An Improved Static-Priority Scheduling Algorithm for Multi-Processor Real-Time Systems

Master of Science Thesis in Secure and Dependable Computer System

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Göteborg, Sweden, June 2010

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Abstract

This thesis deals with the problem of designing a new real-time scheduling algorithm for independent periodic tasks with static priority on multi-processor platforms called IBSP-TS (Interval Based Semi-Partitioned Task Splitting). The widely implemented priority policy Rate-Monotonic is applied in the algorithm. IBSP-TS combines interval-based semi-partition technique and another multi-processor scheduling algorithm SPA2 to achieve the highest possible worst-case utilization bound to ln2 while meeting the deadlines.

The assignment of IBSP is divided into two parts. In the first part, tasks are categorized into several interval groups. Each group has its own assignment policy except for the last interval. In most cases, there are some tasks residual after applying all the policies. All the residual tasks are handled along with the tasks from last interval in the second part of the algorithm. The schedulability can be ensured by feasibility tests.

The simulation experiment shows IBSP-TS has some good properties compared to the best static-priority multi-processor scheduling algorithm at this moment. It generally has higher success ratio, less sorted tasks and also less task migrations. In the best case, it can achieve the break-down utilization point to 76% in simulation. Additionally, this algorithm can let system designer to choose the number of intervals in the algorithm. The more intervals, the less number of sorted tasks there are.

Keywords: Real-Time Scheduling, Utilization Bound, Multi-Processor, Static-Priority, Rate-Monotonic, Periodic, Preemptive, Semi-Partitioned, Task Splitting

Acknowledgement

First of all, we would like to thank our thesis examiner Docent Jan Jonsson. It is because of his excellent teaching in real-time system and his deep and wide knowledge in this field which leaded us into the research of this thesis.

Equivalently, we owe our greatest gratitude to Mr. Risat Mahmud Pathan for his cordial help. Without him, this thesis would not have been possible. He had made available his support in number of ways, such as his invaluable ideas, his timely feedback and his guidance. All these not only provide us inspirations, but also give us confidence in our research.

Additionally, it is a pleasure to thank all the friends around us who had inspired us and gave us suggestions.

Finally, we would like to show our gratitude to our parents for their encouragement and understanding.

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Content

I

Introduction

It has already become the truth that people's life relies more and more on computers. Computers systems are ubiquitous. It can range from the smallest PDA device to super-computer. Some of them need high performance; some of them may require fault tolerance; and some of them have strong timing constraints. For a computer system with strong timing constraints, it is called *real-time systems*. The chief design goal of real-time system is not high throughput, but rather a guarantee of the consistency concerning the amount of time its applications take. Therefore, the correctness of a real-time system is not only logical and functional, but also temporal.

In real world, there are always some critical issues that have to be processed within a fixed amount of time whenever invoked. For example, mechanical systems on production lines, breaking systems in vehicles, flying systems in aircrafts and space shuttles, E-commerce systems in stock exchanges and consoles in nuclear power plants, all these need real-time system to ensure the temporal correctness.

All the real-time system can be categorized into two groups, *hard real-time systems* and *soft real-time systems*. In hard real-time systems, the consequences of not fulfilling a time constraint may be catastrophic, such as medical systems. Hence, predictability is paramount among all concerns. On the other hand, for soft real-time systems, single failures of not fulfilling a time constraint is acceptable, examples of soft real-time systems are multimedia systems and communication systems.

Moore's Law mentioned that the number of transistors per area unit on a integrate circuit doubles approximately every two years. It is certain that, with faster processors, system designers can use the increased capacity to deliver better services. Nevertheless, a fast computer is not enough to ensure real-time properties. Therefore, utilizing the resource as much as possible and ensuring real-time processing lead to a research of *real-time scheduling*. For the time being, real-time scheduling for uniprocessor is quite mature. Meanwhile, the development for multi-core processor is impressive these years. In 2009, even a processor with 100-core was released [1]. However, those well-developed scheduling algorithms for uniprocessor perform poorly in multi-processor systems in the term of worst-case utilization bound. It is proved that only 50% can be achieved [2]. Therefore, developing new algorithms for multi-processor received considerable attention these years.

Recently, there is a static-priority scheduling algorithm for multi-processor systems – IBPS [3] has been proposed. Tasks with the range (0, 1] are categorized into seven intervals to achieve a worst-case utilization bound as 55.2%.Later, another scheduling algorithm for multi-processor systems called the SPA2 [4], has theoretically proved that the worst-case utilization bound for static-priority multi-processor scheduling can achieve to ln2, which is the highest possible value. However, the SPA2 algorithm has to sort all the tasks in a task set which is not applicable for online scheduling. Additionally, SPA2 suffers from the number of subtasks from an individual task, which can bring considerable context switch in a system.

In this paper, a new static-priority scheduling algorithm for multi-processor systems scheduling called IBSP-TS is proposed, which is an interval based semi-partitioned scheduling algorithm. It tries to assign as many tasks as possible to a single processor. However, if a task cannot be fully assigned to a single processor, it will be split into one or more subtasks. IBSP-TS combinations the ideas from IBPS algorithm and SPA2 algorithm to reach the same worst-case schedulable utilization bound as ln2. Meanwhile, it reduces the number of preemptions and migrations for practical use and holds a higher schedulable rate than SPA2.

1.1 Contributions

The main contributions of this thesis are as follows:

1. IBSP-TS achieve the highest possible worst-case utilization bound of a static-priority multi-processor scheduling to ln2.
2. Online scheduling is possible with IBSP-TS
3. There are less overhead for IBSP compared to SPA2 algorithm.
4. For mixed tasks and heavy utilization tasks, IBSP-TS has better schedulability than SPA2.
5. It is possible for the system designer to choose the number of intervals in IBSP-TS. The more intervals, the less sorted tasks are.

1.2 Thesis Outline

The rest of this thesis is organized as follows:

- Chapter II describes the related background of real-time scheduling.
- Chapter III presents the necessary assumption and models of tasks and the system be used in new algorithm IBSP-TS.
- The design details of IBSP-TS algorithm are shown in Chapter IV.
- In Chapter V, the performance of IBSP-TS is estimated by comparing with another algorithm SPA2.
- Finally, Chapter VI concludes this thesis with a discussion on the applicability and extendibility of IBSP-TS algorithm.

II

Related Background

In this section, the related background of real-time scheduling is presented, such as what are real-time tasks, including its parameters and priorities; how is utilization bound defined; what is scheduling feasibility test; what is the difference between different scheduling schemes; and various multi-processor scheduling algorithms.

2.1 Real-Time Tasks

A *task* is unit of work such as a program or code-block that when executed provides some service of an application. It can be either *dependent* or *independent*. For a dependent task, its execution may require an exclusive access to a *shared resource* (e.g., a file, a data structure in shared memory or an external device other than processor time) or it has some precedence constraints. If a resource is shared among multiple tasks, then some tasks may be blocked from being executed until the shared resource is free. Similarly, if tasks have precedence constraints, then one task may need to wait until another task finishes its execution. A region of code with such a requirement is called a *critical section*. Tasks are said to be *independent* when they have no critical sections. In this thesis, each task is assumed to be independent in the sense that it does not interact in any manner (accessing shared data, exchanging messages, etc.) with other tasks. The only resource the tasks share is the processor platform.

A *real-time task* is a task running in real-time system. In general, a real-time task may require a specific amount of particular resource during a specific period of time. A real-time task system can be classified in as *periodic task, aperiodic tasks* or *sporadic tasks.*

A periodic task is a task that arrives with a continuous and deterministic pattern of time interval. That is, it continuously requests resources at time values. In addition to this requirement, a real-time periodic task must complete processing by a specified deadline relative to the time that it acquires the processor.

An *aperiodic task* is a stream of jobs arriving irregularly. There may either be no bound or only a statistical bound on the arrival period. It requests a resource during non-deterministic request periods. Each task job is also associated with a specified deadline, which represents the time necessary for it to complete its execution.

A *sporadic task* is an aperiodic task with a hard deadline and a minimum inter-arrival time.

In this thesis, it is assumed that only periodic tasks are considered.

2.1.1 Task Parameters

A task set is a set of *n* independent periodic tasks denoted as $\Gamma = \{\tau_1, \tau_2, \tau_3, \cdots, \tau_n\}$. An independent periodic task τ_i can be fully characterized by the following parameters.

- *Period* (T_i): Each task in periodic task system has an inter-arrival time of occurrence, called the *period* of the task. In each period, a *job* of the task, which is the recurrent copy of the task, is released.

- *Offset* ($\emptyset_i$): A task is ready to execute at the beginning of each period, called the *released time*, of the task. The first job of a task may arrive at any time-instant; an offset defines the release time of the first job. If the relative deadline of each task in a task set is less than or equal to its period, then the task set is called a *constrained* deadline periodic task system. If the relative deadline of each task in a constrained deadline task set is exactly equal to its period, then the task set is called an *implicit* deadline periodic task system. If a periodic task system is neither constrained nor implicit, then it is called an *arbitrary* deadline periodic task system. In this thesis, scheduling of implicit deadline periodic task system is considered.

- *Deadline* (D_i): Each job of a task has a *relative deadline* that that defines the time window in which the job has to be executed since its release time. The relative deadlines of all the jobs of a particular periodic task are same. The *absolute deadline* of a job is the time instant equal to released time plus the relative deadline.

- *WCET (worst-case execution time)* (C_i): Each periodic task has a WCET that is the maximum execution time that each job of the task requires.

- *Utilization* (U_i): The utilization of a task is actually the utilization of processor for a task, which is the ratio of the execution time of a task to its period.

2.1.2 Task Priorities

In a periodic task system, all that are released but have not complete their individual execution by the time *t* are called *active tasks*. When two or more active tasks compete for a same processor, some rules must be applied to for the scheduling dispatcher to allocate the use of the processor. It assigns priorities to all tasks that are eligible for execution and then selects the highest priority one on a processor. The priority of a task can be *static* or *dynamic*.

For static-priority scheduling, priorities never change once they are assigned. Hence, each job of a task inherits its initial priority. Static-priority scheduling dispatchers are inherently memoryless in that the prioritization of tasks is independent of previous scheduling decisions. *Rate-monotonic (RM) scheduling* [5] is an example of such a policy for periodic and sporadic tasks. Under the RM prioritization, tasks with shorter periods are given higher priority.

Under dynamic-priority discipline, the priority order of an active task may be changed during its execution. For different jobs of a task may have different priorities relative to other tasks in the system. *Earliest-deadline-first (EDF) scheduling* [5] is a dynamic-priority policy often considered for scheduling periodic and sporadic tasks. It gives jobs with earlier absolute deadlines with higher priority.

When a task is released at time *t*, its execution may be delayed due to other higher priority tasks running in the system. In RM scheduling, a task with smaller period has higher priority. In case, two tasks have exactly the same period, they have equal priority, which means the scheduling dispatcher can randomly pick either of them.

In this thesis, the RM scheduling priority is adopted.

2.2 Utilization Bound

A processor is said to be *fully utilized* when an increase in the computation time of any of the tasks in a task set will make the task set unschedulable. The least upper bound of the total utilization is the minimum of all total utilizations over all sets of tasks that fully utilize the processor. This least upper bound of a scheduling algorithm is called the *worst-case utilization bound (minimum achievable utilization bound)* or simply *utilization bound* of the scheduling algorithm. A scheduling algorithm can feasibly schedule any set of tasks on a processor if the total utilization of the tasks is less than or equal to the utilization bound of the scheduling algorithm. For example, in uniprocessor scheduling, the utilization bound of RM is 69% and the utilization bound of EDF is 100% [5].

2.3 Scheduling Feasibility Analysis

To predict the temporal behavior and to determine whether the timing constraints of an application tasks will be met during runtime, *feasibility analysis* of scheduling algorithm is conducted. A schedule is said to be *feasible* if it fulfills all application constraints for a given set of tasks. A set of tasks is said to be *schedulable* if there exists at least one scheduling algorithm that can generate a feasible schedule. A scheduling algorithm is said to be *optimal* with respect to schedulability if it can always find a feasible schedule whenever any other scheduling algorithm can do so.

Feasibility analysis of a hard real-time system refers to the process of determining offline whether the specified system will meet all deadlines at runtime. A *feasibility test* is introduced by doing feasibility analysis. It uses one or several conditions to determine whether a task set is feasible for a given scheduling algorithm. The feasibility test can be *necessary and sufficient* or only *sufficient.*

For a sufficient feasibility test, a task set pass the test shows that it is definitely schedulable. However, if a task set does not pass the sufficient feasibility test, it may still be schedulable. While, for a necessary and sufficient feasibility test, if and only if a task set pass test, its tasks can meet their individual deadlines.

Processor utilization analysis is one of the techniques to perform the feasibility test. It uses the sum of utilizations of all the tasks belong to the set, that is $\sum_{i=1}^{n} U_i$. While, in multi-processor system, *system utilization* is often used which represent the utilization of a task set on *m* processors.

In 1973, *Liu and Layland* derived a sufficient feasibility test for RM scheduling and an exact test for EDF scheduling on uniprocessor.

A set of *n* tasks is schedulable by the RM algorithm if

$$\sum_{i=1}^{n} U_i \leq n(2^{\frac{1}{n}} - 1) \ = \ln 2 \text{ when } n = \infty \text{ [5].}$$

For the EDF algorithm, a set of *n* periodic tasks is schedulable if and only if

$$\sum_{i=1}^{n} U_i \leq 1 \text{ [5].}$$

In 2001, another test is presented which is called *hyperbolic test* [6] for Rate-Monotonic scheduling. It dominates the bound of Liu & Layland for RM, but it has the same effect as Liu & Layland Test if there are infinite numbers of tasks on a processor.

$$\prod_{i=1}^{n}(U_i + 1) \leq 2$$

2.4 Scheduling Features

When the number of tasks does not exceed the number of processors, each task can simply be assigned a dedicated processor. However, usually this is not the case. At least one processor must be shared among multiple tasks. Therefore, the most important part of real-time system design is to choose a good scheduling scheme which can fulfill the application constraints. In the following part, various scheduling schemes are presented. In this thesis, IBSP-TS are defined as a preemptive static priority multi-processor scheduling algorithm.

2.4.1 Preemptive vs. Non-Preemptive

A scheduling algorithm is *preemptive* if the release of a new job of a higher priority task can preempt the job of a currently running lower priority task. During runtime, task scheduling is essentially determining the highest priority active tasks and executing them in the free processor. For example, RM and EDF are examples of preemptive scheduling algorithm.

On the contrary, in *non-preemptive* scheme, a currently executing task always completes its execution before another active task starts execution. Therefore, in non-preemptive scheduling a higher priority active task may need to wait in the ready queue until the currently executing task (may be of lower priority) completes its execution.

A preemptive scheduling algorithm can succeed in meeting deadlines where a non-preemptive scheduling algorithm fails but a non-preemptive scheduling algorithm has naturally the advantage of no run-time overhead caused by preemptions. In this thesis, only preemptive scheduling is considered.

2.4.2 Static vs. Dynamic

The scheduling can be generated offline or online, which represent *static (offline) scheduling* and *dynamic (online) scheduling* separately.

When the complete schedulability analysis of a task system can be done before the system is put in mission, the scheduling is considered as static scheduling. Scheduling dispatcher holds a *time table,* which contains explicit start and completion times for each task job that controls the order of execution at run-time. In order to predict feasibility of a task set, static scheduling analysis requires the availability of all static task parameters, like periods, execution time, and deadlines.

Dynamic scheduling makes scheduling decisions at each time-instant based upon the characteristics of the task that have arrived so far. It has no knowledge of tasks that may arrive in the future. Since a newly arriving task can interfere with the execution of already existing tasks in the system, an *admission controller* is needed to determine whether to accept a new task that arrives online. The feasibility test of a scheduling algorithm can be used as the basis for designing an admission controller for dynamic systems. However, evaluating the feasibility test when a new task arrives must not take too long time. This is because using processing capacity to do the feasibility test could detrimentally affect the timing constraints of the existing tasks in the system.

2.4.3 Uniprocessor vs. Multi-Processor

Real-time scheduling theorists have extensively studied for hard real-time scheduling with uniprocessor. Uniprocessor scheduling algorithm executes tasks on a single processor. The schedulability of a given set of tasks on uniprocessor platform can be determined by using Liu and Layland Test. It is well known that RM and EDF are optimal algorithms for uniprocessor scheduling.

At this point, it is worth to mention that the RM algorithm is widely used in industry because of its simplicity, flexibility and its ease of implementation [7, 8]. It can be used to satisfy the stringent timing constraints of tasks; while at the same time it can also support execution of a-periodic tasks and meet the deadlines of the periodic tasks. RM can be modified easily, for example, to implement priority inheritance protocol for synchronization purpose [9]. The conclusion of the study in [7] is that "*...the Rate-Monotonic algorithm is a simple algorithm which is not only easy to implement but also very versatile*".

Multi-core processors are quickly emerging as the dominant technology in the microprocessor industry. Major chip manufacturers have already adopted multi-core technologies due to the thermal problems that distress traditional single-core chip designs in terms of processor performance and power consumption. In 2009, even a 100-core processor had been released [1]. Recently, many steps have been taken towards obtaining a better understanding of hard real-time scheduling on multi-processors.

As multi-processor systems have faster computational power and fault-tolerance feature, use of multi-processor and distributed systems in real-time applications is becoming popular. As compared to scheduling real-time tasks on a uniprocessor, scheduling tasks on multi-processor and distributed systems is a much more challenging problem.

2.5 Multi-Processor Scheduling

Multi-processor scheduling algorithms are often categorized as *global scheduling* and *partitioned scheduling*. However, it has been proved that neither global nor partitioned static-priority multi-processor scheduling algorithm can achieve a utilization bound greater than 50% [2]. Therefore, in order to achieve utilization bound greater than 50%, another new category called semi-partitioned scheduling comes out recently, which introduces the technique *task splitting* into the traditional partitioned scheduling algorithm. In the following part, each of them is presented.

2.5.1 Global Scheduling

Global scheduling algorithms store tasks which have arrived but execution unfinished in one queue that is shared by all processors. In other words, the highest priority task is always selected to be executed whenever the scheduling dispatcher is invoked, regardless of which processor is being scheduled. At every moment，the *m* highest priority tasks among those are selected for execution on the *m* processors. In addition, task preemptions and migrations are permitted, that is a task preempted on a particular processor may resume execution on the same or on a different processor. Due to this, global scheduling on average utilizes computing resource well.

Proportionate-fair (Pfair) scheduling [10, 11] is a global scheduling approach proposed as a means for optimally scheduling periodic tasks on multi-processors. Pfair scheduling uses a quantum-based model. In Pfair scheduling, although a task has started executing, lower priority tasks receive a guaranteed time quantum per time unit for execution, all tasks hence make some kind of progress per time unit. It is known that *PF* [10], *PD* [11] and *PD2* [12] are optimal Pfair algorithms which can theoretically achieve 100% schedulable system utilization. The PD2 is known to be the most efficient in the optimal Pfair algorithms. *LLREF algorithm* [13] which is based on a different technique relying on the original notation called *T-L Plane (Time and Local Execution Time Domain Plane)*, can also achieve 100% schedulable system utilization.

However, these algorithms will generate a number of task preemptions and migration overhead and they are very complex to implement. There are some simple global dynamic priority scheduling algorithms which perform fairly well with a small number of task preemptions and migration overhead, such as *EDF-US[1/2]* [14, 15] scheduling policy. It gives only a few high-utilization tasks top priority; all other tasks are scheduled according to deadlines. *EDZL (Earliest Deadline Zero Laxity)* [16] is a hybrid preemptive dynamic priority scheduling scheme in which tasks with zero laxity are given highest priority and other tasks are ranked by deadline. Nevertheless, their utilization bounds for schedulable systems are down to 50%.

As applies to static priority scheduling, it is known that Rate-Monotonic priority assignment scheme is not optimal for multi-processor systems, because global RM can miss deadline even utilization approaches zero [17].

RM-US [m/(3m-2)] [18] *algorithm* (*m* is the number of processors) can guarantee schedulability as long as the multiprocessor utilization is below 33%. Later, an improved version *RM-US [0.37482]* [19] guarantees schedulability for all systems up to 37.482% for the system utilization. These two algorithms categorize a task as heavy or light. A task is said to be heavy if the utilization exceeds a certain threshold number and a task is said to be light otherwise. Heavy tasks are assigned with higher priority and the light tasks are assigned with lower priority. The relative priority order among light tasks is given by RM.

Nowadays, the best known utilization bound of global static-priority scheduling is 38%, which is reached by *SM-US [2/(3+ √5)]* (Slack Monotonic) [20]. It uses similar priority scheduling scheme which categorizes tasks as heavy and light and assigns the highest priority to heavy tasks. The relative priority order of light tasks is given by SM, which means task τ_j is assigned higher priority than task τ_i if $T_j - C_j < T_i - C_i$.

Later, another global scheduling algorithm *WM (Weight-Monotonic)* [21] was proposed and has been proved that the worst-case utilization bound is 50% [18]. However, the WM algorithm generates a large number of task preemptions due to the characteristic of Pfair scheduling.

2.5.2 Partitioned Scheduling

The alternative to global scheduling is partitioned scheduling. It partition a task set into groups beforehand. Each processor holds a separate ready queue such that each task group which is assigned to a specific processor. In other words, during the run time, tasks may not migrate from one processor to another. Thus, multi-processor scheduling is equivalent to multiple uniprocessor systems.

The predominant approach of scheduling multi-processor hard real-time systems is usually for partitioned scheduling, since it has the virtue of applying efficient uniprocessor techniques on each processor. As no task migrations occur in partitioned scheduling, it is superior to global in practice. Additionally, the schedulability of partitioned scheduling can be verified by using well-understood uniprocessor analysis techniques.

The most well known static priority scheduling algorithm is *RM-FF* [22], which use First-fit bin-packing algorithm. For First-fit, it means that tasks are assigned into in increasing index order visited processors. It has been proved by Oh & Baker that the utilization guarantee bound in RMFF for a system with m processors using the is between $m(\sqrt{2}-1)$ and $(m+1)/(1+2^{1/(m+1)})$ [23]. The lower bound shows that the worst-case utilization bound is 41%. Another algorithm, *RM-FFDU (First-Fit Decreasing Utilization)* [24], which conducts the FF heuristic after sorting the tasks in decreasing utilization order; it usually performs better than RM-FF algorithm.

Thereafter, another static priority scheduling algorithm called *R-BOUND-MP-NFR (multi-processor next-fit-ring)* [2] is presented with the best result of partitioned static-priority scheduling reaching 50%. It introduces the NFR heuristic into the *R-BOUND-MP* algorithm. Here, R-BOUND-MP is a previously known multi-processor scheduling algorithm which combines *R-BOUND* [25] with First-fit bin-packing algorithm and exploits R-BOUND.

Additionally, there are some simple dynamic-priority scheduling algorithms, such as *EDF-FF (Earliest Deadline First-Fit)* [26] and *EDF-BF (Earliest Deadline Best-Fit)* [26]. For best-fit, it means a task is assigned to a processor which verify the schedulability test after assignment and which maximizes the remaining processor capacity. The system utilization bounds of these algorithms are also 50%, but these partitioned scheduling algorithms can reduce more preemptions and task migration overhead than global scheduling algorithms.

2.5.3 Semi-Partitioned Scheduling

It has been proved that neither global nor partitioned static-priority multi-processor scheduling algorithm can achieve a utilization bound greater than 50% [2]. Therefore, nowadays a lot of work has been done on semi-partitioned scheduling in order to achieve utilization bound higher than 50%. Semi-partitioned scheduling algorithms introduce techniques such as *task splitting* into the traditional partitioned scheduling algorithm. When the spare capacity of the individual processor is not enough to fully accept the execution of a task, it split the task into two or more pieces. Each piece is called a *subtask* and to a dedicated processor. In other words, this task execution is allowed to migrate to different processors. Meanwhile, most tasks are statically assigned to one fixed processor as in traditional partitioned scheduling. Here, the most important condition is that no subtasks split from one task run in parallel. In such a way that a task never returns to the same processor within the same period once it is migrated from one processor to another processor.

RMDP (Rate-Monotonic Deferrable Portion) [27] is a static priority scheduling algorithm based on the semi-portioned scheduling technique and reaches to the worst-case utilization bound as 50%. In RMDP, tasks are sorted in increasing period order and assigned to processors sequentially. If a task *t* makes the total utilization of a processor *P* exceed its utilization bound, the task is split into two parts. The first part is assigned to that processor *P*, and the second part is assigned to the next chosen processor.

DM-PM (Deadline Monotonic-Priority Migration), [28] an algorithm based on the concept of semi-partitioned scheduling also achieves utilization bound higher than 50%. In DM-PM algorithm, each task is assigned to a particular processor by using some kinds of bin-packing heuristics, upon which the schedulable condition for DM is satisfied. If there are no such processors, DM-PM shares the task with more than one processor. By doing this, an unfeasible task with classical partitioned approaches can be scheduled.
a task is qualified to migrate only if it cannot be assigned to any individual processors.

Another static-priority scheduling algorithm – *IBPS (Interval Based Partitioned Scheduling)* [3] has been proposed with system utilization bound 55.2%. In IBPS, tasks are grouped into seven utilization intervals and then the tasks from these groups are assigned to processors using different policies. As IBPS has only two subtasks, the total number of migrations caused by split tasks is lower than of any other task-splitting algorithm.

In a later work, a partitioned scheduling algorithm – *PDMS-HPTS (Partitioned Deadline Monotonic Scheduling - Highest Priority Task Splitting)* [29] with task splitting technique used on the highest priority task can lead to a utilization bound of 60%. A specific instance of this class, where tasks are allocated in the decreasing order of sizes using PDMS-HPTS-DS [29] can achieves system utilization bound of 65% theoretically. The utilization bound of 69.3% is achieved when the utilization of each individual task is restrictively less than 41.4%. Additionally, it has shown that the utilization bound of PDMS-HPTS-DS in simulation can reach to 88% in practice

In term of worst-case utilization bound, the best algorithm for multi-processor with static priority so far is *SPA2 (Semi-Partitioned Algorithm 2)*, which has theoretically shows the utilization bound of 69.3% in worst case [4]. In SPA2, tasks are categorized into heavy tasks and light tasks. For heavy tasks, there is a pre-assigning mechanism. For light tasks, the algorithm assigns tasks in decreasing period order, and always selects the processor with the least workload assigned so far among all processors, to assign the next task.

Task splitting technology can also be applied to dynamic-priority scheduling, for example, the well known *EKG (EDF with task splitting and K processors in a Group.)* [12] and *Ehd2-SIP (EDF with Highest priority Deferrable portion-2 task-Sequential assignment in Increasing Period)* [30] which improves schedulability with a few preemptions.

EKG assigns each task to a particular processor like conventional partitioned scheduling algorithms. However, it can split a task into parts if necessary. It assigns the first part to the current processor on which the assignment is going and the second part to the next picked processor. The two parts of a split task are scheduled exclusively. The least upper bound of the schedulable system utilization for EKG depends on the value of a parameter k which should be selected in the range of $1 \leq k \leq M$ where M is the number of processors in a system. A large k results in a higher bound but more preemptions. The bound becomes 66% in the case of k=2 and 100% in the case of k=M. Namely EKG is an optimal algorithm in the case of k=M, although there are more preemptions[30].

The Ehd2-SIP algorithm takes a similar approach to EKG in such a way that each task is classified into a fixed task or a migratable task, and the approach is more simplified for practical use. While EKG utilizes the full capacity of every processor on which a migratable task is executed, Ehd2-SIP does not fully utilize the processor to reduce the computation complexity. Thus, the scheduling of migratable tasks is more straightforward than EKG. Although it can often successfully schedule a task set with system utilization much higher than 50%, the utilization bound is 50%. From the viewpoint of balance between schedulability and complexity, Ehd2-SIP and EKG with small parameter k are attractive.

Another presented algorithm *EDDP* [31] integrates the notions of Ehd2-SIP and EKG. In EDDP, the deadline of a split task is changed to a smaller deadline called “virtual deadline”. EDDP succeeds the design simplicity of Ehd2-SIP for practical use; and at the same time, it partially imitates the approach of EKG but with an improved system utilization bound. The advantage of EDDP is that the implementation cost is not far beyond the traditional partitioned scheduling algorithms, while the worst-case system utilization bound is no less than 65%.

Moreover, an algorithm called *EDHS (Earliest Deadline Highest priority Split)* [32] based on EDF also uses the task splitting techniques. The only difference between EDHS and those previous splitting techniques is that the tasks are never split as long as they can be partitioned. It has been proved that EDHS algorithm improves schedulable multi-processor utilization by 10 to 30% over the traditional partitioning approach.

In this thesis, the presented algorithm IBSP-TS combines the idea of IBPS [3] and SPA2 [4]. It has the highest possible worst-case utilization bound as ln2 and dominates all other static-priority multi-processor scheduling algorithms except for the SPA algorithm which also achieved to ln2. Nevertheless, IBSP-TS has some other advantages over SPA2, such as less task migrations, less number of sorted tasks.

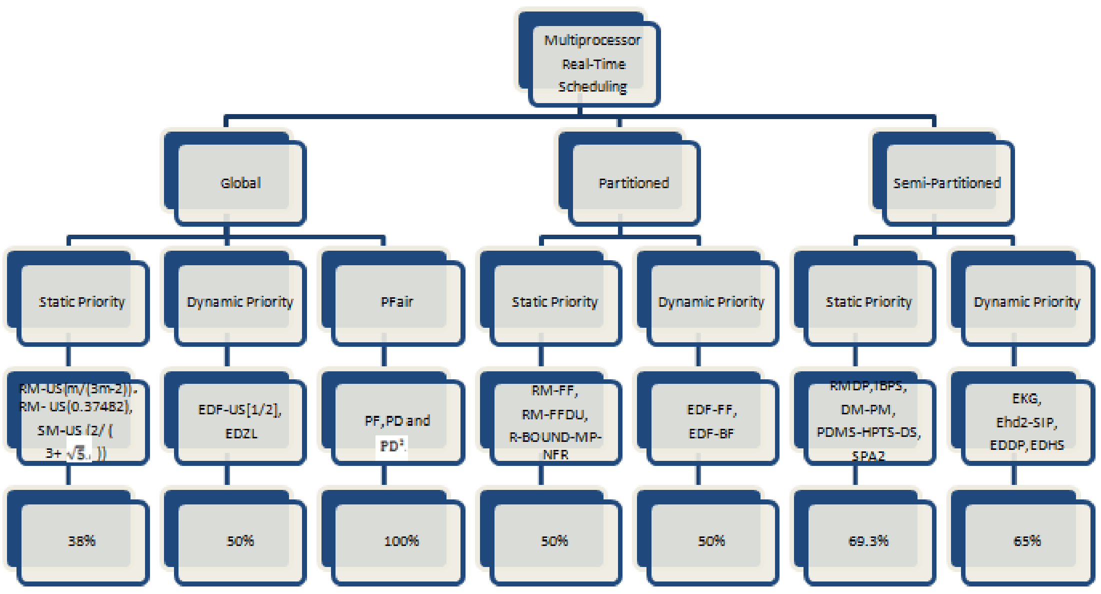


Figure 1: Design space of multi-processor real-time scheduling

Figure 1 shows all the multi-processor real-time scheduling algorithms in existing literature. As it shows that multiprocessor real-time scheduling can be categorized into global scheduling, partitioned scheduling and semi-partitioned scheduling. Each of these categories can be divided into static priority scheduling algorithm and dynamic priority scheduling algorithm. In global scheduling sub-group, there is a special family called Pfair scheduling algorithm. The last layer of this figure shows that the highest utilization bound of that category.

III

Models & Assumptions

In this section, the definition of the system and tasks used by IBSP-TS algorithm is presented. All the assumptions and configuration in the system are critical for designing a real-time scheduling algorithm. In this thesis, the scheduling problem is referring to the assignment of *n* independent, periodically arrived real-time tasks on *m* identical processors in Rate-Monotonic priory.

3.1 System Model

The system used in this thesis is a memory shared multi-processor system composed of *m* processors, P_1, P_2, ... , P_m. Each processor within the multi-processor system is identical. All code and data of tasks are shared among all processors. The overhead of inter-processor is negligible, which means there is no task migration cost for the schedulability test. Whereas, the number of context switches is counted as a metric of algorithm performance.

3.2 Task Model

The system has a task set of *n* periodic tasks denoted as $\Gamma = \{\tau_1, \tau_2, \tau_3, \cdots, \tau_n\}$. All these tasks are independent and preemptive. Moreover, there is no synchronization among tasks. No jobs of a task or subtasks can be executed on two or more processors simultaneously, and a processor cannot execute two or more tasks simultaneously. Jobs of the same task must be executed sequentially which means that every job of τ_i is not allowed to run before the preceding job of τ_i completes.

Each task from this task set is characterized by a pair of parameters $(C_i,\ T_i)$. T_i represents the period of the task and C_i is the WCET $(C_i \leq T_i)$. For periodic tasks, its relative deadlines are equals to its period. Every time a task τ_i arrives, a job $\tau_{i,k}$ of the task is created to denote the k-th copy of the task. A task τ_i is released and ready for execution at every time T_i. The task utilization U_i, is the ration of its execution time to its period, $U_i = \frac{C_i}{T_i}$.

Furthermore, a task τ can be split into two or more subtasks $(\tau_i^1, \tau_i^2, \ldots, \tau_i^m)$ which means a task execution can be migrated to more than one processors. The sum of the execution time of all those subtasks split from one task is exactly equals to the τ's execution time, that is $\sum_{k=1}^{m} C_i^k = C_i$. The period of a subtask inherits from its original task. Thus, the utilization of a subtask becomes $U_i = \frac{C_i^k}{T_i}$

The total utilization of a task set (system utilization) which is defined as the sum of utilizations of all the tasks belong to the set (in a system), that is $\sum_{i=1}^{n} U_i$. All these parameters of a task are not allowed to modify except for subtasks, which split the execution time to several parts.

IV

IBSP-TS Algorithm

In this section, assignment approach and task splitting in IBSP-TS algorithm are presented in detail. Moreover, non-parallel execution of subtasks is shown step by step with mathematic proof. Additionally, pseudo-code and a task assignment example are listed in order to give a more clear understanding of the algorithm.

4.1 Overview

The task assignment is divided into two phases so as to reach the best possible worst-case utilization bound as ln2. Task utilization set (0, 1] is divided into *i* disjoint subsets called utilization *intervals* $\mathrm{I}_1 \sim \mathrm{I}_i$. Tasks from each interval $\mathrm{I}_k, (1 \le k < i)$ are assigned to some processors using a particular *policy* for I_k. By doing this, not only the utilization bound can be achieved to ln2 in each processor, but also the number of tasks left *unassigned* after applying these policies can be reduced. Any task unassigned in $\mathrm{I}_1 \sim \mathrm{I}_{i-1}$ and tasks from the last interval I_i are assigned to processors in by using SPA2 [4]. The more intervals, the less left unassigned tasks left for sorting which is good for online scheduling. Therefore, it is a trade-off between the number of intervals and the number of sorting tasks.

In this thesis, 27 disjoint utilization intervals $\mathrm{I}_1 \sim \mathrm{I}_{27}$. The unassigned task in intervals $\mathrm{I}_1 \sim \mathrm{I}_{26}$ and tasks from the last interval I_{27} are assigned by using SPA2.

4.2 Phase One

4.2.1 Intervals and Policies

Lemma 1: *In Phase One, each processor has the worst-case utilization strictly* *greater than ln2 and all the tasks on the processor can meet their deadlines.*

Proof: The utilization range (0, 1] is divided into twenty-seven disjoint utilization intervals $I_1 \sim I_{27}$, which is described in the following Table 1. Each interval defined by $(U_{low}, U_{up}]$, which means when a task belongs to this interval, its utilization is less or equal to U_{up} and strictly greater than U_{low}.

I_i	U_{low}	U_{up}
I_1	$\ln 2$	1
I_2	$\frac{4}{5}\ln 2$	$\ln 2$
I_3	$\frac{2}{3}\ln 2$	$\frac{4}{5}\ln 2$
I_4	$\frac{3}{5}\ln 2$	$\frac{2}{3}\ln 2$
I_5	$\frac{4}{7}\ln 2$	$\frac{3}{5}\ln 2$
I_6	$\frac{1}{2}\ln 2$	$\frac{4}{7}\ln 2$
I_7	$\frac{4}{9}\ln 2$	$\frac{1}{2}\ln 2$
I_8	$\frac{2}{5}\ln 2$	$\frac{4}{9}\ln 2$
I_9	$\frac{4}{11}\ln 2$	$\frac{2}{5}\ln 2$
I_{10}	$\frac{1}{3}\ln 2$	$\frac{4}{11}\ln 2$
I_{11}	$\frac{4}{13}\ln 2$	$\frac{1}{3}\ln 2$
I_{12}	$\frac{2}{7}\ln 2$	$\frac{4}{13}\ln 2$
I_{13}	$\frac{3}{11}\ln 2$	$\frac{2}{7}\ln 2$
I_{14}	$\frac{1}{4}\ln 2$	$\frac{3}{11}\ln 2$
I_{15}	$\frac{4}{17}\ln 2$	$\frac{1}{4}\ln 2$
I_{16}	$\frac{2}{9}\ln 2$	$\frac{4}{17}\ln 2$
I_{17}	$\frac{3}{14}\ln 2$	$\frac{2}{9}\ln 2$
I_{18}	$\frac{1}{5}\ln 2$	$\frac{3}{14}\ln 2$
I_{19}	$\frac{4}{21}\ln 2$	$\frac{1}{5}\ln 2$
I_{20}	$\frac{2}{11}\ln 2$	$\frac{4}{21}\ln 2$
I_{21}	$\frac{3}{17}\ln 2$	$\frac{2}{11}\ln 2$
I_{22}	$\frac{1}{6}\ln 2$	$\frac{3}{17}\ln 2$
I_{23}	$\frac{4}{25}\ln 2$	$\frac{1}{6}\ln 2$
I_{24}	$\frac{2}{13}\ln 2$	$\frac{4}{25}\ln 2$
I_{25}	$\frac{3}{20}\ln 2$	$\frac{2}{13}\ln 2$
I_{26}	$\frac{1}{7}\ln 2$	$\frac{3}{20}\ln 2$
I_{27}	0	$\frac{1}{7}\ln 2$

Table 1: 27 disjoint utilization intervals I_1-I_{27}

Furthermore, twenty-seven different policies applied in intervals $I_1 \sim I_{27}$. The upper bound of an interval equals to the lower bound of latest previous interval. According to Hyperbolic Test for RM scheduling , if

$$\prod_{i=1}^{n}(U_i + 1) \leq 2, \text{when } n = \infty \text{ it equals to } \ln 2$$

all n tasks on a processor can meet their deadlines. The details of policies are defined are as follow.

1. All tasks with utilization greater than ln2 are assigned to one processor exclusively. Thus, $\mathrm{I}_1 = (\ln 2, 1]$. Therefore, each task $\tau_i \in \mathrm{I}_1 = (\ln 2\,, 1]$ is assigned to one dedicated processor and no more tasks left from this interval.

2. Exactly five tasks $\tau_i \in \mathrm{I}_2 = (\frac{4}{5}\ln 2\,, \ln 2]$ are assigned to four processors. Among these five tasks, the highest priority one is selected and split it into four subtasks. Each task set $\{\tau_1, \frac{1}{4}\tau_i\}$ is assigned to one processor.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:
   $$(\ln 2 + 1) \times \left(\frac{1}{4} \times \ln 2 + 1\right) < 2$$

   All the processors used in this policy maintain utilization bound greater than ln2:

   $$\frac{4}{5}\ln 2 + \frac{1}{4} \times \frac{4}{5}\ln 2 = \ln 2$$

   Thus, there may be 0~4 tasks left in this interval after Phase One.

3. Exactly three tasks $\tau_i \in \mathrm{I}_3 = (\frac{2}{3}\ln 2\,, \frac{4}{5}\ln 2]$ are assigned to two processors. Among these three tasks, the highest priority one is selected and split it into two subtasks. Each task set $\{\tau_1, \frac{1}{2}\tau_i\}$ is assigned to one processor.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:
   $$\left(\frac{4}{5}\ln 2 + 1\right) \times \left(\frac{1}{2} \times \frac{4}{5}\ln 2 + 1\right) < 2$$

   All the processors used in this policy maintain utilization bound greater than ln2:

   $$\frac{2}{3}\ln 2 + \frac{1}{2} \times \frac{2}{3}\ln 2 = \ln 2$$

   Thus, there may be 0~2 tasks left in this interval after Phase One.

4. Exactly five tasks $\tau_i \in \mathrm{I}_4 = (\frac{3}{5}\ln 2\,, \frac{2}{3}\ln 2]$ are assigned to three processors. Two tasks out of these five which has higher priority are split into two subtasks, one is $\frac{1}{3}\tau$ and the other is $\frac{2}{3}\tau$. Task sets $\{\tau_1, \frac{2}{3}\tau_i\}$, $\{\tau_2, \frac{2}{3}\tau_j\}$ and $\{\tau_3, \frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ are assigned to a single processor separately.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{2}{3}\ln 2+1\right)\times\left(\frac{2}{3}\times\frac{2}{3}\ln 2+1\right)<2$$

Since, it has a set $\{\frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ in a processor. Thus, another test is needed to ensure the schedulability.

$$\left(\frac{2}{3}\ln 2+1\right)\times\left(\frac{1}{3}\times\frac{2}{3}\ln 2+1\right)^2<2$$

All the processors used in this policy maintain utilization bound greater than ln2:

$$\frac{3}{5}\ln 2+\frac{2}{3}\times\frac{3}{5}\ln 2=\ln 2$$

Thus, there may be 0~4 tasks left in this interval after Phase One.

5. Exactly seven tasks $\tau_i \in \mathrm{I}_5 = (\frac{4}{7}\ln 2, \frac{3}{5}\ln 2]$ are assigned to four processors. Three tasks out of these seven which has higher priority are split into two subtasks, one is $\frac{1}{4}\tau$ and the other is $\frac{3}{4}\tau$. Task sets $\{\tau_1, \frac{3}{4}\tau_i\}$, $\{\tau_2, \frac{3}{4}\tau_j\}$, $\{\tau_3, \frac{3}{4}\tau_k\}$ and $\{\tau_4, \frac{1}{4}\tau_i, \frac{1}{4}\tau_j, \frac{1}{4}\tau_k\}$ are assigned to a single processor separately.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{3}{5}\ln 2+1\right)\times\left(\frac{3}{4}\times\frac{3}{5}\ln 2+1\right)<2$$

   Since $x=\frac{3}{4}$, it has a set $\{\frac{1}{4}\tau_i, \frac{1}{4}\tau_j, \frac{1}{4}\tau_k\}$ in a processor. Thus, another test is needed.

$$\left(\frac{3}{5}\ln 2+1\right)\times\left(\frac{1}{4}\times\frac{3}{5}\ln 2+1\right)^3<2$$

   All the processors used in this policy maintain utilization bound greater than ln2:

$$\frac{4}{7}\ln 2+\frac{3}{4}\times\frac{4}{7}\ln 2=\ln 2$$

   Thus, there may be 0~6 tasks left in this interval after Phase One.

6. Exactly two tasks $\tau_i \in \mathrm{I}_6 = (\frac{1}{2}\ln 2, \frac{4}{7}\ln 2]$ are assigned to one processor.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{4}{7}\ln 2+1\right)^2<2$$

   All the processors used in this policy maintain utilization bound greater than ln2:

$$2\times\frac{1}{2}\ln 2=\ln 2$$

Thus, there may be 0∼1 tasks left in this interval after Phase One.

7. Exactly nine tasks $\tau_i \in \mathrm{I}_7 = (\frac{4}{9}\ln 2\,,\frac{1}{2}\ln 2]$ are assigned to four processors. Among these nine tasks, the highest priority one is selected and split it into four subtasks. Each task set $\{\tau_1, \tau_2, \frac{1}{4}\tau_i\}$ is assigned to one processor.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:
   $$\left(\frac{1}{2}\ln 2 + 1\right)^2 \times \left(\frac{1}{4} \times \frac{1}{2}\ln 2 + 1\right) < 2$$

   All the processors used in this policy maintain utilization bound greater than ln2:
   $$2 \times \frac{4}{9}\ln 2 + \frac{1}{4} \times \frac{4}{9}\ln 2 = \ln 2$$

   Thus, there may be 0~8 tasks left in this interval after Phase One.

8. Exactly five tasks $\tau_i \in \mathrm{I}_8 = (\frac{2}{5}\ln 2\,,\frac{4}{9}\ln 2]$ are assigned to two processors. Among these five tasks, the highest priority one is selected and split it into two subtasks. Each task set $\{\tau_1, \tau_2, \frac{1}{2}\tau_i\}$ is assigned to one processor.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:
   $$\left(\frac{4}{9}\ln 2 + 1\right)^2 \times \left(\frac{1}{2} \times \frac{4}{9}\ln 2 + 1\right) < 2$$

   All the processors used in this policy maintain utilization bound greater than ln2:
   $$2 \times \frac{2}{5}\ln 2 + \frac{1}{2} \times \frac{2}{5}\ln 2 = \ln 2$$

   Thus, there may be 0∼4 tasks left in this interval after Phase One.

9. Exactly eleven tasks $\tau_i \in \mathrm{I}_9 = (\frac{4}{11}\ln 2\,,\frac{2}{5}\ln 2]$ are assigned to four processors. Three tasks out of these eleven which has higher priority are split into two subtasks, one is $\frac{1}{4}\tau$ and the other is $\frac{3}{4}\tau$. Task sets $\{\tau_1, \tau_2, \frac{3}{4}\tau_i\}$, $\{\tau_3, \tau_4, \frac{3}{4}\tau_j\}$, $\{\tau_5, \tau_6, \frac{3}{4}\tau_k\}$ and $\{\tau_7, \tau_8, \frac{1}{4}\tau_i, \frac{1}{4}\tau_j, \frac{1}{4}\tau_k\}$ are assigned to a single processor separately.

   All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:
   $$\left(\frac{2}{5}\ln 2 + 1\right)^2 \times \left(\frac{3}{4} \times \frac{2}{5}\ln 2 + 1\right) < 2$$

   Since $x = \frac{3}{4}$, it has a set $\{\frac{1}{4}\tau_i, \frac{1}{4}\tau_j, \frac{1}{4}\tau_k\}$ in a processor. Thus, another test is needed.

$$\left(\frac{2}{5}\ln 2 + 1\right)^2 \times \left(\frac{1}{4} \times \frac{2}{5}\ln 2 + 1\right)^3 < 2$$

All the processors used in this policy maintain utilization bound greater than ln2:

$$2 \times \frac{4}{11}\ln 2 + \frac{3}{4} \times \frac{4}{11}\ln 2 = \ln 2$$

Thus, there may be 0~10 tasks left in this interval after Phase One.

10. Exactly three tasks $\tau_i \in \mathrm{I}_{10} = (\frac{1}{3}\ln 2, \frac{4}{11}\ln 2]$ are assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{4}{11}\ln 2 + 1\right)^3 < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$3 \times \frac{1}{3}\ln 2 = \ln 2$$

    Thus, there may be 0~2 tasks left in this interval after Phase One.

11. Exactly thirteen tasks $\tau_i \in \mathrm{I}_{11} = (\frac{4}{13}\ln 2, \frac{1}{3}\ln 2]$ are assigned to four processors. Among these thirteen tasks, the highest priority one is selected and split it into four subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \frac{1}{4}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{1}{3}\ln 2 + 1\right)^3 \times \left(\frac{1}{4} \times \frac{1}{3}\ln 2 + 1\right) < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$3 \times \frac{4}{13}\ln 2 + \frac{1}{4} \times \frac{4}{13}\ln 2 = \ln 2$$

    Thus, there may be 0~12 tasks left in this interval after Phase One.

12. Exactly seven tasks $\tau_i \in \mathrm{I}_{12} = (\frac{2}{7}\ln 2, \frac{4}{13}\ln 2]$ are assigned to two processors. Among these seven tasks, the highest priority one is selected and split it into two subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \frac{1}{2}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{4}{13}\ln 2 + 1\right)^3 \times \left(\frac{1}{2} \times \frac{4}{13}\ln 2 + 1\right) < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$3 \times \frac{2}{7}\ln 2 + \frac{1}{2} \times \frac{2}{7}\ln 2 = \ln 2$$

Thus, there may be 0∼6 tasks left in this interval after Phase One.

13. Exactly eleven tasks $\tau_i \in \mathrm{I}_{13} = (\frac{3}{11}\ln 2, \frac{2}{7}\ln 2]$ are assigned to three processors. Two tasks out of these five which has higher priority are split into two subtasks, one is $\frac{1}{3}\tau$ and the other is $\frac{2}{3}\tau$. Task sets $\{\tau_1, \tau_2, \tau_3, \frac{2}{3}\tau_i\}$, $\{\tau_4, \tau_5, \tau_6, \frac{2}{3}\tau_j\}$ and $\{\tau_7, \tau_8, \tau_9, \frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ are assigned to a single processor separately.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{2}{7}\ln 2 + 1\right)^3 \times \left(\frac{2}{3} \times \frac{2}{7}\ln 2 + 1\right) < 2$$

    Since $x = \frac{2}{3}$, it has a set $\{\frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ in a processor. Thus, another test is needed.

$$\left(\frac{2}{7}\ln 2 + 1\right)^3 \times \left(\frac{1}{3} \times \frac{2}{7}\ln 2 + 1\right)^2 < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$3 \times \frac{3}{11}\ln 2 + \frac{2}{3} \times \frac{3}{11}\ln 2 = \ln 2$$

    Thus, there may be 0∼10 tasks left in this interval after Phase One.

14. Exactly four tasks $\tau_i \in \mathrm{I}_{14} = (\frac{1}{4}\ln 2, \frac{3}{11}\ln 2]$ are assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{3}{11}\ln 2 + 1\right)^4 < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$4 \times \frac{1}{4}\ln 2 = \ln 2$$

    Thus, there may be $0 \sim 3$ tasks left in this interval after Phase One.

15. Exactly seventeen tasks $\tau_i \in \mathrm{I}_{15} = (\frac{4}{17}\ln 2, \frac{1}{4}\ln 2]$ are assigned to four processors. Among these seventeen tasks, the highest priority one is selected and split it into four subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \tau_4, \frac{1}{4}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{1}{4}\ln 2+1\right)^4 \times\left(\frac{1}{4}\times\frac{1}{4}\ln 2+1\right)<2$$

All the processors used in this policy maintain utilization bound greater than ln2:

$$4\times\frac{4}{17}\ln 2+\frac{1}{4}\times\frac{4}{17}\ln 2=\ln 2$$

Thus, there may be 0~16 tasks left in this interval after Phase One.

16. Exactly nine tasks $\tau_i \in \mathrm{I}_{16}=(\frac{2}{9}\ln 2,\frac{4}{17}\ln 2]$ are assigned to two processors. Among these nine tasks, the highest priority one is selected and split it into two subtasks. Each task set $\{\tau_1,\tau_2,\tau_3,\tau_4,\frac{1}{2}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{4}{17}\ln 2+1\right)^4 \times\left(\frac{1}{2}\times\frac{4}{17}\ln 2+1\right)<2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$4\times\frac{2}{9}\ln 2+\frac{1}{2}\times\frac{2}{9}\ln 2=\ln 2$$

    Thus, there may be 0~8 tasks left in this interval after Phase One.

17. Exactly fourteen tasks $\tau_i \in \mathrm{I}_{17}=(\frac{3}{14}\ln 2,\frac{2}{9}\ln 2]$ are assigned to three processors. Two tasks out of these fourteen which has higher priority are split into two subtasks, one is $\frac{1}{3}\tau$ and the other is $\frac{2}{3}\tau$. Task sets $\{\tau_1,\tau_2,\tau_3,\tau_4,\frac{2}{3}\tau_i\}$, $\{\tau_5,\tau_6,\tau_7,\tau_8,\frac{2}{3}\tau_j\}$ and $\{\tau_9,\tau_{10},\tau_{11},\tau_{12},\frac{1}{3}\tau_i,\frac{1}{3}\tau_j\}$ are assigned to a single processor separately.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{2}{9}\ln 2+1\right)^4 \times\left(\frac{2}{3}\times\frac{2}{9}\ln 2+1\right)<2$$

    Since $x=\frac{2}{3}$, it has a set $\{\frac{1}{3}\tau_i,\frac{1}{3}\tau_j\}$ in a processor. Thus, another test is needed.

$$\left(\frac{2}{9}\ln 2+1\right)^4 \times\left(\frac{1}{3}\times\frac{2}{9}\ln 2+1\right)^2<2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$4\times\frac{3}{14}\ln 2+\frac{2}{3}\times\frac{3}{14}\ln 2=\ln 2$$

    Thus, there may be 0~13 tasks left in this interval after Phase One.

18. Exactly five tasks $\tau_i \in \mathrm{I}_{18} = (\frac{1}{5}\ln 2, \frac{3}{14}\ln 2]$ are assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

    $$\left(\frac{3}{14}\ln 2 + 1\right)^5 < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

    $$5 \times \frac{1}{5}\ln 2 = \ln 2$$

    Thus, there may be 0~4 tasks left in this interval after Phase One.

19. Exactly twenty one tasks $\tau_i \in \mathrm{I}_{19} = (\frac{4}{21}\ln 2, \frac{1}{5}\ln 2]$ are assigned to four processors. Among these twenty one tasks, the highest priority one is selected and split it into four subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \frac{1}{4}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

    $$\left(\frac{1}{5}\ln 2 + 1\right)^5 \times \left(\frac{1}{4} \times \frac{1}{5}\ln 2 + 1\right) < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

    $$5 \times \frac{4}{21}\ln 2 + \frac{1}{4} \times \frac{4}{21}\ln 2 = \ln 2$$

    Thus, there may be 0~20 tasks left in this interval after Phase One.

20. Exactly eleven tasks $\tau_i \in \mathrm{I}_{20}(\frac{2}{11}\ln 2, \frac{4}{21}\ln 2]$ are assigned to two processors. Among these eleven tasks, the highest priority one is selected and split it into two subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \frac{1}{2}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

    $$\left(\frac{4}{21}\ln 2 + 1\right)^5 \times \left(\frac{1}{2} \times \frac{4}{21}\ln 2 + 1\right) < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

    $$5\,\frac{2}{11}\ln 2 + \frac{1}{2} \times \frac{2}{11}\ln 2 = \ln 2$$

    Thus, there may be 0~10 tasks left in this interval after Phase One.

21. Exactly seventeen tasks $\tau_i \in \mathrm{I}_{21} = (\frac{3}{17}\ln 2, \frac{2}{11}\ln 2]$ are assigned to three processors. Two tasks out of these five which has higher priority are split into two

subtasks, one is $\frac{1}{3}\tau$ and the other is $\frac{2}{3}\tau$. Task sets $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \frac{2}{3}\tau_i\}$, $\{\tau_6, \tau_7, \tau_8, \tau_9, \tau_{10}, \frac{2}{3}\tau_j\}$ and $\{\tau_{11}, \tau_{12}, \tau_{13}, \tau_{14}, \tau_{15}, \frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ are assigned to a single processor.

All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{2}{11}\ln 2 + 1\right)^5 \times \left(\frac{2}{3} \times \frac{2}{11}\ln 2 + 1\right) < 2$$

Since $x = \frac{2}{3}$, it has a set $\{\frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ in a processor. Thus, another test is needed.

$$\left(\frac{2}{11}\ln 2 + 1\right)^5 \times \left(\frac{1}{3} \times \frac{2}{11}\ln 2 + 1\right)^2 < 2$$

All the processors used in this policy maintain utilization bound greater than ln2:

$$5 \times \frac{3}{17}\ln 2 + \frac{2}{3} \times \frac{3}{17}\ln 2 = \ln 2$$

Thus, there may be 0~16 tasks left in this interval after Phase One.

22. Exactly six tasks $\tau_i \in \mathrm{I}_{22} = (\frac{1}{6}\ln 2, \frac{3}{17}\ln 2]$ are assigned to one processor.

All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{3}{17}\ln 2 + 1\right)^6 < 2$$

All the processors used in this policy maintain utilization bound greater than ln2:

$$6 \times \frac{1}{6}\ln 2 = \ln 2$$

Thus, there may be 0~5 tasks left in this interval after Phase One.

23. Exactly twenty five tasks $\tau_i \in \mathrm{I}_{23} = (\frac{4}{25}\ln 2, \frac{1}{6}\ln 2]$ are assigned to four processors. Among these twenty five tasks, the highest priority one is selected and split it into four subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \frac{1}{4}\tau_i\}$ is assigned to one processor.

All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{1}{6}\ln 2 + 1\right)^6 \times \left(\frac{1}{4} \times \frac{1}{6}\ln 2 + 1\right) < 2$$

All the processors used in this policy maintain utilization bound greater than ln2:

$$6 \times \frac{4}{25}\ln 2 + \frac{1}{4} \times \frac{4}{25}\ln 2 = \ln 2$$

Thus, there may be 0~24 tasks left in this interval after Phase One.

24. Exactly thirteen tasks $\tau_i \in I_{24} = (\frac{2}{13}\ln 2, \frac{4}{25}\ln 2]$ are assigned to two processors. Among these eleven tasks, the highest priority one is selected and split it into two subtasks. Each task set $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \frac{1}{2}\tau_i\}$ is assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{4}{25}\ln 2 + 1\right)^6 \times \left(\frac{1}{2} \times \frac{4}{25}\ln 2 + 1\right) < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$6 \times \frac{2}{13}\ln 2 + \frac{1}{2} \times \frac{2}{13}\ln 2 = \ln 2$$

    Thus, there may be 0~12 tasks left in this interval after Phase One.

25. Exactly twenty tasks $\tau_i \in I_{25} = (\frac{3}{20}\ln 2, \frac{2}{13}\ln 2]$ are assigned to three processors. Two tasks out of these five which has higher priority are split into two subtasks, one is $\frac{1}{3}U$ and the other is $\frac{2}{3}U$. Task sets $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \frac{2}{3}\tau_i\}$, $\{\tau_7, \tau_8, \tau_9, \tau_{10}, \tau_{11}, \tau_{12}, \frac{2}{3}\tau_j\}$ and $\{\tau_{13}, \tau_{14}, \tau_{15}, \tau_{16}, \tau_{17}, \tau_{18}, \frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ are assigned to a single processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{2}{13}\ln 2 + 1\right)^6 \times \left(\frac{2}{3} \times \frac{2}{13}\ln 2 + 1\right) < 2$$

    Since $x = \frac{2}{3}$, it has a set $\{\frac{1}{3}\tau_i, \frac{1}{3}\tau_j\}$ in a processor. Thus, another test is needed.

$$\left(\frac{2}{13}\ln 2 + 1\right)^6 \times \left(\frac{1}{3} \times \frac{2}{13}\ln 2 + 1\right)^2 < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$6 \times \frac{3}{20}\ln 2 + \frac{2}{3} \times \frac{3}{20}\ln 2 = \ln 2$$

    Thus, there may be 0~19 tasks left in this interval after Phase One.

26. Exactly seven tasks $\tau_i \in I_{26} = (\frac{1}{7}\ln 2, \frac{3}{20}\ln 2]$ are assigned to one processor.

    All the tasks assigned by this policy meet their deadlines according to Hyperbolic Bound Test:

$$\left(\frac{3}{20}\ln 2 + 1\right)^7 < 2$$

    All the processors used in this policy maintain utilization bound greater than ln2:

$$7 \times \frac{1}{7} \ln 2 = \ln 2$$

Thus, there may be 0~6 tasks left in this interval after Phase One.

27. The left interval which is $I_{27} = (0, \frac{1}{7} \ln 2]$. Assign tasks which have utilization less than $\frac{1}{7} \ln 2$ along with all the unassigned tasks left which are called *residual tasks* after Phase One in Phase Two by using SPA2.

4.2.2 Non-Parallel Execution

Proposition 1**:** *In Phase One, no subtask split from the same task has parallel execution.*

Proof: There are three cases considered. An assumption is needed that each non-split task has an offset $\emptyset_i = 0$, so that an *critical instance* will occur where the response time of tasks are maximized [5]. When a task $\tau_i(C_i, T_i)$ is split, it is considered as two subtasks, τ_i' has execution time C_i' and period equals to T_i, while τ_i'' has execution time C_i'' and period equals to T_i $(C_i = C_i' + C_i'')$. The first subtask τ_i' has an offset $\emptyset_i' = 0$. However, in order to get non-parallel execution for subtask τ_i'', it has to be given an offset $\emptyset_i'' = C_i'$.

1. There is only one task split in a processor, by picking the highest priority task as split task can ensure non-parallel execution.

2. There are two tasks split in a processor in Policy 4, 13, 17, 21and 25. It is proved by a contradiction.

   Select the highest two tasks within a subset and define them as τ_H and $\tau_L (T_H < T_L)$, so τ_H has higher priority than τ_L. Two tasks are split into two subtasks $\frac{1}{3}\tau_H$, $\frac{2}{3}\tau_H$ and $\frac{1}{3}\tau_L$, $\frac{2}{3}\tau_L$ respectively. $\frac{2}{3}\tau_H$ and $\frac{2}{3}\tau_L$ are assigned into two different processors. Since each of them has the highest priority tasks on those processors, non-parallel execution can be guaranteed. The rest part, $\frac{1}{3}\tau_H$ and $\frac{1}{3}\tau_L$ will be executed on the same processor.

   It is easy to see from Figure 2, the task set $\{\tau_{1i}, \tau_{2i}, \cdots, \tau_{ni}, \frac{2}{3}\tau_H\}$ is assigned to P_1; the task set $\{\tau_{1j}, \tau_{2j}, \cdots, \tau_{nj}, \frac{2}{3}\tau_L\}$ is assigned to P_2; while the other task set $\{\tau_{1k}, \tau_{2k}, \cdots, \tau_{nk}, \frac{1}{3}\tau_H, \frac{1}{3}\tau_L\}$ is assigned to P_3. On P_3, the subtask of τ_H may preempt the subtask of τ_L, which makes τ_L miss its deadline.

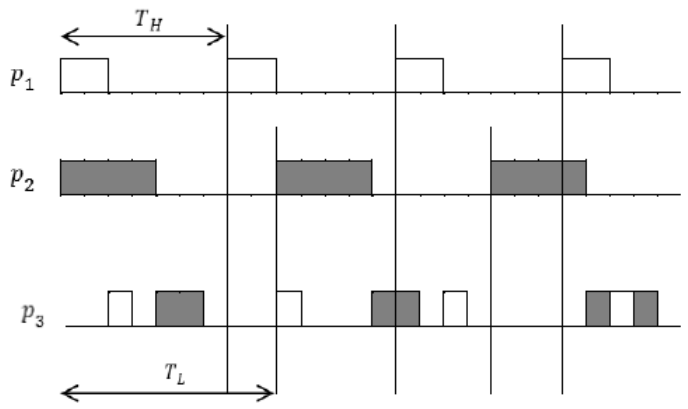


Figure 2: Example of scheduling two $\frac{1}{3}\tau_H$ and $\frac{1}{3}\tau_L$ on one processor

Assume that the subtask of τ_L misses its deadline, the execution time of these two split tasks is respectively represented as C_H and C_L. The response time of $\frac{1}{3}C_L$ goes as follows

$$\frac{1}{3}C_L \geq T_L - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{3}C_H - \frac{2}{3}C_L$$

$$C_L \geq T_L - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{3}C_H$$

$$\frac{C_L}{T_L} \geq \frac{T_L - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{3}C_H}{T_L}$$

$$U_L > 1 - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{3}\frac{C_H}{T_L}$$

$$\because \left\lceil\frac{T_L}{T_H}\right\rceil \leq \frac{T_L}{T_H} + 1$$

$$\therefore U_L > 1 - \frac{T_L}{T_H} \times \frac{1}{3}\frac{C_H}{T_L} - \frac{1}{3}\frac{C_H}{T_L}$$

$$U_L > 1 - \frac{1}{3}\frac{C_H}{T_H} - \frac{1}{3}\frac{C_H}{T_L}$$

$$U_L > 1 - \frac{1}{3}U_H - \frac{1}{3}\frac{C_H}{T_L}$$

$$\because T_H < T_L$$

$$\therefore U_L > 1 - \frac{2}{3}U_H$$

Intervals	Conditions	Testing Procedure
$\mathrm{I}_4: \left(\frac{3}{5}\ln 2, \frac{2}{3}\ln 2\right] \approx (0.416, 0.462]$	$U_L > 1 - \frac{2}{3}U_H$	$\because U_H \in (0.416, 0.462]$ $\therefore U_L \in (0.692, 0.723]$ $However, U_L \in (0.416, 0.462]$ A contradiction is derived, so τ_L will meet its deadline.

I_{11}: $(\frac{3}{11}\ln 2, \frac{2}{7}\ln 2]$ $\approx (0.189, 0.198]$	$U_L > 1 - \frac{2}{3}U_H$	$\because U_H \in (0.189, 0.198]$ $\therefore U_L \in (0.868, 0.874]$ $However,\ U_L \in (0.189, 0.198]$ A contradiction is derived, so τ_L will meet its deadline.
I_{17}: $(\frac{3}{14}\ln 2, \frac{2}{9}\ln 2]$ $\approx (0.149, 0.154]$	$U_L > 1 - \frac{2}{3}U_H$	$\because U_H \in (0.149, 0.154]$ $\therefore U_L \in (0.897, 0.901]$ $However,\ U_L \in (0.149, 0.154]$ A contradiction is derived, so τ_L will meet its deadline.
I_{21}: $(\frac{3}{17}\ln 2, \frac{2}{11}\ln$ $\approx (0.122, 0.126]$	$U_L > 1 - \frac{2}{3}U_H$	$\because U_H \in (0.122, 0.126]$ $\therefore U_L \in (0.916, 0.919]$ $However,\ U_L \in (0.122, 0.126]$ A contradiction is derived, so τ_L will meet its deadline.
I_{25}: $(\frac{3}{20}\ln 2, \frac{2}{13}\ln$ $\approx (0.104, 0.106]$	$U_L > 1 - \frac{2}{3}U_H$	$\because U_H \in (0.104, 0.106]$ $\therefore U_L \in (0.929, 0.931]$ $However,\ U_L \in (0.104, 0.106]$ A contradiction is derived, so τ_L will meet its deadline.

Table 2: Non-parallel execution proof for two split tasks

Table 2 shows the contradictions in each different interval. It is clear that

$$\nvdash U_L > 1 - \frac{2}{3}U_H$$

3. There are three tasks split in a processor in Policy 5 and 9. It is proved by a contradiction.

   Take the highest three tasks in a subset and define them as τ_H, τ_M and $\tau_L (T_H < T_M < T_L)$. τ_H has the highest priority, τ_M has the second higher priority, and τ_L has the lowest priority. Three tasks are split into three subtasks $\frac{1}{4}\tau_H$, $\frac{3}{4}\tau_H$, $\frac{1}{4}\tau_M$, $\frac{3}{4}\tau_M$ and $\frac{1}{4}\tau_L$, $\frac{3}{4}\tau_L$ respectively. Each $\frac{3}{4}\tau_H$, $\frac{3}{4}\tau_M$ and $\frac{3}{3}\tau_L$ are assigned to a dedicated processor. Since each of them has the highest priority on that processor, no parallel running occurs. $\frac{1}{4}\tau_H$, $\frac{1}{4}\tau_M$ and $\frac{1}{3}\tau_L$ will be executed on the same processor.

   In Figure 3, the task set $\{\tau_{1i}, \tau_{2i}, \cdots, \tau_{ni}, \frac{4}{4}\tau_H\}$ is assigned to P_1; the task set

$\{\tau_{1j}, \tau_{2j}, \cdots, \tau_{nj}, \frac{3}{4}\tau_M\}$ is assigned to P_2; another task set $\{\tau_{1k}, \tau_{2k}, \cdots, \tau_{nk}, \frac{3}{4}\tau_L\}$ is assigned to P_3; while the last task set $\{\tau_{1s}, \tau_{2s}, \cdots, \tau_{ns}, \frac{1}{4}\tau_H, \frac{1}{4}\tau_M, \frac{1}{4}\tau_L\}$ is assigned to P_4. On P_4, (a.) the subtask of τ_H may preempt the subtask of τ_M and causes τ_M misses its deadline; (b.) the subtask of τ_H and τ_M may preempt the subtask of τ_L, and causes τ_L misses its deadline.

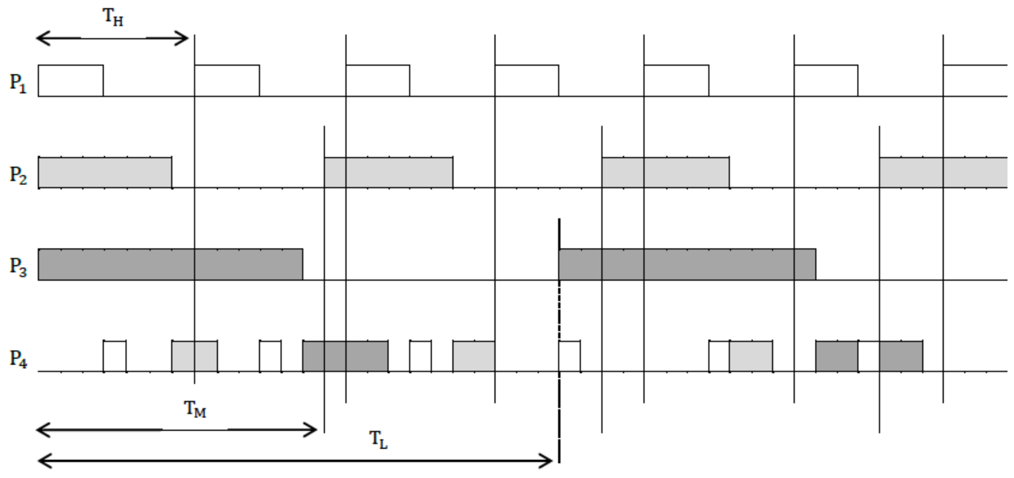


Figure 3: Example of scheduling three $\frac{1}{3}\tau_H$, $\frac{1}{3}\tau_M$ and $\frac{1}{3}\tau_L$ on one processor

a) Assume that the subtask of τ_M miss its deadline, the execution time of these three split tasks is respectively represented as C_H, C_M and C_L. The response time of $\frac{1}{4}C_M$ goes as follows

$$\frac{1}{4}C_M \geq T_M - \left\lceil \frac{T_M}{T_H} \right\rceil \times \frac{1}{4}C_H - \frac{3}{4}C_M$$

$$C_M \geq T_M - \left\lceil \frac{T_M}{T_H} \right\rceil \times \frac{1}{4}C_H$$

$$\frac{C_M}{T_M} \geq \frac{T_L - \left\lceil \frac{T_M}{T_H} \right\rceil \times \frac{1}{4}C_H}{T_M}$$

$$U_M \geq 1 - \left\lceil \frac{T_M}{T_H} \right\rceil \times \frac{1}{4}\frac{C_H}{T_M}$$

$$\because \left\lceil \frac{T_M}{T_H} \right\rceil \leq \frac{T_M}{T_H} + 1$$

$$\therefore U_M > 1 - \frac{T_M}{T_H} \times \frac{1}{4}\frac{C_H}{T_M} - \frac{1}{4}\frac{C_H}{T_M}$$

$$U_M > 1 - \frac{1}{4}\frac{C_H}{T_H} - \frac{1}{4}\frac{C_H}{T_M}$$

$$U_M > 1 - \frac{1}{4}U_H - \frac{1}{4}\frac{C_H}{T_M}$$

$$\because T_H < T_M$$

$$\therefore U_M > 1 - \frac{1}{2}U_H$$

b) Assume that the subtask of $\tau_L$miss its deadline. The response time of $\frac{1}{4}C_L$ goes as follows

$$\frac{1}{4}C_L \geq T_L - \left\lceil\frac{T_L}{T_M}\right\rceil \times \frac{1}{4}C_M - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{4}C_H - \frac{3}{4}C_L$$

$$C_L \geq T_L - \left\lceil\frac{T_L}{T_M}\right\rceil \times \frac{1}{4}C_M - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{4}C_H$$

$$\frac{C_L}{T_L} \geq \frac{T_L - \left\lceil\frac{T_L}{T_M}\right\rceil \times \frac{1}{4}C_M - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{4}C_H}{T_L}$$

$$U_L \geq 1 - \left\lceil\frac{T_L}{T_M}\right\rceil \times \frac{1}{4}\frac{C_M}{T_L} - \left\lceil\frac{T_L}{T_H}\right\rceil \times \frac{1}{4}\frac{C_H}{T_L}$$

$$\because \left\lceil\frac{T_L}{T_H}\right\rceil \leq \frac{T_L}{T_H} + 1, \left\lceil\frac{T_L}{T_M}\right\rceil \leq \frac{T_L}{T_M} + 1$$

$$\therefore U_L > 1 - \frac{T_L}{T_M} \times \frac{1}{4}\frac{C_M}{T_L} - \frac{1}{4}\frac{C_M}{T_L} - \frac{T_L}{T_H} \times \frac{1}{4}\frac{C_H}{T_L} - \frac{1}{4}\frac{C_H}{T_L}$$

$$U_L > 1 - \frac{1}{4}\frac{C_M}{T_M} - \frac{1}{4}\frac{C_M}{T_L} - \frac{1}{4}\frac{C_H}{T_H} - \frac{1}{4}\frac{C_H}{T_L}$$

$$U_L > 1 - \frac{1}{4}U_M - \frac{1}{4}\frac{C_M}{T_L} - \frac{1}{4}U_H - \frac{1}{4}\frac{C_H}{T_L}$$

$$\because T_H < T_L,\ \ T_M < T_L$$

$$\therefore U_L > 1 - \frac{1}{2}U_M - \frac{1}{2}U_H$$

Intervals	**Conditions**	**Testing Procedure**
I_5: $\left(\frac{4}{7}\ln 2, \frac{3}{5}\ln 2\right]$ $\approx (0.396, 0.416]$	$U_M > 1 - \frac{1}{2}U_H$ && $U_L > 1 - \frac{1}{2}U_M - \frac{1}{2}U_H$	$\because\ U_H \in\ (0.396, 0.416]$ $\therefore U_L \in (0.792, 0.802]$ $However, U_M \in\ (0.396, 0.416]$ A contradiction is derived, so τ_M will meet its deadline.
		$\because\ U_H \in\ (0.396, 0.416]$ $\therefore U_L \in (0.584, 0.604]$ $However,\ U_L \in\ (0.396, 0.416]$ A contradiction is derived, so τ_L will meet its deadline.

(a)

Intervals	Conditions	Testing Procedure
I_9: $(\frac{4}{11}\ln 2, \frac{2}{5}\ln 2]$ $\approx (0.252, 0.277]$	$U_M > 1 - \frac{1}{2}U_H$ && $U_L > 1 - \frac{1}{2}U_M - \frac{1}{2}U_H$	$\because U_H \in (0.252, 0.277]$ $\therefore U_L \in (0.861, 0.874]$ $However, U_M \in (0.252, 0.277]$ A contradiction is derived, so τ_M will meet its deadline.
		$\because U_H \in (0.252, 0.277]$ $\therefore U_L \in (0.723, 0.748]$ $However,\ U_L \in (0.252, 0.277]$ A contradiction is derived, so τ_L will meet its deadline.

(b)

Table 3: Non-parallel execution proof for three split tasks

From Table 3, it is clear that

$$\nvdash U_L > 1 - \frac{1}{2}U_M - \frac{1}{2}U_H$$

$$\nvdash U_M > 1 - \frac{1}{2}U_H$$

Thus, all the subtasks do not run simultaneously.

4.3 Phase Two

In Phase Two, all the residual tasks in $\mathrm{I}_1 \sim \mathrm{I}_{26}$ along with those tasks from the last interval I_{27} constitute a task set called unassigned tasks. By using the idea of SPA2 algorithm, those unassigned tasks are assigned into the processors left after Phase One.

4.3.1 Assignment Procedure

Lemma 2: *A Unassigned task set Γ_{unass} with N tasks is schedulable on m processors if $U(\Gamma_{unass}) \leq m\Theta(N)$* [4].

The assignment procedure in SPA2 ensures the bound of ln2 in worst case. It contains following steps:

1. Sort all the unassigned tasks in decreasing priority order.

2. Define a task τ_i is a heavy task if its utilization

$$U_i > \frac{\Theta(N)}{\Theta(N)+1}, \quad \Theta(N) = LLB\,(n) = n\left(2^{\frac{1}{n}} - 1\right) = \ln 2 \text{ when } n = \infty$$

3. A heavy task is determined to be a pre-assigned to a single processor if the following condition satisfies, $Count_{LP}$ is the number of unused processors left at the time being.

$$\sum_{j>i} U_j \le (Count_{LP} - 1) * \Theta(N)$$

   These processors are called pre-assigned processors. All the rest processors are called normal processors. The remaining tasks after pre-assignment step become normal tasks.

4. Sort all the normal processors based on system utilization (the sum of utilization of all the tasks have been assigned to a processor).

5. Assign a normal task τ_n to a normal processor whose utilization is minimal among all normal processors.

6. If a normal task τ_n cannot be fully assigned to a normal processor P_{norm}, split τ_n and assign as much as possible to P_{norm}. Now P_{norm} is fully utilized which means its utilization equals to $\Theta(N)$. The rest split part of τ_n becomes a new normal task and put back in the unassigned normal task queue.

7. Go to Step 4 until all normal processors are fully utilized.

8. Sort all the pre-assigned processors in increasing priority order.

9. Assign as many normal tasks as possible to a pre-assigned processor P_{pre} until it is fully utilized. Then select next pre-assigned processor.

10. If a normal task cannot be fully assigned to P_{pre}, assign the split part to the next selected pre-assigned processor.

4.3.2 Non-Parallel Execution

Proposition 2: *No subtasks in Phase Two suffers from parallel execution problem.*

SPA2 has already proved that their algorithm do not suffer the problem of subtasks running in parallel. There are some facts they have shown.

1. Each pre-assigned task has the lowest priority on its host processor.
2. Each body subtask (all the subtasks split from a normal task excluding the last split part) has the highest priority on its host processor.

4.4 Utilization Bound

Theorem: *If a task set Γ has utilization* $U(\Gamma) \leq m\ln2$, *all the tasks can be scheduled in m processors.*

Proof: Let U_1 be the utilization of the tasks which have been assigned in Phase One. Denote m_1 as the number of processors used in Phase One. By Lemma 1, it is certain that

$$U_1 > m_1\ln2$$

Let U_2 be the utilization of the tasks which have been assigned in Phase Two. There must exist some integer *x* such that

$$x\ln2 < U_2 \leq (x+1)\ln2, \qquad x \in \mathbb{N}$$

According to Lemma 2, all the unassigned tasks in Phase Two can be scheduled on *x*+1 processors if $U_2 \leq (x+1)\ln2$.

$$\because U = U_1 + U_2 \leq m\ln2, \qquad m = m_1 + m_2$$

$$U_1 + U_2 \leq m\ln2, \qquad U_1 > m_1\ln2$$

$$m_1\ln2 + U_2 < m\ln2$$

$$U_2 < (m - m_1)\ln2$$

$$\therefore U_2 < m_2\ln2$$

$$\because x\ln2 < U_2 \leq (x+1)\ln2$$

$$x\ln2 < U_2 \leq m_2\ln2$$

$$\therefore x < m_2$$

$$\because x \in \mathbb{N}$$

$$\therefore x + 1 \leq m_2$$

4.5 Pseudo Code

There are some notifications for the algorithm shown in Figure 4:

- $Uti(\tau_i)$ is the utilization of a task and $SysUti(P_n)$ is the utilization of a processor.
- $List_{[1,2,\ldots 27]}$ represents the set of tasks belonging to different intervals and $Policy_i$ is the assignment policy for $Interval_i$.
- $Residual_i$ means the set of residual tasks in $Interval_i$ after applying $Policy_i$; and $UnassignQ$ is the queue for unassigned tasks sorted in decreasing priority order.

- $Count_{LP}$ is the count number of how many processors left after applying all the policies.

- P_{pre} and P_{norm} the sets represent processors contains pre-assigned tasks and processors with no pre-assigned tasks respectively. P_{pre} is sorted in increasing order of its pre-assigned tasks priority.

- $NormQ$ is the list of unassigned tasks after pre-assignment step, initial to be empty.

```
1.  **for** $i \coloneqq 1$ to $N$ **do**
2.      **if** $Uti(\tau_i) \in Interval_{[1,2,\ldots 27]}$ **then**
3.          push($\tau_i, List_{[1,2,\ldots 27]}$)
4.      **end if**
5.  **end for**
6.  **for** $i \coloneqq 1$ to 26 **do**
7.      apply($Policy_i, List_i$)
8.      $UnassignQ \coloneqq \sum Residual_i \cup List_{27}$
9.  **end for**
10. **if** $Count_{LP} \leq 0$ **then** "Assignment failed"
11. sort $UnassignQ$ (priority ↓)
12. **for** $i \coloneqq 1$ to $N$ **do**
13.     **if** $Uti(\tau_i) > \Theta(N)/(\Theta(N)+1) \wedge \sum_{j>i} Uti(\tau_j) \leq (Count_{LP} - 1) \times \Theta(N)$ **then**
14.         pre- assign($\tau_i, P_{pre_n}$)
15.         $Count_{LP} - 1$
16.         $SysUti(P_{pre_n}) = Uti(\tau_i)$
17.     **else**
18.         push($\tau_i, NormQ$)
19.     **end if**
20. **end for**
21. sort $P_{pre}$(priority ↑)
22. **while** NormQ $\neq \emptyset$ **do**
23.     $\tau_i \coloneqq$ pop($NormQ$)
24.     **if** $\exists\ P_{norm_i} \neq \Theta(N)$ **then**
25.         $P_n \coloneqq$ (mini $SysUti(P_{norm})$)
26.     **else**
27.         $P_n \coloneqq$ pop($P_{pre}$)
28.     **end if**
29.     **if** $SysUti(P_n) + Uti(\tau_i) \leq \Theta(N)$ **then**
30.         assign($\tau_i, P_n$)
31.         $SysUti(P_n) := SysUti(P_n) + Uti(\tau_i)$
32.         $|NormQ| - 1$
33.     **else**
34.         split $\tau_i$ into two subtasks, $\tau_i^k$ and $\tau_i^{k+1}$
```

```
35.         $Uti\ (\tau_i^k) := \Theta(N) - SysUti(P_{norm_n})$
36.         $\tau_i^{k+1} := Uti(\tau_i) - Uti\ (\tau_i^k)$
37.         assign($\tau_i^k,\ P_{norm_n}$)
38.         $SysUti(P_{norm_n}) := \Theta(N)$
39.         push($\tau_i^{k+1}, NormQ$)
40.     end if
41.     if $\forall\ P_n = \Theta(N)$ then "Assignment failed"
42. end while
```

Figure 4: Pseudo code of IBSP-TS

1. Line 1~5: Assign a task to an interval list based on the utilization of the task.

2. Line 6~9: Apply tasks in each interval with a different policy. Put all the unassigned first 26 intervals together with tasks in last interval into the unassigned task queue.

3. Line 10: If the number of processors left after Phase One (all 26 policies) is less or equal to 0, assignment failed.

4. Line 11: Sort all the tasks in the unassigned queue in decreasing priority order.

5. Line 12~20: Determine whether a unassigned task is a pre-assigned task or a normal task. If the condition is satisfied, the task is a pre-assigned task. Assign the task into a processor namely the pre-assigned processor. If the condition is not satisfied, put the task into a queue called normal task queue. All the left processors other than pre-assigned processors are called normal processors.

6. Line 21: Sort the index of pre-assigned processors in increasing priority order in term of the pre-assigned tasks on those processors.

7. Line 22~42: If there is a normal task unassigned, try to assign it to a normal processor first which has the minimal utilization among all normal processor. If it cannot be fully assigned to a normal processor, split the task and assign a subtask which can exactly make the normal processor fully utilized. If all normal processors are fully utilized, assign the task to a pre-assigned processor which is sorted in increasing priority order based on the pre-assigned task on that processor. If the task is assigned and the pre-assigned processor is not fully utilized, put the pre-assigned processor back to the list with its original index. If a task cannot be fully assigned to a pre-assigned processor, split the task and assign a subtask which can exactly make the pre-assigned processor fully utilized.

4.6 An Assignment Example

Here, an example of how a task is assigned by IBSP-TS is given in order to give a better understand of the algorithm.

12 tasks in a task set $\Gamma\ \{\tau_1, \tau_2, \tau_3, \dots, \tau_{12}\}$ are going to be assigned on 8 processors $P\ \{P_1, P_2, P_3, \dots, P_8\}$. The utilization range of these tasks is (0, 1] and the periods of the tasks are randomly generated within 0 and 1000 time unit.

All the 12 tasks are listed as follows. U_i and T_i are the utilization, period of the task; I_n is the interval which the task belongs to. Table 4 shows the 12 tasks in the task set.

$\tau_1 \in I_2$	U_1=0.652583	T_1=550
$\tau_2 \in I_{23}$	U_2=0.113183	T_2=528
$\tau_3 \in I_2$	U_3=0.578995	T_3=508
$\tau_4 \in I_3$	U_4=0.485351	T_4=89
$\tau_5 \in I_{27}$	U_5=0.038732	T_5=235
$\tau_6 \in I_1$	U_6=0.976029	T_6=720
$\tau_7 \in I_6$	U_7=0.372674	T_7= 671
$\tau_8 \in I_1$	U_8=0.823314	T_8=210
$\tau_9 \in I_7$	U_9=0.317253	T_9=941
$\tau_{10} \in I_3$	U_{10}=0.477383	T_{10}=221
$\tau_{11} \in I_1$	U_{11}=0.743396	T_{11}=838
$\tau_{12} \in I_3$	U_{12}=0.463743	T_{12}=110

Table 4: Example of a task set

In Phase One, τ_6, τ_8 and τ_{11} are assigned to three dedicated processors, P_1, P_2 and P_3. According to Policy I_3, the highest priority task in I_3, τ_4 is picked and equally split into τ_4^1 and τ_4^2. τ_4^1 along with τ_{10} are assigned to P_4; τ_4^2 and τ_{12} together are assigned to P_5. Table 5 is the processors assigned in Phase One.

P_1	τ_6
P_2	τ_8
P_3	τ_{11}

(a)

P_4	τ_{10}	τ_4^1
P_5	τ_{12}	τ_4^2

(b)

Table 5: The assignment in Phase One

After Phase One, only three processor P_6, P_7 and P_8 are left. Since there are 6 tasks left, $\Theta(6) = 0.734772$. All the remaining unassigned tasks are sorted in decreasing priority order. UnassignQ $\{\tau_5, \tau_3, \tau_2, \tau_1, \tau_7, \tau_9\}$

According to the condition, τ_i is a pre-assigned task if

$$U_i > \Theta(6)/(\Theta(6)+1)\ , \ \sum\nolimits_{j>i} U_j \leq (Count_{LP}-1) \times \Theta(6)$$

Pre-assigned tasks are PreQ $\{\tau_1, \tau_3\}$ sorted in increasing priority order. Normal tasks are NormQ $\{\tau_5, \tau_2, \tau_7, \tau_9\}$.

Assign the first two pre-assign tasks τ_1 and τ_3 in two processors, P_6 and P_7. Thereafter, normal tasks are assigned. Firstly, assign the τ_5 to the left processor P_8. Since there is only one normal processor, the minimal utilization processor picked in next assignment is still P_8 until it becomes fully utilized. Therefore, τ_5, τ_2 and τ_7 are directly assigned to P_8. However, the processor utilization do not allow τ_9 to be fully assigned to P_8. It split τ_9 into two parts, τ_9^1 and τ_9' so that τ_9^1 can be assigned to P_8 and make P_8 precisely fully utilized. Next, the scheduling dispatcher picks the first pre-assigned processor to assign the rest part of τ_9. It select P_6 and try to assign as much as possible, but still τ_9' cannot be fully assigned into P_6. Then, τ_9' is again split into two parts τ_9^2 and τ_9'' so that τ_9^2 can be assigned to P_6 and fully utilize P_6.Finally, the last part of τ_9, τ_9'' can be assigned to P_7 and named as τ_9^3. Table 6 is the processors assigned in Phase Two.

P_8	τ_5	τ_2	τ_7	τ_9^1
U_i	0.038732	0.113183	0.372674	0.210182
Uti of P_8	0.038732	0.151916	0.524590	0.734772

(a)

P_6	τ_1	τ_9^2
U_i	0.652583	0.082189
Uti of P_6	0.652583	0.734772

(b)

P_7	τ_3	τ_9^3
U_i	0.578995	0.024881
Uti of P_7	0.578995	0.603877

(c)

Table 7: The assignment in Phase Two

V

Evaluation

In this section, the simulation results are shown to evaluate the performance of IBSP-TS, and compare it to the pure SPA2 algorithm. To the best of our knowledge, SPA2 is the only one algorithm at this moment which has already proved the utilization bound of 69.3% based on semi-partition fixed-priority scheduling. By running the simulation, it shows that IBSP-TS performs much better than the SPA2 algorithms in most cases.

5.1 Simulation Setup

In all the simulations, there is a set of three parameters: U_{min}, U_{max} and m. The value m means the number of processors in the system. The utilization of a randomly generated task uniformly distributed within (U_{min}, U_{max}].

There are six sets of parameter m, the number of processors, in the simulation, which are m=4, m=8, m=16, m=32, m=64 and m=128. This parameter m is used to estimate the impact of increasing number of processors and the schedulability. For the utilization range, (U_{min}, U_{max}], there are five sets, (0, 1], (0, 0.25], (0, 0.5], (0.25, 0.75] and (0.5, 1] to represent mixed, extra light, normal light, medium and heavy task utilization respectively. Thus, it is easy to observe whether there is any impact because of different individual task utilization. Therefore, totally there are $6 \times 5 = 30$ different parameter sets for simulation experiments.

For each parameter set, 1,000,000 task sets are generated. Each task set contains a number of randomly generated tasks. A task set is said to be schedulable if all the tasks in a task set can be successfully assigned to no more than the number of processors provided in the system which is m. The 1,000,000 task sets are generated according to the following procedure:

1. Initially, m+1 tasks are generated in the first task set.

2. Calculate whether the task set is an infeasible task set $\Gamma(U) > m$

3. If it is an infeasible task set, reset the number of tasks in a task set to m+1.

4. Verify whether this task set can be scheduled by either IBSP-TS or SPA2.

5. If the task set is schedulable by either of those two algorithms, then calculate the necessary metrics (e.g. count the number of total split tasks, number of sorted tasks and maximum number of subtasks in a task) to evaluate the performance of the experiment.

6. Increase the number of tasks in next task set by 1.

To evaluate the performance of an algorithm, several measures are needed in the simulation.

For each experiment, the schedulability of an algorithm can be estimated by the term of success ratio, which is defined as follows.

$$\text{Success Ratio} = \frac{\text{the number of schedulable task sets}}{\text{the number of total task sets}} \times 100\%$$

The greater value of success ratio an algorithm has the better performance of schedulability the algorithm is. In addition, 100 buckets (scales) are set to represent the system utilization of a task set which is from 0 to 100%. In each bucket, an individual success ratio (for example, the success ratio for task set with system utilization 69%) is calculated. Since both algorithms has theoretically proved the schedulability bound for task sets with ln2, all the individual success ratio should be 100% if the system utilization is below 69% (excluding 69%). Bucket 69% can contain utilization 69.9% which is greater than Ln2. Moreover, it is good for an algorithm to have a higher Break-down Point (the individual success ratio becomes no more 100%), which can actually extend the utilization bound in real cases.

Meanwhile, since in real situation, task migration cost is not negligible, it is better for an algorithm to have as smaller number of single task being split and subtasks (split tasks from a single task) as possible. Thus, average number of split tasks (Avg. Split), the average number of sorted tasks (Avg. Sort) and the maximum number of subtasks split from a single task (Max Sub) are also important properties for an algorithm.

$$\text{Avg. Split} = \frac{\text{the total number of split tasks in all schedulable task sets}}{\text{the number of schedulable task sets}}$$

$$\text{Avg. Sort} = \frac{\text{the total number of sorted tasks in all schedulable task sets}}{\text{the number of schedulable task sets}}$$

However, due to many factors, one algorithm may have worse Success Ration but it perform better schedulability in some cases. Another measure is defined, which is called Superiority. It means that a task set can be schedulable by one algorithm, but not schedulable by the other one.

$$\text{Superority} = \frac{\text{the number of task sets only schedulable by this algoritm}}{\text{the number of task sets schedulable by both algorithms}} \times 100\%$$

The simulation code is shown in Appendix.

5.2 Simulation Result

The results of the thirty experiments for both algorithms are grouped into six categories which are based on different sets for parameters (U_{min}, U_{max}]. The statistics for every five different *m* is shown in tables and figures.

Table 7 gives the results of simulation for mixed tasks within (0, 1]. The difference of success ratio between IBSP-TS and SPA2 increases along with the increment of number of processors. It starts from around 6% to more than 18%. Moreover, there are some critical advantages for IBSP-TS. The maximum number of subtasks for IBSP-TS is always less or equal to that of SPA2. In the worst case for SPA2, it even makes a task migrate 116 times. Additionally, the less average number of sorted tasks in IBSP-TS is a great property for online scheduling, and this property is always the same in all five groups of simulations. The reason why IBSP-TS has these advantages is Phase One in IBSP-TS, when the number of tasks in each task set increases, the chance of more tasks can be handled in different first 26 intervals is becoming bigger and bigger. However, this will make IBSP-TS has more split tasks if there are more processors available (*m*>8). Furthermore, the superiority of SPA2 only happens when *m* is small, for example, *m*=4 or *m*=8, but for IBSP-TS, it can have 26% in best case.

	IBSP-TS	SPA2
Success Ratio	62.5893%	56.1998%
Avg. Split	0.2850	0.4477
Avg. Sort	4.3916	5.7735
Max. Sub	4	4
Superiority	10.2334%	0.0274%

(a) *m*=4

	IBSP-TS	SPA2
Success Ratio	63.2001%	54.3822%
Avg. Split	0.5233	0.5968
Avg. Sort	7.5195	10.6983
Max. Sub	7	8
Superiority	13.9526%	0.0003%

(b) *m*=8

	IBSP-TS	SPA2
Success Ratio	64.2393%	52.1397%
Avg. Split	1.0550	0.7539
Avg. Sort	4.3916	5.7735
Max. Sub	9	16
Superiority	18.8352%	0%

(c) *m*=16

	IBSP-TS	SPA2
Success Ratio	64.8203%	49.5652%
Avg. Split	2.447456	0.9710
Avg. Sort	20.8337	39.8876
Max. Sub	10	32
Superiority	23.5344%	0%

(d) *m*=32

	IBSP-TS	SPA2
Success Ratio	64.4618%	47.3246%
Avg. Split	5.6462	1.3309
Avg. Sort	34.9671	78.5207
Max. Sub	12	62
Superiority	26.5851%	0%

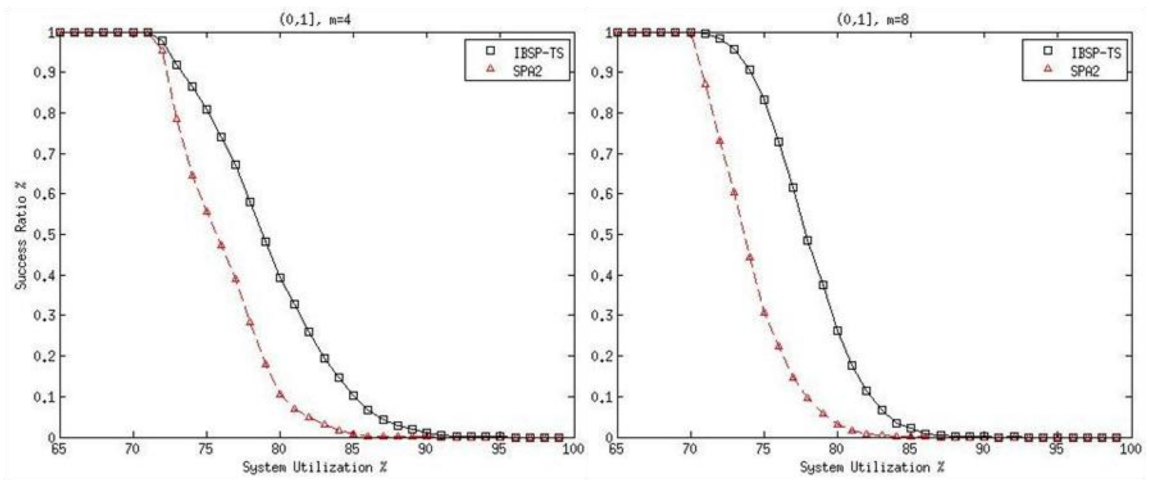
(e) *m*=64

	IBSP-TS	SPA2
Success Ratio	63.8920%	45.5303%
Avg. Split	12.9382	1.9672
Avg. Sort	57.6213	155.6472
Max. Sub	15	117
Superiority	23.7387%	0%

(f) *m*=128

Table 7: Simulation results within (0, 1]

From Figure 5, it is quite clear to see that IBSP-TS always has a higher success ratio than SPA2 in every scale of system utilization. Figure 6 presents the break-down point in utilization for both algorithms. Once again, the difference of both algorithms becomes more and more obvious.



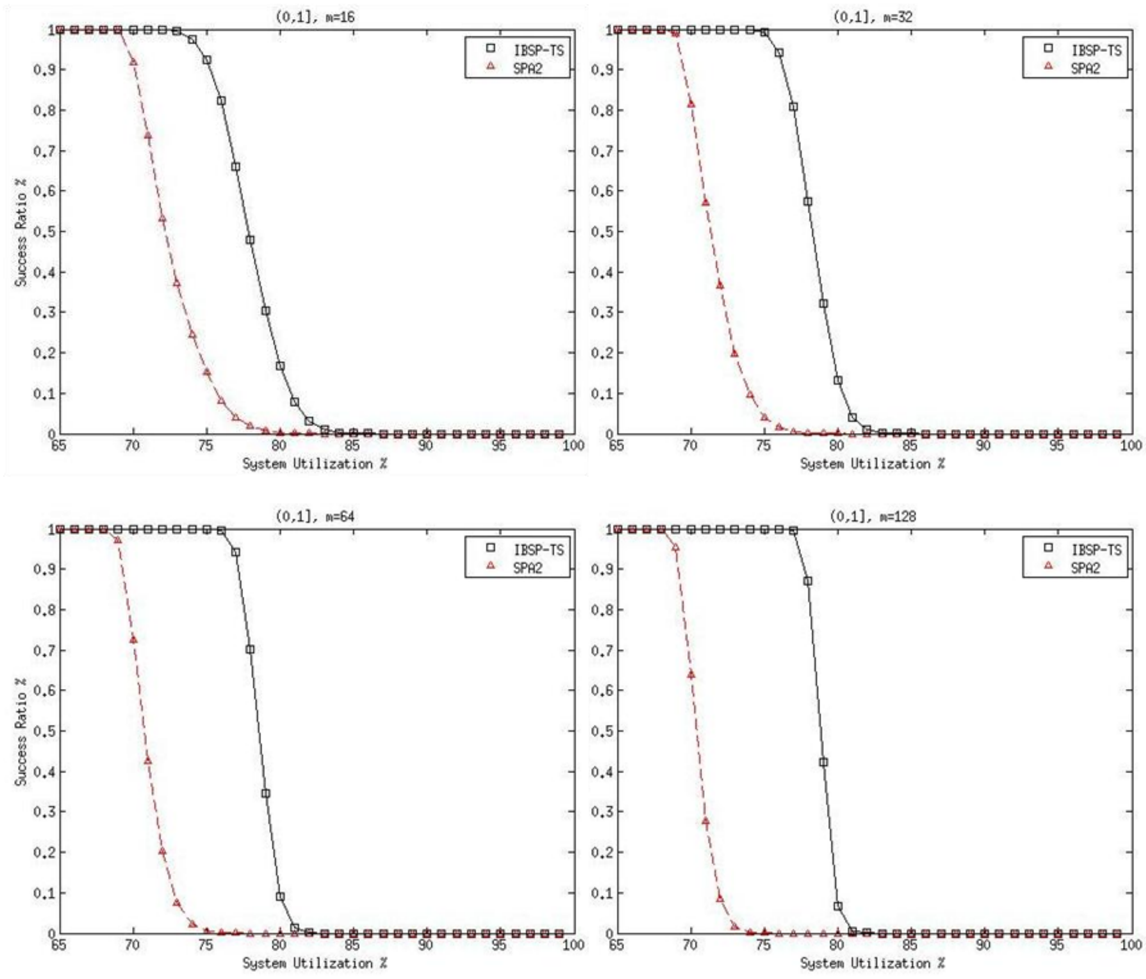


Figure 5: Success ratio in simulations within (0, 1] with different *m*

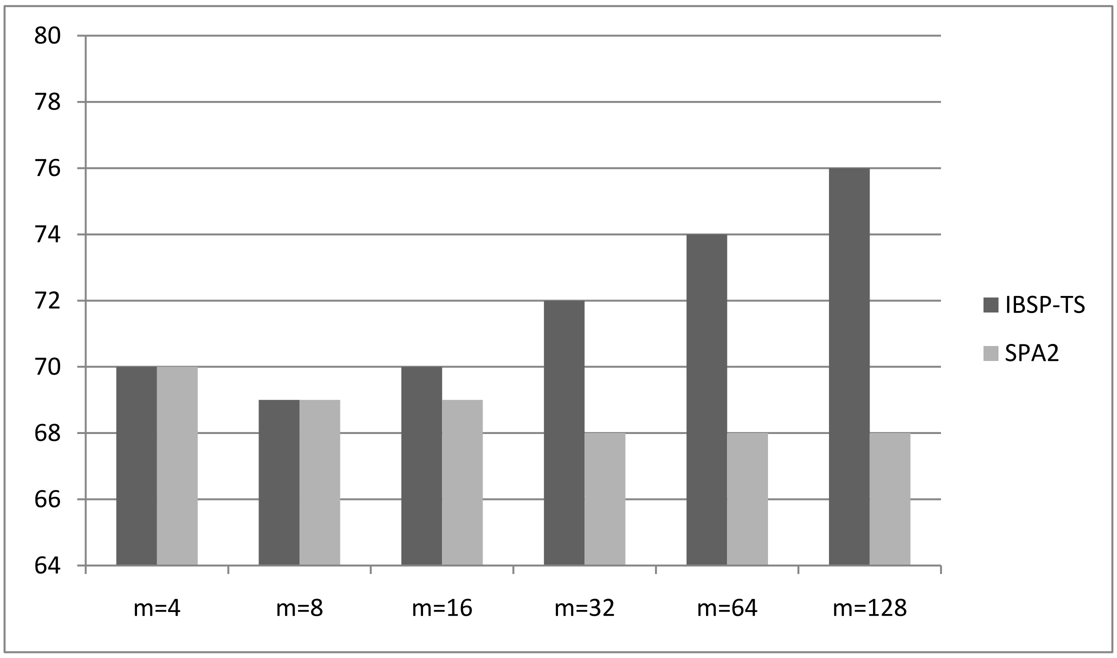


Figure 6: Break-down point in simulations within (0, 1]

Table 8 shows the results of simulation for extra light tasks within (0, 0.25]. The difference of success ratio between two algorithms increases from 0.2% to 2% when *m* increases. However, due to the light task (0, 0.25], there are less intervals used in Phase One, which causes IBSP-TS becomes more or less as same as SPA2. Thus, the Avg. Sort for IBSP-TS is quite close to that of SPA2. For Max. Sub, IBSP-TS may suffer the same problem as SPA2 because of no pre-assigned tasks and the assignments for last several very light tasks, but when *m*=128, IBSP-TS has 7 less subtasks than SPA2. However, IBSP-TS still perform a bit better than SPA2 which is listed by superiority, and even when *m*=4, the superiority of SPA2 is negligible.

	IBSP-TS	SPA2
Success Ratio	71.5340%	71.3367%
Avg. Split	0.1492	0.1701
Avg. Sort	13.5280	13.786403
Max. Sub	4	4
Superiority	0.2769%	0.0003%

(a) *m*=4

	IBSP-TS	SPA2
Success Ratio	70.6056%	70.1482%
Avg. Split	0.2696	0.3640
Avg. Sort	25.4879	26.8990
Max. Sub	7	7
Superiority	0.6476%	0%

(b) *m*=8

	IBSP-TS	SPA2
Success Ratio	69.9035%	69.0946%
Avg. Split	0.5105	0.7491
Avg. Sort	47.5833	53.0866
Max. Sub	10	10
Superiority	1.1572%	0%

(c) *m*=16

	IBSP-TS	SPA2
Success Ratio	69.6106%	68.2750%
Avg. Split	1.2082	1.5135
Avg. Sort	86.0277	105.4322
Max. Sub	11	15
Superiority	1.9187%	0%

(d) *m*=32

	IBSP-TS	SPA2
Success Ratio	69.3125%	67.5807%
Avg. Split	3.6731	3.0477
Avg. Sort	148.6568	210.1630
Max. Sub	14	19
Superiority	2.4985%	0%

(e) *m*=64

	IBSP-TS	SPA2
Success Ratio	68.9200%	66.8760%
Avg. Split	10.4517	6.1051
Avg. Sort	252.4917	419.4665
Max. Sub	13	22
Superiority	2.9658%	0%

(f) *m*=128

Table 8: Simulation results for tasks within (0, 0.25]

For extra light tasks, generally, both algorithms have more or less the same performance. However, it is still the truth that IBSP-TS has higher success ratio and break-down point as shown in Figure 7 and Figure 8.

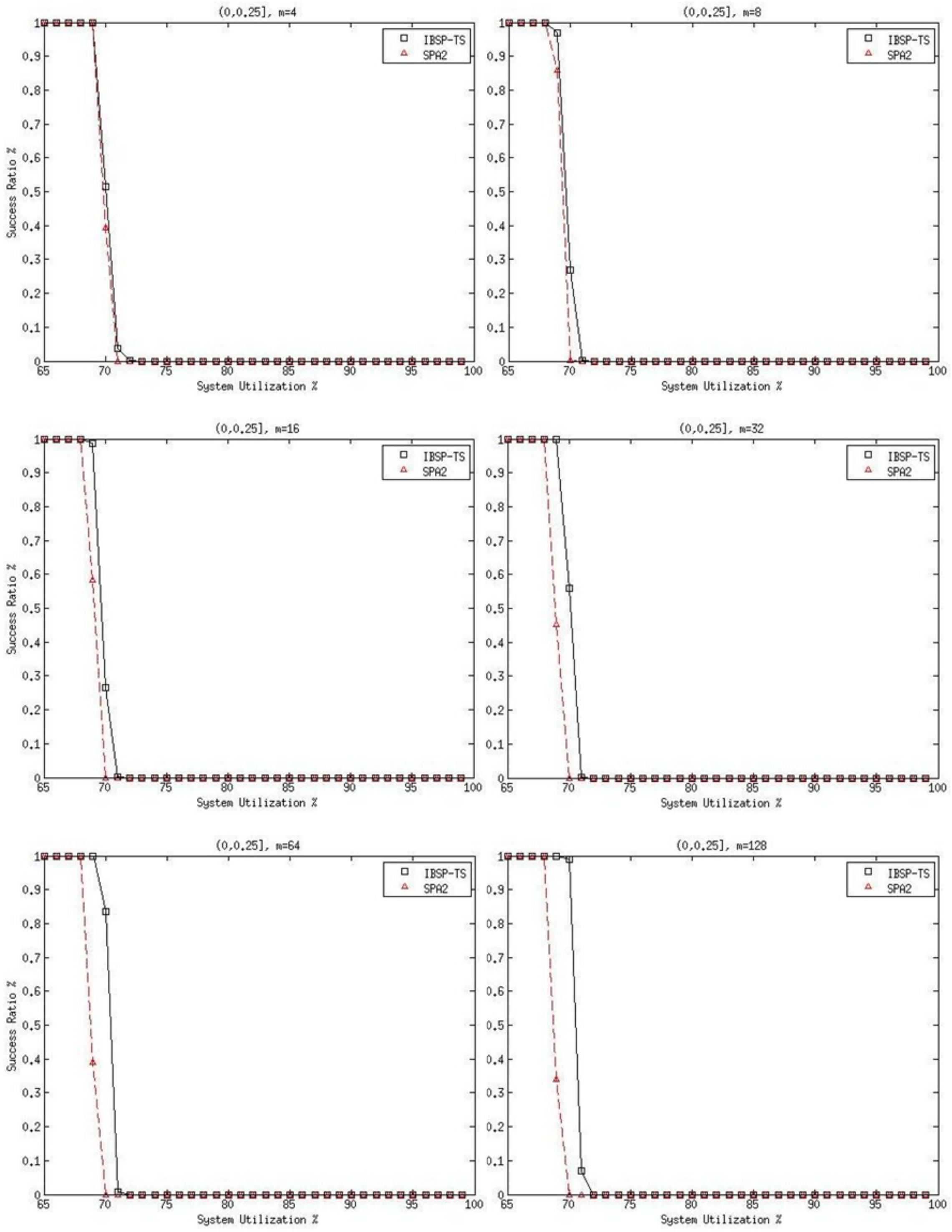


Figure 7: Success ratio in simulations within (0, 0.25] with different *m*

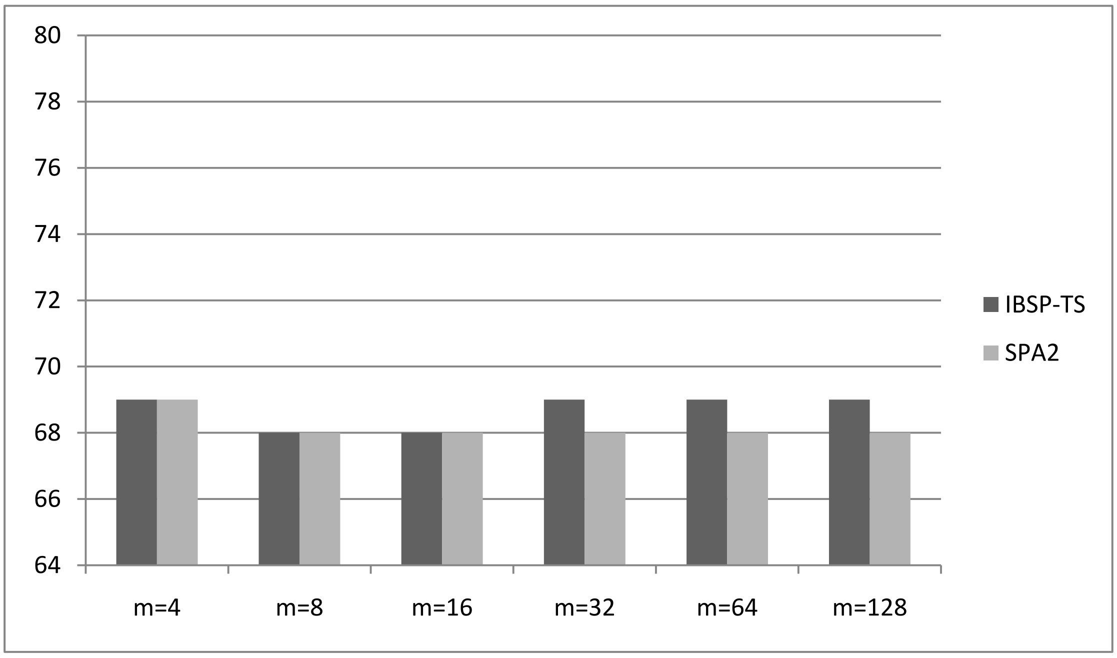


Figure 8: Break-down point in simulations within (0, 0.25]

Table 9 shows the results of simulation for normal light tasks within (0, 0.5]. The difference of success ratio between two algorithms rises from around 0.6% to 4% with increment value *m*. Since it is still considered to be light tasks, in some cases, the performance are similar for both algorithms. IBSP-TS is again better than SPA2 for all the statistics. The superiority of SPA2 when *m*=4 is still too small to be noticed.

	IBSP-TS	SPA2
Success Ratio	67.9117%	67.3575%
Avg. Split	0.4683	0.5297
Avg. Sort	7.8863	8.2655
Max. Sub	4	4
Superiority	0.8180%	0.0002%

(a) *m*=4

	IBSP-TS	SPA2
Success Ratio	67.0182%	65.9863%
Avg. Split	0.9414	1.1221
Avg. Sort	14.4441	15.8448
Max. Sub	8	8
Superiority	1.5397%	0%

(b) *m*=8

	IBSP-TS	SPA2
Success Ratio	66.5590%	64.7012%
Avg. Split	1.8953	2.2901
Avg. Sort	25.8697	30.9503
Max. Sub	10	12
Superiority	2.7912%	0%

(c) *m*=16

	IBSP-TS	SPA2
Success Ratio	66.2644%	63.5591%
Avg. Split	4.3302	4.5985
Avg. Sort	44.5071	61.1194
Max. Sub	13	15
Superiority	4.0826%	0%

(d) *m*=32

	IBSP-TS	SPA2
Success Ratio	66.0109%	62.5631%
Avg. Split	10.4161	9.2292
Avg. Sort	72.9510	121.4573
Max. Sub	15	20
Superiority	5.2231%	0%

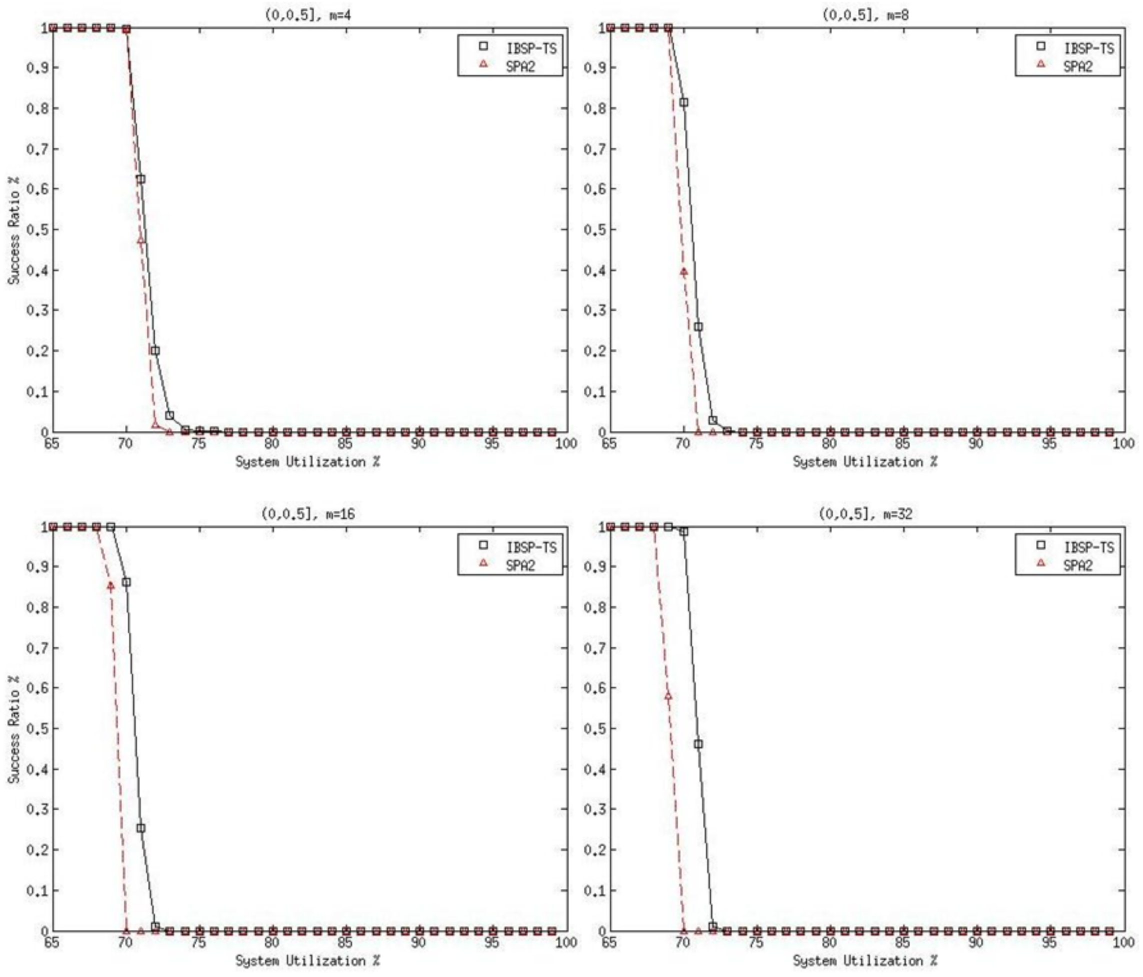
(e) *m*=64

	IBSP-TS	SPA2
Success Ratio	65.7586%	61.8576%
Avg. Split	25.2838	18.5089
Avg. Sort	166.2817	242.2193
Max. Sub	19	27
Superiority	5.9323%	0%

(f) *m*=128

Table 9: Simulation results for tasks within (0, 0.5]

In Figure 9, the trend of success ratio is the same as previous observation for mixed tasks and normal tasks. For the break-down point shown in Figure 10, IBSP-TS dominates the other.



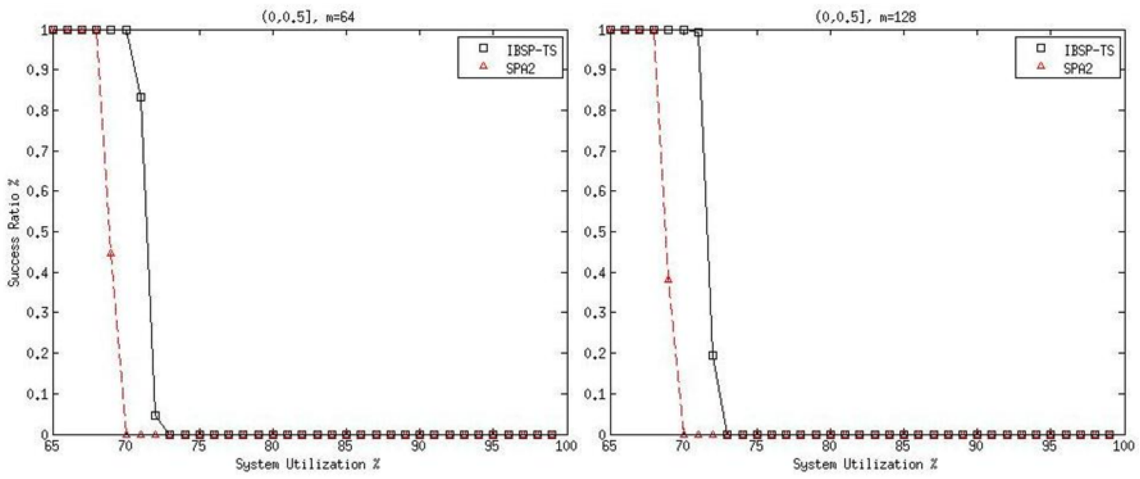


Figure 9: Success ratio in simulations within (0, 0.5] with different *m*

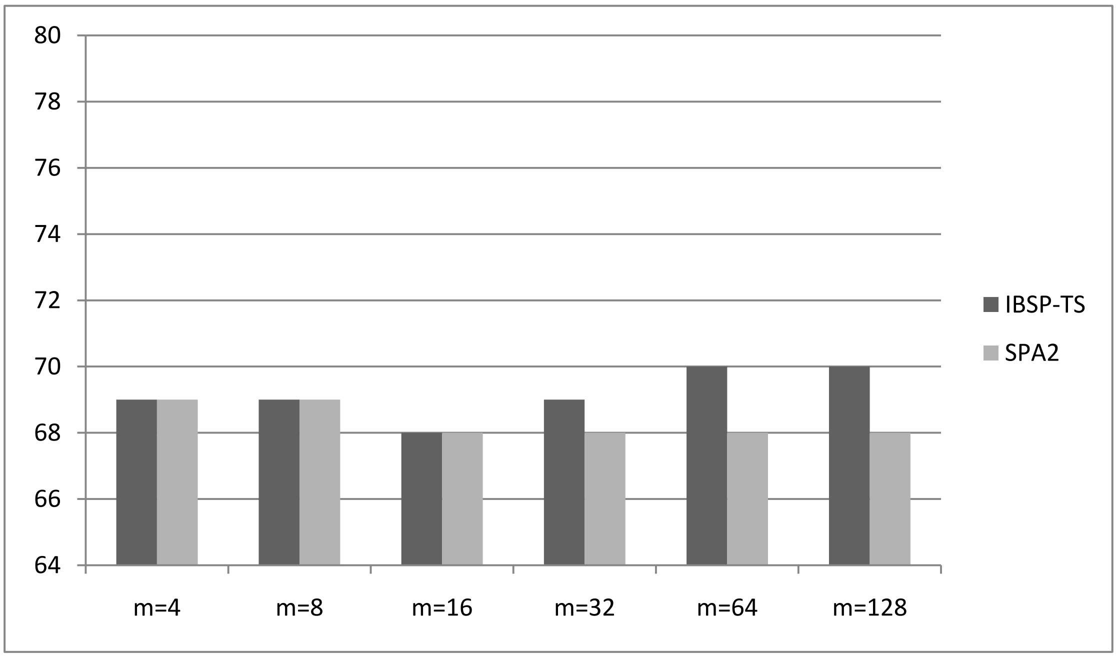


Figure 10: Break-down point in simulations within (0, 0.5]

Table 10 is the statistics of simulation for medium tasks within (0.25, 0.75]. The difference of success ratio between two algorithms increases from around 3% to 17%. Moreover, for superiority, IBSP-TS performs twice as good as SPA2 when *m* is relatively big. This shows that IBSP-TS is better for higher utilization tasks. It is because in IBSP-TS, the idea of a part of the algorithm is from SPA2, thus, it makes IBSP suffer the same problem as SPA2 when all the tasks are light. The price of higher success ratio and superiority is a more split tasks. However, considered the great improvement of IBSP-TS, it is acceptable.

	IBSP-TS	SPA2
Success Ratio	12.9744%	9.3209%
Avg. Split	1.0005	1.0000
Avg. Sort	2.7858	5.0000
Max. Sub	4	4
Superiority	28.3906%	0.3219%

(a) *m*=4

	IBSP-TS	SPA2
Success Ratio	28.5884%	17.6567%
Avg. Split	1.3275	1.0172
Avg. Sort	2.5347	9.0169
Max. Sub	5	8
Superiority	38.2382%	0%

(b) *m*=8

	IBSP-TS	SPA2
Success Ratio	33.2281%	17.7830%
Avg. Split	2.7815	1.4055
Avg. Sort	2.8645	17.4052
Max. Sub	5	16
Superiority	46.4820%	0%

(c) *m*=16

	IBSP-TS	SPA2
Success Ratio	35.5960%	18.0860%
Avg. Split	5.9868	2.2608
Avg. Sort	2.9442	34.2604
Max. Sub	5	30
Superiority	49.1909%	0%

(d) *m*=32

	IBSP-TS	SPA2
Success Ratio	36.7294%	18.2315%
Avg. Split	12.4927	3.9909
Avg. Sort	2.9792	67.9989
Max. Sub	5	36
Superiority	50.3627%	0%

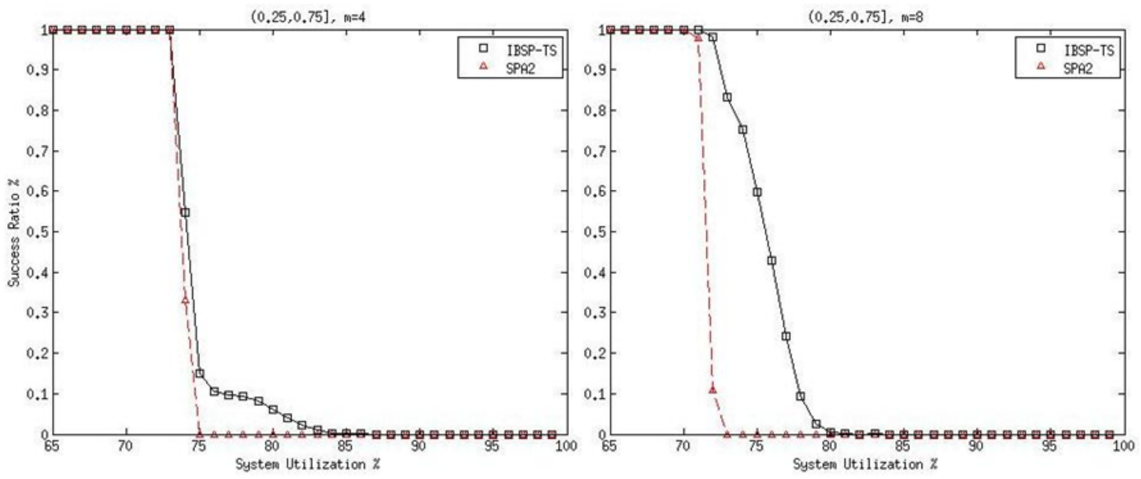
(e) *m*=64

	IBSP-TS	SPA2
Success Ratio	37.3115%	18.2986%
Avg. Split	25.5589	7.4895
Avg. Sort	2.9906	135.4894
Max. Sub	5	37
Superiority	50.9572%	0%

(f) *m*=128

Table 10: Simulation results for tasks within (0.25, 0.75]

Figure 11 and Figure 12 are the graphs for success ratio and break-down point.



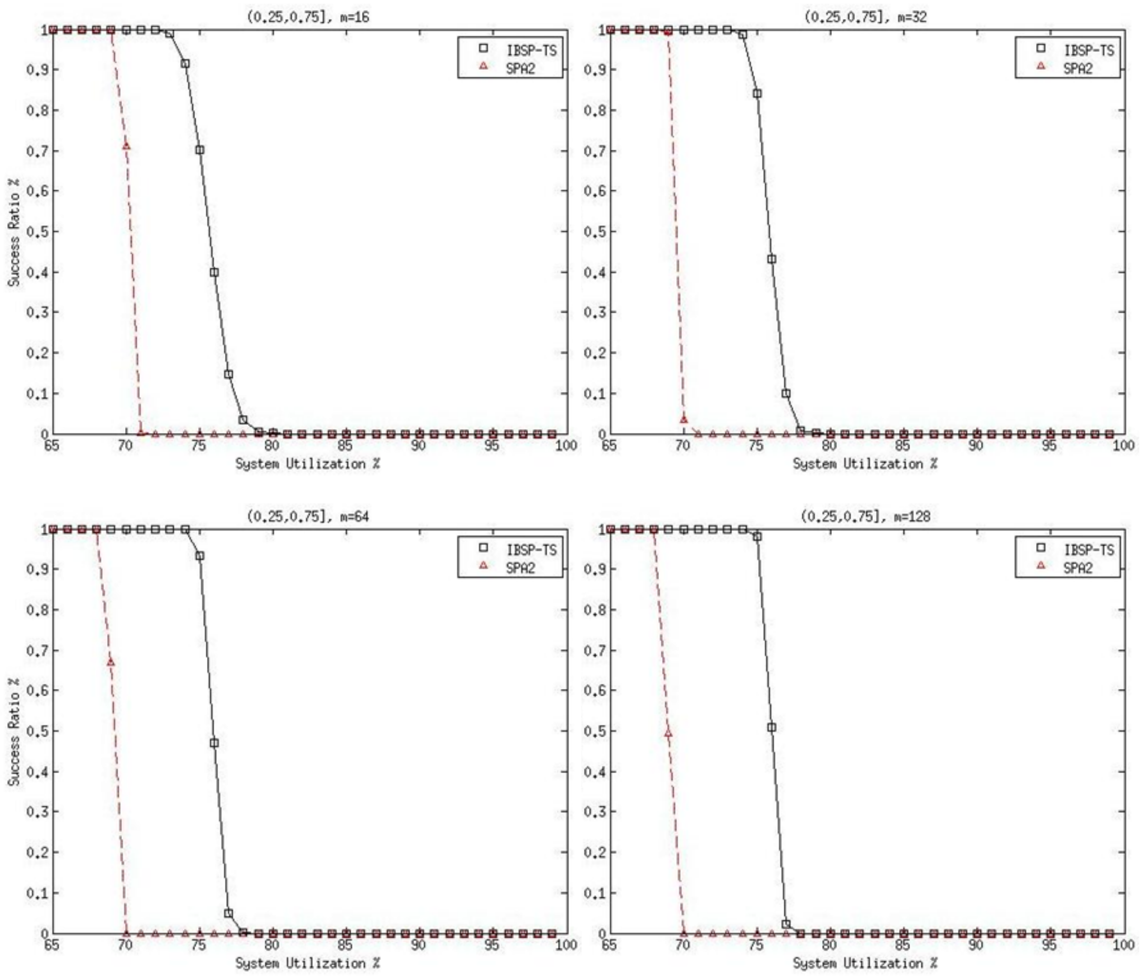


Figure 11: Success ratio in simulations within (0.25, 0.75] with different *m*

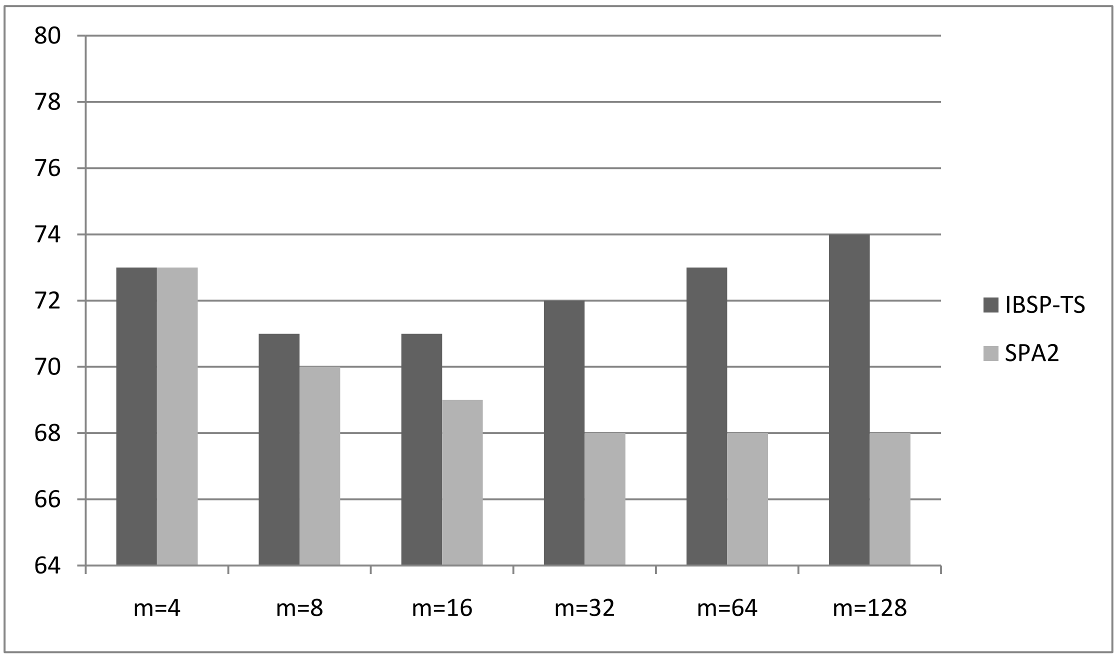


Figure 12: Break-down point in simulations within (0.25, 0.75]

Table 11 presents the simulation results for heavy tasks within (5, 1]. Because of high utilization of each individual tasks, the success ratio for both algorithms is quite low. SPA2 can barely schedule the task sets, which leads the superiority of IBSP-TS to more than 90% and even close to 100%. This fully shows the bad performance of SPA2 when it deals with high utilization task sets. In both average sorted tasks and maximum subtasks, IBSP-TS has huge advantage over SPA2 when *m* are relatively large.

	IBSP-TS	SPA2
Success Ratio	2.5584%	0.9197%
Avg. Split	1.0001	1.0000
Avg. Sort	2.1216	5.0000
Max. Sub	4	4
Superiority	64.0896%	0.1054%

(a) *m*=4

	IBSP-TS	SPA2
Success Ratio	10.9078%	0.9402%
Avg. Split	1.0203	1.0257
Avg. Sort	2.5347	9.0169
Max. Sub	5	8
Superiority	91.3805%	0%

(b) *m*=8

	IBSP-TS	SPA2
Success Ratio	22.1257%	0.6156%
Avg. Split	1.3598	1.1556
Avg. Sort	2.6832	17.0101
Max. Sub	5	16
Superiority	97.2177%	0%

(c) *m*=16

	IBSP-TS	SPA2
Success Ratio	27.1188%	0.2832%
Avg. Split	2.6948	1.5448
Avg. Sort	2.8902	33.0230
Max. Sub	5	32
Superiority	98.9557%	0%

(d) *m*=32

	IBSP-TS	SPA2
Success Ratio	29.3367%	0.1099%
Avg. Split	5.6993	2.5960
Avg. Sort	2.9517	65.0246
Max. Sub	5	62
Superiority	99.6254%	0%

(e) *m*=64

	IBSP-TS	SPA2
Success Ratio	30.3594%	0.0395%
Avg. Split	11.8472	5.1089
Avg. Sort	2.9788	129.0203
Max. Sub	5	52
Superiority	99.8699%	0%

(f) *m*=128

Table 11: Simulation results for tasks within (0.25, 0.75]

Figure 13 and Figure 14 shows the success ratio and break-down point for both algorithms.

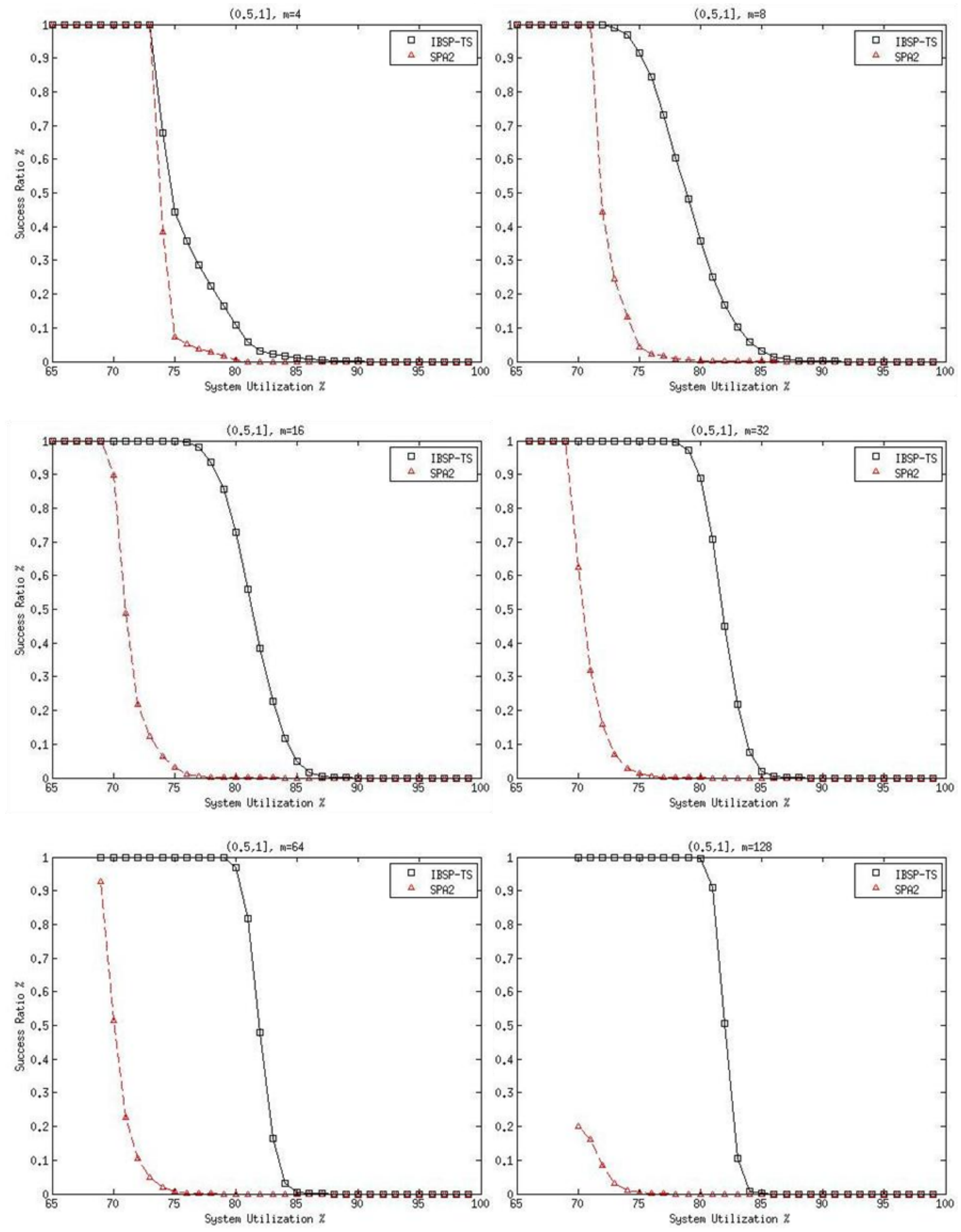


Figure 13: Success ratio in simulations within (0.5, 1] with different *m*

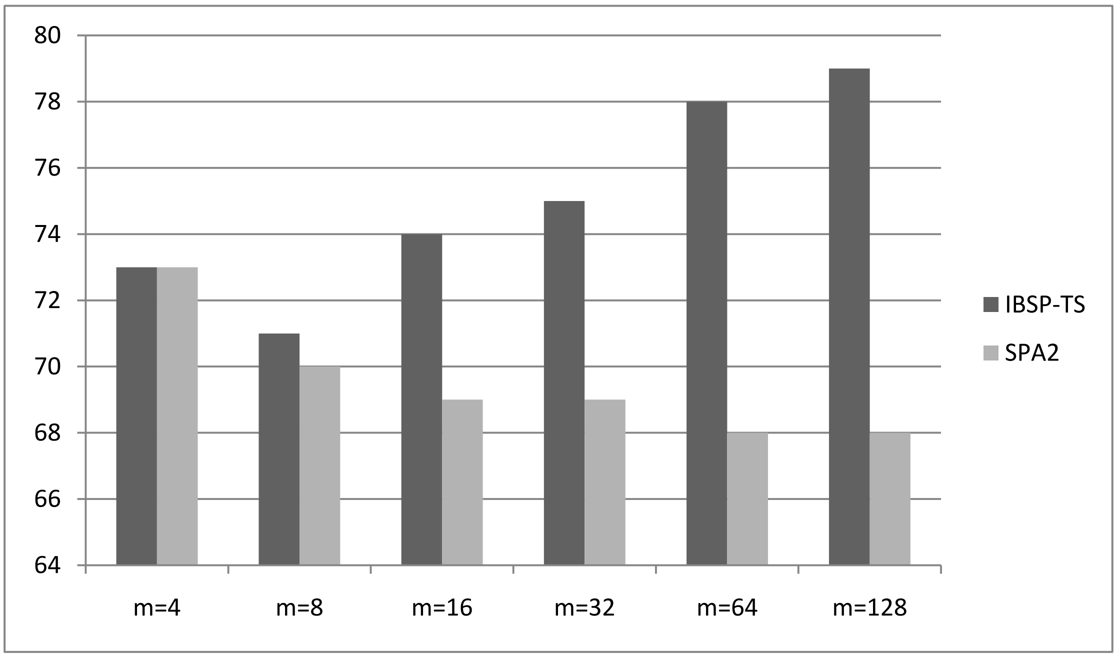


Figure 14: Break-down point in simulations within (0.5, 1]

VI

Conclusion

The research in this thesis is to design a static-priority scheduling algorithm in a multi-processor real-time system which can assign a set of independent periodic tasks to a number of processors in the system and ensure each task does not miss its deadline. Therefore, the IBSP-TS (Interval Based Semi-Partitioned Task Splitting) algorithm is derived. It combines the idea of the IBPS algorithm and the SPA algorithm with semi-partition technique and RM priority policy to achieve the highest possible worst-case utilization bound to ln2.

Note: there are some assumptions in IBSP-TS for systems and tasks. The multi-processor system used for IBSP-TS assumes that all the processors are identical and data-shareable; also there is no task migration do not have cost. Moreover, tasks are independent and preemptive. There is no synchronization among tasks. No jobs of a task or subtasks can be executed on two or more processors simultaneously, and a processor cannot execute two or more tasks simultaneously

The schedulability test of IBSP is divided into two parts together with the assignment procedure. In Phase One, by calculating the worst-case processor work load within each interval, the Hyperbolic Bound test can guarantee the feasibility. In Phase Two, the Liu & Layland test ensure the assignment procedure only schedule a feasible number of tasks on a processor. Therefore, if ln2 which is 69.3% capacity of processors in a system is used, all the tasks in the task set can meet their deadlines.

Although IBSP-TS has the same worst-case utilization bound of SPA, it has some advantages over SPA2. First of all, there are less sorted tasks in the assignment, which is suitable for online scheduling for multi-processor system. Certainly, the sorting itself is also time consuming. Besides, IBSP-TS leaves the algorithm implementation decision to the system designers. They can come up with their preferred choice by selecting the number of intervals in the system implementation. The more intervals, the less number of sorted tasks there are.

From simulation, IBSP-TS also shows its superiority to SPA2. In most cases, if a task can be scheduled by SPA2, it can be scheduled by IBSP-TS as well. However, only when the number of processors in the system is quite small, there may be few task sets which are only schedulable by SPA2. Nevertheless, in terms of task sets success ratio, average number of split tasks and maximum number of subtasks, IBSP-TS is much better than SPA2 in general utilization tasks or heavy utilization tasks. The lower number of task migrations, the less preemption cost there is. Additionally, the break-down point in utilization for IBSP-TS can reach to 76% for general tasks; and for heavy tasks, it can be 79%.

In summary, IBSP-TS provides a feasibility condition with the highest possible worst-case utilization bound condition as ln2, less task migrations and higher success ratio compared to the best static-priority multi-processor scheduling at this time being. Moreover, IBSP-TS can adopt other priority polices as well, for example deadline-monotonic. Additionally, it leaves the implementation decision in IBSP-TS to system designers. All these properties make IBSP-TS more efficiently to schedule tasks in multi-processor real-time systems compared to many other competing scheduling algorithms.

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Appendix

The simulation C code for the performance comparison between IBSP-TS and SPA is attached as follows.

```
#include<stdio.h>
#include<stdlib.h>
#include<time.h>
#include<math.h>
#include<malloc.h>

#define SET  1000000
#define PROC 128
#define N 1
#define Ln logl(2)


typedef struct{
    double uti;
    int period;
} task;

int task_num=PROC+N;
double weight, theta;
task arr_task[PROC*10];
double tot_uti;
int chaox_split;
int chaox_sub;
int chaox_sort;
int neu_split;
int neu_sub;
int neu_sort;
int chaox_return;
int neu_return;

double init()
{
    int i;
    double tmp;
    tot_uti=0;
    for(i=0;i<task_num;i++){
        tmp=rand();
        if(tmp==0)
            tmp=rand();
        arr_task[i].uti=tmp/(4.0*RAND_MAX);
        tot_uti=tot_uti+arr_task[i].uti;
    }
    while(tot_uti>PROC){
        task_num=PROC+N;
        printf("Regenerate & task_num %d\n", task_num);
        tot_uti=0;
        for(i=0;i<task_num;i++){
            tmp=rand();
            if(tmp==0)
                tmp=rand();
            arr_task[i].uti=tmp/(4.0*RAND_MAX);
            tot_uti=tot_uti+arr_task[i].uti;
        }
    }
    for(i=0;i<task_num;i++){
        arr_task[i].period=rand()%1000;
        //printf("period %d\n", arr_task[i].period);
        //printf("uti %f\n", arr_task[i].uti);
    }
    theta=(pow(2,(1.0/task_num))-1)*task_num;
    weight=theta/(theta+1);
    printf("task_num %d theta %f weight %f\n",task_num, theta, weight);
}

double chaox()
{
    int i, j, x, m, n, p, k, t, flg, proc_num, count_x;
    double    x_uti, tmp, new_weight, new_theta, c;
    int *subtask;
    double *uti, *pre_cpu, *norm_cpu;
    int count_1, count_2, count_3, count_4, count_5, count_6, count_7, count_8,
count_9, count_10, count_11, count_12, count_13, count_14, count_15, count_16,
count_17, count_18, count_19, count_20, count_21, count_22, count_23, count_24,
count_25, count_26, count_27, count_pre, count_norm;
    task *a2, *a3, *a4, *a5, *a6, *a7, *a8, *a9, *a10, *a11, *a12, *a13, *a14, *a15,
*a16, *a17, *a18, *a19, *a20, *a21, *a22, *a23, *a24, *a25, *a26, *ax, *pre, *norm;

    int rem_2, rem_3, rem_4, rem_5, rem_6, rem_7, rem_8, rem_9, rem_10,
rem_11, rem_12, rem_13, rem_14, rem_15, rem_16, rem_17, rem_18, rem_19,
rem_20, rem_21, rem_22, rem_23, rem_24, rem_25, rem_26, rem_x;

    count_1=0;
    count_2=0;
    count_3=0;
    count_4=0;
    count_5=0;
    count_6=0;
    count_7=0;
    count_8=0;
    count_9=0;
    count_10=0;
    count_11=0;
    count_12=0;
    count_13=0;
    count_14=0;
    count_15=0;
    count_16=0;
    count_17=0;
    count_18=0;
    count_19=0;
    count_20=0;
    count_21=0;
    count_22=0;
    count_23=0;
    count_24=0;
    count_25=0;
    count_26=0;
    count_27=0;
    chaox_return=0;

    for(i=0;i<task_num;i++){
        if(arr_task[i].uti>Ln) count_1++;
        else if(arr_task[i].uti<=Ln && arr_task[i].uti>(4.0/5*Ln)) count_2++;
```

```
            else if(arr_task[i].uti<=(4.0/5*Ln) && arr_task[i].uti>(2.0/3*Ln)) count_3++;
            else if(arr_task[i].uti<=(2.0/3*Ln) && arr_task[i].uti>(3.0/5*Ln)) count_4++;
            else if(arr_task[i].uti<=(3.0/5*Ln) && arr_task[i].uti>(4.0/7*Ln)) count_5++;
            else if(arr_task[i].uti<=(4.0/7*Ln) && arr_task[i].uti>(1.0/2*Ln)) count_6++;
            else if(arr_task[i].uti<=(1.0/2*Ln) && arr_task[i].uti>(4.0/9*Ln)) count_7++;
            else if(arr_task[i].uti<=(4.0/9*Ln) && arr_task[i].uti>(2.0/5*Ln)) count_8++;
            else if(arr_task[i].uti<=(2.0/5*Ln) && arr_task[i].uti>(4.0/11*Ln)) count_9++;
            else if(arr_task[i].uti<=(4.0/11*Ln) && arr_task[i].uti>(1.0/3*Ln))
count_10++;
            else if(arr_task[i].uti<=(1.0/3*Ln) && arr_task[i].uti>(4.0/13*Ln))
count_11++;
            else if(arr_task[i].uti<=(4.0/13*Ln) && arr_task[i].uti>(2.0/7*Ln))
count_12++;
            else if(arr_task[i].uti<=(2.0/7*Ln) && arr_task[i].uti>(3.0/11*Ln))
count_13++;
            else if(arr_task[i].uti<=(3.0/11*Ln) && arr_task[i].uti>(1.0/4*Ln))
count_14++;
            else if(arr_task[i].uti<=(1.0/4*Ln) && arr_task[i].uti>(4.0/17*Ln))
count_15++;
            else if(arr_task[i].uti<=(4.0/17*Ln) && arr_task[i].uti>(2.0/9*Ln))
count_16++;
            else if(arr_task[i].uti<=(2.0/9*Ln) && arr_task[i].uti>(3.0/14*Ln))
count_17++;
            else if(arr_task[i].uti<=(3.0/14*Ln) && arr_task[i].uti>(1.0/5*Ln))
count_18++;
            else if(arr_task[i].uti<=(1.0/5*Ln) && arr_task[i].uti>(4.0/21*Ln))
count_19++;
            else if(arr_task[i].uti<=(4.0/21*Ln) && arr_task[i].uti>(2.0/11*Ln))
count_20++;
            else if(arr_task[i].uti<=(2.0/11*Ln) && arr_task[i].uti>(3.0/17*Ln))
count_21++;
            else if(arr_task[i].uti<=(3.0/17*Ln) && arr_task[i].uti>(1.0/6*Ln))
count_22++;
            else if(arr_task[i].uti<=(1.0/6*Ln) && arr_task[i].uti>(4.0/25*Ln))
count_23++;
            else if(arr_task[i].uti<=(4.0/25*Ln) && arr_task[i].uti>(2.0/13*Ln))
count_24++;
            else if(arr_task[i].uti<=(2.0/13*Ln) && arr_task[i].uti>(3.0/20*Ln))
count_25++;
            else if(arr_task[i].uti<=(3.0/20*Ln) && arr_task[i].uti>(1.0/7*Ln))
count_26++;
            else count_27++;
      }
```

```
    rem_2=count_2%5;
    rem_3=count_3%3;
    rem_4=count_4%5;
    rem_5=count_5%7;
    rem_6=count_6%2;
    rem_7=count_7%9;
    rem_8=count_8%5;
    rem_9=count_9%11;
    rem_10=count_10%3;
    rem_11=count_11%13;
    rem_12=count_12%7;
    rem_13=count_13%11;
    rem_14=count_14%4;
    rem_15=count_15%17;
    rem_16=count_16%9;
    rem_17=count_17%14;
    rem_18=count_18%5;
    rem_19=count_19%21;
    rem_20=count_20%11;
    rem_21=count_21%17;
    rem_22=count_22%6;
    rem_23=count_23%25;
    rem_24=count_24%13;
    rem_25=count_25%20;
    rem_26=count_26%7;

    if(count_2 || count_5 || count_7 || count_9 || count_11 || count_15 ||
count_19 || count_23)
        chaox_sub=4;
    else if(count_4 || count_13 || count_17 || count_21 || count_25)
        chaox_sub=3;
    else if(count_3 || count_8 || count_12 || count_16 || count_20 || count_24)
        chaox_sub=2;
    else chaox_sub=0;

    chaox_split=count_2/5+count_3/3+count_4/5*2+count_5/7*3+count_7/9+coun
t_8/5+count_9/11*3+count_11/13+count_12/7+count_13/11*2+count_15/17+coun
t_16/9+count_17/14*2+count_19/21+count_20/11+count_21/17*2+count_23/25+c
ount_24/13+count_25/20*2;

    count_x=rem_2+rem_3+rem_4+rem_5+rem_6+rem_7+rem_8+rem_9+rem_10+
rem_11+rem_12+rem_13+rem_14+rem_15+rem_16+rem_17+rem_18+rem_19+rem
_20+rem_21+rem_22+rem_23+rem_24+rem_25+rem_26+count_27;
    chaox_sort=count_x;
    x=count_x;
    new_theta=(pow(2,(1.0/x))-1)*x;
    new_weight=new_theta/(new_theta+1);
    printf("new_theta %f new_weight %f\n", new_theta, new_weight);

    proc_num=PROC-count_1-count_2/5*4-count_3/3*2-count_4/5*3-count_5/7*4
-count_6/2-count_7/9*4-count_8/5*2-count_9/11*4-count_10/3-count_11/13*4-co
unt_12/7*2-count_13/11*3-count_14/4-count_15/17*4-count_16/9*2-count_17/14
*3-count_18/5-count_19/21*4-count_20/11*2-count_21/17*3-count_22/6-count_2
3/25*4-count_24/13*2-count_25/20*3-count_26/7;

    printf("proc_num %d\n", proc_num);
    if(proc_num>0 && x>0){
        a2=(task*)malloc(count_2*sizeof(task));
        a3=(task*)malloc(count_3*sizeof(task));
        a4=(task*)malloc(count_4*sizeof(task));
        a5=(task*)malloc(count_5*sizeof(task));
        a6=(task*)malloc(count_6*sizeof(task));
        a7=(task*)malloc(count_7*sizeof(task));
        a8=(task*)malloc(count_8*sizeof(task));
        a9=(task*)malloc(count_9*sizeof(task));
        a10=(task*)malloc(count_10*sizeof(task));
        a11=(task*)malloc(count_11*sizeof(task));
        a12=(task*)malloc(count_12*sizeof(task));
        a13=(task*)malloc(count_13*sizeof(task));
        a14=(task*)malloc(count_14*sizeof(task));
        a15=(task*)malloc(count_15*sizeof(task));
        a16=(task*)malloc(count_16*sizeof(task));
        a17=(task*)malloc(count_17*sizeof(task));
        a18=(task*)malloc(count_18*sizeof(task));
        a19=(task*)malloc(count_19*sizeof(task));
        a20=(task*)malloc(count_20*sizeof(task));
        a21=(task*)malloc(count_21*sizeof(task));
        a22=(task*)malloc(count_22*sizeof(task));
        a23=(task*)malloc(count_23*sizeof(task));
        a24=(task*)malloc(count_24*sizeof(task));
        a25=(task*)malloc(count_25*sizeof(task));
        a26=(task*)malloc(count_26*sizeof(task));
        ax=(task*)malloc(count_x*sizeof(task));

        for(i=0;i<task_num;i++){
            if(arr_task[i].uti>Ln) continue;
            else if(arr_task[i].uti<=Ln && arr_task[i].uti>(4.0/5*Ln)){
                a2[count_2-1].uti=arr_task[i].uti;
                a2[count_2-1].period=arr_task[i].period;
                //printf("a2 %f\n", a2[count_2-1]);
                count_2--;
            }
            else if(arr_task[i].uti<=(4.0/5*Ln) && arr_task[i].uti>(2.0/3*Ln)){
                a3[count_3-1].uti=arr_task[i].uti;
                a3[count_3-1].period=arr_task[i].period;
                //printf("a3 %f\n", a3[count_3-1]);
                count_3--;
            }
            else if(arr_task[i].uti<=(2.0/3*Ln) && arr_task[i].uti>(3.0/5*Ln)){
                a4[count_4-1].uti=arr_task[i].uti;
                a4[count_4-1].period=arr_task[i].period;
                //printf("a4 %f\n", a4[count_4-1]);
                count_4--;
            }
            else if(arr_task[i].uti<=(3.0/5*Ln) && arr_task[i].uti>(4.0/7*Ln)){
                a5[count_5-1].uti=arr_task[i].uti;
                a5[count_5-1].period=arr_task[i].period;
                //printf("a5 %f\n", a5[count_5-1]);
                count_5--;
            }
            else if(arr_task[i].uti<=(4.0/7*Ln) && arr_task[i].uti>(1.0/2*Ln)){
                a6[count_6-1].uti=arr_task[i].uti;
                a6[count_6-1].period=arr_task[i].period;
                //printf("a6 %f\n", a6[count_6-1]);
                count_6--;
            }
            else if(arr_task[i].uti<=(1.0/2*Ln) && arr_task[i].uti>(4.0/9*Ln)){
                a7[count_7-1].uti=arr_task[i].uti;
                a7[count_7-1].period=arr_task[i].period;
                //printf("a7 %f\n", a7[count_7-1]);
                count_7--;
            }
            else if(arr_task[i].uti<=(4.0/9*Ln) && arr_task[i].uti>(2.0/5*Ln)){
                a8[count_8-1].uti=arr_task[i].uti;
                a8[count_8-1].period=arr_task[i].period;
                //printf("a8 %f\n", a8[count_8-1]);
                count_8--;
            }
            else if(arr_task[i].uti<=(2.0/5*Ln) && arr_task[i].uti>(4.0/11*Ln)){
                a9[count_9-1].uti=arr_task[i].uti;
                a9[count_9-1].period=arr_task[i].period;
                //printf("a9 %f\n", a9[count_9-1]);
                count_9--;
            }
            else if(arr_task[i].uti<=(4.0/11*Ln) && arr_task[i].uti>(1.0/3*Ln)){
                a10[count_10-1].uti=arr_task[i].uti;
                a10[count_10-1].period=arr_task[i].period;
                //printf("a10 %f\n", a10[count_10-1]);
                count_10--;
            }
            else if(arr_task[i].uti<=(1.0/3*Ln) && arr_task[i].uti>(4.0/13*Ln)){
                a11[count_11-1].uti=arr_task[i].uti;
                a11[count_11-1].period=arr_task[i].period;
                //printf("a11 %f\n", a11[count_11-1]);
                count_11--;
            }
            else if(arr_task[i].uti<=(4.0/13*Ln) && arr_task[i].uti>(2.0/7*Ln)){
                a12[count_12-1].uti=arr_task[i].uti;
                a12[count_12-1].period=arr_task[i].period;
                //printf("a12 %f\n", a12[count_12-1]);
                count_12--;
            }
            else if(arr_task[i].uti<=(2.0/7*Ln) && arr_task[i].uti>(3.0/11*Ln)){
                a13[count_13-1].uti=arr_task[i].uti;
                a13[count_13-1].period=arr_task[i].period;
                //printf("a13 %f\n", a13[count_13-1]);
                count_13--;
            }
            else if(arr_task[i].uti<=(3.0/11*Ln) && arr_task[i].uti>(1.0/4*Ln)){
                a14[count_14-1].uti=arr_task[i].uti;
                a14[count_14-1].period=arr_task[i].period;
                //printf("a14 %f\n", a14[count_14-1]);
                count_14--;
            }
            else if(arr_task[i].uti<=(1.0/4*Ln) && arr_task[i].uti>(4.0/17*Ln)){
                a15[count_15-1].uti=arr_task[i].uti;
                a15[count_15-1].period=arr_task[i].period;
                //printf("a15 %f\n", a15[count_15-1]);
                count_15--;
            }
            else if(arr_task[i].uti<=(4.0/17*Ln) && arr_task[i].uti>(2.0/9*Ln)){
                a16[count_16-1].uti=arr_task[i].uti;
                a16[count_16-1].period=arr_task[i].period;
                //printf("a16 %f\n", a16[count_16-1]);
                count_16--;
            }
```

```
else if(arr_task[i].uti<=(2.0/9*Ln) && arr_task[i].uti>(3.0/14*Ln)){
      a17[count_17-1].uti=arr_task[i].uti;
      a17[count_17-1].period=arr_task[i].period;
      //printf("a17 %f\n", a17[count_17-1]);
      count_17--;
}
else if(arr_task[i].uti<=(3.0/14*Ln) && arr_task[i].uti>(1.0/5*Ln)){
      a18[count_18-1].uti=arr_task[i].uti;
      a18[count_18-1].period=arr_task[i].period;
      //printf("a18 %f\n", a18[count_18-1]);
      count_18--;
}
else if(arr_task[i].uti<=(1.0/5*Ln) && arr_task[i].uti>(4.0/21*Ln)){
      a19[count_19-1].uti=arr_task[i].uti;
      a19[count_19-1].period=arr_task[i].period;
      //printf("a19 %f\n", a19[count_19-1]);
      count_19--;
}
else if(arr_task[i].uti<=(4.0/21*Ln) && arr_task[i].uti>(2.0/11*Ln)){
      a20[count_20-1].uti=arr_task[i].uti;
      a20[count_20-1].period=arr_task[i].period;
      //printf("a20 %f\n", a20[count_20-1]);
      count_20--;
}
else if(arr_task[i].uti<=(2.0/11*Ln) && arr_task[i].uti>(3.0/17*Ln)){
      a21[count_21-1].uti=arr_task[i].uti;
      a21[count_21-1].period=arr_task[i].period;
      //printf("a21 %f\n", a21[count_21-1]);
      count_21--;
}
else if(arr_task[i].uti<=(3.0/17*Ln) && arr_task[i].uti>(1.0/6*Ln)){
      a22[count_22-1].uti=arr_task[i].uti;
      a22[count_22-1].period=arr_task[i].period;
      //printf("a22 %f\n", a22[count_22-1]);
      count_22--;
}
else if(arr_task[i].uti<=(1.0/6*Ln) && arr_task[i].uti>(4.0/25*Ln)){
      a23[count_23-1].uti=arr_task[i].uti;
      a23[count_23-1].period=arr_task[i].period;
      //printf("a23 %f\n", a23[count_23-1]);
      count_23--;
}
else if(arr_task[i].uti<=(4.0/25*Ln) && arr_task[i].uti>(2.0/13*Ln)){
      a24[count_24-1].uti=arr_task[i].uti;
```

```
            a24[count_24-1].period=arr_task[i].period;
            //printf("a24 %f\n", a24[count_24-1]);
            count_24--;
        }
        else if(arr_task[i].uti<=(2.0/13*Ln) && arr_task[i].uti>(3.0/20*Ln)){
            a25[count_25-1].uti=arr_task[i].uti;
            a25[count_25-1].period=arr_task[i].period;
            //printf("a25 %f\n", a25[count_25-1]);
            count_25--;
        }
        else if(arr_task[i].uti<=(3.0/20*Ln) && arr_task[i].uti>(1.0/7*Ln)){
            a26[count_26-1].uti=arr_task[i].uti;
            a26[count_26-1].period=arr_task[i].period;
            //printf("a26 %f\n", a26[count_26-1]);
            count_26--;
        }
        else {
            ax[count_x-1].uti=arr_task[i].uti;
            ax[count_x-1].period=arr_task[i].period;
            //printf("a27 %f\n", ax[count_x-1]);
            count_x--;
        }
    }

    for(i=1;i<=rem_2;i++){
        ax[count_x-1].uti=a2[rem_2-i].uti;
        ax[count_x-1].period=a2[rem_2-i].period;
        //printf("a2x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_3;i++){
        ax[count_x-1].uti=a3[rem_3-i].uti;
        ax[count_x-1].period=a3[rem_3-i].period;
        //printf("a3x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_4;i++){
        ax[count_x-1].uti=a4[rem_4-i].uti;
        ax[count_x-1].period=a4[rem_4-i].period;
        //printf("a4x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_5;i++){
        ax[count_x-1].uti=a5[rem_5-i].uti;
        ax[count_x-1].period=a5[rem_5-i].period;
        //printf("a5x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_6;i++){
        ax[count_x-1].uti=a6[rem_6-i].uti;
        ax[count_x-1].period=a6[rem_6-i].period;
        //printf("a6x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_7;i++){
        ax[count_x-1].uti=a7[rem_7-i].uti;
        ax[count_x-1].period=a7[rem_7-i].period;
        //printf("a7x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_8;i++){
        ax[count_x-1].uti=a8[rem_8-i].uti;
        ax[count_x-1].period=a8[rem_8-i].period;
        //printf("a8x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_9;i++){
        ax[count_x-1].uti=a9[rem_9-i].uti;
        ax[count_x-1].period=a9[rem_9-i].period;
        //printf("a9x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_10;i++){
        ax[count_x-1].uti=a10[rem_10-i].uti;
        ax[count_x-1].period=a10[rem_10-i].period;
        //printf("a10x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_11;i++){
        ax[count_x-1].uti=a11[rem_11-i].uti;
        ax[count_x-1].period=a11[rem_11-i].period;
        //printf("a11x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_12;i++){
        ax[count_x-1].uti=a12[rem_12-i].uti;
        ax[count_x-1].period=a12[rem_12-i].period;
        //printf("a12x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_13;i++){
        ax[count_x-1].uti=a13[rem_13-i].uti;
        ax[count_x-1].period=a13[rem_13-i].period;
        //printf("a13x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_14;i++){
        ax[count_x-1].uti=a14[rem_14-i].uti;
        ax[count_x-1].period=a14[rem_14-i].period;
        //printf("a14x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_15;i++){
        ax[count_x-1].uti=a15[rem_15-i].uti;
        ax[count_x-1].period=a15[rem_15-i].period;
        //printf("a15x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_16;i++){
        ax[count_x-1].uti=a16[rem_16-i].uti;
        ax[count_x-1].period=a16[rem_16-i].period;
        //printf("a16x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_17;i++){
        ax[count_x-1].uti=a17[rem_17-i].uti;
        ax[count_x-1].period=a17[rem_17-i].period;
        //printf("a17x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_18;i++){
        ax[count_x-1].uti=a18[rem_18-i].uti;
        ax[count_x-1].period=a18[rem_18-i].period;
        //printf("a18x %f\n", ax[count_x-1]);
        count_x--;
    }
    for(i=1;i<=rem_19;i++){
        ax[count_x-1].uti=a19[rem_19-i].uti;
        ax[count_x-1].period=a19[rem_19-i].period;
        //printf("a19x %f\n", ax[count_x-1]);
        count_x--;
    }
```

```
for(i=1;i<=rem_20;i++){
        ax[count_x-1].uti=a20[rem_20-i].uti;
        ax[count_x-1].period=a20[rem_20-i].period;
        //printf("a20x %f\n", ax[count_x-1]);
        count_x--;
}
for(i=1;i<=rem_21;i++){
        ax[count_x-1].uti=a21[rem_21-i].uti;
        ax[count_x-1].period=a21[rem_21-i].period;
        //printf("a21x %f\n", ax[count_x-1]);
        count_x--;
}
for(i=1;i<=rem_22;i++){
        ax[count_x-1].uti=a22[rem_22-i].uti;
        ax[count_x-1].period=a22[rem_22-i].period;
        //printf("a22x %f\n", ax[count_x-1]);
        count_x--;
}
for(i=1;i<=rem_23;i++){
        ax[count_x-1].uti=a23[rem_23-i].uti;
        ax[count_x-1].period=a23[rem_23-i].period;
        //printf("a23x %f\n", ax[count_x-1]);
        count_x--;
}
for(i=1;i<=rem_24;i++){
        ax[count_x-1].uti=a24[rem_24-i].uti;
        ax[count_x-1].period=a24[rem_24-i].period;
        //printf("a24x %f\n", ax[count_x-1]);
        count_x--;
}
for(i=1;i<=rem_25;i++){
        ax[count_x-1].uti=a25[rem_25-i].uti;
        ax[count_x-1].period=a25[rem_25-i].period;
        //printf("a25x %f\n", ax[count_x-1]);
        count_x--;
}
for(i=1;i<=rem_26;i++){
        ax[count_x-1].uti=a26[rem_26-i].uti;
        ax[count_x-1].period=a26[rem_26-i].period;
        //printf("a26x %f\n", ax[count_x-1]);
        count_x--;
}

free(a2);
```

```
a2=NULL;
free(a3);
a3=NULL;
free(a4);
a4=NULL;
free(a5);
a5=NULL;
free(a6);
a6=NULL;
free(a7);
a7=NULL;
free(a8);
a8=NULL;
free(a9);
a9=NULL;
free(a10);
a10=NULL;
free(a11);
a11=NULL;
free(a12);
a12=NULL;
free(a13);
a13=NULL;
free(a14);
a14=NULL;
free(a15);
a15=NULL;
free(a16);
a16=NULL;
free(a17);
a17=NULL;
free(a18);
a18=NULL;
free(a19);
a19=NULL;
free(a20);
a20=NULL;
free(a21);
a21=NULL;
free(a22);
a22=NULL;
free(a23);
a23=NULL;
free(a24);
```

```
a24=NULL;
free(a25);
a25=NULL;
free(a26);
a26=NULL;

x_uti=0;
for(i=0;i<x;i++) x_uti=x_uti+ax[i].uti;
//printf("x_uti %f\n",x_uti);

count_pre=0;
count_norm=0;


uti=(double*)malloc(x*sizeof(double));
pre=(task*)malloc(x*sizeof(task));
norm=(task*)malloc(x*sizeof(task));
for(i=0;i<x-1;i++){
        for(j=i+1;j<x;j++){
                if(ax[j].period<ax[i].period){
                        tmp=ax[i].period;
                        ax[i].period=ax[j].period;
                        ax[j].period=tmp;
                        tmp=ax[i].uti;
                        ax[i].uti=ax[j].uti;
                        ax[j].uti=tmp;
                }
        }
}

//for(i=0;i<x;i++) printf("ax %f\n",ax[i]);
tmp=0;
for(i=0;i<x;i++){
        tmp=tmp+ax[i].uti;
        uti[i]=x_uti-tmp;
        //printf("uti[] %f\n", uti[i]);
        //printf("proc_num %d\n", proc_num);
        c=(proc_num-1)*new_theta;
        //printf("c %f\n", c);
        if(i!=x-1 && ax[i].uti>new_weight && uti[i]<=c){
                pre[count_pre].uti=ax[i].uti;
                //printf("ax[] %f pre[] %f\n", ax[i].uti, pre[count_pre].uti);
                pre[count_pre].period=ax[i].period;
                count_pre++;
```

```
            proc_num--;
        }
        else {
            norm[count_norm].uti=ax[i].uti;
            norm[count_norm].period=ax[i].period;
            count_norm++;
        }
    }

    free(uti);
    uti=NULL;
    free(ax);
    ax=NULL;

    for(i=0;i<count_pre-1;i++){
        for(j=i+1;j<count_pre;j++){
            if(pre[j].period>pre[i].period){
                tmp=pre[i].period;
                pre[i].period=pre[j].period;
                pre[j].period=tmp;
                tmp=pre[i].uti;
                pre[i].uti=pre[j].uti;
                pre[j].uti=tmp;
            }
        }
    }

    //for(i=0;i<count_pre;i++) printf("pre[] %f\n",pre[i].uti);
    //for(i=0;i<count_norm;i++) printf("norm[] %f\n",norm[i].uti);

    pre_cpu=(double*)malloc(count_pre*sizeof(double));
    norm_cpu=(double*)malloc(proc_num*sizeof(double));
    subtask=(int*)malloc(count_norm*sizeof(int));

    for(i=0;i<count_pre;i++) pre_cpu[i]=pre[i].uti;
    free(pre);
    pre=NULL;

    for(i=0;i<proc_num;i++) norm_cpu[i]=0;
    for(i=0;i<count_norm;i++) subtask[i]=1;
    n=proc_num;
    m=count_pre;
    p=count_norm;
    i=0;
```

```
t=0;
flg=0;
printf("CHAOX pre %d, norm %d, x %d, proc_num %d\n", m, p, x, n);

while(count_norm>0){
    while(proc_num>0){
        if((norm_cpu[0]+norm[i].uti)<=new_theta){
            norm_cpu[0]=norm_cpu[0]+norm[i].uti;
            //printf("norm[i].uti %f\n", norm[i].uti);
            //printf("norm_cpu[0] %f\n",norm_cpu[0]);
            count_norm--;
            if(norm_cpu[0]==new_theta) {
                proc_num--;
                printf("Retard\n");
            }
            for(j=0;j<n-1;j++){
                for(k=j+1;k<n;k++){
                    if(norm_cpu[k]<norm_cpu[j]){
                        tmp=norm_cpu[j];
                        norm_cpu[j]=norm_cpu[k];
                        norm_cpu[k]=tmp;
                    }
                }
            }
            i++;
            if(count_norm==0) break;
        }
        else {
            //printf("~~1norm[i].uti %f\n", norm[i].uti);
            norm[i].uti=norm[i].uti-(new_theta-norm_cpu[0]);
            //printf("~~2norm[i].uti %f\n", norm[i].uti);
            norm_cpu[0]=new_theta;
            //printf("~~norm_cpu[0] %f\n",norm_cpu[0]);
            proc_num--;
            subtask[i]++;
            for(j=0;j<n-1;j++){
                for(k=j+1;k<n;k++){
                    if(norm_cpu[k]<norm_cpu[j]){
                        tmp=norm_cpu[j];
                        norm_cpu[j]=norm_cpu[k];
                        norm_cpu[k]=tmp;
                    }
                }
            }
            if(proc_num==0) break;
        }
    }
    if(count_norm==0) break;
    while(count_pre>0){
        while(t<m){
            if((pre_cpu[t]+norm[i].uti)<=new_theta){
                //printf("i %d\n", i);
                //printf("t %d\n", t);
                pre_cpu[t]=pre_cpu[t]+norm[i].uti;
                //printf("``norm[i].uti %f\n", norm[i].uti);
                //printf("``pre_cpu[0] %f\n",pre_cpu[t]);
                count_norm--;
                i++;
                if(pre_cpu[t]==new_theta){
                    t++;
                    count_pre--;
                    printf("Retard\n");
                }
                if(count_norm==0) break;
            }
            else {
                //printf("^^1norm[i].uti %f\n", norm[i].uti);
                norm[i].uti=norm[i].uti-(new_theta-pre_cpu[t]);
                //printf("^^2norm[i].uti %f\n", norm[i].uti);
                pre_cpu[t]=new_theta;
                //printf("^^pre_cpu[0] %f\n",pre_cpu[t]);
                t++;
                //printf("t %d\n", t);
                count_pre--;
                subtask[i]++;
            }
        }
        if(count_norm==0) break;
    }
    if(count_pre==0 && proc_num==0) {
        printf("! CHAOX ASSIGNMENT FAILED 1\n");
        flg=1;
        chaox_return=1;
        break;
    }
}
free(norm);
norm=NULL;
```

```
        for(i=0;i<p-1;i++){
            for(j=i+1;j<p;j++){
                if(subtask[j]>subtask[i]){
                    tmp=subtask[i];
                    subtask[i]=subtask[j];
                    subtask[j]=tmp;
                }
            }
        }
        for(i=0;i<p;i++){
            //printf("subtask[] %d\n", subtask[i]);
            if(subtask[i]!=1) chaox_split++;
        }

        if(chaox_sub<subtask[0]) chaox_sub=subtask[0];
        if(flg==1) {
            chaox_split=0;
            chaox_sort=0;
            chaox_sub=0;
        }
        if(flg!=1){
            double temp=0;
            for(i=0;i<m;i++) temp=temp+pre_cpu[i];
            for(i=0;i<n;i++) temp=temp+norm_cpu[i];
            printf("CHAOX Uti CPU %f,    Uti Tot %f\n",temp, x_uti);

        }
        free(pre_cpu);
        pre_cpu=NULL;
        free(norm_cpu);
        norm_cpu=NULL;
        free(subtask);
        subtask=NULL;
    }
    else if(proc_num==0 && x>0){
        printf("! CHAOX ASSIGNMENT FAILED 2\n");
        chaox_return=1;
        chaox_split=0;
        chaox_sort=0;
        chaox_sub=0;
    }
    else if(proc_num<0){
        printf("! CHAOX ASSIGNMENT FAILED 3\n");
        chaox_return=1;
        chaox_split=0;
        chaox_sort=0;
        chaox_sub=0;
    }
}

double neu()
{
    int i, j, k, t, m, n, p, flg, norm_num, count_pre, count_norm;
    double tmp, tsk_uti, c;
    task *tsk,    *pre, *norm;
    double *uti, *pre_cpu, *norm_cpu;
    int *subtask;
    neu_return=0;
    neu_split=0;
    neu_sort=task_num;
    tsk=(task*)malloc(task_num*sizeof(task));

    for(i=0;i<task_num;i++){
        tsk[i].uti=arr_task[i].uti;
        tsk[i].period=arr_task[i].period;
    }

    tsk_uti=0;
    for(i=0;i<task_num;i++) tsk_uti=tsk_uti+tsk[i].uti;

    count_pre=0;
    count_norm=0;
    norm_num=PROC;
    pre=(task*)malloc(task_num*sizeof(task));
    norm=(task*)malloc(task_num*sizeof(task));
    uti=(double*)malloc(task_num*sizeof(double));

    for(i=0;i<task_num-1;i++){
        for(j=i+1;j<task_num;j++){
            if(tsk[j].period<tsk[i].period){
                tmp=tsk[i].period;
                tsk[i].period=tsk[j].period;
                tsk[j].period=tmp;
                tmp=tsk[i].uti;
                tsk[i].uti=tsk[j].uti;
                tsk[j].uti=tmp;
            }
        }
    }
    //for(i=0;i<task_num;i++) printf("tsk[] %f\n",tsk[i]);

    tmp=0;
    for(i=0;i<task_num;i++){
        tmp=tmp+tsk[i].uti;
        uti[i]=tsk_uti-tmp;
        c=(norm_num-1)*theta;
        if(i!=task_num-1 && tsk[i].uti>weight && uti[i]<=c){
            pre[count_pre].uti=tsk[i].uti;
            pre[count_pre].period=tsk[i].period;
            //printf("pre %f\n",pre[count_pre]);
            count_pre++;
            norm_num--;
        }
        else {
            norm[count_norm].uti=tsk[i].uti;
            norm[count_norm].period=tsk[i].period;
            //printf("norm %f\n",norm[count_norm]);
            count_norm++;
        }
    }

    for(i=0;i<count_pre-1;i++){
        for(j=i+1;j<count_pre;j++){
            if(pre[j].period>pre[i].period){
                tmp=pre[i].period;
                pre[i].period=pre[j].period;
                pre[j].period=tmp;
                tmp=pre[i].uti;
                pre[i].uti=pre[j].uti;
                pre[j].uti=tmp;
            }
        }
    }


    free(tsk);
    tsk=NULL;
    free(uti);
    uti=NULL;
    pre_cpu=(double*)malloc(count_pre*sizeof(double));
    norm_cpu=(double*)malloc(norm_num*sizeof(double));
```

```
for(i=0;i<count_pre;i++) pre_cpu[i]=pre[i].uti;
free(pre);
pre=NULL;
subtask=(int*)malloc(count_norm*sizeof(int));
for(i=0;i<norm_num;i++) norm_cpu[i]=0;
for(i=0;i<count_norm;i++) subtask[i]=1;

n=norm_num;
m=count_pre;
p=count_norm;
i=0;
t=0;
flg=0;

while(count_norm>0){
      while(norm_num>0){
            if((norm_cpu[0]+norm[i].uti)<=theta){
                  norm_cpu[0]=norm_cpu[0]+norm[i].uti;
                  //printf("norm[i].uti %f\n", norm[i].uti);
                  //printf("norm_cpu[0] %f\n",norm_cpu[0]);
                  count_norm--;
                  if(norm_cpu[0]==theta) norm_num--;
                  for(j=0;j<n-1;j++){
                        for(k=j+1;k<n;k++){
                              if(norm_cpu[k]<norm_cpu[j]){
                                    tmp=norm_cpu[j];
                                    norm_cpu[j]=norm_cpu[k];
                                    norm_cpu[k]=tmp;
                              }
                        }
                  }
                  i++;
                  if(count_norm==0) break;
            }
            else {
                  //printf("~~1norm[i].uti %f\n", norm[i].uti);
                  norm[i].uti=norm[i].uti-(theta-norm_cpu[0]);
                  //printf("~~2norm[i].uti %f\n", norm[i].uti);
                  norm_cpu[0]=theta;
                  //printf("~~norm_cpu[0] %f\n",norm_cpu[0]);
                  norm_num--;
                  subtask[i]++;
                  for(j=0;j<n-1;j++){
                        for(k=j+1;k<n;k++){
```

```
                if(norm_cpu[k]<norm_cpu[j]){
                    tmp=norm_cpu[j];
                    norm_cpu[j]=norm_cpu[k];
                    norm_cpu[k]=tmp;
                }
            }
        }
        if(norm_num==0) break;
    }
}
if(count_norm==0) break;
while(count_pre>0){
    while(t<m){
        if((pre_cpu[t]+norm[i].uti)<=theta){
            //printf("i %d\n", i);
            //printf("t %d\n", t);
            pre_cpu[t]=pre_cpu[t]+norm[i].uti;
            //printf("``norm[i].uti %f\n", norm[i].uti);
            //printf("``pre_cpu[0] %f\n",pre_cpu[t]);
            count_norm--;
            i++;
            if(pre_cpu[t]==theta){
                t++;
                count_pre--;
            }
            if(count_norm==0) break;
        }
        else {
            //printf("^^1norm[i].uti %f\n", norm[i].uti);
            norm[i].uti=norm[i].uti-(theta-pre_cpu[t]);
            //printf("^^2norm[i].uti %f\n", norm[i].uti);
            pre_cpu[t]=theta;
            //printf("^^pre_cpu[0] %f\n",pre_cpu[t]);
            t++;
            //printf("t %d\n", t);
            count_pre--;
            subtask[i]++;
        }
    }
    if(count_norm==0) break;
}
if(count_pre==0 && norm_num==0) {
    printf("! NEU ASSIGNMENT FAILED\n");
    flg=1;
```

```
                neu_return=1;
                break;
            }
        }

        free(norm);
        pre=NULL;

        if(flg==0){
            for(i=0;i<p-1;i++){
                for(j=i+1;j<p;j++){
                    if(subtask[j]>subtask[i]){
                        tmp=subtask[i];
                        subtask[i]=subtask[j];
                        subtask[j]=tmp;
                    }
                }
            }

            for(i=0;i<p;i++){
                //printf("subtask[] %d\n", subtask[i]);
                if(subtask[i]!=1) neu_split++;
            }
            if(neu_sub<subtask[0]) neu_sub=subtask[0];

            double temp=0;
            for(i=0;i<m;i++) temp=temp+pre_cpu[i];
            for(i=0;i<n;i++) temp=temp+norm_cpu[i];
            printf("NEU Uti CPU %f,    Uti Tot %f\n",temp, tsk_uti);
            printf("NEU pre %d, norm %d\n", m, n);
        }
        else {
            neu_split=0;
            neu_sort=0;
            neu_sub=0;
        }

        free(pre_cpu);
        pre_cpu=NULL;
        free(norm_cpu);
        norm_cpu=NULL;
        free(subtask);
        subtask=NULL;
    }
```

```
main()
{
    int i, j;
    int chaox_tot_split=0;
    int chaox_max_subtask=0;
    int chaox_tot_sort=0;
    double chaox_tot_task=0;
    double chaox_ok=0;
    int neu_tot_split=0;
    int neu_max_subtask=0;
    int neu_tot_sort=0;
    double neu_ok=0;
    double sys_uti, ratio;
    int n=0;
    int m=0;
    typedef struct{
        int succ;
        int fail;
    } bucket;
    bucket chaox_bucket[100], neu_bucket[100];
    double chaox_graph[100], neu_graph[100];
    for(i=0;i<100;i++){
        chaox_bucket[i].succ=0;
        chaox_bucket[i].fail=0;
        neu_bucket[i].succ=0;
        neu_bucket[i].fail=0;
    }
    srand((unsigned)time(0));
    for(i=0;i<SET;i++){
        init();
        sys_uti=(tot_uti/PROC)*100;
        for(j=0;j<100;j++){
            if(sys_uti<j+1) break;
        }
        //printf("j %d\n",j);
        chaox();
        chaox_tot_split=chaox_tot_split+chaox_split;
        if(chaox_max_subtask<chaox_sub) chaox_max_subtask=chaox_sub;
        chaox_tot_sort=chaox_tot_sort+chaox_sort;
        neu();
        neu_tot_split=neu_tot_split+neu_split;
        neu_tot_sort=neu_tot_sort+neu_sort;
        if(neu_max_subtask<neu_sub) neu_max_subtask=neu_sub;
        if(chaox_return==0) {
            chaox_ok++;
            chaox_bucket[j].succ++;
        }
        if(neu_return==0) {
            neu_ok++;
            neu_bucket[j].succ++;
        }
        if(chaox_return) chaox_bucket[j].fail++;
        if(neu_return) neu_bucket[j].fail++;
        printf("chx succ %d, chx fail %d\n", chaox_bucket[j].succ,
chaox_bucket[j].fail);
        printf("neu succ %d, neu fail %d\n", neu_bucket[j].succ, neu_bucket[j].fail);
        if(chaox_return==0 && neu_return) n++;
        if(chaox_return && neu_return==0) m++;
        //if(m>0) sleep(100000);
        /*if(chaox_return && neu_return){
            task_num=PROC+N;
            i--;
            printf("TASK RESET~~~~~~~~~~\n");
            //sleep(1);
        }
        else*/
        task_num++;
        printf("CHAOX_OK %f       NEU_OK %f\n", chaox_ok, neu_ok);
        printf("No.SET %d      TASK NUM %d\n", i+1, task_num);
        printf("------------------------------------------\n");
        printf("------------------------------------------\n");
        //if(chaox_bucket[68].fail>0) sleep(100000);
    }

    for(i=0;i<100;i++){

    chaox_graph[i]=((double)chaox_bucket[i].succ)/(chaox_bucket[i].succ+chaox_bu
cket[i].fail);

    neu_graph[i]=((double)neu_bucket[i].succ)/(neu_bucket[i].succ+neu_bucket[i].f
ail);
        printf("%d %f\n",i, chaox_graph[i]);
    }
    printf("++++++++++++++++++++++++++++++++++++++++++++\n");
    for(i=0;i<100;i++) printf("%d %f\n",i, neu_graph[i]);
    printf("CHAOX\nOK %f\nTot Split %d\nAvg Split %f\nMax. Subtask %d\nTotal
Sorted %d\nAvg Sorted %f\nAverage Ratio %f\n", chaox_ok, chaox_tot_split,
```

```
chaox_tot_split/chaox_ok, chaox_max_subtask, chaox_tot_sort,
chaox_tot_sort/chaox_ok, chaox_ok/SET);
	printf("++++++++++++++++++++++++++++++++++++++++++++\n");
	printf("NEU\nOK %f\nTot Split %d\nAvg Split %f\nMax. Subtask %d\nTotal
Sorted %d\nAvg Sorted %f\nAverage Ratio %f\n", neu_ok, neu_tot_split,
neu_tot_split/neu_ok, neu_max_subtask, neu_tot_sort, neu_tot_sort/neu_ok,
neu_ok/SET);
	printf("++++++++++++++++++++++++++++++++++++++++++++\n");
	printf("Dominant Value n %d m %d\n", n, m);
	printf("++++++++++++++++++++++++++++++++++++++++++++\n");
}
```