | | | | | |
| *Reinforcement* | [redacted] | | | | | | | | |
| *Tension* | 5.49 | 2.09 | 1.15 | 0.57 | 0.31 | 0.19 | 0.10 | 0.05 | 0.01 |
| *Concrete* | $5.26 \cdot 10^{-4}$ | $4.73 \cdot 10^{-5}$ | $1.47 \cdot 10^{-5}$ | $4.28 \cdot 10^{-6}$ | $1.52 \cdot 10^{-6}$ | $6.63 \cdot 10^{-7}$ | $1.12 \cdot 10^{-7}$ | $2.83 \cdot 10^{-8}$ | $2.73 \cdot 10^{-9}$ |

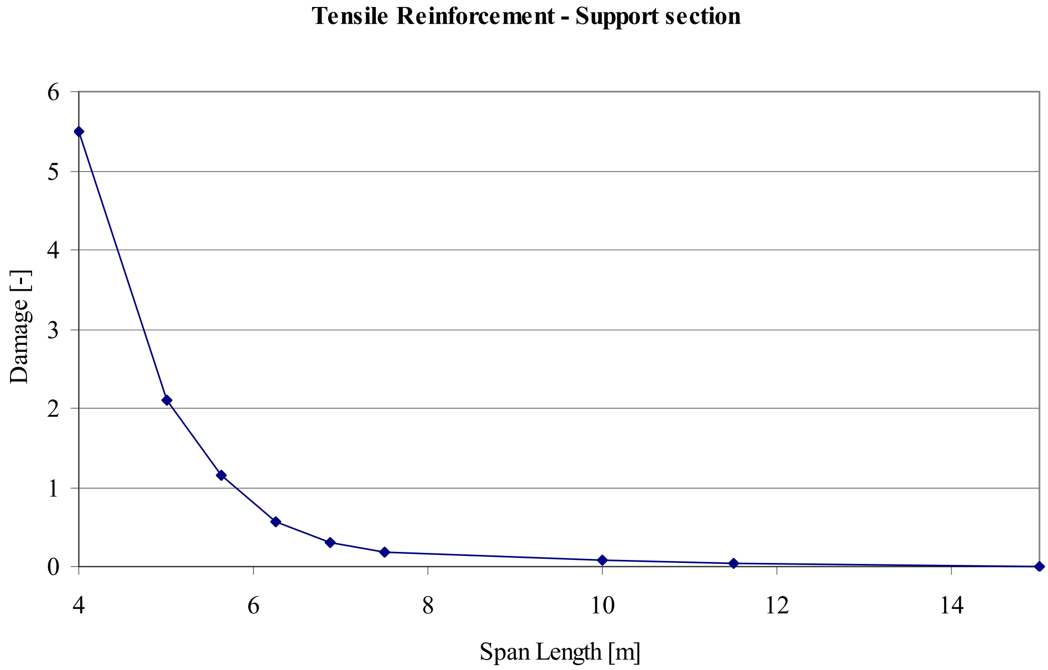



*Figure A.7 Result from fatigue assessments calculation for the Cumulative Damage Method for reinforcement in tension in support section*

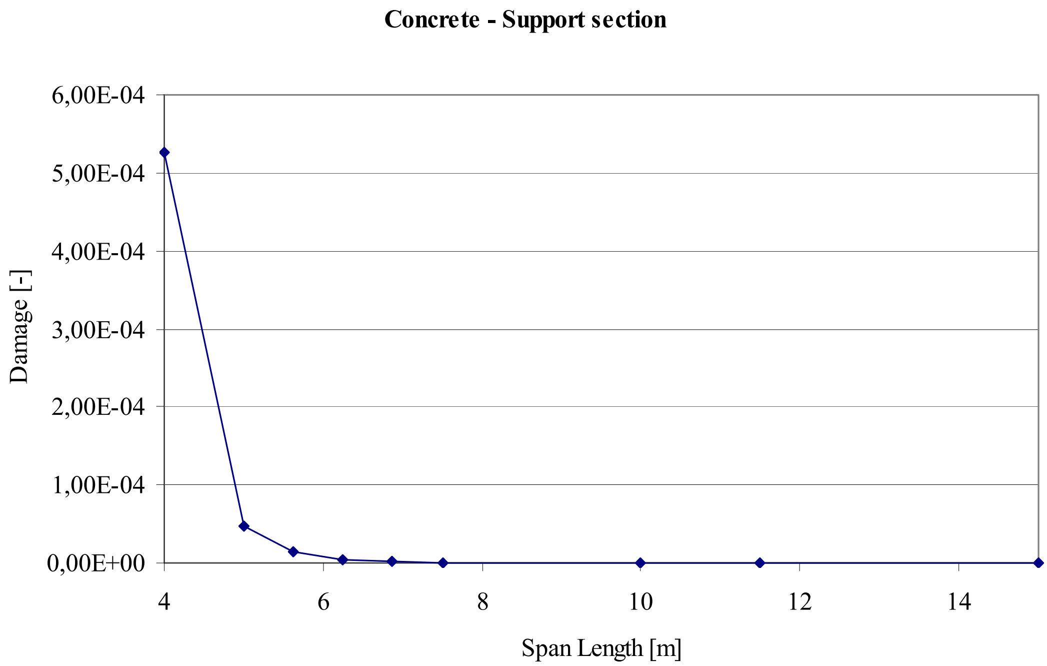



*Figure A.8 Result from fatigue assessments calculation for the Cumulative Damage Method for concrete in support section*

# Appendix B. Results from the study with varying span for the Simply Supported Bridge model

Before a fatigue assessment can be performed needs e.g. geometrical properties and bending moments in maximum moment section be known, these parameters are presented in Table B.1.

*Table B.1 Geometrical properties and bending moments extracted from StripStep2 in maximum moment section.*

| | *Height of cross-section* | *Moment ULS* | *Moment due to permanent load* |
|---|---|---|---|
| *L* *[m]* | *H* *[m]* | $M_{Ed}$ *[kNm]* | $M_{Ed.Perm}$ *[kNm]* |
| 2 | 0.20 | 308 | 39 |
| 2.5 | 0.20 | 489 | 61 |
| 3.5 | 0.27 | 974 | 131 |
| 5 | 0.39 | 1904 | 308 |
| 7.5 | 0.58 | 4151 | 846 |
| 10 | 0.71 | 7164 | 1775 |

The amount of reinforcement bars which are used in fatigue assessment for the simply supported bridge model is presented in Table B.2. The amount of reinforcement bars are taken from ULS design.

*Table B.2 The amount of reinforcement bars for the simply supported bridge model*

| *L [m]* | 2 | 2.5 | 3.5 | 5 | 7.5 | 10 |
|---|---|---|---|---|---|---|
| *Amount of reinforcement bars* | | | | | | |
| *Tension* | 12.81 | 22.66 | 28.75 | 33.90 | 44.33 | 54.54 |
| *Compression* | 11.25 | 11.25 | 11.25 | 11.25 | 11.25 | 11.20 |

The λ - Coefficient Method depends on different parameters e.g. λ – factors, traffic load induced moments. Some of theses parameters are presented in Table B.3. The maximum and minimum moments induced by traffic load, LM71, are only governing for the λ - Coefficient Method. The minimum moment is not presented, because it is set to zero for all cases. The total λ - factors for steel and concrete, including the modified α – factor, are presented in Table B.3 for the simply supported bridge model .

The dynamic factor and the final creep factor are governing for both fatigue assessments methods, which are varying with chancing span, see Table B.3.

*Table B.3 Factors which are influenced by the span for the simply supported bridge model*

| | Dynamic factor | Final creep factor | Maximum moment induced by traffic load LM71 | Total λ - factor | |
|---|---|---|---|---|---|
| *L [m]* | $\Phi_2$ | $\varphi_{cr}$ | $M_{max}$ *[kNm]* | $\lambda_s$ | $\lambda_c$ |
| 2 | 1.67 | 1.55 | 125 | 1.23 | 0.89 |
| 2.5 | 1.67 | 1.55 | 137.4 | 1.18 | 0.89 |
| 3.5 | 1.67 | 1.52 | 244.8 | 1.10 | 0.87 |
| 5 | 1.53 | 1.48 | 482.4 | 1.01 | 0.85 |
| 7.5 | 1.39 | 1.44 | 1099.5 | 0.89 | 0.80 |
| 10 | 1.31 | 1.42 | 1855.6 | 0.79 | 0.74 |

The results are considered for the tensile reinforcement and concrete. The results from the λ -Coefficient Method for the Simply Supported Bridge Model are presented in Table B.4 and visualised in Figure B.1 and Figure B.2.

*Table B.4 The results from λ - Coefficient Method for varying spans for the simply supported bridge model*

| *L [m]* | 2 | 2.5 | 3.5 | 5 | 7.5 | 10 |
|---|---|---|---|---|---|---|
| *λ - Coefficient Method* | | | | | | |
| *Reinforcement* | | | | | | |
| *Tension* | 2.28 | 1.42 | 1.24 | 1.10 | 0.96 | 0.79 |
| *Concrete* | 1.27 | 1.20 | 1.08 | 0.97 | 0.90 | 0.85 |

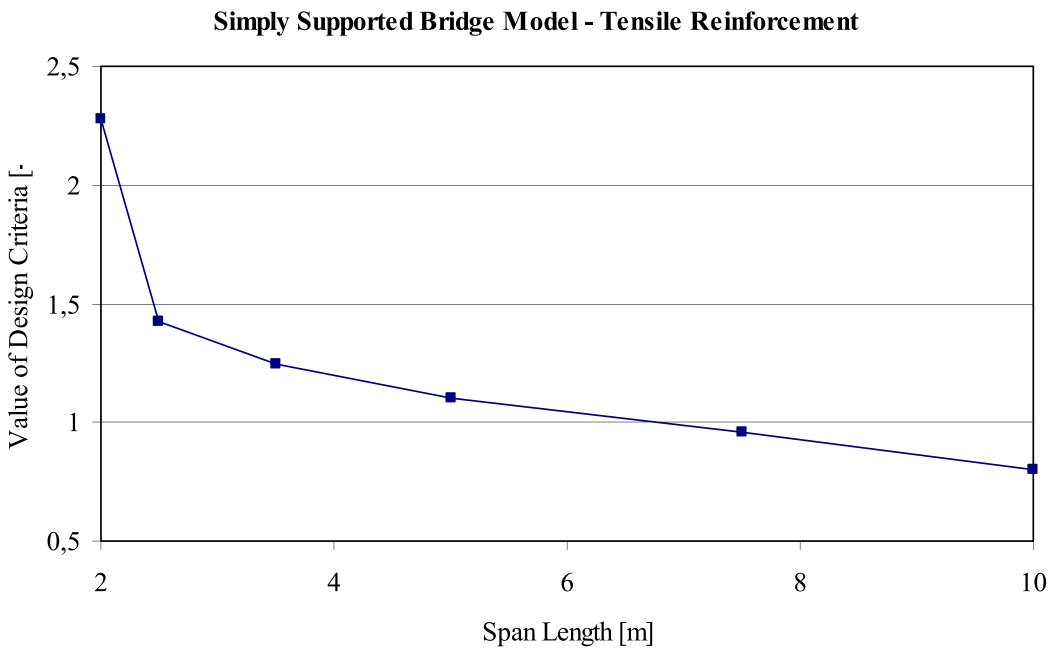



*Figure B.1 Result from fatigue assessments calculation for the λ-Coefficient Method for tensile reinforcement for the simply supported bridge model*

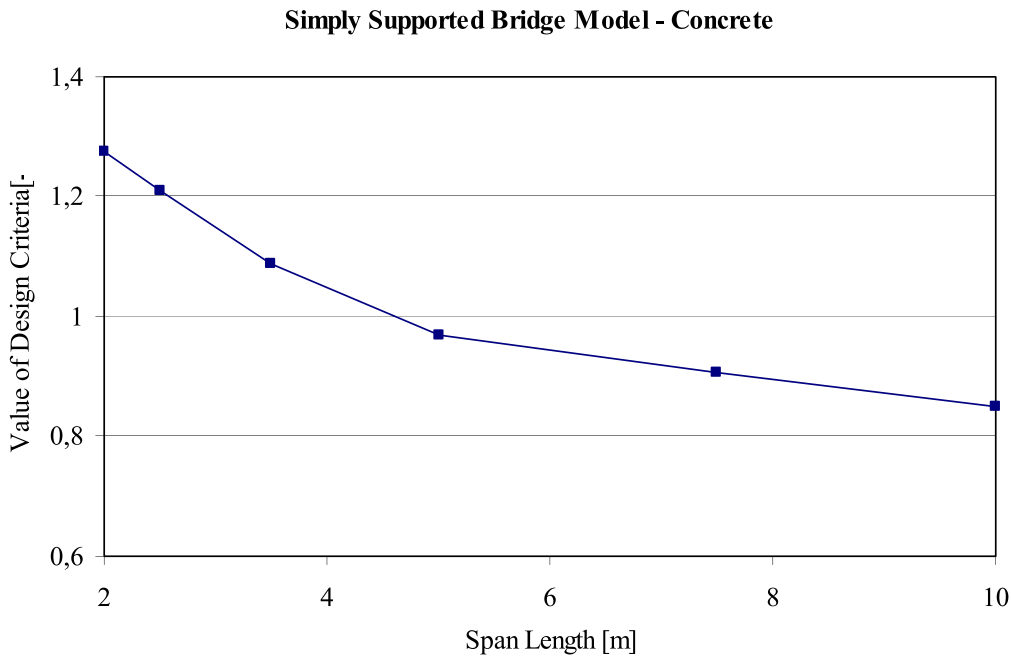



*Figure B.2 Result from fatigue assessments calculation for the λ-Coefficient Method for concrete for the simply supported bridge model*

The results for the tensile reinforcement and concrete are presented for the Cumulative Damage Method for field and support section in Table B.5. The results are also visualised in Figure B.3 and Figure B.4.

*Table B.5 Presenting the damage in considered section for the Cumulative Damage Method*

| *L [m]* | 2 | 2.5 | 3.5 | 5 | 7.5 | 10 |
|---|---|---|---|---|---|---|
| *Cumulative Damage Method* | | | | | | |
| *Reinforcement* | | | | | | |
| *Tension* | 92.32 | 18.49 | 4.34 | 1.42 | 0.17 | 0.05 |
| *Concrete* | 698.76 | $2.95 \cdot 10^{4}$ | 29.82 | 0.06 | $3.74 \cdot 10^{-4}$ | $3.50 \cdot 10^{-5}$ |

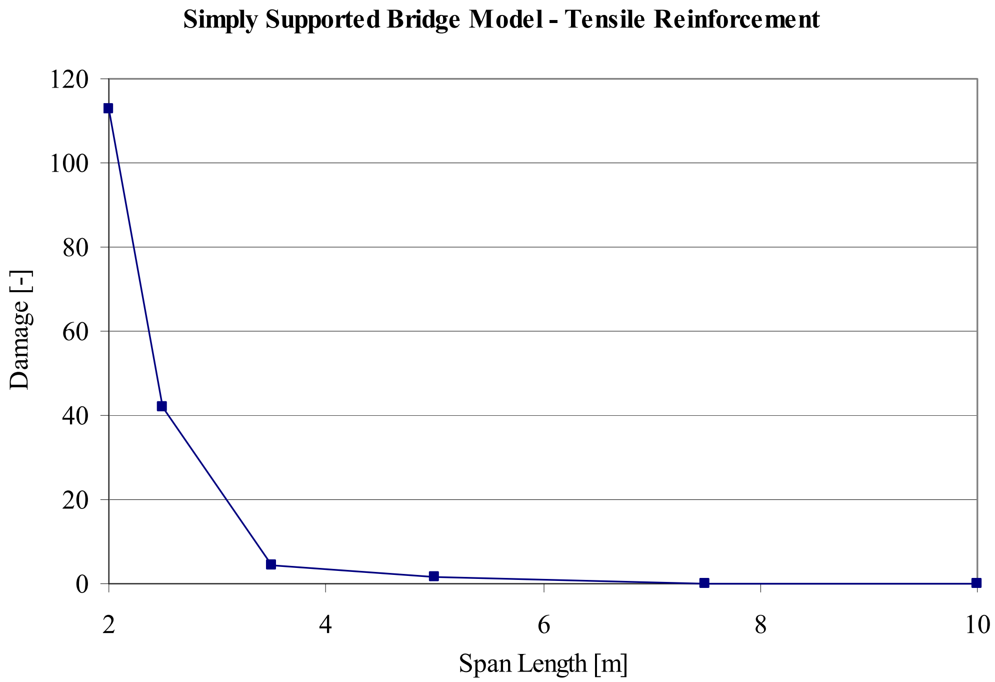



*Figure B.3 Result from fatigue assessments calculation for the Cumulative Damage Method for reinforcement in tension in the simply supported bridge model*

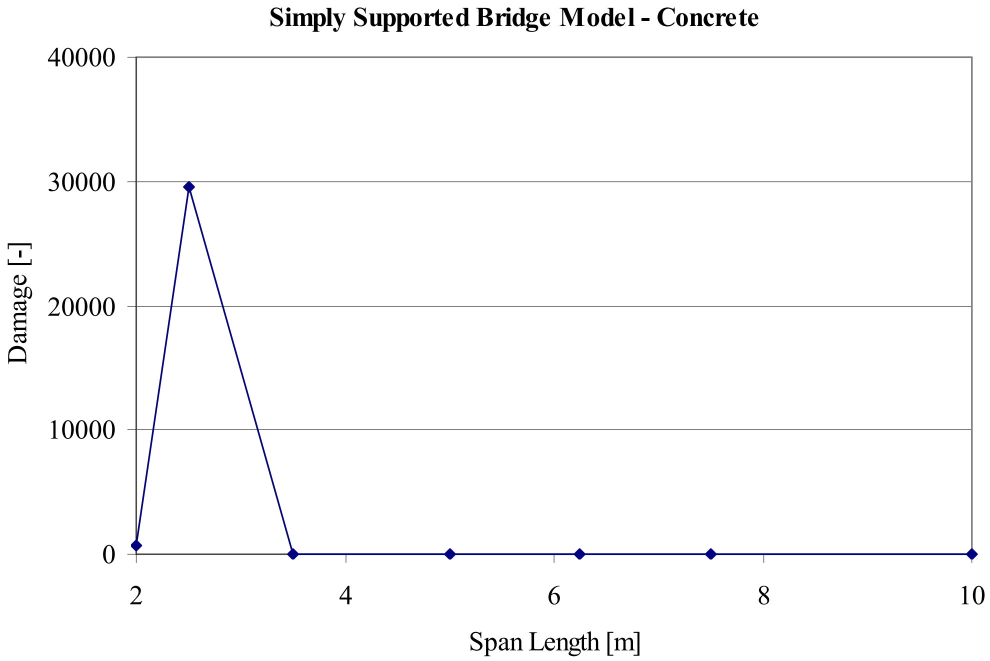



*Figure B.4 Result from fatigue assessments calculation for the Cumulative Damage Method for concrete for the simply supported bridge model*

# Appendix C. Results from the study of the sectional stresses for the Continuous Bridge model

Sectional stresses obtained from calculations, used in the Cumulative Damage Method are presented in Figure C.1 and Figure C.2 for the field and support sections for the Continuous Bridge model. These figures visualises the entire series of stress ranges that occurs during one day of loading with the fatigue load models. There is one range value for each cycle and the series for four different spans are showed.

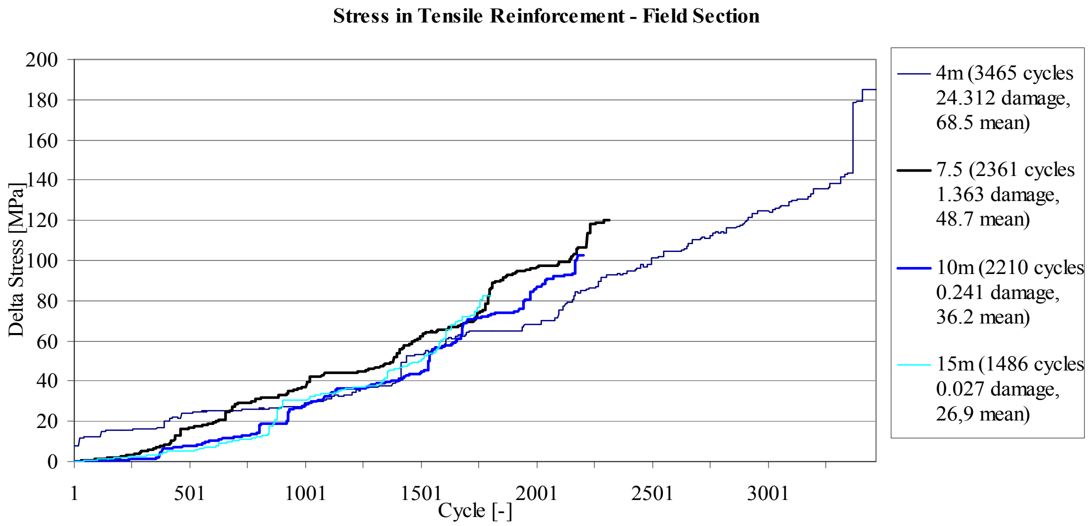



*Figure C.1 The stress range in the tensile reinforcement for each cycle during one day. The values are sorted by size, not as they appear.*

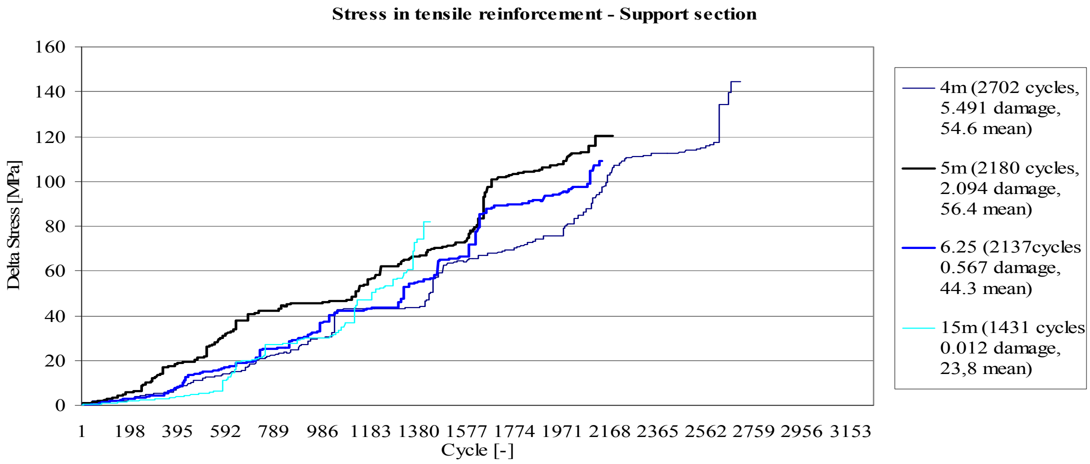



*Figure C.2 The stress range in the tensile reinforcement for each cycle during one day. The values are sorted by size, not as they appear.*

To show the influence from the different stress ranges, Figure C.1 and Figure C.2 are redone to histograms for span 4 meters and 15 meters. Figure C.4 and Figure C.6

visualises the stress range with regard to the number of cycles together with the corresponding value of the Damage. It also shows the mean value of all stress ranges.

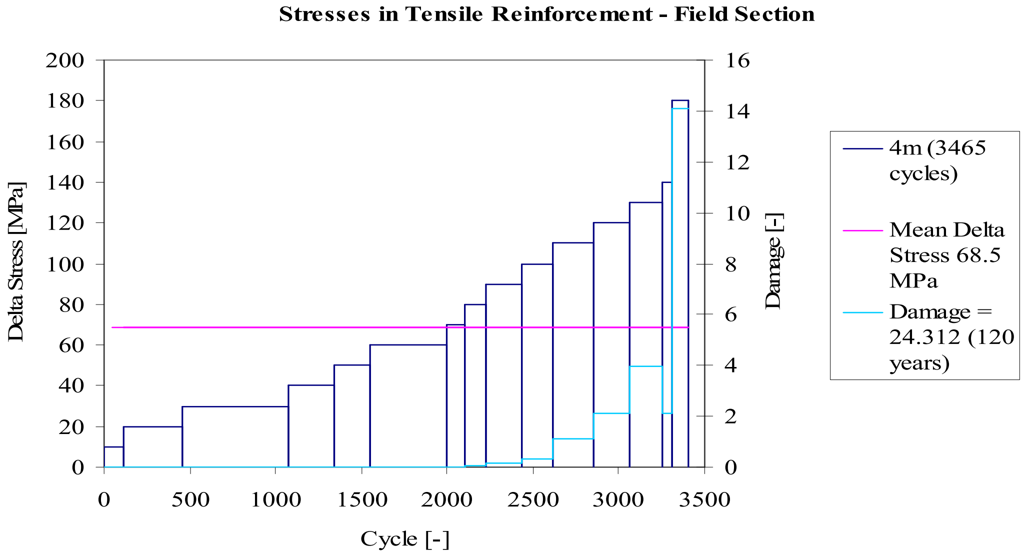



*Figure C.3* *The delta stress with regard to the number of cycles in field section for the continuous bridge model for the tensile reinforcement for the Cumulative Damage Method.*

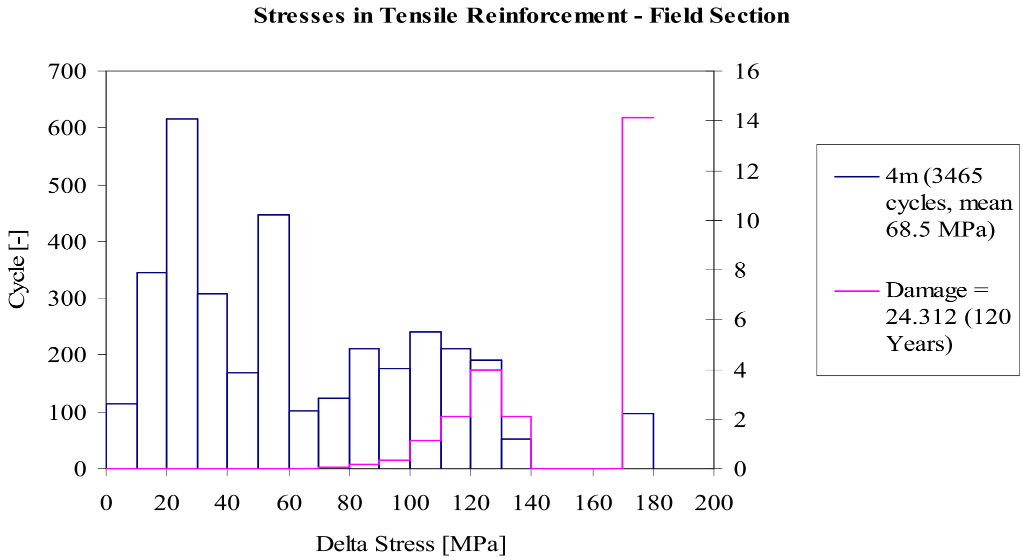



*Figure C.4* *The number of cycles with regard to the delta stress in field section for the continuous bridge model for the tensile reinforcement for the Cumulative Damage Method.*

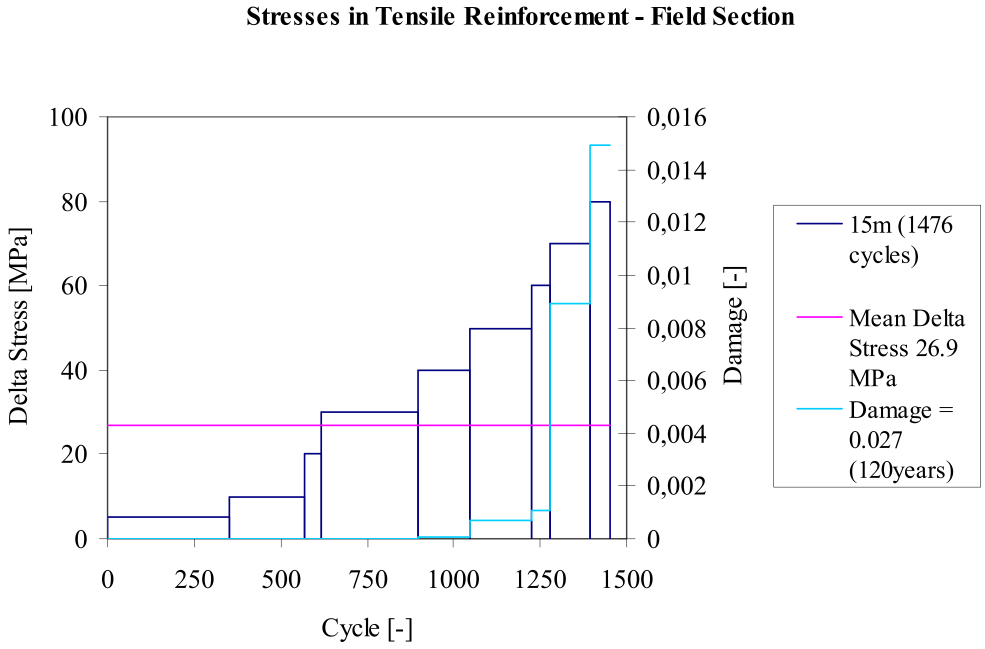



*Figure C.5* *The delta stress with regard to the number of cycles in field section for the continuous bridge model for the tensile reinforcement for the Cumulative Damage Method.*

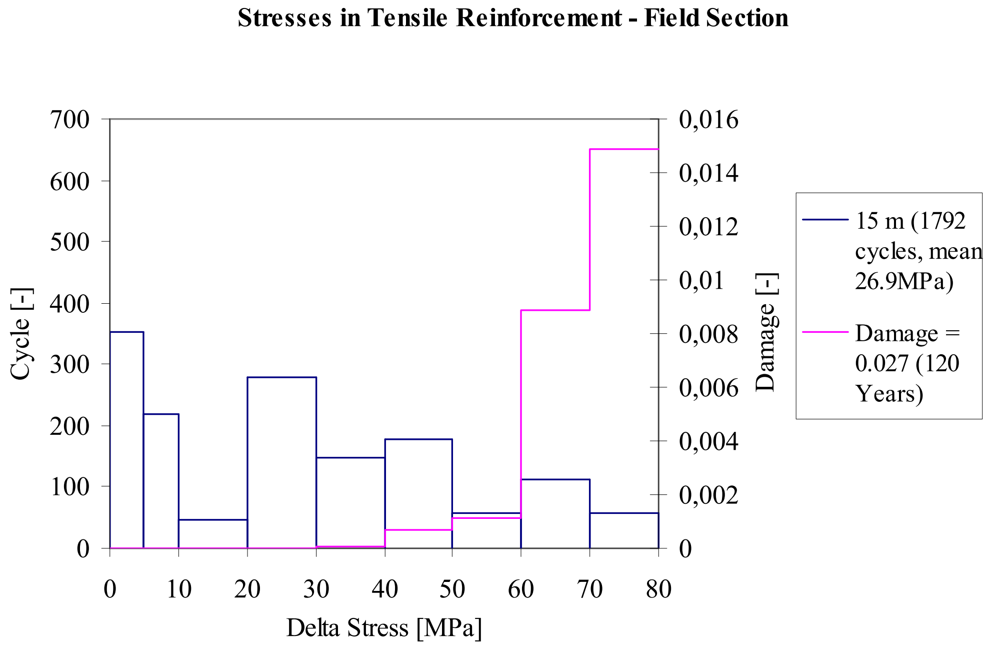



*Figure C.6* *The number of cycles with regard to the delta stress in field section for the continuous bridge model for the tensile reinforcement for the Cumulative Damage Method.*

# Appendix D. Results from the study regarding the compressive reinforcement for the Continuous Bridge model

Table D.1 shows the amount of reinforcement bars in tension and in compression for the continuous bridge model in field section.

*Table D.1 Amount of reinforcement bars in field section for the continuous bridge model*

| *Number of reinforcement bars* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 7.75 | 8 | 8.75 | 9.375 | 10 | 11.5 | 15 |
| *Tension* | 11.24 | 12.99 | 15.24 | 17.60 | 18.10 | 18.61 | 19.90 | 21.25 | 22.44 | 25.38 | 32.59 |
| *With compressive reinforcement* | 11.25 | 11.25 | 11.25 | 12.80 | 13.26 | 13.71 | 15.07 | 16.20 | 17.33 | 20.04 | 26.37 |
| *Without compressive reinforcement* | - | - | - | - | - | - | - | - | - | - | - |

Table D.2 and Table D.3 shows the result from the fatigue assessment for the λ - Coefficient Method and Cumulative Damage Method in field section when the compressive reinforcement is removed. The results are also visualised in Figure D.1 to Figure D.4. To be able to compare the new established result with the old results, the results with compressive reinforcement is presented in Appendix A.

*Table D.2 The result for the λ - Coefficient Method for field section without compressive reinforcement*

| *Field* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 7.75 | 8 | 8.75 | 9.375 | 10 | 11.5 | 15 |
| *λ - Coefficient Method* | | | | | | | | | | | |
| *Reinforcement* | | | | | | | | | | | |
| *Tension* | 1.56 | 1.25 | 1.11 | 1.05 | 0.97 | 0.96 | 0.89 | 0.83 | 0.80 | 0.77 | 0.64 |
| *Concrete* | 0.88 | 0.78 | 0.73 | 0.72 | 0.69 | 0.69 | 0.67 | 0.66 | 0.65 | 0.64 | 0.62 |

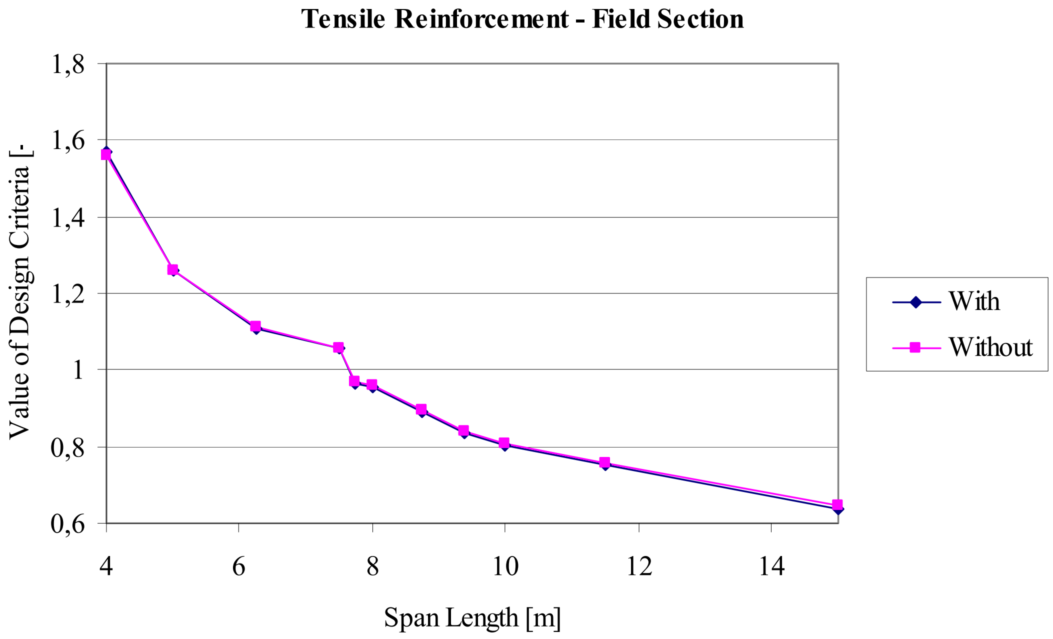



*Figure D.1* *Diagram for fatigue assessments calculations for reinforcement in tension according to the λ - Coefficient Method in field section, with and without compressive reinforcement*

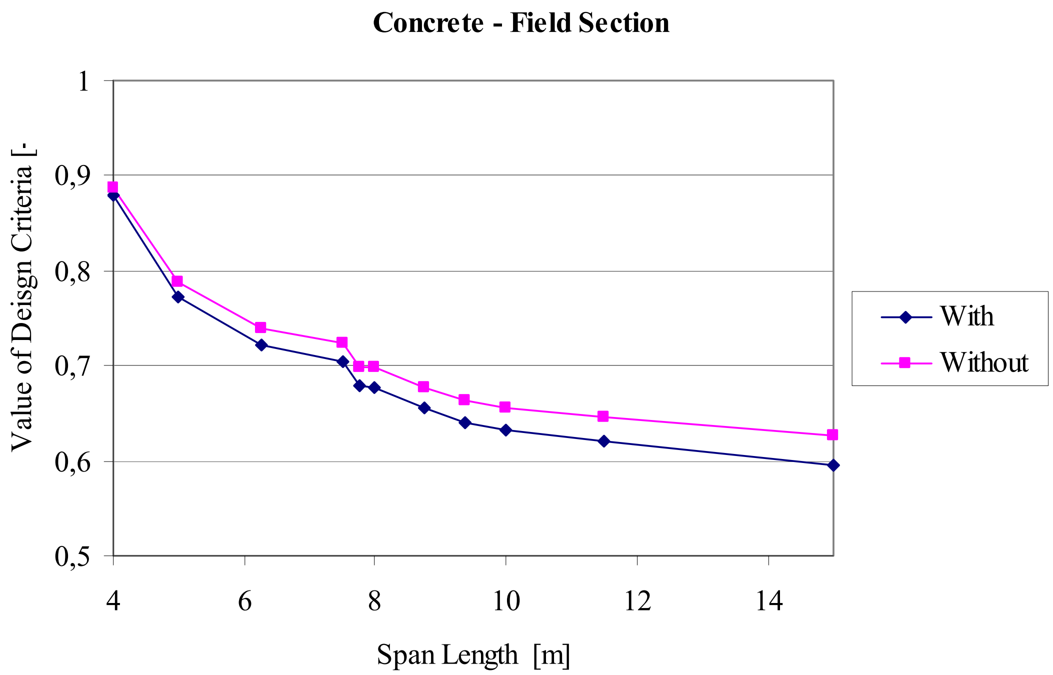



*Figure D.2* *Diagram for fatigue assessments calculations for concrete according to the λ - Coefficient Method in field section, with and without compressive reinforcement*

*Table D.3 The damage for the Cumulative Damage Method for field section without compressive reinforcement*

| *Field* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 7.75 | 8 | 8.75 | 9.375 | 10 | 11.5 | 15 |
| *Cumulative Damage Method* | | | | | | | | | | | |
| *Reinforcement* | [redacted] | | | | | | | | | | |
| *Tension* | 23.53 | 9.53 | 3.55 | 1.41 | 1.05 | 0.88 | 0.57 | 0.36 | 0.25 | 0.10 | 0.02 |
| *Concrete* | $5.99 \cdot 10^{-3}$ | $8.29 \cdot 10^{-4}$ | $1.60 \cdot 10^{-4}$ | $5.94 \cdot 10^{-6}$ | $3.40 \cdot 10^{-6}$ | $2.45 \cdot 10^{-6}$ | $9.00 \cdot 10^{-7}$ | $3.96 \cdot 10^{-7}$ | $1.65 \cdot 10^{-5}$ | $9.40 \cdot 10^{-6}$ | $3.69 \cdot 10^{-6}$ |

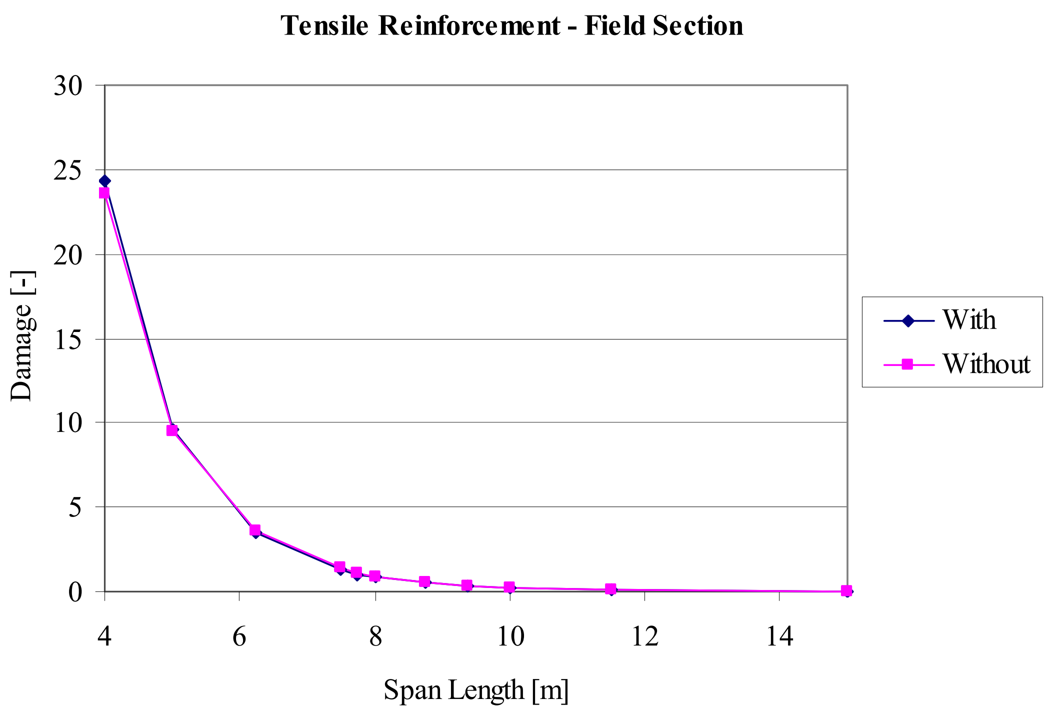



*Figure D.3 Result from fatigue assessments calculation for the Cumulative Damage Method for reinforcement in tension in field section, with and without compressive reinforcement*

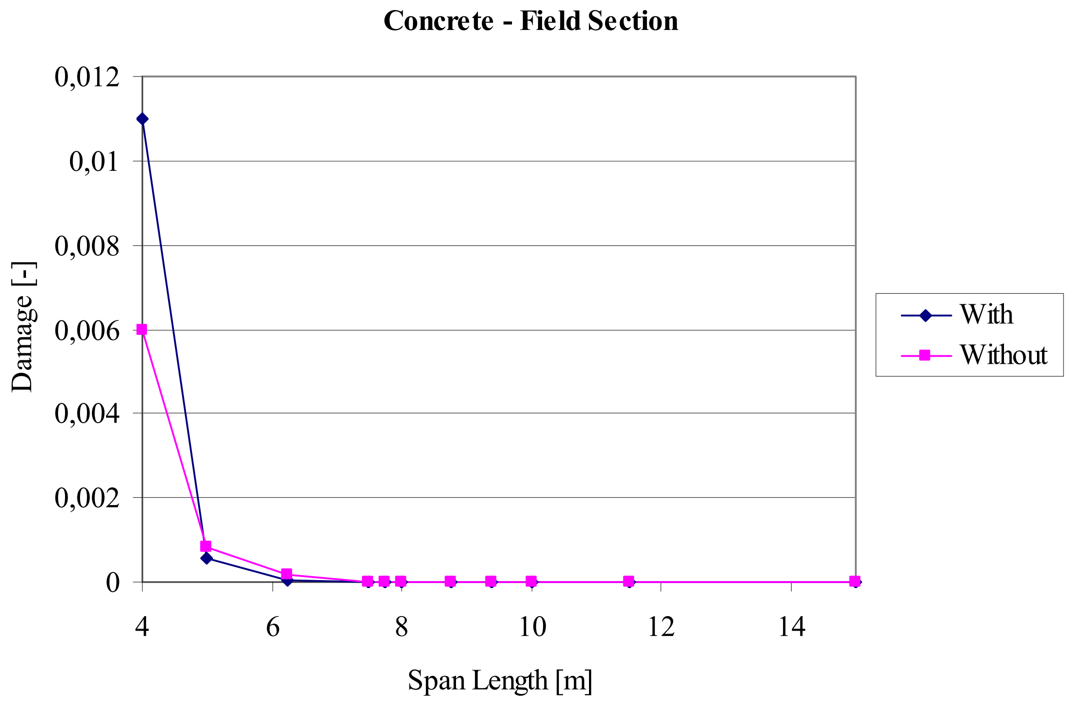



*Figure D.4* *Result from fatigue assessments calculation for the Cumulative Damage Method for concrete in field section, with and without compressive reinforcement*

# Appendix E. Results from the study regarding the Design criteria's with adjusted Continuous Bridge models

*Table E.1 Damage obtained for the Cumulative Damage Method when different design criteria govern the design.*

| *Support section* | | | | | |
|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 10 |
| *Damage - Cumulative Damage Method* | | | | | |
| *ULS Design* | 5.49 | 2.09 | 0.57 | 0.19 | 0.09 |
| *λ = 1* | 0.75 | 0.47 | 0.34 | 0.23 | 0.59 |
| *Cumulative Damage Method= 1* | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

*Table E.2 Values obtained for the λ-Coefficient Method when different design criteria govern the design.*

| *Support section* | | | | | |
|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 10 |
| *Value of design criteria - λ-Coefficient Method* | | | | | |
| *ULS Design* | 1.25 | 1.18 | 1.06 | 0.98 | 0.81 |
| *Cumulative Damage Method= 1* | 1.03 | 1.09 | 1.13 | 1.18 | 0.92 |
| *λ = 1* | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

*Table E.3 The amount of reinforcement obtained when the different design criteria's govern the design.*

| *Support section* | | | | | |
|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 10 |
| *Amount of reinforcement* | | | | | |
| *ULS Design* | 20.46 | 23.87 | 28.72 | 33.32 | 41.96 |
| *Intermediate* | 22.74 | 24.94 | 27.80 | 30.39 | 36.86 |
| *Cumulative Damage Method= 1* | 25.03 | 26.01 | 26.89 | 27.45 | 31.77 |
| *λ = 1* | 25.90 | 28.40 | 30.45 | 32.47 | 33.75 |

*Table E.4 Damage obtained for the Cumulative Damage Method when different design criteria govern the design.*

| Field section | | | | | |
|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 8 |
| *Damage - Cumulative Damage Method* | | | | | |
| *ULS Design* | 24.31 | 9.64 | 3.51 | 1.36 | 0.85 |
| *λ = 1* | 0.77 | 1.72 | 1.39 | 0.84 | 1.27 |
| *Cumulative Damage Method= 1* | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

*Table E.5 Values obtained for the λ-Coefficient Method when different design criteria govern the design.*

| Field section | | | | | |
|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 8 |
| *Value of design criteria - λ-Coefficient Method* | | | | | |
| *ULS Design* | 1.57 | 1.26 | 1.11 | 1.06 | 0.96 |
| *Cumulative Damage Method= 1* | 1.03 | 0.94 | 0.97 | 1.02 | 0.97 |
| *λ = 1* | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

*Table E.6 The amount of reinforcement obtained when the different design criteria's govern the design.*

| Field section | | | | | |
|---|---|---|---|---|---|
| *L [m]* | 4 | 5 | 6.25 | 7.5 | 8 |
| *Amount of reinforcement* | | | | | |
| *ULS Design* | 11.24 | 12.99 | 15.24 | 17.60 | 18.61 |
| *Intermediate* | 14.37 | 15.29 | 16.43 | 17.92 | 18.44 |
| *Cumulative Damage Method= 1* | 17.51 | 17.60 | 17.62 | 18.24 | 18.27 |
| *λ = 1* | 18.05 | 16.53 | 16.96 | 18.60 | 17.78 |

# Appendix F. Inquiry of the influence of the effective creep factor in stress calculation

Since fatigue assessment of concrete not only depends on the stress cycles but also on the stress mean level, a method of combining the long term permanent load and the short term fatigue load must be derived. This due to that the creep is considered only for the permanent load and it is done by using an effective creep factor. In this section an inquiry by which the influence of the method of determining the $\varphi_{ef}$-factor is performed.

When the neutral axis, moment of inertia and finally the stresses in a section is to be calculated, a modular ratio is needed. The ratio, or further on called the $\alpha_{ef}$-factor, determines the distribution of stresses between the concrete and the reinforcing steel. The $\alpha_{ef}$-factor is calculated with regard to the $\varphi_{ef}$-factor and takes into account the creep properties and the young's modulus of the materials, see equation (F.1). Hereafter some alternative cases are presented in order to determine what impact different approaches have on the fatigue assessment. The bridge used for this inquiry is a slab bridge with 5m span, one side simply supported and one side fully fixed. It is loaded with fatigue loading according to Eurocode.

$$\alpha_{ef} = \frac{E_s}{E_{cm}}(1 + \varphi_{ef}) \tag{F.1}$$

Where $E_s$ is the modulus of elasticity for steel,

$E_{cm}$ is the modulus of elasticity for concrete.

The first approach, called case 1, is the method used for calculating the $\varphi_{ef}$-factor further on in this thesis. It determines a $\varphi_{ef}$-factor for each extreme value in every cycle, see Figure F.1, i.e. the maximum value and the minimum value of a fatigue load cycle are added to the permanent value and the $\varphi_{ef}$-factor is determined according to equation (F.2).

$$\varphi_{ef} = \frac{\varphi(\infty, t_0) \cdot M_{perm}}{M_{perm} + M_{fatigue}} \tag{F.2}$$

Where $\varphi(\infty, t_0)$ is the final creep coefficient, according to EN 1992-1-1: SIS (2005),

$M_{perm}$ is the first order bending moment of the permanent load,

$M_{fatigue}$ is the first order bending moment of the fatigue load.

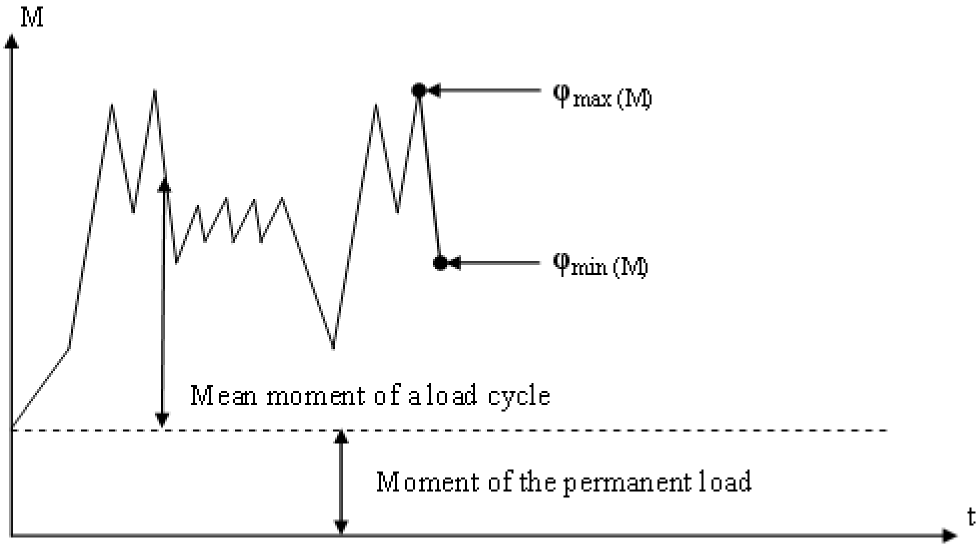



*Figure F.1 Values from each cycle for which the effective creep factor is calculated, Case 1.*

The second case is when a mean value of the $\varphi_{ef}$-factor is used for each cycle. The mean value of the fatigue load plus the permanent load gives the $\varphi_{ef}$-factor which is used for in the calculations for both the maximum and minimum values, see Figure F.2.

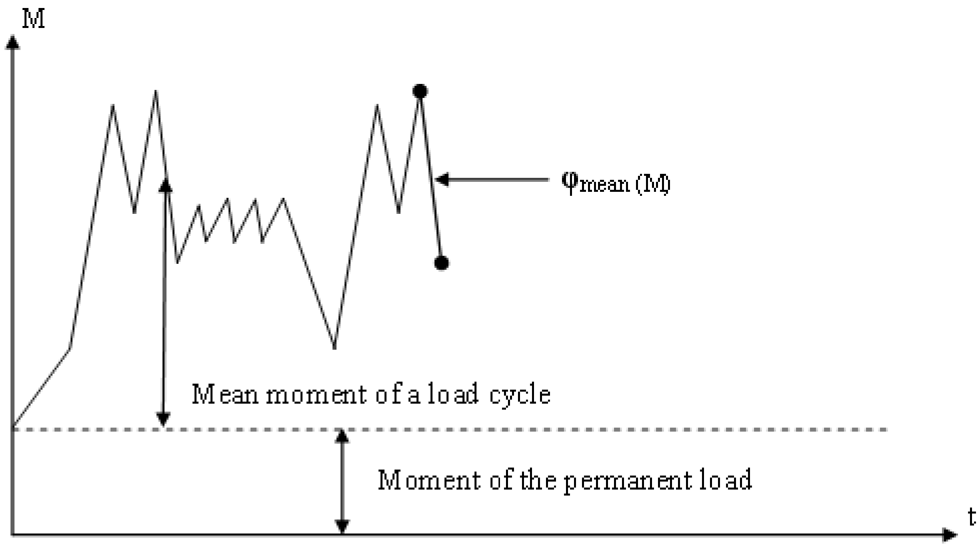



*Figure F.2 Value from each cycle for which the effective creep factor is calculated, Case 2.*

The third case also uses a mean value of the $\varphi_{ef}$-factor, but here a single value for the entire series of fatigue loading is determined, see Figure F.3.

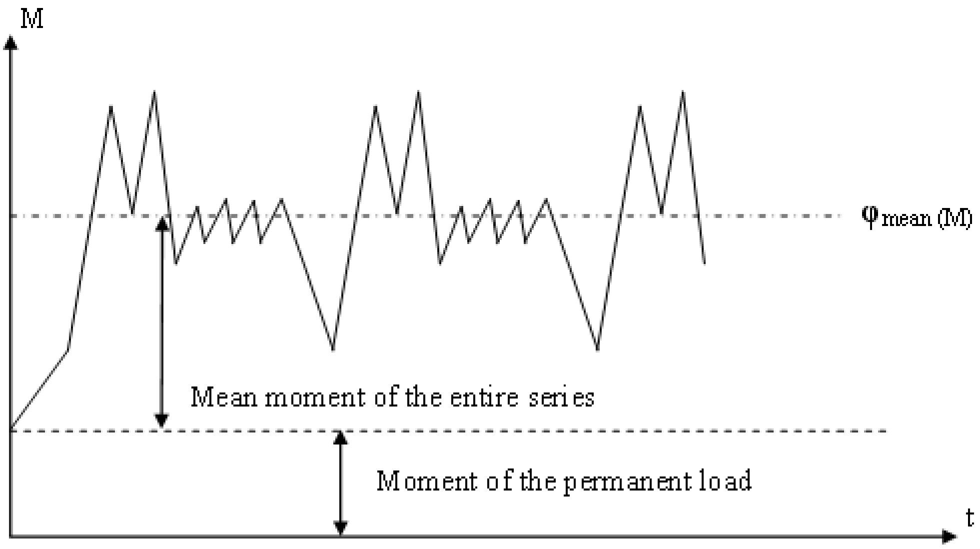



*Figure F.3 The mean value of the entire fatigue loading series, for which the effective creep factor is calculated, Case 3.*

The fourth case is taking a mean value between the $\varphi_{ef}$-factor calculated for the permanent load only, i.e. the long term value, and the mean value used in case three, see Figure F.4.

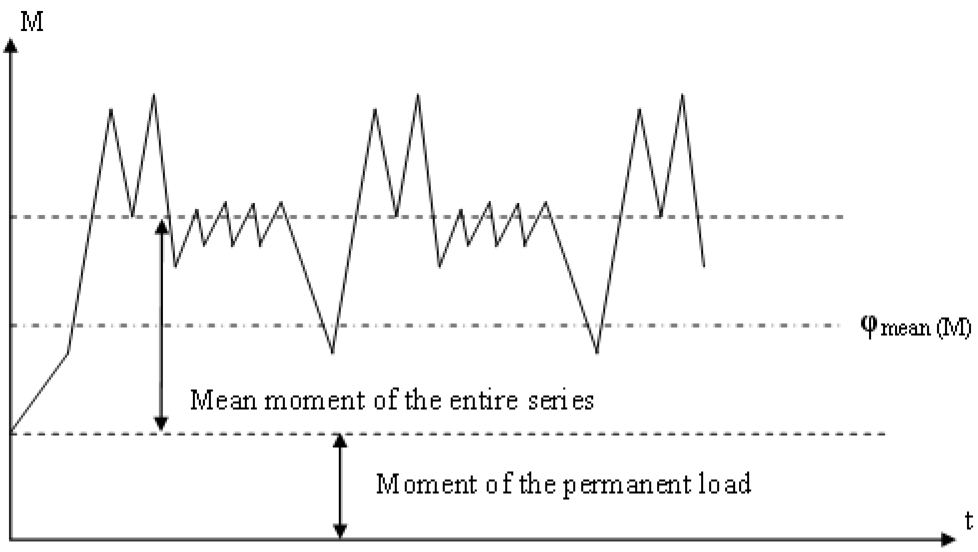



*Figure F.4 The mean value between the permanent value and the entire fatigue loading series value, for which the effective creep factor is calculated, Case 4.*

The cases are calculated for both the λ-Coefficient Method and Cumulative Damage Method and the results can be seen in Table F.1 and Table F2. It can be concluded that the method of determining the $\varphi_{ef}$-factor does not have a big influence on the results obtained by the Cumulative Damage Method and λ-Coefficient Method. Therefore the decision regarding the use of the method in Case 1 can be justified.

*Table F.1 Results from the inquiry regarding the effective creep coefficient for the Cumulative Damage Method.*

| *Cumulative Damage Method* | | | | |
|---|---|---|---|---|
| *Neutral axis* | *Case 1* | *Case 2* | *Case 3* | *Case 4* |
| *x [m]* | | | 0.148 | 0.154 |
| $x_{min}$ *[m]* | 0.134 | 0.136 | | |
| $x_{max}$ *[m]* | 0.159 | 0.156 | | |
| *Stress Range Tensile Reinforcement [MPa]* | | | | |
| $\Delta\sigma_{st\ max}$ | 120.484 | 119.513 | 119.512 | 119.960 |
| $\Delta\sigma_{st\ min}$ | 0.191 | 0.191 | 0.191 | 0.192 |
| *Concrete stress [MPa]* | | | | |
| *Concrete top level* | | | | |
| $\sigma_{c\ max}$ | -2.806 | -2.743 | -2.904 | -2.770 |
| $\sigma_{c\ min}$ | -6.834 | -6.254 | -6.102 | -5.819 |
| *Concrete bottom level* | | | | |
| $\sigma_{c\ max}$ | -2.123 | -2.183 | -2.324 | -2.216 |
| $\sigma_{c\ min}$ | -5.959 | -5.973 | -5.425 | -5.174 |
| *Calculated Damage* | | | | |
| *Reinforcement* | 2.094 | 1.960 | 1.957 | 2.024 |
| *Concrete* | $2.014\cdot10^{-4}$ | $3.391\cdot10^{-5}$ | $3.361\cdot10^{-5}$ | $3.287\cdot10^{-5}$ |

*Table F2 Results from the inquiry regarding the effective creep coefficient for the λ-Coefficient Method.*

| *λ-Coefficient Method* | | | | |
|---|---|---|---|---|
| *Neutral axis* | *Case 1* | *Case 2* | *Case 3* | *Case 4* |
| *x [m]* | | 0.148 | 0.148 | 0.154 |
| *$x_{min}$ [m]* | 0.134 | | | |
| *$x_{max}$ [m]* | 0.159 | | | |
| *Stress Range Tensile Reinforcement [MPa]* | | | | |
| *$\Delta\sigma_{st}$* | 118.706 | 117.762 | 117.762 | 118.200 |
| *Concrete stress [MPa]* | | | | |
| *$\sigma_c$ max* | -6.762 | -6.042 | -6.042 | -5.764 |
| *$\sigma_c$ min* | -2.124 | -2.323 | -2.323 | -2.216 |
| *Calculated Damage* | | | | |
| *Reinforcement* | 1.181 | 1.172 | 1.172 | 1.176 |
| *Concrete* | 0.817 | 0.749 | 0.749 | 0.730 |

# Appendix G. Design of reinforcement and detailing for a concrete slab bridge according to Eurocode

***Geometrical properties:***

$kNm := kN \cdot m$

One edge simply supported and the other fully fixed

Bridge span: $L := 7m$

Width of slab: $w := 4.5m$

Length of haunch: $l_h := 2.5m$

Height left end of slab: $h_l := 540mm$

Height right end of slab: $h_r := 830mm$

Thickness ballast: $h_B := 600mm$

Reinforcement bar area: $\phi := 25mm \qquad A_{si25} := \frac{\phi^2}{4} \cdot \pi = 490.874 \cdot mm^2$

$\phi_{16} := 16mm \qquad A_{si16} := \frac{\phi_{16}^2}{4} \cdot \pi = 201.062 \cdot mm^2$

$\phi_{10} := 10mm \qquad A_{si10} := \frac{\phi_{10}^2}{4} \cdot \pi = 78.54 \cdot mm^2$

Sectional constants, State I (Plain concrete):

| | Left support | Right support (Haunch) |
|---|---|---|
| Concrete Area: | $A_{I.l} := w \cdot h_l = 2.43\,m^2$ | $A_{Ir} := w \cdot h_r = 3.735\,m^2$ |
| Moment of Inertia: | $I_{I.l} := \frac{w \cdot h_l^3}{12} = 0.059\,m^4$ | $I_{Ir} := \frac{w \cdot h_r^3}{12} = 0.214\,m^4$ |

***Material properties:*** [EN 1992-1-1:2005 3.1.3 table 3.1 and 3.2]

Concrete C35/45:

$f_{ck} := 35MPa$

$$f_{cm} := \frac{f_{ck} + 8MPa}{MPa} = 43$$

$f_{ctm} := 3.2MPa$

$f_{ctk.0.05} := 2.2MPa$

$E_{cm} := 34GPa$

$\alpha_{cc} := 1$

$\gamma_C := 1.5$

$$f_{cd} := \alpha_{cc} \frac{f_{ck}}{\gamma_C} = 23.333 \cdot MPa$$

Steel B500B:

$f_{yk} := 500MPa$

$E_{sm} := 200GPa$

$\gamma_S := 1.15$

$$f_{yd} := \frac{f_{yk}}{\gamma_S} = 434.783 \cdot MPa$$

Creep coefficient: [EN 1992-1-1:2005 3.1.4] and [Annex B.1]

Concrete age at first loading (days): $t_o := 28$

Concrete age at considered time (days): $t := 70 \cdot 365 = 2.555 \times 10^4$

Ambient relative humidity [%]: $RH := 80$

Perimeter of the member in contact with the atmosphere: $u := 2 \cdot w + 2 \cdot h_1 = 10.08\,m$

Notional size of the member [mm]: $h_o := \frac{2 \cdot A_{I.1}}{u} \cdot \frac{1000}{m} = 482.143$

Coefficient depending on the relative humidity and the notional size:

$$\beta_H := \begin{cases} 1.5 \cdot \left[1 + (0.012 \cdot 80)^{18}\right] \cdot h_o \ldots + 250 \cdot \left(\frac{35}{f_{cm}}\right)^{0.5} & \text{if } 1.5 \cdot \left[1 + (0.012 \cdot 80)^{18}\right] \cdot h_o \ldots + 250 \cdot \left(\frac{35}{f_{cm}}\right)^{0.5} \le 1500 \cdot \left(\frac{35}{f_{cm}}\right)^{0.5} \\ 1500 \cdot \left(\frac{35}{f_{cm}}\right)^{0.5} & \text{otherwise} \end{cases} = 1.296 \times 10^3$$

Coefficient that describes the development of creep with time after loading:

$$\beta_{c.t.to} := \left[\frac{(t - t_o)}{\beta_H + (t - t_o)}\right]^{0.3} = 0.985$$

A factor to allow for the effect of relative humidity on the notional creep coefficient:

$$\varphi_{RH} := \left[1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_o}} \left(\frac{35}{f_{cm}}\right)^{0.7}\right] \cdot \left(\frac{35}{f_{cm}}\right)^{0.2} = 1.172$$

A factor to allow for the effect of concrete strength on the notional creep coefficient:

$$\beta_{fcm} := \frac{16.8}{\sqrt{f_{cm}}} = 2.562$$

A factor to allow for the effect of concrete age at loading on the notional creep coefficient:

$$\beta_{to} := \frac{1}{0.1 + t_o^{\,0.2}} = 0.488$$

The notional creep coefficient: $\varphi_o := \varphi_{RH} \cdot \beta_{fcm} \cdot \beta_{to} = 1.466$

Creep coefficient: $\varphi_{cr} := \beta_{c.t.to} \cdot \varphi_o = 1.445$

***Loads:***

Permanent loads:

Weight reinforced concrete: $\gamma_{gC} := 25\frac{kN}{m^3}$

At the free end: $g_{d.l} := \gamma_{gC} \cdot w \cdot h_l = 60.75 \cdot \frac{kN}{m}$

At the fixed end: $g_{d.r} := \gamma_{gC} \cdot w \cdot h_r = 93.375 \cdot \frac{kN}{m}$

Weight ballast: $\gamma_{gB} := 20\frac{kN}{m^3}$

As uniform load: $g_B := \gamma_{gB} \cdot w \cdot h_B = 54 \cdot \frac{kN}{m}$

Weight of Rail as uniform load: $g_R := 1.2\frac{kN}{m}$

Partial factor for a permanent load in ULS: $\gamma_G := 1.35$ [EN 1990/A1:2005 Table A2.4(B)]

Variable Loads LM71: [EN 1991-2:2003 6]

Uniformly distribute train load: $q_{71} := 80\frac{kN}{m}$

Concentrated axle load: $P_{71} := 250kN$

Length of locomotive: $l_t := 6.4m$

Locomotive load as a uniform load: $q_{l.71} := \frac{4 \cdot P_{71}}{l_t} = 156.25 \cdot \frac{kN}{m}$

Classification factor: $\alpha := 1.33$

Partial factor for a variable load in ULS: $\gamma_Q := 1.5$ [EN 1990/A1:2005 Table A2.4(B)]

## Dynamic amplification factor: [EN 1991-2:2005 6.4.5]

$L_\phi$ - determinant length according to [EN 1991-2 Table 6.2 case 5.2]

$$L_1 := L \quad L_2 := L \quad k := 1.2 \quad n := 2 \quad L_m := \frac{1}{n}(L_1 + L_2)$$

$$L_\phi := L_m \cdot \frac{k}{m} = 8.4$$

Carefully maintained track; [EN 1991-2 6.4.5.2 (3)P], [BFS 2009:16 6§]

$$\phi_2 := \begin{cases} 1.00 & \text{if } \frac{1.44}{\sqrt{L_\phi} - 0.2} + 0.82 < 1.00 \\ \frac{1.44}{\sqrt{L_\phi} - 0.2} + 0.82 & \text{if } 1.00 \le \frac{1.44}{\sqrt{L_\phi} - 0.2} + 0.82 \le 1.67 \\ 1.67 & \text{if } \frac{1.44}{\sqrt{L_\phi} - 0.2} + 0.82 > 1.67 \end{cases} = 1.354$$

## Reduction of the dynamic factor: [EN 1991-2:2005 Annex D]

The reduction factor takes into account the fact that $\phi_2$ gives a value that is not reflecting the mean effects during the structures service life.

Speed limit of the different train models [km/h]: $v := \begin{pmatrix} 80 \\ 100 \\ 120 \\ 160 \\ 200 \end{pmatrix}$

$$K := \begin{cases} \frac{v}{160} & \text{if } L_\phi \le 20 \\ \frac{v}{47.16 \cdot L_\phi^{0.408}} & \text{if } L_\phi > 20 \\ 0 & \text{otherwise} \end{cases} \quad \varphi' := \frac{K}{1 - K + K^4} = \begin{pmatrix} 0.889 \\ 1.185 \\ 1.324 \\ 1 \\ 0.57 \end{pmatrix} \quad \varphi'' := 0.56 e^{-\left(\frac{L_\phi^2}{100}\right)} = 1.134$$

The reduction factor: $\varphi_{fat} := 1 + \frac{1}{2}\left(\varphi' + \frac{1}{2} \cdot \varphi''\right) = \begin{pmatrix} 1.728 \\ 1.876 \\ 1.946 \\ 1.784 \\ 1.569 \end{pmatrix}$

## Multiplicators used in calculations by Stripstep2:

This factor is used for LM71 in the ultimate state (ULS). It takes into account the dynamic factor,classification factor and the partial factor for a variable load: $\alpha \cdot \phi_2 \cdot \gamma_Q = 2.701$

This factor is used for LM71 in the service state (SLS) Includes classification and dynamic factor: $\alpha \cdot \phi_2 = 1.8$

### *Approximation of reinforcement*

In the result file from StripStep2 the closest coordinates to the maximum moment section is put in the equation below together with the corresponding shear values in order to approximate the load divider. The achieved length is then once again put into StripStep2 and a new calculation is performed:

Approximation of the load divider: $x_0 := 0.313 \cdot L + (0.375 - 0.313) \cdot \frac{29}{29 + 60} \cdot L = 2.332\,m$

Design moment accieved from calculations in StripStep2:

Field section: $M_{Ed.f} := 1571 \cdot kNm$

Support section: $M_{Ed.s} := 4781kNm$

for $\begin{pmatrix} x_0 \\ L \end{pmatrix} = \begin{pmatrix} 2.332 \\ 7 \end{pmatrix} m$

Assumed distance from centre reinforcement to edge: $d_e := 50mm$

Height of considered section: Field section: $d_f := h_l - d_e = 490 \cdot mm$

Support section: $d_s := h_r - d_e = 780 \cdot mm$

Aproximation of internal lever arm: $z_f := 0.9 \cdot d_f = 441 \cdot mm$

$z_s := 0.9 \cdot d_s = 702 \cdot mm$

## Approximate reinforcement in support section:

Assuming yielding of reinforcement $\sigma_s := f_{yd} = 434.783 \cdot MPa$

Required amount of steel area: $A_{supp} := \frac{M_{Ed.s}}{f_{yd} \cdot z_s} = 0.016\,m^2$

Assuming bar dimension: $A_{si25} = 490.874 \cdot mm^2$

Number of bars required: $n_s := ceil\left(\frac{A_{supp}}{A_{si25}}\right) = 32$

## Approximate reinforcement in field section:

Assuming yielding of reinforcement $\sigma_s = 434.783 \cdot MPa$

Required amount of steel area: $A_{field} := \frac{M_{Ed.f}}{f_{yd} \cdot z_f} = 8.193 \times 10^3 \cdot mm^2$

Number of bars required: $n_f := ceil\left(\frac{A_{field}}{A_{si25}}\right) = 17$

Placing of reinforcement with regard to minimum concrete cover:

Bar diameter: $c_{min.b} := \phi = 25 \cdot mm$ [EN 1992-1-1:2005 4.4.1.2(3)]

Assuming exposure class XS1 , XF1, XD1 and XC4 Table 4.1
Exposure classes gives a recommended structural classification Table 4.3N
The enviromental requirement for $c_{min.dur}$ is taken for table 4.4(S) with regard to exposure class

$c_{min.dur} := 30mm$ Decisive case: L100, XS1 [BFS 2009:16 Table 4.4(S)]

$\Delta c_{dur} := 0mm$ [EN 1992-1-1:2005 4.4.1.1(6)]

$\Delta c_{dur.st} := 0mm$ [EN 1992-1-1:2005 4.4.1.1(7)]

$\Delta c_{dur.add} := 0mm$ [EN 1992-1-1:2005 4.4.1.1(8)]

$$c_{min} := \max\left(c_{min.b}, c_{min.dur} + \Delta c_{dur} - \Delta c_{dur.st} - \Delta c_{dur.add}, 10mm\right) = 30 \cdot mm$$

$\Delta c_{dev} := 10mm$ [EN 1992-1-1:2005 4.4.1.3]

Minimum layer of concrete cover thickness:

$c_{nom.c} := c_{min} + \Delta c_{dev} = 40 \cdot mm$ [EN 1992-1-1:2005, 4.4.1.1 Eq.4.1]

With regard to anchorage: $c_{nom.a} := c_{nom.c}$

Distance to the edge; center of the bar: $d_e := c_{nom.a} + \frac{\phi}{2} = 52.5 \cdot mm$

Distance between bars:

$k_1 := 1$ [BFS 2009:16]

$k_2 := 5mm$

$d_g := 20mm$ Assumed

$\max\left(k_1 \cdot \phi, d_g + k_2, 20mm\right) = 25 \cdot mm$ [EN 1992-1-1:2005 8.2(2)]

Reinforcement center to center distance: $cc := \max\left(k_1 \cdot \phi, d_g + k_2, 20mm\right) + \phi = 50 \cdot mm$

Maximum number of bars in one layer: $n_{max} := \frac{w - d_e \cdot 2}{cc} = 87.9$

## Minimum amount of reinforcement: [EN 1992-1-1:2005 9.3.1.1 (1)]

To know the minumum number of bars that will continue through the whole slab at the field and support sections.

Amount of minimum steel area required - Field section:

$$A_{sf.min} := \begin{cases} 0.26 \cdot \dfrac{f_{ctm}}{f_{yk}} \cdot w \cdot d_f & \text{if } 0.26 \cdot \dfrac{f_{ctm}}{f_{yk}} \cdot w \cdot d_f \geq 0.0013 \cdot w \cdot d_f \\ 0.0013 \cdot w \cdot d_f & \text{otherwise} \end{cases} = 3.669 \times 10^3 \cdot mm^2$$

Number of bars required: $n_{min.f} := \dfrac{A_{sf.min}}{A_{si25}} = 7.475$

Spacing between bars: $s_{sf.min} := \dfrac{w}{n_{min.f}} = 0.602\,m$

Amount of minimum steel area required - Support section:

$$A_{ss.min} := \begin{cases} 0.26 \cdot \dfrac{f_{ctm}}{f_{yk}} \cdot w \cdot d_s & \text{if } 0.26 \cdot \dfrac{f_{ctm}}{f_{yk}} \cdot w \cdot d_s \geq 0.0013 \cdot w \cdot d_s \\ 0.0013 \cdot w \cdot d_s & \text{otherwise} \end{cases} = 5.841 \times 10^3 \cdot mm^2$$

Number of bars required: $n_{min.s} := \dfrac{A_{ss.min}}{A_{si25}} = 11.898$

Spacing between bars: $s_{ss.min} := \dfrac{w}{n_{min.s}} = 0.378\,m$

## *Design of the slab with regard to moment capacity:*

Stress block factors for fully developed stress block:

Factor for average stress: $\alpha_R := 0.81$

Factor for location of stress resultant: $\beta_R := 0.416$

Ultimate strain of concrete: $\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

### Field section:

Assuming yielding in the tensile reinforcement:

Approximated number of bars: $n_f = 17$

New assumption if needed (See end of this section): $n_f := 16$

$x_f := 46mm$

$$x_f := \mathrm{root}\left(\alpha_R \cdot f_{cd} \cdot w \cdot x_f + E_{sm} \cdot \frac{x_f - d_e}{x_f} \cdot \varepsilon_{cu} \cdot A_{sf.min} - f_{yd} n_f \cdot A_{si25}, x_f\right) = 45.103 \cdot mm$$

Check of the assumptions in the equation:

Compression side: $\varepsilon_{s.c} := \frac{x_f - d_e}{x_f} \cdot \varepsilon_{cu} = -5.74 \times 10^{-4}$

$\varepsilon_{sy} := \frac{f_{yd}}{E_{sm}} = 2.174 \times 10^{-3}$

no yielding: $\varepsilon_{s.c} \le \varepsilon_{sy} = 1$ yielding: $\varepsilon_{s.c} \ge \varepsilon_{sy} = 0$

Tension side: $\varepsilon_{s.t} := \frac{(h_1 - d_e) - x_f}{x_f} \cdot \varepsilon_{cu} = 0.034$

no yielding: $\varepsilon_{s.t} \le \varepsilon_{sy} = 0$ yielding: $\varepsilon_{s.t} \ge \varepsilon_{sy} = 1$

$$M_{Rd.f} := \alpha_R \cdot f_{cd} \cdot w \cdot x_f \cdot \left[(h_1 - d_e) - \beta_R \cdot x_f\right] \dots + \varepsilon_{s.c} \cdot E_{sm} \cdot A_{sf.min} \cdot \left[(h_1 - d_e) - d_e\right] = 1.615 \times 10^3 \cdot kNm$$

Check that the moment capacity is correct according to the design moment. If not, the approximation of number of bars in the tensile area needs to be modified at the top of this section:

$M_{Rd.f} \ge M_{Ed.f} = 1$ $M_{Ed.f} = 1.571 \times 10^3 \cdot kNm$

## Support section:

Assuming yielding in the tensile reinforcement:

Approximated number of bars: $n_s = 32$

New assumption if needed (See end of this section): $n_s := 30$

$x_s := 65\text{mm}$

$$x_s := \text{root}\left(\alpha_R \cdot f_{cd} \cdot w \cdot x_s + E_{sm} \cdot \frac{x_s - d_e}{x_s} \cdot \varepsilon_{cu} \cdot A_{ss.min} - f_{yd} \cdot n_s \cdot A_{si25}, x_s\right) = 65.652 \cdot \text{mm}$$

Check of the assumptions in the equation:

Compression side: $\varepsilon_{s.c} := \dfrac{x_s - d_e}{x_s} \cdot \varepsilon_{cu} = 7.011 \times 10^{-4}$

$\varepsilon_{sy} := \dfrac{f_{yd}}{E_{sm}} = 2.174 \times 10^{-3}$

no yielding: $\varepsilon_{s.c} \le \varepsilon_{sy} = 1$ yielding: $\varepsilon_{s.c} \ge \varepsilon_{sy} = 0$

Tension side: $\varepsilon_{s.t} := \dfrac{(h_r - d_e) - x_s}{x_s} \cdot \varepsilon_{cu} = 0.038$

no yielding: $\varepsilon_{s.t} \le \varepsilon_{sy} = 0$ yielding: $\varepsilon_{s.t} \ge \varepsilon_{sy} = 1$

Moment capacity in the section:

$$M_{Rd.s} := \alpha_R \cdot f_{cd} \cdot w \cdot x_s \cdot \left[(h_r - d_e) - \beta_R \cdot x_s\right] \ldots + \varepsilon_{s.c} \cdot E_{sm} \cdot A_{ss.min} \cdot \left[(h_r - d_e) - d_e\right] = 4.783 \times 10^3 \cdot \text{kNm}$$

Check that the moment capacity is correct according to the design moment. If not, the approximation of number of bars in the tensile area needs to be modified at the top of this section:

$M_{Rd.s} \ge M_{Ed.s} = 1$ $M_{Ed.s} = 4.781 \times 10^3 \cdot \text{kNm}$

## Additional demands regarding maximum spacing:

The spacing with the calculated number of bars:

Spacing between bars, support section: $s_s := \frac{w}{n_s} = 0.15\,m$

Spacing between bars, field section: $s_f := \frac{w}{n_f} = 0.281\,m$

Maximum spacing of bars valid in slabs: [EN 1992-1-1:2005 9.3.1.1 (3)]

$$s_{max.slabs.s} := \left| \begin{array}{l} 3 \cdot h_r \text{ if } 3 \cdot h_r \le 400mm \\ 400mm \text{ otherwise} \end{array} \right. = 0.4\,m$$

$$s_{max.slabs.f} := \left| \begin{array}{l} 3 \cdot h_l \text{ if } 3 \cdot h_l \le 400mm \\ 400mm \text{ otherwise} \end{array} \right. = 0.4\,m$$

Check of demands regarding maximum spacing:

$s_s \le s_{max.slabs.s} = 1$

$$s_s := \left| \begin{array}{l} s_s \text{ if } (s_s \le s_{max.slabs.s}) \\ s_{max.slabs.s} \text{ otherwise} \end{array} \right. = 0.15\,m$$

$s_f \le s_{max.slabs.f} = 1$

$$s_f := \left| \begin{array}{l} s_f \text{ if } (s_f \le s_{max.slabs.f}) \\ s_{max.slabs.f} \text{ otherwise} \end{array} \right. = 0.281\,m$$

Correspondingly for minimum amount of reinforcement:

$s_{ss.min} \le s_{max.slabs.s} = 1$

$$s_{ss.min} := \left| \begin{array}{l} s_{ss.min} \text{ if } (s_{ss.min} \le s_{max.slabs.s}) \\ s_{max.slabs.s} \text{ otherwise} \end{array} \right. = 0.378\,m$$

$s_{sf.min} \le s_{max.slabs.f} = 0$

$$s_{sf.min} := \left| \begin{array}{l} s_{sf.min} \text{ if } (s_{sf.min} \le s_{max.slabs.f}) \\ s_{max.slabs.f} \text{ otherwise} \end{array} \right. = 0.4\,m$$

Maximum spacing of bars in the area of maximum moment section: [EN 1992-1-1:2005 9.3.1.1 (3)]

$$s_{max.slabs.sm} := \left| \begin{array}{l} 2 \cdot h_r \text{ if } 2 \cdot h_r \le 250mm \\ 250mm \text{ otherwise} \end{array} \right. = 0.25\,m$$

$$s_{max.slabs.fm} := \left| \begin{array}{l} 2 \cdot h_l \text{ if } 2 \cdot h_l \le 250mm \\ 250mm \text{ otherwise} \end{array} \right. = 0.25\,m$$

Check of demands regarding maximum spacing in the area of maximum moment section:

$s_s \le s_{max.slabs.sm} = 1$

$$s_{sm} := \begin{cases} s_s & \text{if } s_s \le s_{max.slabs.sm} \\ s_{max.slabs.sm} & \text{otherwise} \end{cases} = 0.15\,m$$

$s_f \le s_{max.slabs.fm} = 0$

$$s_{fm} := \begin{cases} s_f & \text{if } s_f \le s_{max.slabs.fm} \\ s_{max.slabs.fm} & \text{otherwise} \end{cases} = 0.25\,m$$

In accordance with the demands on maximum spacing the number of bars in the maximum moment sections is as follows:

Support section: $n_{sm} := \dfrac{w}{s_{sm}} = 30$

Field section: $n_{fm} := \dfrac{w}{s_{fm}} = 18$

Maximum number of bar that fits in one row: $n_{max} = 87.9$

Sections which is in compression and areas other than maximum moment sections:

Support section: $n_{s.min} := \dfrac{w}{s_{ss.min}} = 11.898$

Field section: $n_{f.min} := \dfrac{w}{s_{sf.min}} = 11.25$

## Calculation of tensile capacity demand on the longitudinal reinforcement:

Additional tensile force from shear reinforcement: [EN 1992-1-1:2005 6.2.3 (7)]

Moment and Shear force envelopes and span coordinates imported from MS Excel worksheet "Sectional Forces". Calculated in Stripstep2.

$V_{pos} :=$ ...\Figure Se[illegible]al forces.xls

$V_{Ed.pos} := V_{pos} \cdot kN$

$M_{pos} :=$ ...\Figure Se[illegible]al forces.xls

$M_{Ed.pos} := M_{pos} \cdot kNm$

$Lx :=$ ...\Figure Se[illegible]al forces.xls

$Lx := Lx \cdot m$

$V_{neg} :=$ ...\Figure Se[illegible]al forces.xls

$V_{Ed.neg} := V_{neg} \cdot kN$

$M_{neg} :=$ ...\Figure Se[illegible]al forces.xls

$M_{Ed.neg} := M_{neg} \cdot kNm$

$z :=$ ...\Figure Se[illegible]al forces.xls

$i := 0 .. (\text{length}(z) - 1)$

$$z_i := \begin{cases} z_i \cdot \left[ (h_l - d_e - x_f) \dfrac{1}{m} \right] & \text{if } 0 \cdot m \le Lx_i \le L - l_h \\ z_i \cdot \left[ (h_l - d_e - x_s) + \left[ Lx_i - (L - l_h) \right] \cdot \dfrac{h_r - h_l}{l_h} \right] \cdot \dfrac{1}{m} & \text{if } L - l_h < Lx_i \le L \\ 1 & \text{otherwise} \end{cases}$$

Needed tensile capacity due to the moment:

$$M_{Ed.z.pos} := \frac{M_{Ed.pos}}{z \cdot m} = \cdots \cdot kN \qquad M_{Ed.z.neg} := \frac{M_{Ed.neg}}{z \cdot m} = \cdots \cdot kN$$

$M_{Ed.z.pos}$ (kN):

| | 0 |
|---|---|
| 0 | 0 |
| 1 | $1.216 \cdot 10^3$ |
| 2 | $1.343 \cdot 10^3$ |
| 3 | $1.808 \cdot 10^3$ |
| 4 | $2.177 \cdot 10^3$ |
| 5 | ... |

$M_{Ed.z.neg}$ (kN):

| | 0 |
|---|---|
| 0 | 0 |
| 1 | 327.76 |
| 2 | 359.406 |
| 3 | 490.51 |
| 4 | 585.447 |
| 5 | ... |

The additional tensile force needed in the longitudinal reinforcement due to the shear force:

Assumed angle of the shear crack in design: $\theta := 22\mathrm{deg}$

Angle between longitudinal and the shear reinforcement: $\alpha_{sr} := 90\mathrm{deg}$

$$\Delta F_{td.pos} := \frac{V_{Ed.pos} \cdot \left(\cot(\theta) - \cot(\alpha_{sr})\right)}{2} = \cdot \mathrm{kN}$$

| | 0 |
|---|---|
| 0 | 1.666·10³ |
| 1 | 1.353·10³ |
| 2 | 1.317·10³ |
| 3 | 1.165·10³ |
| 4 | 1.04·10³ |
| 5 | ... |

$$\Delta F_{td.neg} := \frac{V_{Ed.neg} \cdot \left(\cot(\theta) - \cot(\alpha_{sr})\right)}{2} = \cdot \mathrm{kN}$$

| | 0 |
|---|---|
| 0 | -451.703 |
| 1 | -366.313 |
| 2 | -356.413 |
| 3 | -315.574 |
| 4 | -282.16 |
| 5 | ... |

Total demand of tensile capacity in the longitudinal reinforcement:

$$F_{td.pos} := M_{Ed.z.pos} + \Delta F_{td.pos} = \cdot \mathrm{kN}$$

| | 0 |
|---|---|
| 0 | 1.666·10³ |
| 1 | 2.569·10³ |
| 2 | 2.659·10³ |
| 3 | 2.973·10³ |
| 4 | 3.216·10³ |
| 5 | ... |

$$F_{td.neg} := M_{Ed.z.neg} + \Delta F_{td.neg} = \cdot \mathrm{kN}$$

| | 0 |
|---|---|
| 0 | -451.703 |
| 1 | -38.553 |
| 2 | 2.993 |
| 3 | 174.936 |
| 4 | 303.287 |
| 5 | ... |

Number of bars in accordance with the demands for tensile capacity:

Assumed angle of shear crack in design: $\theta = 22 \cdot deg$

Demand in the maximum moment sections:

Support:

$$n_{ss} := -\frac{\min(M_{Ed.z.neg})}{A_{si25} \cdot f_{yd}} = 31.469$$

Field:

$$n_{ff} := \frac{\max(M_{Ed.z.pos})}{A_{si25} \cdot f_{yd}} = 16.639$$

Minimum amount of bars in tension and compression:

$n_{s.min} = 11.898$ $\qquad$ $n_{f.min} = 11.25$

Demand with regart to the moment and the additional tensile force from the shear reinforcement:

Free support: $\qquad$ Fixed support:

$x_{45}$

$$n_{free.45} := \frac{F_{td.pos_1}}{A_{si25} \cdot f_{yd}} = 12.036 \qquad n_{fix.45} := \frac{-F_{td.neg}(\mathrm{length}(F_{td.neg})-2)}{A_{si25} \cdot f_{yd}} = 40.176$$

$$n_{free45} := \min(n_{ff}, n_{free.45}) = 12.036 \qquad n_{fix45} := \min(n_{ss}, n_{fix.45}) = 31.469$$

$x_{22}$

$$n_{free.22} := \frac{F_{td.pos_4}}{A_{si25} \cdot f_{yd}} = 15.07 \qquad n_{fix.22} := \frac{-F_{td.neg}(\mathrm{length}(F_{td.neg})-6)}{A_{si25} \cdot f_{yd}} = 25.607$$

$$n_{free22} := \min(n_{ff}, n_{free.22}) = 15.07 \qquad n_{fix22} := \min(n_{ss}, n_{fix.22}) = 25.607$$

$$i := 0 .. \left(\frac{L}{m} \cdot 100\right)$$

$$Lx2 := 0m, 0.01m .. L = \boxed{\ \ldots\ }\ m$$

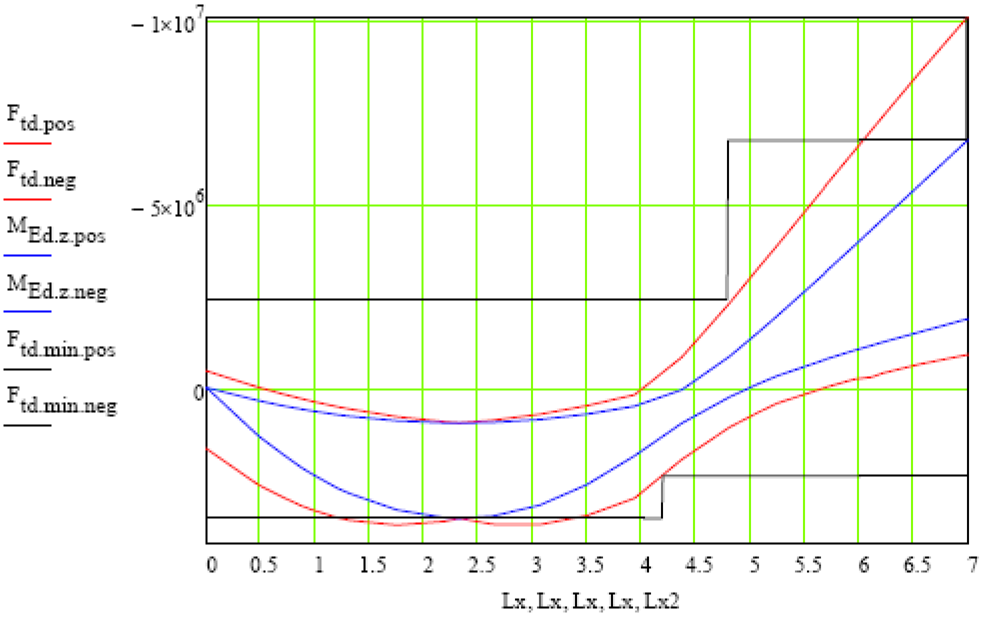
$$F_{td.min.neg_i} := \begin{cases} -(A_{si25} \cdot f_{yd} \cdot n_{f.min}) & \text{if } -0.5 \cdot m \le Lx2_i < 4.8m \\ -(A_{si25} \cdot f_{yd} \cdot n_{ss}) & \text{if } 4.8m \le Lx2_i < L \\ -10^8 N & \text{otherwise} \end{cases}$$

$$F_{td.min.pos_i} := \begin{cases} (A_{si25} \cdot f_{yd} \cdot n_{ff}) & \text{if } 0 \cdot m \le Lx2_i < 4.2m \\ (A_{si25} \cdot f_{yd} \cdot n_{f.min}) & \text{if } 4.2 \cdot m \le Lx2_i \le 7m \\ 10^8 \cdot N & \text{otherwise} \end{cases}$$

$$A_{si25} \cdot f_{yd} = 2.134 \times 10^5 N$$

Figure with the tensile demands on the longitudinal reinforcement:

Created according to figure 9.2 in EN 1992-1-1:2005




The Blue lines are the tensile demand from the sectional moment only.

The Red lines also includes the additional tensile demand from the sectional shear force.

The Black lines represent the minimum amount of reinforcement in a section together with the amount of reinforcement in the maximum moment sections which governs the maximum amount of reinforcement needed in the entire slab.

# Appendix H. Cumulative Damage Method Fatigue Assessment of railway bridge

Sectional forces in the considered section:

Permanent moment:
(Calculated in StripStep2)

$M_{perm} := 983kNm$

Inserting tables from textfiles:
(Calculated in AFB)

$$M_a := \begin{pmatrix} 2000 \\ 1500 \\ 1000 \\ 500 \end{pmatrix} \qquad M_m := \begin{pmatrix} 1500 \\ 15 \\ 10 \\ 5 \end{pmatrix} \qquad n_n := \begin{pmatrix} 1 \\ 1 \\ 0.5 \\ 1 \end{pmatrix}$$

Moment amplitude for each cycle:

$$M_A := M_a \cdot kNm = \begin{pmatrix} 2 \times 10^3 \\ 1.5 \times 10^3 \\ 1 \times 10^3 \\ 500 \end{pmatrix} \cdot kNm$$

Mean moment for each cycle:

$$M_M := -M_m \cdot kNm = \begin{pmatrix} -1.5 \times 10^3 \\ -15 \\ -10 \\ -5 \end{pmatrix} \cdot kNm$$

These values should have a positive sign regardless of the value of the input data in order for the calculations to be correct!

Number of cycles:

$N_{year} := 120 \qquad days := 365$

$$n := n_n \cdot N_{year} \cdot days = \begin{pmatrix} 4.38 \times 10^4 \\ 4.38 \times 10^4 \\ 2.19 \times 10^4 \\ 4.38 \times 10^4 \end{pmatrix}$$

Minimum moment in a load cycle:

$$M_{min} := M_M - \frac{M_A}{2} = \begin{pmatrix} -2.5 \times 10^3 \\ -765 \\ -510 \\ -255 \end{pmatrix} \cdot kNm$$

Maximum moment in a load cycle:

$$M_{max} := M_M + \frac{M_A}{2} = \begin{pmatrix} -500 \\ 735 \\ 490 \\ 245 \end{pmatrix} \cdot kNm$$

## Arrangement of reinforcement:

Mimimum distances are calculated in Mathcad - Bridge design
To be able to do as general calculations as possible, three layers of reinforcement on the tension side and one layer on the compressive side are used in the calculations. If there are fewer layers in the current case the number of bars is set to zero.

Compression side:

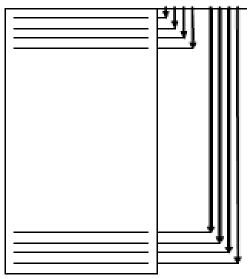
First layer: $d_1 := d_e = 52.5 \cdot mm$

Tension side:

First layer: $d'_1 := h_r - d_e = 777.5 \cdot mm$

Second layer: $d'_2 := 0mm$

Third layer: $d'_3 := 0mm$



## Amount of reinforcement: (Calculated in Mathcad - Bridge Design)

### Compression side

Layer 1:

Number of bars first layer: $n_1 := n_{s.min} = 11.898$

$A_{s1} := A_{si25} \cdot n_1 = 5.841 \times 10^{-3}\,m^2$

### Tension side:

Layer 1:

Number of bars first layer: $n'_1 := 30$

$A'_{s1} := A_{si25} \cdot n'_1 = 0.015\,m^2$

Layer 2:

Number of bars second layer: $n'_2 := 0$

$A'_{s2} := A_{si25} \cdot n'_2 = 0$

Layer 3:

Number of bars third layer: $n'_3 := 0$

$A'_{s3} := A_{si25} \cdot n'_3 = 0$

## Modular ratios based on effective creep functions:

The final creep function: (Calculated in Mathcad - Bridge Design) $\varphi_{cr} = 1.445$

The creep function for variable load in the quasi-permanent combination: $\varphi_{qp} := 0$

[Engström B (2008).*Restraint cracking of reinforced concrete structures.* Chalmers unisversity of technology,Göteborg 2008, Section 6.4.3]

Effective creep function for the maximum sectional moments due to permanent and fatigue load:

$$\varphi_{ef.max} := \frac{\varphi_{cr} \cdot M_{perm} + \varphi_{qp} \cdot (M_{max})}{M_{perm} + M_{max}} = \begin{pmatrix} 2.94 \\ 0.827 \\ 0.964 \\ 1.156 \end{pmatrix}$$

$$\alpha_{s.ef.max} := (1 + \varphi_{ef.max}) \cdot \frac{E_{sm}}{E_{cm}} = \begin{pmatrix} 23.176 \\ 10.744 \\ 11.553 \\ 12.684 \end{pmatrix}$$

$\min(\alpha_{s.ef.max}) = 10.744$

$\max(\alpha_{s.ef.max}) = 23.176$

Effective creep function for the minimum sectional moments due to permanent and fatigue load:

$$\varphi_{ef.min} := \frac{\varphi_{cr} \cdot M_{perm} + \varphi_{qp} \cdot (M_{min})}{M_{perm} + M_{min}} = \begin{pmatrix} -0.936 \\ 6.514 \\ 3.002 \\ 1.95 \end{pmatrix}$$

$$\alpha_{s.ef.min} := (1 + \varphi_{ef.min}) \cdot \frac{E_{sm}}{E_{cm}} = \begin{pmatrix} 0.376 \\ 44.197 \\ 23.541 \\ 17.356 \end{pmatrix}$$

$\min(\alpha_{s.ef.min}) = 0.376$

$\max(\alpha_{s.ef.min}) = 44.197$

Effective creep function for the minimum sectional moments due to permanent load:

$$\varphi_{ef.perm} := \frac{\varphi_{cr} \cdot M_{perm}}{M_{perm}} = 1.445$$

$$\alpha_{s.ef.perm} := \left(1 + \varphi_{ef.perm}\right) \cdot \frac{E_{sm}}{E_{cm}} = 14.379$$

## Sectional constants for the maximum sectional moment:

[Engström B (2008). *Design and analysis of continuous beams and colums*. Chalmers unisversity of technology,Göteborg 2008, Chapter 7]

Neutral axis of the effective transformed concrete section: $i := 0 .. \left(\text{length}\left(\alpha_{s.ef.max}\right) - 1\right)$

$x_{II.max} := 0.20m$

$$x_{II.max_i} := \text{root}\left[\begin{array}{l} \frac{w \cdot \left(x_{II.max}\right)^2}{2} + \left(\alpha_{s.ef.max_i} - 1\right) \cdot A_{s1} \cdot \left(x_{II.max} - d_1\right) \cdots \\ + -\alpha_{s.ef.max_i} \cdot A'_{s1} \cdot \left(d'_1 - x_{II.max}\right) - \alpha_{s.ef.max_i} \cdot A'_{s2} \cdot \left(d'_2 - x_{II.max}\right) \cdots \\ + -\alpha_{s.ef.max_i} \cdot A'_{s3} \cdot \left(d'_3 - x_{II.max}\right) \end{array}, x_{II.max}\right]$$

$$x_{II.max} = \begin{pmatrix} 0.259 \\ 0.194 \\ 0.199 \\ 0.207 \end{pmatrix} m$$

$\min\left(x_{II.max}\right) = 0.194\,m$

$\max\left(x_{II.max}\right) = 0.259\,m$

Moment of inertia for the effective transformed concrete section:

$$I_{II.max_i} := \frac{w \cdot \left(x_{II.max_i}\right)^3}{3} + \left(\alpha_{s.ef.max_i} - 1\right) \cdot A_{s1} \cdot \left(x_{II.max_i} - d_1\right)^2 + \alpha_{s.ef.max_i} \cdot A'_{s1} \cdot \left(d'_1 - x_{II.max_i}\right)^2 \cdots$$
$$+ \alpha_{s.ef.max_i} \cdot A'_{s2} \cdot \left(d'_2 - x_{II.max_i}\right)^2 + \alpha_{s.ef.max_i} \cdot A'_{s3} \cdot \left(d'_3 - x_{II.max_i}\right)^2$$

$$I_{II.max} = \begin{pmatrix} 0.123 \\ 0.066 \\ 0.07 \\ 0.076 \end{pmatrix} m^4$$

## Sectional constants for the minimum sectional moment:

Neutral axis of the effective transformed concrete section:

$x_{II.min} := 0.22\text{m}$

$$x_{II.min_i} := \text{root}\left[\frac{w \cdot x_{II.min}^2}{2} + \left(\alpha_{s.ef.min_i} - 1\right)\cdot A_{s1}\cdot\left(x_{II.min} - d_1\right) \cdots + -\alpha_{s.ef.min_i}\cdot A'_{s1}\cdot\left(d'_1 - x_{II.min}\right) - \alpha_{s.ef.min_i}\cdot A'_{s2}\cdot\left(d'_2 - x_{II.min}\right) \cdots + -\alpha_{s.ef.min_i}\cdot A'_{s3}\cdot\left(d'_3 - x_{II.min}\right), x_{II.min}\right]$$

$$x_{II.min} = \begin{pmatrix} 0.042 \\ 0.32 \\ 0.26 \\ 0.233 \end{pmatrix}\text{m}$$

$\min\left(x_{II.min}\right) = 0.042\text{ m}$

$\max\left(x_{II.min}\right) = 0.32\text{ m}$

Moment of inertia for the effective transformed concrete section:

$$I_{II.min_i} := \frac{w\cdot\left(x_{II.min_i}\right)^3}{3} + \left(\alpha_{s.ef.min_i} - 1\right)\cdot A_{s1}\cdot\left(x_{II.min_i} - d_1\right)^2 + \alpha_{s.ef.min_i}\cdot A'_{s1}\cdot\left(d'_1 - x_{II.min_i}\right)^2 \cdots + \alpha_{s.ef.min_i}\cdot A'_{s2}\cdot\left(d'_2 - x_{II.min_i}\right)^2 + \alpha_{s.ef.min_i}\cdot A'_{s3}\cdot\left(d'_3 - x_{II.min_i}\right)^2$$

$$I_{II.min} = \begin{pmatrix} 3.109\times 10^{-3} \\ 0.203 \\ 0.125 \\ 0.098 \end{pmatrix}\text{m}^4$$

## Sectional constants for the permanent sectional moment:

Neutral axis of the effective transformed concrete section:

$x_{II.perm} := 0.22m$

$$x_{II.perm} := \mathrm{root}\left[\frac{w \cdot x_{II.perm}^{2}}{2} + (\alpha_{s.ef.perm} - 1) \cdot A_{s1} \cdot (x_{II.perm} - d_1) \ldots \right. \\ + -\alpha_{s.ef.perm} \cdot A'_{s1} \cdot (d'_1 - x_{II.perm}) - \alpha_{s.ef.perm} \cdot A'_{s2} \cdot (d'_2 - x_{II.perm}) \ldots \\ \left. + -\alpha_{s.ef.perm} \cdot A'_{s3} \cdot (d'_3 - x_{II.perm}) \quad , x_{II.perm}\right]$$

$x_{II.perm} = 0.217\,m$

Moment of inertia for the effective transformed concrete section:

$$I_{II.perm} := \frac{w \cdot x_{II.perm}^{3}}{3} + (\alpha_{s.ef.perm} - 1) \cdot A_{s1} \cdot (x_{II.perm} - d_1)^2 \ldots \\ + \alpha_{s.ef.perm} \cdot A'_{s1} \cdot (d'_1 - x_{II.perm})^2 + \alpha_{s.ef.perm} \cdot A'_{s2} \cdot (d'_2 - x_{II.perm})^2 \ldots \\ + \alpha_{s.ef.perm} \cdot A'_{s3} \cdot (d'_3 - x_{II.perm})^2 \quad = 0.084\,m^4$$

### *Calculating stresses due to permanent load and fatigue load:*

When calculating the stresses the first index describes which material that is considered (s - steel, c - concrete), and the second indicates on which reinforcement level the stress is calculated (t - tensile reinforcement, c - compressive reinforcement).

## Concrete stresses

Maximum tensile concrete stress in a cycle:
(Uncracked state I)

$$\sigma_{ct.I.max_i} := \frac{M_{perm} + M_{max_i}}{I_{Ir}} \cdot \frac{h_r}{2} \qquad \sigma_{ct.I.max} = \begin{pmatrix} 0.935 \\ 3.325 \\ 2.851 \\ 2.377 \end{pmatrix} \cdot MPa$$

$$\min(\sigma_{ct.I.max}) = 0.935 \cdot MPa$$

$$\max(\sigma_{ct.I.max}) = 3.325 \cdot MPa$$

Check if the section is cracked:
(If one value is equal to one the section can be considered as cracked)

$$\sigma_{ct.I.max} \geq f_{ctm} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Maximum compressive concrete stress in a cycle:

$$\sigma_{c.II.max_i} := \frac{M_{perm} + M_{max_i}}{I_{II.max_i}} \cdot \left(-x_{II.max}\right)_i \qquad \sigma_{c.II.max} = \begin{pmatrix} -1.013 \\ -5.043 \\ -4.188 \\ -3.351 \end{pmatrix} \cdot MPa$$

Minimum compressive concrete stress in a cycle:

$$\sigma_{c.II.min_i} := \frac{M_{perm} + M_{min_i}}{I_{II.min_i}} \cdot -x_{II.min_i} \qquad \sigma_{c.II.min} = \begin{pmatrix} 20.671 \\ -0.343 \\ -0.985 \\ -1.732 \end{pmatrix} \cdot MPa$$

Concrete compressive stress at permanent load:

$$\sigma_{c.II.perm} := \frac{M_{perm}}{I_{II.perm}} \cdot -x_{II.perm} = -2.539 \cdot MPa$$

## Steel stresses:

Stresse range in the compression reinforcement

$$\Delta\sigma_{sc_i} := \left(\frac{\alpha_{s.ef.max_i} + \alpha_{s.ef.min_i}}{2}\right) \cdot \left[\frac{M_{max_i} - M_{min_i}}{\left(\frac{I_{II.max_i} + I_{II.min_i}}{2}\right)} \cdot \left[d_1 - \left(\frac{x_{II.max_i} + x_{II.min_i}}{2}\right)\right]\right]$$

$$\Delta\sigma_{sc} = \begin{pmatrix} -36.492 \\ -62.494 \\ -31.884 \\ -14.472 \end{pmatrix} \cdot MPa$$

Stresse range in the tension reinforcement

$$\Delta\sigma_{st_i} := \left(\frac{\alpha_{s.ef.max_i} + \alpha_{s.ef.min_i}}{2}\right) \cdot \left[\frac{M_{max_i} - M_{min_i}}{\left(\frac{I_{II.max_i} + I_{II.min_i}}{2}\right)} \cdot \left[d'_1 - \left(\frac{x_{II.max_i} + x_{II.min_i}}{2}\right)\right]\right]$$

$$\Delta\sigma_{st} = \begin{pmatrix} 233.587 \\ 159.295 \\ 98.625 \\ 48.259 \end{pmatrix} \cdot MPa$$

$$\min(\Delta\sigma_{st}) = 48.259 \cdot MPa$$

$$\max(\Delta\sigma_{st}) = 233.587 \cdot MPa$$

## *Fatigue calculation of the reinforcing steel*

[EN 1992-1-1:2005 Section 6.8.4]

Partial factor taking material uncertainties into account:

$\gamma_{S.fat} := 1.15$ [EN 1992-1-1:2005 2.4.2.4(1)]

Partial factor taking the uncertainties in the fatigue load model into account:

$\gamma_{F.fat} := 1$ [EN 1992-1-1:2005 2.4.2.3(1)]

Resisting stress range at N* cycles in [MPa]:

$\Delta\sigma_{Rsk} := 162.5\text{MPa}$ [EN 1992-1-1:2005 6.8.4, table 6.3N]

Exponent defining the of slopes of the S-N relation:

$$k_1 := \left\| \begin{array}{l} 5 \;\text{ if }\; \Delta\sigma_{Rsk} = 162.5\text{MPa} = 5 \\ 3 \;\text{ if }\; \Delta\sigma_{Rsk} = 58.5\text{MPa} \\ 3 \;\text{ if }\; \Delta\sigma_{Rsk} = 35\text{MPa} \end{array} \right.$$

[EN 1992-1-1:2005 6.8.4, Table 6.3N]

Exponent defining the of slopes of the S-N relation:

$$k_2 := \left\| \begin{array}{l} 9 \;\text{ if }\; \Delta\sigma_{Rsk} = 162.5\text{MPa} = 9 \\ 5 \;\text{ if }\; \Delta\sigma_{Rsk} = 58.5\text{MPa} \\ 5 \;\text{ if }\; \Delta\sigma_{Rsk} = 35\text{MPa} \end{array} \right.$$

[EN 1992-1-1:2005 6.8.4, Table 6.3N]

## Damage calculation of the tensile reinforcement:

Current stress range:

$$\Delta\sigma_{st} = \begin{pmatrix} 233.587 \\ 159.295 \\ 98.625 \\ 48.259 \end{pmatrix} \cdot \text{MPa}$$

*Calculated before*

## Caluclating resisting number of cycles for the given stresses [EN 1992-1-1:2005 6.8.5(3)]

Reference amount of cycles until failure, depending on the steel type which is verified:

$N := 10^6$ [EN 1992-1-1:2005 Table 6.3N]

$i := 0 .. \left(\text{length}(\Delta\sigma_{st}) - 1\right)$

$$N_{st_i} := \left\| \begin{array}{ll} N \cdot \left( \dfrac{\dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}}{\gamma_{F.fat} \cdot \Delta\sigma_{st_i}} \right)^{k_1} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{st_i} \geq \dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \\ N \cdot \left( \dfrac{\dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}}{\gamma_{F.fat} \cdot \Delta\sigma_{st_i}} \right)^{k_2} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{st_i} < \dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \end{array} \right.$$

$$N_{st} = \begin{pmatrix} 8.1009 \times 10^4 \\ 5.4925 \times 10^5 \\ 2.5441 \times 10^7 \\ 1.5819 \times 10^{10} \end{pmatrix}$$

Total damage inflicted on the tensile reinforcement:

$j := \left(\text{length}(N_{st}) - 1\right) = 3$

$$D_{st} := \sum_{i=0}^{j} \frac{n_i}{N_{st_i}} \qquad \boxed{D_{st} = 0.621}$$

[EN 1992-1-1:2005 6.8.4 Eq (6.70)]

$$\text{Check} := \left\| \begin{array}{ll} \text{"OK"} & \text{if } D_{st} \leq 1 \\ \text{"NOT OK"} & \text{otherwise} \end{array} \right. = \text{"OK"}$$

$$D_{st.i} := \frac{n}{N_{st}} = \begin{pmatrix} 0.5407 \\ 0.0797 \\ 8.6083 \times 10^{-4} \\ 2.7688 \times 10^{-6} \end{pmatrix}$$

$\min(D_{st.i}) = 2.769 \times 10^{-6}$

$\max(D_{st.i}) = 0.541$

## Damage calculation of the compression reinforcement:

Current stress range:
(Should have a positive sign)

*Calculated before* $\Delta\sigma_{sc} := -1 \cdot \Delta\sigma_{sc}$

$$\Delta\sigma_{sc} = \begin{pmatrix} 36.492 \\ 62.494 \\ 31.884 \\ 14.472 \end{pmatrix} \cdot MPa$$

## Caluclating resisting number of cycles for the given stresses[EN 1992-1-1:2005 6.8.5(3)]

Reference amount of cycles until failure, depending on the steel type which is verified:

$N = 1 \times 10^{6}$ [EN 1992-1-1:2005 Table 6.3N]

$i := 0 .. \left(\mathrm{length}(\Delta\sigma_{sc}) - 1\right)$

$$N_{sc_i} := \begin{cases} N \cdot \left(\dfrac{\dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}}{\gamma_{F.fat} \cdot \Delta\sigma_{sc_i}}\right)^{k_1} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{sc_i} \ge \dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \\ N \cdot \left(\dfrac{\dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}}{\gamma_{F.fat} \cdot \Delta\sigma_{sc_i}}\right)^{k_2} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{sc_i} < \dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \end{cases} \qquad N_{sc} = \begin{pmatrix} 1.957 \times 10^{11} \\ 1.545 \times 10^{9} \\ 6.595 \times 10^{11} \\ 8.067 \times 10^{14} \end{pmatrix}$$

Total damage inflicted on the compression reinforcement:

$j := \left(\mathrm{length}(N_{sc}) - 1\right)$

$$D_{sc} := \sum_{i=0}^{j} \frac{n_i}{N_{sc_i}} \qquad D_{sc} = 2.861 \times 10^{-5}$$

[EN 1992-1-1:2005 6.8.4 Eq (6.70)]

$$Check := \begin{cases} \text{"OK"} & \text{if } D_{sc} \le 1 \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"OK"}$$

$$D_{sc.i} := \frac{n}{N_{sc}} = \begin{pmatrix} 2.238 \times 10^{-7} \\ 2.835 \times 10^{-5} \\ 3.321 \times 10^{-8} \\ 5.43 \times 10^{-11} \end{pmatrix}$$

## *Fatigue calculation of Concrete under compression*

[EN 1992-2:2005 Section 6.8.7]

Design partial factor taking material uncertainties into account:

$\gamma_{C.fat} := 1.5$ [EN 1992-1-1:2005 2.4.2.4(1)]

Characteristic compressive concrete strength in [MPa]:

$f_{ck} = 35 \cdot MPa$ [EN 1992-1-1:2005 3.1.3 table 3.1]

Design compressive concrete strength in [MPa]:

$f_{cdfat} := \dfrac{f_{ck}}{\gamma_{C.fat}} = 23.333 \cdot MPa$ [EN 1992-1-1:2005 3.1.3 table 3.1]

In NA recommended value:

$k_1 := 1.0$ [BFS 2009:16 6.8.7(1)]

Coefficient which depend on the type of cement:

$s := 0.25$ [EN 1992-2:2005 6.8.7 (1)]

The time of first cyclic load application on concrete in days:

$t_0 := 28$ [EN 1992-2:2005 6.8.7 (1)]

Coefficient for concrete strength at first load application:

$\beta_{cc} := e^{s \cdot \left(1 - \sqrt{\frac{28}{t_0}}\right)} = 1$ [EN 1992-2:2005 6.8.7 (1)]

Design fatigue strength of concrete:

$f_{cd.fat} := k_1 \cdot \beta_{cc} \cdot f_{cdfat} \cdot \left(1 - \dfrac{f_{ck}}{250MPa}\right) = 20.067 \cdot MPa$ [EN 1992-2:2005 Eq.(6.76)]

Minimum concrete stress taken into account both permanent loads and traffic load

$$\sigma_{cd.min} := -\sigma_{c.II.min} = \begin{pmatrix} -20.671 \\ 0.343 \\ 0.985 \\ 1.732 \end{pmatrix} \cdot \text{MPa}$$

These values should have a positive sign regardless of the value of the input data in order for the calculations to be correct!

Maximum concrete stress taken into account both permanent loads and traffic load

$$\sigma_{cd.max} := -\sigma_{c.II.max} = \begin{pmatrix} 1.013 \\ 5.043 \\ 4.188 \\ 3.351 \end{pmatrix} \cdot \text{MPa}$$

Mimimum and maximum compressive stress level in the concrete, taking both permanent and traffic loads into account

$$E_{cd.max} := \frac{\sigma_{cd.max}}{f_{cd.fat}} = \begin{pmatrix} 0.05 \\ 0.251 \\ 0.209 \\ 0.167 \end{pmatrix} \qquad E_{cd.min} := \frac{\sigma_{cd.min}}{f_{cd.fat}} = \begin{pmatrix} -1.03 \\ 0.017 \\ 0.049 \\ 0.086 \end{pmatrix}$$

Stress ratio between minimum and maximum concrete stress level due to permanent and traffic loads:

If the stress ratio is larger or equal to one, or smaller than zero, the ultimate number of constant amplitue cycles can not be calculated. This is because of the square root in the equation. If the stress ratio is larger then 1 or one, or smaller than zero will the value be imaginare. Therefore is a demand set to put these values to zero. This because the compressive concrete will be in tension instead.

$$i := 0 .. \left(\text{length}\left(E_{cd.max}\right) - 1\right)$$

$$R_i := \begin{cases} 0 & \text{if } \left(\dfrac{E_{cd.min}}{E_{cd.max}}\right)_i < 0 \\ 0 & \text{if } \left(\dfrac{E_{cd.min}}{E_{cd.max}}\right)_i \geq 1 \\ \left(\dfrac{E_{cd.min}}{E_{cd.max}}\right)_i & \text{otherwise} \end{cases} \qquad R = \begin{pmatrix} 0 \\ 0.068 \\ 0.235 \\ 0.517 \end{pmatrix}$$

Ultimate number of constant amplitude cycles in interval "i" that can be carried before failure:

$$N_c := 10^{\left(14 \cdot \frac{1 - E_{cd.max}}{\sqrt{1-R}}\right)} = \begin{pmatrix} 1.966 \times 10^{13} \\ 7.198 \times 10^{10} \\ 4.645 \times 10^{12} \\ 5.986 \times 10^{16} \end{pmatrix}$$

[EN 1992-2:2005 6.8.7(101)]

Total damage inflicted on the concrete under compression:

$$j := \left(\text{length}\left(N_c\right) - 1\right)$$

$$D_c := \sum_{i=0}^{j} \frac{n_i}{N_{c_i}} \qquad \boxed{D_c = 6.154 \times 10^{-7}}$$

[EN 1992-2:2005 6.8.7(101)]

$$\text{Check} := \begin{cases} \text{"OK"} & \text{if } D_c \leq 1 \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"OK"}$$

$$D_{c.i} := \frac{n}{N_c} = \begin{pmatrix} 2.228 \times 10^{-9} \\ 6.085 \times 10^{-7} \\ 4.715 \times 10^{-9} \\ 7.317 \times 10^{-13} \end{pmatrix}$$

# Appendix I. λ-Coefficient Method Fatigue Assessment of railway bridges

Sectional forces in the considered section:

Permanent moment:
(Calculated in StripStep2)

$M_{perm} := 983kNm$

Maximum moment for traffic load:
(Calculated in AFB)

$M_{max} := 1303kNm$

Minimum moment for traffic load:
(Calculated in AFB)

$M_{min} := 0kNm$

Arrangement of reinforcement:

Mimimum distances are calculated in Mathcad - Bridge design
To be able to do as general calculations as possible, four layers of reinforcement is used in the calculations. If there is fewer layers in the current case the number of bars is set to zero.

Compression side:

First layer: $d_1 := d_e = 52.5 \cdot mm$

Tension side:

First layer: $d'_1 := h_r - d_e = 777.5 \cdot mm$

Second layer: $d'_2 := 0mm$

Third layer: $d'_3 := 0mm$

## Amount of reinforcement (Calculated in Bridge Re-Design)

Compression side

Layer 1:

Number of bars first layer: $n_1 := n_{f.min} = 11.25$

$A_{s1} := A_{si25} \cdot n_1 = 5.522 \times 10^{-3} \, m^2$

Tension side

Layer 1:

Number of bars first layer: $n'_1 := n_{ss} = 31.469$

$A'_{s1} := A_{si25} \cdot n'_1 = 0.015 \, m^2$

Layer 2:

Number of bars second layer: $n'_2 := 0$

$A'_{s2} := A_{si25} \cdot n'_2 = 0$

Layer 3:

Number of bars third layer: $n'_3 := 0$

$A'_{s3} := A_{si25} \cdot n'_3 = 0$

### ***Modular rations based on effective creep functions:***

The final creep coefficient: (Calculated in Mathcad - Bridge Design) $\varphi_{cr} = 1.445$

The creep function for variable load in the quasi-permanent combination: $\varphi_{qp} := 0$

[Engström B (2008). *Restraint cracking of reinforced concrete structures.* Chalmers unisversity of technology, Göteborg 2008, Section 6.4.3]

Effective creep function for the maximum sectional moments due to permanent and fatigue load:

$$\varphi_{ef.max} := \frac{\varphi_{cr} \cdot M_{perm} + \varphi_{qp} \cdot (M_{max})}{M_{perm} + M_{max}} = 0.621$$

$$\alpha_{s.ef.max} := (1 + \varphi_{ef.max}) \cdot \frac{E_{sm}}{E_{cm}} = 9.536$$

Effective creep function for the minimum sectional moments due to permanent and fatigue load:

$$\varphi_{ef.min} := \frac{\varphi_{cr} \cdot M_{perm} + \varphi_{qp} \cdot (M_{min})}{M_{perm} + M_{min}} = 1.445$$

$$\alpha_{s.ef.min} := (1 + \varphi_{ef.min}) \cdot \frac{E_{sm}}{E_{cm}} = 14.379$$

Effective creep function for the minimum sectional moments due to permanent load:

$$\varphi_{ef.perm} := \frac{\varphi_{cr} \cdot M_{perm}}{M_{perm}} = 1.445$$

$$\alpha_{s.ef.perm} := (1 + \varphi_{ef.perm}) \cdot \frac{E_{sm}}{E_{cm}} = 14.379$$

## Sectional constants for the maximum sectional moment:

[Engström B (2008). *Design and analysis of continuous beams and colums.* Chalmers unisversity of technology, Göteborg 2008 Chapter 7]

Neutral axis of the effective transformed concrete section:

$x_{II.max} := 0.19\text{m}$

$$x_{II.max} := \text{root}\left[\begin{array}{l} \frac{w \cdot x_{II.max}^{2}}{2} + (\alpha_{s.ef.max} - 1) \cdot A_{s1} \cdot (x_{II.max} - d_1) - \alpha_{s.ef.max} \cdot A'_{s1} \cdot (d'_1 - x_{II.max}) \cdots, x_{II.m} \\ + -\alpha_{s.ef.max} \cdot A'_{s2} \cdot (d'_2 - x_{II.max}) - \alpha_{s.ef.max} \cdot A'_{s3} \cdot (d'_3 - x_{II.max}) \end{array}\right.$$

$x_{II.max} = 0.189\text{ m}$

Moment of inertia for the effective transformed concrete section:

$$I_{II.max} := \frac{w \cdot x_{II.max}^{3}}{3} + (\alpha_{s.ef.max} - 1) \cdot A_{s1} \cdot (x_{II.max} - d_1)^2 + \alpha_{s.ef.max} \cdot A'_{s1} \cdot (d'_1 - x_{II.max})^2 \dots = 0.062\,\mathrm{m}$$
$$+ \alpha_{s.ef.max} \cdot A'_{s2} \cdot (d'_2 - x_{II.max})^2 + \alpha_{s.ef.max} \cdot A'_{s3} \cdot (d'_3 - x_{II.max})^2$$

## Sectional constants for the minimum sectional moment:

Neutral axis of the effective transformed concrete section:

$$x_{II.min} := 0.22\mathrm{m}$$

$$x_{II.min} := \mathrm{root}\left[\frac{w \cdot x_{II.min}^{2}}{2} + (\alpha_{s.ef.min} - 1) \cdot A_{s1} \cdot (x_{II.min} - d_1) - \alpha_{s.ef.min} \cdot A'_{s1} \cdot (d'_1 - x_{II.min}) \dots, x_{II.min} \right.$$
$$\left. + -\alpha_{s.ef.min} \cdot A'_{s2} \cdot (d'_2 - x_{II.min}) - \alpha_{s.ef.min} \cdot A'_{s3} \cdot (d'_3 - x_{II.min}) \right]$$

$$x_{II.min} = 0.222\,\mathrm{m}$$

Moment of inertia for the effective transformed concrete section:

$$I_{II.min} := \frac{w \cdot x_{II.min}^{3}}{3} + (\alpha_{s.ef.min} - 1) \cdot A_{s1} \cdot (x_{II.min} - d_1)^2 + \alpha_{s.ef.min} \cdot A'_{s1} \cdot (d'_1 - x_{II.min})^2 \dots = 0.087\,\mathrm{m}^4$$
$$+ \alpha_{s.ef.min} \cdot A'_{s2} \cdot (d'_2 - x_{II.min})^2 + \alpha_{s.ef.min} \cdot A'_{s3} \cdot (d'_3 - x_{II.min})^2$$

## Sectional constants for the permanent sectional moment:

Neutral axis of the effective transformed concrete section:

$$x_{II.perm} := 0.22\mathrm{m}$$

$$x_{II.perm} := \mathrm{root}\left[\frac{w \cdot x_{II.perm}^{2}}{2} + (\alpha_{s.ef.perm} - 1) \cdot A_{s1} \cdot (x_{II.perm} - d_1) \dots \quad , x_{II.perm} \right.$$
$$+ -\alpha_{s.ef.perm} \cdot A'_{s1} \cdot (d'_1 - x_{II.perm}) \dots$$
$$\left. + -\alpha_{s.ef.perm} \cdot A'_{s2} \cdot (d'_2 - x_{II.perm}) - \alpha_{s.ef.perm} \cdot A'_{s3} \cdot (d'_3 - x_{II.perm}) \right]$$

$$x_{II.perm} = 0.222\,\mathrm{m}$$

Moment of inertia for the effective transformed concrete section:

$$I_{II.perm} := \frac{w \cdot x_{II.perm}^{3}}{3} + (\alpha_{s.ef.perm} - 1) \cdot A_{s1} \cdot (x_{II.perm} - d_1)^2 + \alpha_{s.ef.perm} \cdot A'_{s1} \cdot (d'_1 - x_{II.perm})^2 \dots$$
$$+ \alpha_{s.ef.perm} \cdot A'_{s2} \cdot (d'_2 - x_{II.perm})^2 + \alpha_{s.ef.perm} \cdot A'_{s3} \cdot (d'_3 - x_{II.perm})^2$$

$$I_{II.perm} = 0.087\,\mathrm{m}^4$$

### *Calculating stresses due to permanent load and fatigue load:*

When calculating the stresses the first index describes which material that is considered (s - steel, c - concrete), and the second indicates on which reinforcement level the stress is calculated (t - tensile reinforcement, c - compressive reinforcement).

## Concrete stresses:

Maximum tensile concrete stress in a cycle:
(Uncracked state I)

$$\sigma_{ct.II.max} := \frac{M_{perm} + M_{max}}{I_{Ir}} \cdot \frac{h_r}{2} = 4.424 \cdot MPa$$

Check if the section is cracked:
(If one value is equal to one the section can be considered as cracked)

$$\sigma_{ct.II.max} \geq f_{ctm} = 1$$

Maximum compressive concrete stress:

$$\sigma_{c.II.max} := \frac{M_{perm} + M_{max}}{I_{II.max}} \cdot -x_{II.max} = -6.962 \cdot MPa$$

$$\sigma_{c.II.min} := \frac{M_{perm} + M_{min}}{I_{II.min}} \cdot -x_{II.min} = -2.506 \cdot MPa$$

$$\sigma_{c.II.perm} := \frac{M_{perm}}{I_{II.perm}} \cdot -x_{II.perm} = -2.506 \cdot MPa$$

## Steel stresses:

Stresse range in the compressive reinforcement:

$$\Delta\sigma_{sc} := \left(\frac{\alpha_{s.ef.max} + \alpha_{s.ef.min}}{2}\right) \cdot \left[\frac{M_{max} - M_{min}}{\left(\frac{I_{II.max} + I_{II.min}}{2}\right)} \cdot \left[d_1 - \left(\frac{x_{II.max} + x_{II.min}}{2}\right)\right]\right] = -31.964 \cdot MPa$$

Stresse range in the tensile reinforcement:

$$\Delta\sigma_{st} := \left(\frac{\alpha_{s.ef.max} + \alpha_{s.ef.min}}{2}\right) \cdot \left[\frac{M_{max} - M_{min}}{\left(\frac{I_{II.max} + I_{II.min}}{2}\right)} \cdot \left[d'_1 - \left(\frac{x_{II.max} + x_{II.min}}{2}\right)\right]\right] = 119.56 \cdot MPa$$

## *Fatigue calculation of the reinforcing steel*

[EN 1992 -2:2005 Annex NN.3.1]

Partial factor taking material uncertainties into account:

$\gamma_{S.fat} := 1.15$ [EN 1992-1-1:2005 2.4.2.4(1)]

Partial factor taking the uncertainties in the fatigue load model into account:

$\gamma_{F.fat} := 1$ [EN 1992-1-1:2005 2.4.2.3(1)]

Resisting stress range at N* cycles in [MPa]:

$\Delta\sigma_{Rsk} := 162.5\text{MPa}$ [EN 1992-1-1:2005 6.8.4, table 6.3N]

Exponent defining the of slopes of the S-N relation:

$$k_1 := \begin{cases} 5 & \text{if } \Delta\sigma_{Rsk} = 162.5\text{MPa} \\ 3 & \text{if } \Delta\sigma_{Rsk} = 58.5\text{MPa} \\ 3 & \text{if } \Delta\sigma_{Rsk} = 35\text{MPa} \end{cases} = 5$$

[EN 1992-1-1:2005 6.8.4, Table 6.3N]

### *Determining the λ-coefficients for steel:*

$\lambda_{s.1}$ is a function of the critical length of linfluence line and traffic:

Heavy traffic mix and reinforcing steel, Continuous beam, Intermediate support area [EN 1992-2:2005 Table NN.2]

$L = 7\,\text{m}$ L - critical length of the influence line [EN 1993-2:2006 9.5.3(4)]

According to TK Bro, when using the simplified method in Annex NN the following modification can be used. The value $\lambda_{s1}$ and $\lambda_{c1}$ may be multiplied with a factor α when using heavy traffic mix.

The factor α will vary as seen below:

$$\alpha := \begin{cases} 1.33 & \text{if } L = 0\text{m} \\ 1 & \text{if } L \ge 10\text{m} \\ 1.33 - 0.33\cdot\dfrac{L}{10\cdot\text{m}} & \text{if } 0\text{m} \le L \le 10\text{m} \end{cases} = 1.099$$

[TK Bro D.2.1 (g)]

$$\lambda_{s1} := \begin{cases} 0.85 & \text{if } L \le 2\cdot\text{m} \\ 0.75 & \text{if } L \ge 20\cdot\text{m} \\ 0.85 + (0.75 - 0.85)\cdot\left(\log\left(\dfrac{L}{\text{m}}\right) - 0.3\right) & \text{if } 2\cdot\text{m} < L < 20\cdot\text{m} \end{cases} = 0.795$$

[EN 1992-2:2005 Eq (NN.108)]

Modified factor $\lambda_{s.1}$: $\lambda_{s.1} := \lambda_{s1}\cdot\alpha = 0.874$

$\lambda_{s.2}$ value denotes the influence of annual traffic volume

Volume of traffic (tonnes/year/track)

$Vol := 25 \cdot 10^6$ [TK Bro:2009 B.3.4.1.5 (w)]

The slope of the appropriate S-N line:

$k_2 := 9$ [EN 1992-1-1:2005 Table 6.3N]

$$\lambda_{s.2} := \sqrt[k_2]{\frac{Vol}{25 \cdot 10^6}} = 1$$ [EN 1992-2:2005 Eq (NN.109)]

$\lambda_{s.3}$ value denotes the influence of the service life:

Design life of bridge in years:

$N_{years} := 120$ [BFS 2009:16 6.9 (6)]

$$\lambda_{s.3} := \sqrt[k_2]{\frac{N_{years}}{100}} = 1.02$$ [EN 1992-2:2005 Eq (NN.110)]

$\lambda_{s.4}$ value denotes the effect of loading from more than one track:

Used if there is more than one track, in this case only one track is considered.

$\lambda_{s.4} := 1$ [EN 1992-2:2005 Eq (NN.111)]

$\lambda_s$ - correction factor to calculate the damage equivalent stress range:

$\lambda_s := \lambda_{s.1} \cdot \lambda_{s.2} \cdot \lambda_{s.3} \cdot \lambda_{s.4} = 0.892$ EN 1992-2:2005 Eq (NN.107)]

$\Delta\sigma_{s.71}$ - steel stress range due to load model 71:

Compressive reinforcement:

$$\Delta\sigma_{sc.71} := \left|\Delta\sigma_{sc}\right| = 31.964 \cdot MPa$$

Tensile reinforcement:

$$\Delta\sigma_{st.71} := \left|\Delta\sigma_{st}\right| = 119.56 \cdot MPa$$

## Equivalent steel stress range:

$$\Delta\sigma_{s.equ} = \lambda_s \cdot \phi \cdot \Delta\sigma_{s.71}$$ [EN 1992-2:2005 Eq (NN.106)]

At the level of the compressive reinforcement:

$$\Delta\sigma_{sc.equ} := \lambda_s \cdot \phi_2 \cdot \left|\Delta\sigma_{sc.71}\right| = 38.601 \cdot MPa$$

At the level of the tensile reinforcement:

$$\Delta\sigma_{st.equ} := \lambda_s \cdot \phi_2 \cdot \left|\Delta\sigma_{st.71}\right| = 144.388 \cdot MPa$$

## Compressive reinforcement:

Rewritten from [EN 1992-1-1:2005 6.8.5]

$$D_{sc.\lambda} := \frac{\gamma_{F.fat} \cdot \Delta\sigma_{sc.equ}}{\dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}} = 0.273$$

$D_{sc.\lambda} = 0.273$

$$Check := \begin{cases} \text{"OK"} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{sc.equ} \le \dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"OK"}$$

## Tensile reinforcement:

$$D_{st.\lambda} := \frac{\gamma_{F.fat} \cdot \Delta\sigma_{st.equ}}{\dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}} = 1.022$$

$D_{st.\lambda} = 1.022$

$$Check := \begin{cases} \text{"OK"} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{st.equ} \le \dfrac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"NOT OK"}$$

## *Fatigue calculation of Concrete under compression*

[EN 1992 -2:2005 Annex NN.3.2]

Design partial factor taking material uncertainties into account:

$$\gamma_{C.fat} := 1.5$$ [EN 1992-1-1:2005 2.4.2.4(1)]

Characteristic compressive concrete strength in [MPa]:

$$f_{ck} = 35 \cdot MPa$$ [EN 1992-1-1:2005 3.1.3 table 3.1]

Design compressive concrete strength in [MPa]:

$$f_{cdfat} := \frac{f_{ck}}{\gamma_{C.fat}} = 23.333 \cdot MPa$$ [EN 1992-1-1:2005 3.1.3 table 3.1]

In NA recommended value:

$$k_1 := 1.0$$ [BFS 2009:16 6.8.7(1)]

Coefficient which depend on the type of cement:

$$s := 0.25$$ [EN 1992-2:2005 6.8.7 (1)]

The time of first cyclic load application on concrete in days:

$$t_0 := 28$$ [EN 1992-2:2005 6.8.7 (1)]

Coefficient for concrete strength at first load application:

$$\beta_{cc} := e^{s \cdot \left(1 - \sqrt{\frac{28}{t_0}}\right)} = 1$$ [EN 1992-2:2005 6.8.7 (1)]

Design fatigue strength of concrete:

$$f_{cd.fat} := k_1 \cdot \beta_{cc} \cdot f_{cdfat} \cdot \left(1 - \frac{f_{ck}}{250MPa}\right) = 20.067 \cdot MPa$$ [EN 1992-2:2005 Eq.(6.76)]

Maximum compressive concrete stress for permanent load:

$$\sigma_{c.perm} := \left|\sigma_{c.II.perm}\right| = 2.506 \cdot MPa$$

***Determining the λ-coefficients for concrete:***

$\lambda_{c.0}$ - factor to take account of permanent stress:

$$\lambda_{c.0} := \begin{cases} 0.94 + 0.2\cdot\dfrac{\sigma_{c.perm}}{f_{cd.fat}} & \text{if } 0.94 + 0.2\cdot\dfrac{\sigma_{c.perm}}{f_{cd.fat}} \geq 1 \\ 1 & \text{otherwise} \end{cases} = 1$$ [EN 1992-2:2005 Eq (NN.115)]

$\lambda_{c.1}$ - factor accounting for element type which considers the damaging effect of traffic depending on the critical length of the influence line or area:

Heavy traffic mix and reinforcing steel, Continuous beam, Intermediate support area

$L = 7\,m$ L - critical length of the influence lline [EN 1993-2:2006 9.5.3 (4)]

According to TK Bro, when using the simplified method in Annex NN can following modification be used. The value $\lambda_{s1}$ and $\lambda_{c1}$ can be multiplied with a factor α when using heavy traffic mix. The factor α will vary as seen below:

$$\alpha_{mod} := \begin{cases} 1.33 & \text{if } L = 0m \\ 1 & \text{if } L \geq 10m \\ 1.33 - 0.33\cdot\dfrac{L}{10\cdot m} & \text{if } 0m \leq L \leq 10m \end{cases} = 1.099$$ [TK Bro D.2.1 (g)]

$$\lambda_{c1} := \begin{cases} 0.75 & \text{if } L \leq 2m \\ 0.85 & \text{if } L \geq 20m \\ 0.75 + (0.85 - 0.75)\cdot\left(\log\left(\dfrac{L}{m}\right) - 0.3\right) & \text{if } 2m < L < 20m \end{cases} = 0.805$$

Modified factor $\lambda_{c.1}$: $\lambda_{c.1} := \lambda_{c1}\cdot\alpha_{mod} = 0.884$ [EN 1992-2:2005 Eq (NN.108)]

$\lambda_{c.2.3}$ - factor to tacke account of the traffic volume and design life of bridge:

Volume of traffic (tonnes/year/track)

$Vol = 2.5 \times 10^{7}$ [TK Bro:2009 B.3.4.1.5 (w)]

Design life of bridge in years:

$N_{years} = 120$ [BFS 2009:16 6.9 (6)]

$$\lambda_{c.2.3} := 1 + \frac{1}{8}\cdot\log\left(\frac{Vol}{25\cdot 10^{6}}\right) + \frac{1}{8}\cdot\log\left(\frac{N_{years}}{100}\right) = 1.01$$ [EN 1992-2:2005 Eq (NN.116)]

$\lambda_{c.4}$ - factor to be applied when the structural element is loaded by more than one track:

Used if there is more than one track, in this case only one track is considered.

$$\lambda_{c.4} := 1$$ [EN 1992-2:2005 Eq (NN.117)]

$\lambda_c$ - Correction factor to calculate the upper and lower stresses in the damage equivalent stress spectrum:

$$\lambda_c := \lambda_{c.0} \cdot \lambda_{c.1} \cdot \lambda_{c.2.3} \cdot \lambda_{c.4} = 0.893$$ [EN 1992-2:2005 Eq (NN.114)]

Concrete stress spectrum:

$$\sigma_{c.perm} = 2.506 \cdot MPa$$

$\sigma_{c.perm}$ - the compressive concrete stress caused by the characteristic combination of actions, without load model 71

$$\sigma_{c.max.71} := \left|\sigma_{c.II.max}\right| \cdot \phi_2 = 9.424 \cdot MPa$$

$\sigma_{c.max.71}$ - maximum compressive stress caused by the caused by the characteristic combination of actions, with load model 71 and dynamic factor

$$\sigma_{c.min.71} := \left|\sigma_{c.II.min}\right| \cdot \phi_2 = 3.392 \cdot MPa$$

$\sigma_{c.min.71}$ - minimum compressive stress caused by the caused by the characteristic combination of actions, with load model 71 and dynamic factor

The upper and lower of the damage equivalent stress spectrum taking both permanent and traffic loads into account with the correction factor:

$$\sigma_{cd.max.equ} := \sigma_{c.perm} + \lambda_c \cdot \left(\sigma_{c.max.71} - \sigma_{c.perm}\right) = 8.683 \cdot MPa$$ [EN 1992 2-2:2005 Eq (NN.113)]

$$\sigma_{cd.min.equ} := \sigma_{c.perm} - \lambda_c \cdot \left(\sigma_{c.perm} - \sigma_{c.min.71}\right) = 3.297 \cdot MPa$$

Partial factor for materials for ULS:

$$\gamma_{sd} := 1$$ [EN 1992-1-1:2005 2.4.2.4(2) table 2.1N]

Reference amount of cycles until failure:

$$N := 10^6$$ [EN 1992 2-2:2005 NN.3.2(101)]

Mimimum and maximum compressive stress level taking into account the permanent and traffic load with the correction factor:

$$E_{cd.min.equ} := \gamma_{sd} \cdot \frac{\sigma_{cd.min.equ}}{f_{cd.fat}} = 0.164 \qquad E_{cd.max.equ} := \gamma_{sd} \cdot \frac{\sigma_{cd.max.equ}}{f_{cd.fat}} = 0.433$$

Concrete stress ratio taking into account the permanent and traffic loads:

$$R_{equ} := \frac{E_{cd.min.equ}}{E_{cd.max.equ}} = 0.38$$

[EN 1992-2:2005 NN.3.2(101)]

Requirement that needs to be fulfilled in order to know that the inflicted damage is ok.

$$Check := \begin{array}{|l} \text{"OK"} \quad \text{if } 14 \cdot \dfrac{1 - E_{cd.max.equ}}{\sqrt{1 - R_{equ}}} \geq 6 \\ \text{"NOT OK"} \quad \text{otherwise} \end{array} = \text{"OK"}$$

[EN 1992-2:2005 Eq (NN.112)]

Writing the value to know how must larger or smaller then six the requirement is:

$$14 \cdot \frac{1 - E_{cd.max.equ}}{\sqrt{1 - R_{equ}}} = 10.084$$

Total damage inflicted on the concrete

$$D_c := E_{cd.max.equ} + \frac{\log(N)}{14} \cdot \sqrt{1 - R_{equ}}$$

$D_c = 0.77$

$$Check := \begin{array}{|l} \text{"OK"} \quad \text{if } D_c \leq 1 \\ \text{"NOT OK"} \quad \text{otherwise} \end{array} = \text{"OK"}$$

# Appendix J. Design of shear reinforcement and detailing for a concrete slab bridge according to Eurocode

***Design of the slab with regard to shear capacity:***

Check if the section need shear reinforcement: [EN 1992-2 Eq.6.2b]

Sections which is designed for shear reinforcement is:
1: The distance d from the face of the supports due to the inclination of the shear crack
2: The distance 2d from the support due to that the reduction factor of the influence of load close to support ends here

Assumed that the angle of the shear crack is 45 degrees:

$$\theta := 45\text{deg}$$

$$d := h_1 - d_e = 0.488\,\text{m}$$

Distance from the support to the crack on the upper side of the slab:

$$x_{45} := \cot(\theta)\cdot d = 0.488\,\text{m}$$

Coordinates used in Stripstep2: $\begin{pmatrix} x_{45} \\ 2\cdot d \end{pmatrix} = \begin{pmatrix} 0.488 \\ 0.975 \end{pmatrix}\text{m}$

$$\sigma_{sd} := f_{yd} = 434.783\cdot\text{MPa}$$

$\eta_1 := 1.0$ Assuming good conditions during concreting

$$\eta_2 := \left| \begin{array}{ll} 1 & \text{if } \phi \le 32\text{mm} \\ \dfrac{\left(132 - \dfrac{\phi}{\text{m}}\right)}{100} & \text{otherwise} \end{array} \right. = 1$$

$$f_{bd} := 2.25\cdot\eta_1\cdot\eta_2\cdot f_{ctk.0.05} = 4.95\cdot\text{MPa}$$

$$l_{bd} := \frac{\phi}{4}\cdot\frac{\sigma_{sd}}{f_{bd}} = 0.549\,\text{m}$$

Anchorage of reinforcement is assumed to be satisfied. Therefore the area of the tensile reinforcement is set to the amount aviliable in the section considered

$A_{sl} := A_{sf.min} = 3.669\times 10^{-3}\,\text{m}^2$ [EN 1992-1-1:2005 9.2.2]

$b_w := w = 4.5\,\text{m}$ $b_w$ - smallest width of the cross-section in the tensile area[mm]

$N_{Ed} := 0\text{kN}$ Axial force in the section due to loading or prestressing

$$\rho_l := \begin{cases} \dfrac{A_{sl}}{b_w \cdot d} & \text{if } \dfrac{A_{sl}}{b_w \cdot d} \le 0.02 \\ 0.02 & \text{otherwise} \end{cases} = 1.673 \times 10^{-3}$$ [EN 1992-2 Eq.6.2b]

$$\sigma_{cp} := \begin{cases} 0 & \text{if } \dfrac{N_{Ed}}{A_{I.1}} \ge 0.2 \cdot f_{cd} \\ \dfrac{N_{Ed}}{A_{I.1}} & \text{otherwise} \end{cases} = 0$$ no pretenisoning or axial forces [EN 1992-2 Eq.6.2b]

$$C_{Rd.c} := \frac{0.18}{\gamma_C} = 0.12$$ [EN 1992-2 Eq.6.2b]; National Annex

$$k_1 := 0.15$$ [EN 1992-2 Eq.6.2b]; National Annex

$$k := \begin{cases} 1 + \sqrt{\dfrac{200 \cdot mm}{d}} & \text{if } 1 + \sqrt{\dfrac{200 \cdot mm}{d}} \le 2.0 \\ 2 & \text{otherwise} \end{cases} = 1.641$$ [EN 1992-2 Eq.6.2b]

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{MPa} \right)^{\frac{1}{2}} = 0.435$$ [EN 1992-2 Eq.6.3N]

Design value for the shear resistance:

$$V_{Rdc} := \begin{cases} \left[ C_{Rd.c} \cdot k \cdot \left( 100 \cdot \rho_l \cdot \dfrac{f_{ck}}{MPa} \right)^{\frac{1}{3}} \cdots + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d & \text{if } \left[ C_{Rd.c} \cdot k \cdot \left( 100 \cdot \rho_l \cdot \dfrac{f_{ck}}{MPa} \right)^{\frac{1}{3}} \cdots + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d \ge \left( v_{min} \cdots + k_1 \cdot \sigma_{cp} \right) \cdot b_w \cdot d \\ \left[ \left( v_{min} + k_1 \cdot \sigma_{cp} \right) \cdot b_w \cdot d \right] & \text{otherwise} \end{cases}$$

$$V_{Rd.c} := V_{Rdc} \cdot \frac{10^6 \cdot N}{m^2} = 954.462 \cdot kN$$ [EN 1992-1-1 section 6.2.1]

Shear force in the sections considered calculated in Stripstep2:

$$V_{Ed.x.45} := 1073kN$$

$$V_{Ed.2d} := 829kN$$

for $\begin{pmatrix} x_{45} \\ 2 \cdot d \end{pmatrix} = \begin{pmatrix} 0.488 \\ 0.975 \end{pmatrix} m$ $\qquad \dfrac{\begin{pmatrix} x_{45} \\ 2 \cdot d \end{pmatrix}}{L} = \begin{pmatrix} 0.07 \\ 0.139 \end{pmatrix}$

## Load reduction factor due to influence of support: [EN 1992-1-1:2005 6.2.2 (6)]

According to EC the shear force may be reduced in a region close to the supports. The reduction is made for concentrated loads by calculating a factor $\beta$ by which the shear force in the considered section is multiplied. For uniformly distributed loads, which is the case for a railway bridge, the reduction can be estimated according to Al-Emrani et.al 2006.

Load model considered in the reduction is the locomotive load LM71 distrubuted as a uniform load:

$$q_{l.71} = 156.25 \cdot \frac{kN}{m}$$

The total load which may be reduced:

$$q_d := \gamma_G \cdot g_{d.1} + \alpha \cdot \phi_2 \cdot \gamma_Q \cdot q_{l.71} = 503.978 \cdot \frac{kN}{m}$$

The reduced shear force:

$$V_{Ed.red} := \begin{vmatrix} \left[ V_{Ed.x.45} - \frac{(2 \cdot d - x_{45})^2}{4 \cdot d} \cdot q_d \right] & \text{if } x_{45} < 2 \cdot d \\ V_{Ed.x.45} & \text{otherwise} \end{vmatrix} = 1.012 \times 10^3 \cdot kN$$

$$\nu := 0.6 \left( 1 - \frac{f_{ck}}{250 \cdot MPa} \right) = 0.516$$

The unreduced shear force should however always satisfy:

$$V_{Ed.x.45} \le 0.5 \cdot w \cdot d \cdot \nu \cdot f_{cd} = 1$$

Check if the section demands shear reinforcement:

$$V_{Rd.c} > V_{Ed.red} = 0$$

$$V_{Rd.c} > V_{Ed.2d} = 1$$

## Design of sections requiring shear reinforcement; EN 1992-1-1:2005 6.2.3

Assumed that the angle of the shear crack is 45 or 22 degrees:

$\theta := 22\text{deg}$ $\qquad d = 0.488\ \text{m}$

Distance from the support to the crack on the upper side of the slab:

$x_\theta := \cot(\theta)\cdot d = 1.207\ \text{m}$

Coordinates used in Stripstep2: $\qquad x_\theta = 1.207\ \text{m}$

If the crack surfaces at a distance greater than 2d the shear force should not be reduced.
If so only the section of the crack originating from the edge of the support is checked.

$x_\theta > 2\cdot d = 1$

The maximum transversal spacing between shear reinforcement:

$s_{t.max} := 0.75\cdot d = 0.366\ \text{m}$ $\qquad$ [EN 1992-1-1:2005 9.2.2 (8)]

Number of legs in the transversal direction: $\qquad n_{t.sr} := \dfrac{w}{s_{t.max}} = 12.308$

Cross sectional area of the shear reinforcement for each spacing in the longitudinal direction: $\qquad A_{sw} := n_{t.sr}\cdot A_{si10} = 9.666\times 10^{-4}\cdot \text{m}^2$

Approximation of the internal lever arm in the cross section: $z := 0.9\cdot d = 0.439\ \text{m}$

Design yield strength of the shear reinforcement: $\qquad f_{ywd} := f_{yd} = 434.783\cdot\text{MPa}$

Shear force in the sections considered calculated in Stripstep2:

$$V_{Ed.x.\theta} := \begin{cases} 723\text{kN} & \text{if } \theta = 22\text{deg} \\ V_{Ed.x.45} & \text{if } \theta = 45\text{deg} \\ 0 & \text{otherwise} \end{cases} = 723\cdot\text{kN}$$

for $\qquad x_\theta = 1.207\ \text{m} \qquad \dfrac{x_\theta}{L} = 0.172$

## Load reduction factor due to influence of support: [EN 1992-1-1:2005 6.2.3 (8)]

According to EC the shear force may be reduced in a region close to the supports. The reduction is made for concentrated loads by calculating a factor $\beta$ by which the shear force in the considered section is multiplied. For uniformly distributed loads, which is the case for a railway bridge, the reduction can be estimated according to Al-Emrani et.al 2006.

Load model considered in the reduction is the distributed load from LM71: $q_{l.71} = 156.25 \cdot \frac{kN}{m}$

The total load which may be reduced: $q_d = 503.978 \cdot \frac{kN}{m}$

The reduced shear force:

$$V_{Ed.red2} := \begin{array}{|l} \left[ V_{Ed.x.\theta} - \frac{(2 \cdot d - x_\theta)^2}{4 \cdot d} \cdot q_d \right] \text{ if } x_\theta < 2 \cdot d \\ V_{Ed.x.\theta} \text{ otherwise} \end{array} = 723 \cdot kN$$

Spacing of the stirrups: $s_x := \frac{A_{sw}}{V_{Ed.red2}} \cdot z \cdot f_{ywd} \cdot 0.80 \cdot \cot(\theta) = 0.505\ m$

Which should satisfy the condition: [EN 1992-1-1:2005 6.2.3 (3)]

$$V_{Ed.red2} \le 0.75 \cdot \frac{x_\theta}{s_x} A_{sw} \cdot f_{ywd} \cdot \sin(\alpha) = 1$$

Design value for the shear resistance: $V_{Rd.s} := \frac{A_{sw}}{s_x} \cdot z \cdot f_{ywd} \cdot \cot(\theta) = 903.75 \cdot kN$

Coefficient for the state of stress in the compressive chord:

Non prestressed structures: $\alpha_{cd} := 1$

For concrete with $f_{ck}$ < 60 MPa $\nu_1 := 0.6$

Upper limit due to concrete crushing in the compressive strut:

$$V_{Rd.max} := \frac{\alpha_{cd} \cdot b_w \cdot z \cdot \nu_1 \cdot f_{cd}}{\cot(\theta) + \tan(\theta)} = 9.601 \times 10^3 \cdot kN$$ [EN 1992-1-1:2005 6.2.3 (3)]

Which the unreduced shear force should be less then:

$$V_{Ed.x.\theta} \le V_{Rd.max} = 1$$

## Detailing of shear reinforcement; [EN 1992-1-1:2005 9.2.2]

Angle between longitudinal and the shear reinforcement: $\alpha_{sr} = 90 \cdot deg$

Ratio of the shear reinforcement:

$$\rho_w := \frac{A_{sw}}{s_x \cdot b_w \cdot \sin(\alpha_{sr})} = 0.043 \cdot \%$$ [EN 1992-1-1:2005 9.2.2 (5)]

Additional demand for bridges: [BFS 2009:16]

$$\rho_w := \begin{cases} \rho_w & \text{if } \rho_w > 0.15\% \\ 0.15\% & \text{otherwise} \end{cases} = 0.15 \cdot \%$$

Minimum ratio of shear reinforcement: [EN 1992-1-1:2005 9.2.2 (5)]

$$\rho_{w.min} := \frac{\left(0.08 \cdot \sqrt{\frac{f_{ck}}{MPa}}\right)}{\frac{f_{yk}}{MPa}} = 0.095 \cdot \%$$

$$\rho_w := \begin{cases} \rho_w & \text{if } \rho_w > \rho_{w.min} \\ \rho_{w.min} & \text{otherwise} \end{cases} = 0.15 \cdot \%$$

When the width of the beam is wider than the height the minimum ration of shear reinforcement can be reduced: [BFS 2009:16]

$w > .h_1 = 1$ $$\rho_{w.min} := \left(0.10 + \frac{0.05 \cdot h_1}{b_w}\right) \cdot \frac{1}{100} = 0.106 \cdot \%$$

$$\rho_w := \begin{cases} \rho_w & \text{if } \rho_w > \rho_{w.min} \\ \rho_{w.min} & \text{otherwise} \end{cases} = 0.15 \cdot \%$$

The maximum longitudinal spacing between shear reinforcement:

$$s_{l.max} := 0.75 \cdot d \cdot \left(1 + \cot\left(\alpha_{sr}\right)\right) = 0.366\,m \qquad \text{[EN 1992-1-1:2005 9.2.2 (6)]}$$

$$s_x := \begin{cases} s_x & \text{if } s_x \le s_{l.max} \\ s_{l.max} & \text{otherwise} \end{cases} = 0.366\,m$$

Number of stirrups in the section with the shear crack:

$$n_{s.s} := \frac{z \cdot \cot(\theta)}{s_x} = 2.97$$

## Requirement of secondary transverse reinforcement:

Field section: [EN 1992-1-1:2005 9.3.1.1 (2)]

A minimum of 20% of the principal reinforcement should be provided as transversal reinforcement:

Area in the tensile side of the slab: $A_{t.f} := 20\% \cdot A_{field} = 1.639 \times 10^{-3}\,m^2$

Area of one stirrup in the transversal direction: $A_{si16} = 2.011 \times 10^{-4}\,m^2$

Number of bars needed in the section: $n_{t.f} := \mathrm{ceil}\left(\dfrac{A_{t.f}}{A_{si16}}\right) = 9$

Spacing of stirrups in the section: $s_{t.f} := \dfrac{L}{n_{t.f}} = 0.778\,m$

Maximum spacing of stirrups in the section: [EN 1992-1-1:2005 9.3.1.1 (3)]

$$s_{max.slabs.t.f} := \begin{cases} 3 \cdot h_1 & \text{if } 3 \cdot h_1 \le 400mm \\ 400mm & \text{otherwise} \end{cases} = 0.4\,m$$

$$s_{t.f} \le s_{max.slabs.t.f} = 0$$

Since support sections not need to be considered and the amount of reinforcement is less on the compression side the spacing is taken as the maximum spacing requirement

# Appendix K. Cumulative Damage Method Fatigue induced by shear stresses

Control for members without Shear reinforcement: [EN 1992-1-1:2005 6.8.7(4)]

Design value for the shear resistance, calculated in Bridge design:

$$V_{Rd.c} = 954.462 \cdot kN$$

Shear forces in the section considered calculated in StripStep2:
The design value of the maximum and minimum shear force under frequent load combination.

Coordinates used in Stripstep2:

$$x_{45} = 0.488\,m$$

$$V_{x45.min} := 214kN + -47kN$$

$$\frac{x_{45}}{L} = 0.07$$

$$V_{x45.max} := 214kN + 727kN = 941 \cdot kN$$

Shear force that the concrete strut should bring to the support:

$$V_{Ed.min} := V_{x45.min} - x_{45} \cdot q_{71} = 128 \cdot kN$$

$$V_{Ed.max} := V_{x45.max} - x_{45} \cdot q_{l.71} = 864.828 \cdot kN$$

For members **not** requiring design shear reinforcement in the ULS it may be assumed that the concrete resists fatigue due to shear effects where the following apply:

$$\text{Check} := \left| \begin{array}{l} \text{"Equation 1"} \quad \text{if} \quad \dfrac{V_{Ed.min}}{V_{Ed.max}} \geq 0 \\ \text{"Equation 2"} \quad \text{if} \quad \dfrac{V_{Ed.min}}{V_{Ed.max}} < 0 \end{array} \right. = \text{"Equation 1"}$$

$$V := \left| \begin{array}{l} \dfrac{|V_{Ed.max}|}{|V_{Rd.c}|} \leq 0.5 + 0.45 \cdot \dfrac{|V_{Ed.min}|}{|V_{Rd.c}|} \quad \text{if Check} = \text{"Equation 1"} \\ \dfrac{|V_{Ed.max}|}{|V_{Rd.c}|} \leq 0.5 - \dfrac{|V_{Ed.min}|}{|V_{Rd.c}|} \quad \text{if Check} = \text{"Equation 2"} \end{array} \right. = 0$$

$$\text{Check} := \left| \begin{array}{l} \text{"OK"} \quad \text{if } V = 1 \\ \text{"NOT OK, need shear reinforcement"} \quad \text{if } V \neq 1 \end{array} \right. = \text{"NOT OK, need shear reinforcement"}$$

### Check of the web, induced by compressive stresses in concrete

If the cross-section is rectangular, this check is seldom determinate.

$\alpha_{cw}$ - is a coefficient taking into account the state of stresses in the compression chord [EN 1992-1-1:2005 6.2.3(3)]

No axial force from prestress: $\sigma_{cp} = 0$

$$\alpha_{cw} := \begin{vmatrix} 1 \text{ if } \sigma_{cp} = 0 \\ \left(1 + \frac{\sigma_{cp}}{f_{cd}}\right) \text{ if } 0 < \sigma_{cp} \le 0.25 \cdot f_{cd} \\ 1.25 \text{ if } 0.25 \cdot f_{cd} < \sigma_{cp} \le 0.5 \cdot f_{cd} \\ 2.5 \cdot \left(1 - \frac{\sigma_{cp}}{f_{cd}}\right) \text{ if } 0.5 \cdot f_{cd} < \sigma_{cp} < 1.0 \cdot f_{cd} \end{vmatrix} = 1$$

When calculating the stresses in the strut the angle may be taken as:

$$\theta_{fat} := \sqrt{\tan(\theta)} = 36.419 \cdot deg$$

Distance from the support to the crack on the upper side of the slab:

$$x_{\theta.fat} := \cot(\theta_{fat}) \cdot (h_1 - d_e) = 0.661\, m$$

Coordinates used in Stripstep2: $x_{\theta.fat} = 0.661\, m$

Sectional height at the shear crack:

$$d_{x.\theta.fat} := h_1 = 0.54\, m \qquad \frac{x_{\theta.fat}}{L} = 0.094$$

Internal lever arm: $z := 0.9 \cdot d_{x.\theta.fat} = 0.486\, m$ [Figure 6.5 and 6.2.3(1) EN 1992-1-1]

### *Fatigue calculation of Concrete under compression*

[EN 1992-2:2005 Section 6.8.7]

Design partial factor taking material uncertainties into account:

$\gamma_{C.fat} := 1.5$ [EN 1992-1-1:2005 2.4.2.4(1)]

Characteristic compressive concrete strength in [MPa]:

$f_{ck} = 35 \cdot MPa$ [EN 1992-1-1:2005 3.1.3 table 3.1]

Design compressive concrete strength in [MPa]:

$f_{cdfat} := \dfrac{f_{ck}}{\gamma_{C.fat}} = 23.333 \cdot MPa$ [EN 1992-1-1:2005 3.1.3 table 3.1]

In NA recommended value:

$k_1 := 1.0$ [BFS 2009:16 6.8.7(1)]

Coefficient which depend on the type of cement:

$s := 0.25$ [EN 1992-2:2005 6.8.7 (1)]

The time of first cyclic load application on concrete in days:

$t_0 := 28$ [EN 1992-2:2005 6.8.7 (1)]

Coefficient for concrete strength at first load application:

$\beta_{cc} := e^{s \cdot \left(1 - \sqrt{\frac{28}{t_0}}\right)} = 1$ [EN 1992-2:2005 6.8.7 (1)]

Design fatigue strength of concrete:

$f_{cd.fat} := k_1 \cdot \beta_{cc} \cdot f_{cdfat} \cdot \left(1 - \dfrac{f_{ck}}{250MPa}\right) = 20.067 \cdot MPa$ [EN 1992-2:2005 Eq.(6.76)]

Fatigue strength reduction factor for concrete cracked in shear:

$\nu := 0.6 \cdot \left(1 - \dfrac{f_{ck}}{250 \cdot MPa}\right) = 0.516$

The reduced fatigue strength for concrete subjected to shear:

$f_{cdw.fat} := \nu \cdot f_{cd.fat} = 10.354 \cdot MPa$

## Sectional forces in the considered section:

Sectional shear force due to permanent load from Stripstep2:

Input with positive sign: $V_g := 189kN$

Shear forces for each cycle in the section considered calculated in AFB:
The design value of the maximum and minimum shear force under frequent load combination

Inserting tables from textfiles:

$$V_a := \begin{pmatrix} 100 \\ 150 \\ 200 \\ 250 \end{pmatrix} \qquad V_m := \begin{pmatrix} 10 \\ 20 \\ 30 \\ 40 \end{pmatrix} \qquad n_V := \begin{pmatrix} 1 \\ 1 \\ 0.5 \\ 1 \end{pmatrix}$$

Shear force amplitude from AFB:

$$V_A := V_a \cdot kN = \begin{pmatrix} 100 \\ 150 \\ 200 \\ 250 \end{pmatrix} \cdot kN$$

Mean shear force from AFB:
(Should have a positive sign)

$$V_M := V_m \cdot kN = \begin{pmatrix} 10 \\ 20 \\ 30 \\ 40 \end{pmatrix} \cdot kN$$

Number of cycles from AFB:

$N_{year} := 120$   $days := 365$   $n := n_V \cdot N_{year} \cdot days = \begin{pmatrix} 4.38 \times 10^4 \\ 4.38 \times 10^4 \\ 2.19 \times 10^4 \\ 4.38 \times 10^4 \end{pmatrix}$

Max shear force:

$$V_{fat.max} := V_g + V_M + \frac{V_A}{2} = \begin{pmatrix} 249 \\ 284 \\ 319 \\ 354 \end{pmatrix} \cdot kN$$

Min shear force:

$$V_{fat.min} := V_g + V_M - \frac{V_A}{2} = \begin{pmatrix} 149 \\ 134 \\ 119 \\ 104 \end{pmatrix} \cdot kN$$

Rewritten version of [EN 1992-1-1:2005 6.2.3(3)] gives for vertical shear reinforcement:

Maximum stress in the compressive chord:

$$\sigma_{cdw.max} := \frac{V_{fat.max}\cdot\left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw}\cdot w\cdot z\cdot \nu} = \begin{pmatrix} 0.462 \\ 0.527 \\ 0.592 \\ 0.657 \end{pmatrix}\cdot MPa$$

Minimum stress in the compressive chord:

$$\sigma_{cdw.min} := \frac{V_{fat.min}\cdot\left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw}\cdot w\cdot z\cdot \nu} = \begin{pmatrix} 0.276 \\ 0.249 \\ 0.221 \\ 0.193 \end{pmatrix}\cdot MPa$$

If the following condition is satisfied the fatigue verification is assumed:

$$\frac{\max\left(\sigma_{cdw.max}\right)}{f_{cdw.fat}} \le 0.5 + 0.45\,\frac{\min\left(\sigma_{cdw.min}\right)}{f_{cdw.fat}} = 1$$

$E_{cd.min.equ}$, $E_{cd.max.equ}$ - Mimimum and maximum compressive stress level

[EN 1992 2-2:2005 NN.3.2(101)]

$$E_{cdw.max} := \frac{\sigma_{cdw.max}}{f_{cdw.fat}} = \begin{pmatrix} 0.045 \\ 0.051 \\ 0.057 \\ 0.063 \end{pmatrix} \qquad E_{cdw.min} := \frac{\sigma_{cdw.min}}{f_{cdw.fat}} = \begin{pmatrix} 0.027 \\ 0.024 \\ 0.021 \\ 0.019 \end{pmatrix}$$

R - stress ratio:

$$R_V := \frac{E_{cdw.min}}{E_{cdw.max}} = \begin{pmatrix} 0.598 \\ 0.472 \\ 0.373 \\ 0.294 \end{pmatrix}$$ [EN 1992 2-2:2005 NN.3.2(101)]

Ultimate number of constant amplitude cycles in interval "i" that can be carried before failure:

$$N_{cw} := 10^{\left(14 \cdot \frac{1 - E_{cdw.max}}{\sqrt{1 - R_V}}\right)} = \begin{pmatrix} 1.277 \times 10^{21} \\ 1.922 \times 10^{18} \\ 4.685 \times 10^{16} \\ 4.008 \times 10^{15} \end{pmatrix}$$ [EN 1992-2:2005 6.8.7(101)]

Total damage inflicted on the concrete under compression:

$$j := \left(\text{length}\left(N_{cw}\right) - 1\right)$$

$$D_{cw} := \sum_{i=0}^{j} \frac{n_i}{N_{cw_i}} \qquad D_{cw} = 1.142 \times 10^{-11}$$ [EN 1992-2:2005 6.8.7(101)]

$$\text{Check} := \begin{cases} \text{"OK"} & \text{if } D_{cw} \le 1 \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"OK"}$$

$$D_{cw.i} := \frac{n}{N_{cw}} = \begin{pmatrix} 0 \\ 2.279 \times 10^{-14} \\ 4.675 \times 10^{-13} \\ 1.093 \times 10^{-11} \end{pmatrix}$$

# Appendix L. λ-Coefficient Method Fatigue induced by shear stresses

The calculations until here is identical with the CDM.

## Sectional forces in the considered section:

Sectional shear force due to permanent load from Stripstep2:

Input with a positive sign: $V_g := 435kN$

$$\frac{x_{\theta.fat}}{L} = 0.094$$

Sectional forces due to traffic load from AFB:

Maximum shear force from the traffic load: $\Delta V_{max} := 326.6kN$

Minimum shear force from the traffic load: $\Delta V_{min} := 0kN$

Min shear force: $V_{fat.min} := V_g + \Delta V_{min} = 435 \cdot kN$

Max shear force: $V_{fat.max} := V_g + \Delta V_{max} = 761.6 \cdot kN$

### *Fatigue calculation of Concrete under compression*

[EN 1992 -2:2005 Annex NN.3.2]

Put $\sigma_{cw.max} = \sigma_{cw.min} = f_{cd}$

Strength reduction factor for concrete cracked in shear:

$\upsilon_1 := 0.6$ for $f_{ck} \leq 60MPa$

Internal lever arm: $z := 0.9 \cdot d_{x.\theta.fat} = 0.486\,m$ [Figure 6.5 and 6.2.3(1) EN 1992-1-1]

Maximum stress in the compressive chord:

$$\sigma_{cw.max} := \frac{V_{fat.max} \cdot \left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw} \cdot w \cdot z \cdot \upsilon_1} = 1.215 \cdot MPa$$

Minimum stress in the compressive chord:

$$\sigma_{cw.min} := \frac{V_{fat.min} \cdot \left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw} \cdot w \cdot z \cdot \upsilon_1} = 0.694 \cdot MPa$$

If the following condition is satisfied the fatigue verification is assumed:

$$\frac{\sigma_{cw.max}}{f_{cdw.fat}} \leq 0.5 + 0.45 \frac{\sigma_{cw.min}}{f_{cdw.fat}} = 1$$

If this requirement is fulfilled, further calculation does not need to be carried out. According to EN 1992-2:2005 6.8.7(2), the fatigue verification can be assumed. However, in order to compare two methods (CDM and LCM), a value of the damage (D) is prefered. Therefore the calculations will be continued after this verification.

Stress from the permanent loads in the compressive chord:

$$\sigma_{cw.perm} := \frac{V_g \cdot \left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw} \cdot w \cdot z \cdot \upsilon_1} = 0.694 \cdot \text{MPa}$$

Maximum and minimum compressive stress caused by the characteristic combination of actions, with load model LM 71 and the dynamic factor:

Maximum stress from LM 71 in the compressive chord:

$$\sigma_{cw.max.71} := \frac{\Delta V_{max} \cdot \left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw} \cdot w \cdot z \cdot \upsilon_1} \cdot \phi_2 = 0.705 \cdot \text{MPa}$$

Minimum stress from LM 71 in the compressive chord:

$$\sigma_{cw.min.71} := \frac{\Delta V_{min} \cdot \left(\cot\left(\theta_{fat}\right) + \tan\left(\theta_{fat}\right)\right)}{\alpha_{cw} \cdot w \cdot z \cdot \upsilon_1} \cdot \phi_2 = 0 \cdot \text{MPa}$$

When calculating the correction factor, $\lambda_{cw}$, for concrete, the factor who takes into account the permanent stresses is the only factor that differs from earlier calculations in "Degerfors - Support LCM". Therefore only the calculation of this factor is shown.

$\lambda_{cw.0}$ - factor to take account of permanent stress [EN 1992-2:2005 NN.3.2(104)]

$$\lambda_{cw.0} := \begin{cases} 0.94 + 0.2 \cdot \dfrac{\sigma_{cw.perm}}{f_{cdw.fat}} & \text{if } 0.94 + 0.2 \cdot \dfrac{\sigma_{cw.perm}}{f_{cdw.fat}} \geq 1 \\ 1 & \text{otherwise} \end{cases} = 1$$

$\lambda_{cw}$ - Correction factor takes account of the permanent stress, the span, annual traffic volume, desgin life and multiple tracks:

$$\lambda_{cw} := \lambda_{cw.0} \cdot \lambda_{c.1} \cdot \lambda_{c.2.3} \cdot \lambda_{c.4} = 0.893$$ [EN 1992-2:2005 NN.3.2(103)]

$\sigma_{cw.max.equ}$, $\sigma_{cw.min.equ}$ - The upper and lower of the damage equivalent stress spectrum [EN 1992 2-2:2005 Annex N.N Eq. NN.113]

$$\sigma_{cw.max.equ} := \sigma_{cw.perm} + \lambda_{cw} \cdot \left(\sigma_{cw.max.71} - \sigma_{cw.perm}\right) = 0.704 \cdot \text{MPa}$$

$$\sigma_{cw.min.equ} := \sigma_{cw.perm} - \lambda_{cw} \cdot \left(\sigma_{cw.perm} - \sigma_{cw.min.71}\right) = 0.074 \cdot \text{MPa}$$

Partial factor for materials for ULS: $\gamma_{sd} = 1$ [EN 1992-1-1:2005 2.4.2.4(2) table 2.1N]

$E_{cd.min.equ}$, $E_{cd.max.equ}$ - Mimimum and maximum compressive stress level taking into account the permanent and traffic load with the correction factor.

Reference amount of cycles until failure: $N = 1 \times 10^{6}$ [EN 1992 2-2:2005 NN.3.2(101)]

$$E_{cw.min.equ} := \gamma_{sd} \cdot \frac{\sigma_{cw.min.equ}}{f_{cd.fat}} = 3.703 \times 10^{-3}$$

$$E_{cw.max.equ} := \gamma_{sd} \cdot \frac{\sigma_{cw.max.equ}}{f_{cd.fat}} = 0.035$$

$R_{equ}$ - Stress ratio taking into account the permanent and traffic loads

If the stress ratio has a negative value, it means that the concrete is in tension instead of being in compression which the model is intended for. Therefore the stress ratio smaller than zero will be set equal to zero.

$$R_{equ} := \begin{cases} 0 & \text{if } \dfrac{E_{cw.min.equ}}{E_{cw.max.equ}} < 0 \\ \dfrac{E_{cw.min.equ}}{E_{cw.max.equ}} & \text{otherwise} \end{cases} = 0.106$$

[EN 1992 2-2:2005 NN.3.2(101)]

Requirement that needs to be fulfilled in order to know that the inflicted damage is ok.

$$Check := \begin{cases} \text{"OK"} & \text{if } 14 \cdot \dfrac{1 - E_{cw.max.equ}}{\sqrt{1 - R_{equ}}} \geq 6 \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"OK"}$$

[EN 1992 2-2:2005 NN.3.2(101)]

Writing the value to know how must larger or smaller than six the requirement is:

$$14 \cdot \frac{1 - E_{cw.max.equ}}{\sqrt{1 - R_{equ}}} = 14.284$$

Total damage inflicted on the concrete

$$D_c := E_{cw.max.equ} + \frac{\log(N)}{14} \cdot \sqrt{1 - R_{equ}}$$

$D_c = 0.44$

$$Check := \begin{cases} \text{"OK"} & \text{if } D_c \leq 1 \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"OK"}$$

## *Fatigue calculation of the reinforcing steel*

[EN 1992 -2:2005 Annex NN.3.1]

$\Delta\sigma_{s.equ} = \lambda_s \cdot \phi \cdot \Delta\sigma_{s.71}$ [EN 1992-2:2005 NN.3.1(101)]

$\lambda_s = \lambda_{s.1} \cdot \lambda_{s.2} \cdot \lambda_{s.3} \cdot \lambda_{s.4}$ [EN 1992-2:2005 NN.3.1(102)]

$\lambda_s$ - correction factor to calculate the damage equivalent stress range

$\Delta\sigma_{s.71}$ - steel stress range due to load model 71

$\phi$ - dynamic factor [EN 1991-2 6.4.5.2 (3)P], [BFS 2009:16 6§]

The correction factor is calculated in "Mathcad - LCM Support"

$\lambda_s = 0.892$

### $\Delta\sigma_{s.71}$ - steel stress range due to load model 71

Cross sectional area of the shear reinforcement for each spacing in the longitudinal direction from calculations in "Mathcad - Bridge design"

$A_{sw} = 9.666 \times 10^{-4}\,m^2$

Maximum transversal spacing between shear reinforcement, taken from "Bridge design"

$s_x = 0.366\,m$

Steel stress range in the shear reinforcement due to load model 71

Rewritten equation from [EN 1992-1-1:2005 6.2.3 (3)]

By putting the design yield strength of the shear reinforcement equal to the shear resistance, the equation can be rewritten in order to get the stress range in the shear reinforcment

Put $\sigma_{sw.71} = f_{ywd}$

$$\Delta\sigma_{sw.71} := \frac{\Delta V_{max} - \Delta V_{min}}{\frac{A_{sw}}{s_{t.max}} \cdot z \cdot \cot(\theta_{fat})} = 187.531 \cdot MPa$$

Equivalent stress in the shear reinforcement due to load model 71, taking into account correction factor and dynamic factor

$$\Delta\sigma_{sw.equ} := \Delta\sigma_{sw.71} \cdot \phi_2 \cdot \lambda_s = 226.473 \cdot MPa$$

Design critetria value for the shear reinforcement:

Input parameters exept the current stress range are defined in previous calculations for LCM

$$D_{sw} := \frac{\gamma_{F.fat} \cdot \Delta\sigma_{sw.equ}}{\frac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}}} = 1.603$$

$D_{sw} = 1.603$

$$Check := \begin{cases} \text{"OK"} & \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{sw.equ} \le \frac{\Delta\sigma_{Rsk}}{\gamma_{S.fat}} \\ \text{"NOT OK"} & \text{otherwise} \end{cases} = \text{"NOT OK"}$$